Absorption

Consider a beam passing through an absorbing medium. Define the **absorption coefficient**, α_ν, by

$$dI_\nu = -\alpha_\nu I_\nu ds$$

i.e., the fractional loss in intensity in travelling a distance ds is $\alpha_\nu\, ds$ (convention: positive α_ν means energy loss).

Suppose the absorption is due to particles (atoms, molecules etc) with number n per unit volume. Each presents an effective absorbing area (or **cross section**) σ_ν to the radiation (units, m^2).

Within a beam of length ds and area dA, the number of such absorbers is $n\ dA\ ds$. The fraction of the beam that is absorbed is:

$$\frac{dI_\nu}{I_\nu} = -\frac{n\, dA\, ds\, \sigma_\nu}{dA}$$

Comparing this with the equation above we have: $\alpha_\nu = n\, \sigma_\nu$

Finally, we can define the **mass absorption coefficient** (or, **opacity** coefficient) κ_ν (m^2 kg^{-1}),

$$\alpha_\nu = \rho\, \kappa_\nu$$

Confusingly, the "absorption" can be positive or negative. This is because stimulated emission, like true absorption, is proportional to I_ν, and so is conveniently combined into the absorption coefficient.

The radiative transfer equation

- Is an ordinary differential equation along a straight line
- Combines the effects of absorption and emission
- Gives
  - variation in the specific intensity along a ray
  - macroscopic description of the radiation field

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

To use it, we need

1. To determine α_ν and j_ν for all physical processes that emit or absorb radiation. We'll consider specific examples later. Details can be complicated and messy.
2. To solve the transfer equation, which looks deceptively simple. The problem is that the emission coefficient j_ν, and sometimes the absorption coefficient α_ν, often depend upon I_ν, and *not just along a ray*. For example, scattering will couple rays in different directions that intersect each other.

First, we consider simple special cases.

Medium with no absorption

In this case, $\alpha_\nu = 0$, and $\frac{dI_\nu}{ds} = j_\nu$

whose solution is :

$$I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')ds'$$

i.e., the increase in brightness is just the integral of the emission coefficient along the line of sight.

Medium with no emission

In this case, $j_\nu = 0$, and $\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$

with solution:

$$I_\nu(s) = I_\nu(s_0)\exp\left[-\int_{s_0}^{s} \alpha_\nu(s')ds'\right]$$

i.e., the brightness decreases by the exponential of the absorption coefficient integrated along the line of sight.

Optical depth

Distance s isn't always a good independent variable to use in the transfer equation.
Emission and absorption change the intensity on length scales that can vary enormously.
So we define a new variable, the optical depth, τ_ν

$$d\tau_\nu = \alpha_\nu\, ds$$

or,

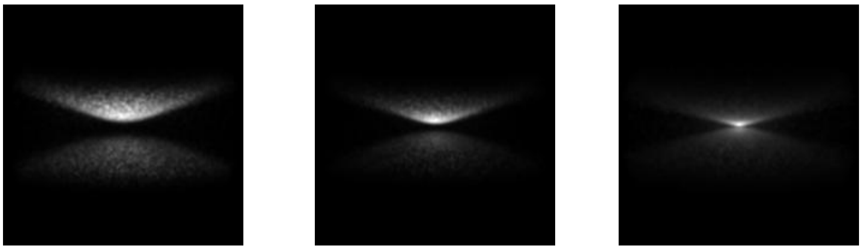
$$\tau_\nu(\mathrm{s}) = \int_{s_0}^{s} \alpha_\nu(s')ds'$$

Here s_0 is an arbitrary point that sets the zero of the optical depth scale
Optical depth increases along the path of a ray
Given a typical ray passing through a medium, we say that:

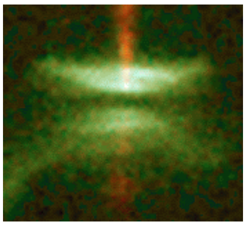
- The medium is optically thick if $\tau_\nu > 1$ (photons usually absorbed)
- The medium is optically thin if $\tau_\nu < 1$ (photons usually pass through)

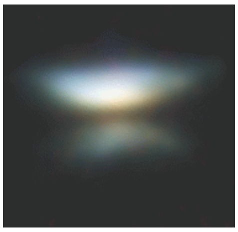
A medium can often be optically thin at some frequencies yet optically thick at others.
e.g., dusty disks around young stars are:

Optically thick to infra-red emission at $\lambda \sim 1\ \mu$m.
Optically thin to emission at mm-wavelengths.



V (0.55μm) I (0.85μm) K (2.25μm)





HH30 protoplanetary disk imaged by HST in optical and near-IR. Width of dust lane decreases towards longer wavelengths as dust is optically thinner. Width decrease less than predicted if dust is same as in ISM, providing evidence for a grayer dust opacity and grain growth in the disk.

Cotera et al. 2001

Source function

Dividing the transfer equation by the absorption coefficient,

$$\frac{dI_\nu}{\alpha_\nu ds} = -I_\nu + \frac{j_\nu}{\alpha_\nu}$$

Since $d\tau_\nu = \alpha_\nu ds$, we can write this as:

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

where we have defined the **source function** $\boldsymbol{S_\nu}$ as the ratio between the emission and absorption coefficients,

$$S_\nu = \frac{j_\nu}{\alpha_\nu}$$

Formal solution of the transfer equation

Start by writing the transfer equation in terms of optical depth & source function:

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

Multiply by the integrating factor : e^{τ_ν}

$$e^{\tau_\nu}\frac{dI_\nu}{d\tau_\nu} + e^{\tau_\nu} I_\nu = \frac{d}{d\tau_\nu}\left[I_\nu e^{\tau_\nu}\right] = e^{\tau_\nu} S_\nu$$

$$I_\nu e^{\tau_\nu} = \int_0^{\tau_\nu} e^{\tau'_\nu} S_\nu \, d\tau'_\nu + \text{constant}$$

When $\tau_\nu = 0$, $I_\nu = I_\nu(0)$. Thus,

$$I_\nu(\tau_\nu) = I_\nu(0)\, e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} S_\nu(\tau'_\nu)\, d\tau'_\nu$$

Recall that $\tau_\nu = 1$ implies absorption by a factor of $1/e$.

Interpretation: the final intensity is

the initial intensity diminished by absorption,

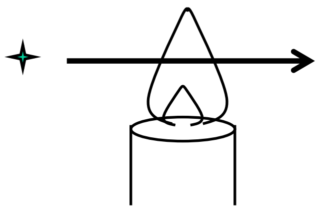
plus the integrated source function likewise diminished by absorption.

A candle flame casts a shadow

For a constant source function, the solution becomes,

$$I_\nu(\tau_\nu) = I_\nu(0)\, e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

and as $\tau_\nu \to \infty$, $I_\nu \to S_\nu$. From the transfer equation it is clear this holds more generally, i.e., the specific intensity approaches the source function at large optical depth.



Soot particles cast shadows, attenuating incident beam, but they also add intensity with a source function (brightness) that depends on soot temperature. Shadow vanishes when T(soot) =T(maglite)

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Lecture 3 revision quiz

- To derive the intensity differential in slide 1, what assumptions must be made about the distribution of absorbers?
- Using a sketch, show how scattering can lead to non local coupling of the radiation field with radiation from different regions of a nebula contributing to the equation of radiation transfer.
- Revise the mathematical technique for solving differential equations using an integrating factor and derive the formal solution for the equation of radiation transfer.

Lecture 3 revision quiz

- Starting from the formal solution of the equation of radiation transfer, derive the solution for a constant source function.
- Make sketches to explain the physical situations described by the solution of the equation of radiation transfer in the cases with no absorption, no emission, absorption and emission.
- Identify the household items Andrew Cameron used for the example in the final slides.

Lecture 3 revision quiz

- In a violent desert sandstorm, the visibility is about 10m and the mean sand particle diameter is about 20 microns.
  - Estimate the number density of sand particles.
  - Estimate the absorption coefficient α,
  - If the sand particles are made of material with density 5000 kg m^{-3}, estimate the mass absorption coefficient κ.