Characteristic temperatures

Effective temperature

Most sources are only approximately blackbodies (if that).

So we integrate the flux over frequency and define:

$$F = \iint I_\nu \cos\theta \, d\Omega \, d\nu = \sigma T_e^4$$

i.e. a source of **effective temperature** T_e is the temperature of a blackbody which produces the same total flux.

Colour temperature

Often we don't know the distance or size of a source

Hence it's easier to measure the *shape* of the spectrum than the specific intensity.

The **colour temperature**, T_c, is the temperature of a blackbody spectrum with the same shape, i.e., ignoring the vertical (intensity) scale.

If the emitter is really a blackbody, then the colour temperature gives the correct blackbody temperature.

Brightness temperature

For a source of specific intensity I_ν, we define the brightness temperature T_b as the temperature of a blackbody which would have the same brightness at frequency ν,

$$I_\nu = B_\nu(T_b)$$

Note that we can do this for an arbitrary spectrum – it's just a way of measuring intensity in temperature units. Unless the spectrum is a blackbody, however, T_b will vary with frequency and may be unrelated to the real temperature of the source.

Brightness temperatures are often used in radio astronomy. For a source with a Rayleigh-Jeans spectrum,

$$I_\nu = \frac{2\nu^2}{c^2} kT_b \qquad T_b = \frac{c^2}{2\nu^2 k} I_\nu$$

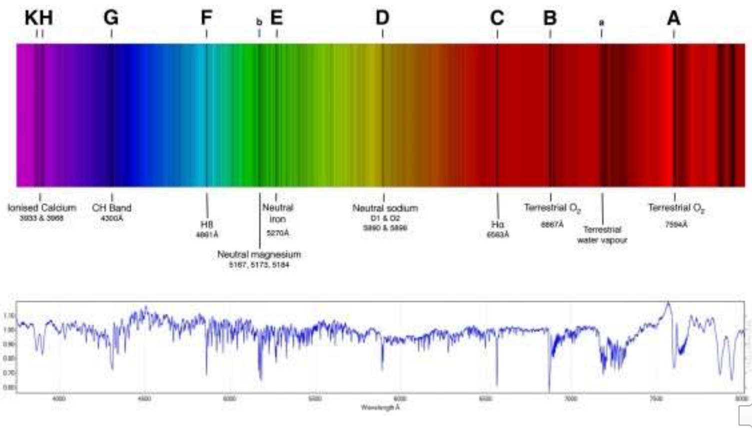
The transfer equation in this limit becomes, $$\frac{dT_b}{d\tau_\nu} = -T_b + T$$

For a constant T we have, $$T_b(\tau_\nu) = T_b(0)e^{-\tau_\nu} + T(1 - e^{-\tau_\nu})$$

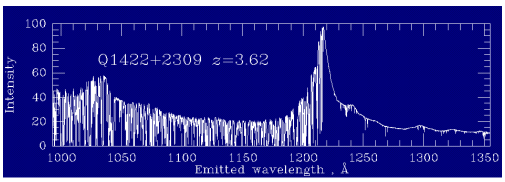
i.e., the brightness temperature of the radiation approaches the temperature at large optical depth.

Spectral lines

Most spectra are more complicated than blackbodies. Some sources show absorption line spectra, e.g., the Sun in the optical.

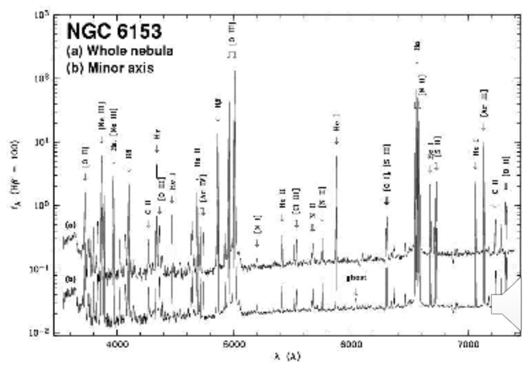


Spectral lines

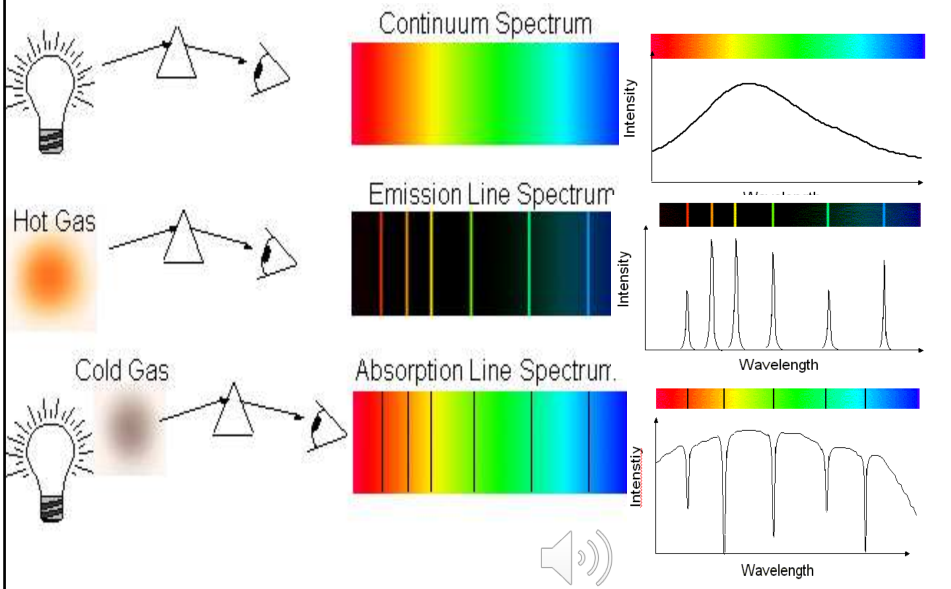


Cold gas along the line of sight also produces absorption features (the Lyman alpha forest) in quasar spectra.

However, nebulae often show emission line spectra.



Why this difference?



Example: HI 21 cm radio absorption/emission

Off-source emission

On-source absorption

Hughes et al 1971, ApJS 23, 323

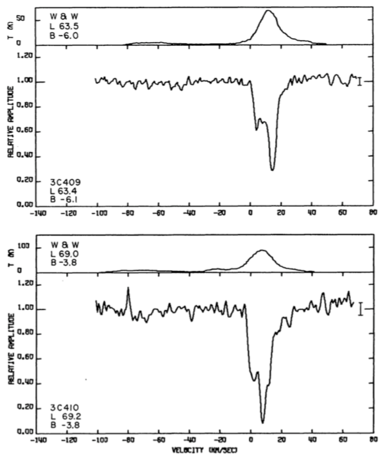


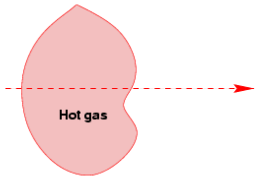
FIG. 1.—*Continued*

Absorption vs emission line spectra

Use the result derived earlier. For a constant source function S_ν,

$$I_\nu(\tau_\nu) = I_\nu(0)\, e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

where $I_\nu(\tau_\nu)$ is the resultant intensity after travelling an optical depth τ_ν through a medium with initial intensity $I_\nu(0)$.



For a nebula, consider a ray passing through the whole volume of hot gas. Thus, $I_\nu(0)=0$.

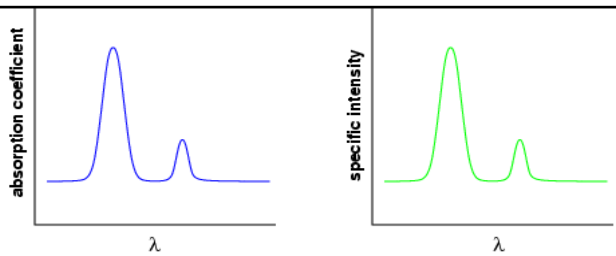
There are two limits. If $\tau_\nu \ll 1$, then $$e^{-\tau_\nu} \cong 1 - \tau_\nu$$

and the emergent intensity is, $$I_\nu(\tau_\nu) = S_\nu(1 - 1 + \tau_\nu) = \tau_\nu S_\nu$$

If the gas is also in local thermodynamic equilibrium (LTE), then $S_\nu = B_\nu$, and

$$I_\nu = \tau_\nu B_\nu \propto \alpha_\nu B_\nu$$

i.e., the intensity is large at frequencies where the absorption coefficient is large.



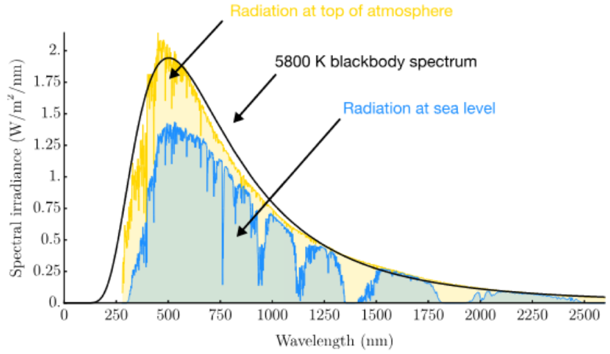
For a hot gas, the absorption coefficient is large at the frequencies of spectral lines.

For an optically thin medium $\tau_\nu \ll 1$, we expect an emission line spectrum with large intensity at the frequencies where α_ν is large. This limit is appropriate:

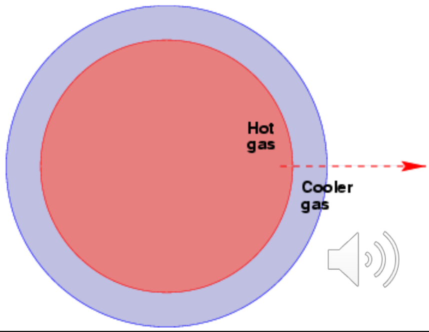
- In many nebulae, which are often optically thin at least in the continuum and line wings.
- In the solar corona, which shows an emission line spectrum visible during solar eclipses.

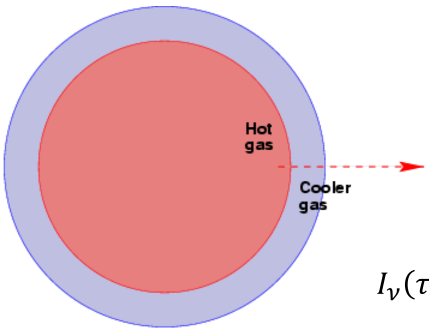
As before, if the medium is instead very optically thick, $\tau_\nu \gg 1$, then

$$S_\nu(1 - e^{-\tau_\nu}) \rightarrow S_\nu \qquad \text{and} \qquad I_\nu = S_\nu = B_\nu \quad \text{(LTE)}$$



Stars are very optically thick. However, they do not show the smooth intensity distribution at all frequencies given by the black body formula. This is because the source function is not constant along the path. Consider a two layer model…





Stars are very optically thick. However, they do not show the smooth intensity distribution at all frequencies given by the black body formula. This is because the source function is not constant along the path. Consider a two layer model,

If $\tau_\nu \ll 1$, then

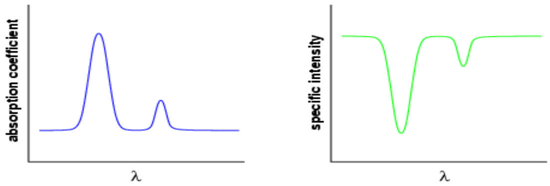
$$\begin{aligned} I_\nu(\tau_\nu) &= I_\nu(0)\, e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu}) \\ &= I_\nu(0)(1 - \tau_\nu) + \tau_\nu\, S_\nu \\ &= I_\nu(0) + \tau_\nu\, [S_\nu - I_\nu(0)] \end{aligned}$$

The sign of the second term depends on whether S_ν or $I_\nu(0)$ is larger.

- $S_\nu > I_\nu(0)$. In this case the emergent intensity is *greater* at frequencies where τ_ν is large. We expect emission lines on top of the background intensity.

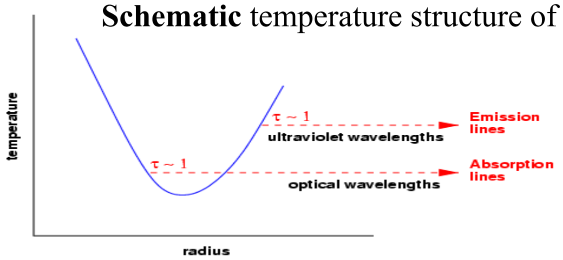
- $S_\nu < I_\nu(0)$. The emergent intensity is *reduced* at frequencies where τ_ν is large. We expect absorption lines.

In LTE, $S_\nu = B_\nu$, which *increases* with increasing temperature. If we see radiation from a layer in the star where $dT/dr < 0$, we are then in the second regime, $S_\nu < I_\nu(0)$. We see an absorption line spectrum,



This is the case for the ***optical*** spectrum of the sun. However, in the ***ultraviolet***, light comes from higher layers for which the temperature is increasing with radius. This gives an emission line spectrum.

Schematic temperature structure of the sun



Summary

Emission line spectra are produced:

- By an optically thin volume of gas with no background light (e.g., an emission line nebula)
- By optically thick gas in which the source function increases outwards (e.g., Sun in the UV)

Absorption line spectra are produced:

- By an optically thin volume of gas illuminated by background radiation whose intensity is greater than the source function (e.g., cold gas in the sight line of a quasar)
- By optically thick gas in which the source function decreases outwards (e.g., Sun in the optical)

If the source function is the Planck function, a decreasing source function corresponds to a decreasing temperature

Lecture 5 revision quiz:

- Describe to a classmate, with the aid of sketches, the distinction between colour temperature, brightness temperature and effective temperature.
- Show that in the limit $I_\nu << kT$, the specific intensity of a blackbody source of temperature T_b reduces to:

$$I_\nu = \frac{2\nu^2}{c^2} kT_b$$

- Make appropriate approximations and substitutions into the formal solution of the equation of radiation transfer to change from intensity to temperature.

Lecture 5 revision quiz:

- Sketch the physical situation that gives rise to the off-source emission and on-source absorption in the example of 21cm emission and absorption lines.
- What theorem is used to derive the approximation for small optical depth?

$$e^{-\tau_\nu} \cong 1 - \tau_\nu$$

- Check that this approximation is correct.