# Widths of spectral lines

- Real spectral lines are broadened because:
  - Energy levels are not infinitely sharp
  - Atoms are moving relative to observer
- **Three mechanisms determine the line profile $\phi(\nu)$**
  - **Quantum mechanical uncertainty in the energy E of levels with finite lifetimes. Determines the natural width of a line (generally very small).**
  - **Collisional broadening. Collisions reduce the effective lifetime of a state, leading to broader lines. High pressure gives more collisions (e.g., stars).**
  - **Doppler or thermal broadening, due to the thermal (or large-scale turbulent) motion of individual atoms in the gas relative to the observer.**
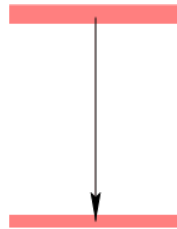
Natural width

- Uncertainty principle. Energy level above ground state with energy E and lifetime Δt, has uncertainty in energy:

$$\Delta E \Delta t \sim \hbar$$

  i.e., short-lived states have large uncertainties in the energy

- A photon emitted in a transition from this level to the ground state will have a range of possible frequencies,

$$\Delta \nu \sim \frac{\Delta E}{h} \sim \frac{1}{2\pi \Delta t}$$

Lorentzian profile

- This effect is called *natural broadening*
- If the spontaneous decay of an atomic state n (to all lower energy levels $n' < n$) proceeds at a rate,

$$\gamma = \sum_{n'} A_{n\,n'}$$

  it can be shown (see damped harmonic oscillator derivations in Chapter 3.6 of Rybicki & Lightman) that the line profile for transitions to the ground state is of the form,

$$\phi(v) = \frac{\gamma/4\pi^2}{\left(v - v_0\right)^2 + \left(\gamma/4\pi\right)^2}$$

- This is a *Lorentzian* or *natural* line profile
- If both the upper and lower states are broadened, sum the γ values for the two states

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Schematic Lorentz profile

- If radiation is present, add stimulated emission rates to spontaneous ones
- Well away from the line centre,

$$\phi(v) \propto \Delta v^{-2}$$

- Decay is much slower than a gaussian
- Natural linewidth isn't often directly observed, except in the line wings in low-pressure (nebular) environments. Other broadening mechanisms usually dominate.
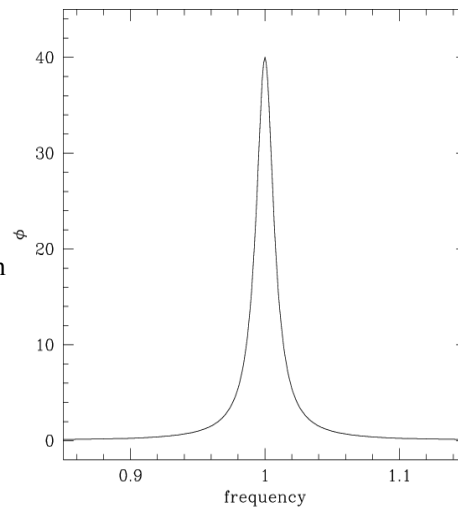
Collisional broadening

- The natural linewidth arises because excited states have a finite lifetime
- Collisions shorten the characteristic time for the emission process
- If the frequency of collisions is ν_{col}, then the profile is

$$\phi(\nu) = \frac{\Gamma/4\pi^2}{(\nu - \nu_0)^2 + (\Gamma/4\pi)^2}$$

where $\Gamma = \gamma + 2\nu_{col}$

- Still a Lorentz profile. Collisions dominate in high density environments, hence get broader lines in dwarfs than giants of the same spectral type.

Thermal broadening

- Random motions of atoms in a gas depend on temperature.
- *Most probable speed u* is

$$\frac{1}{2}mu^2 = kT$$

- Maxwell's Law of velocity distribution gives number of atoms with a given speed or velocity. Important to distinguish between forms of this law for *speed* and for *one component of velocity*:

$$dN(v_x) \propto \exp\left(-\frac{mv_x^2}{2kT}\right) dv_x$$

Speeds and velocities

- For any one component (say v_x), the distribution of *velocities* is described by a Gaussian
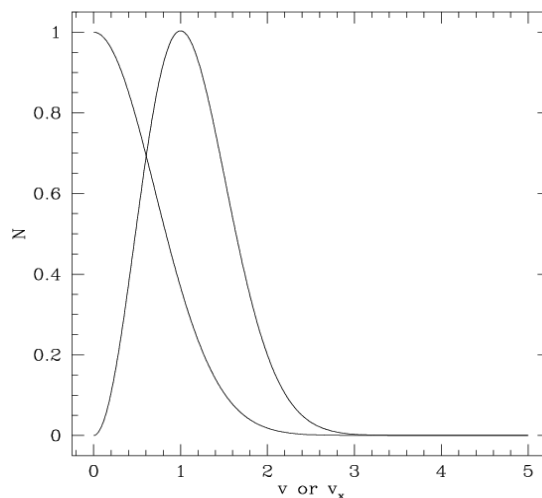- Distribution law for *speeds* has an extra factor of v^2,

$$dN(v) \propto v^2 \exp\left(-\frac{mv^2}{2kT}\right) dv$$

- Prove this for yourself in tutorial sheet 3, Q3
- The one-component version gives the thermal broadening, since we only care about the motions along the line of sight

Distributions of speed and velocity

- Few particles have speeds much greater than the mean.

Thermal Doppler effect

- Doppler-shifted frequency ν for an atom moving at velocity υ_x along the line of sight differs from ν_0 in rest frame of atom:

- $$\frac{\nu - \nu_0}{\nu_0} = \frac{v_x}{c}$$

- Combine Doppler shift with the one-dimensional distribution of velocities to get profile function:

$$\phi(\nu) = \frac{e^{-(\nu - \nu_0)^2 / \Delta \nu_D^2}}{\Delta \nu_D \sqrt{\pi}}$$

- where Doppler width $\Delta \nu_D$ is defined as

$$\Delta \nu_D = \frac{\nu_0}{c} \sqrt{\frac{2kT}{m}}$$

Including turbulent velocities

- If turbulent motions in gas can be described by a similar velocity distribution, the effective Doppler width becomes

$$\Delta \nu_D = \frac{\nu_0}{c} \sqrt{\frac{2kT}{m} + v_{\text{turb}}^2}$$

- where υ_{turb} is a root mean-square measure of the turbulent velocities. This situation occurs, e.g., in observations of star forming regions or in convective stellar photospheres.
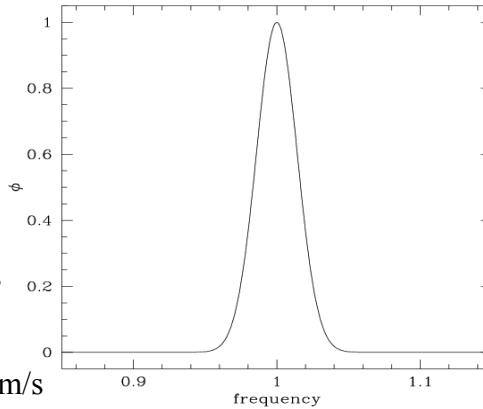
Thermal line profile

- This is just a Gaussian, which falls off very rapidly away from the line centre.

- Numerically, for hydrogen the thermal broadening in velocity units is

$$\frac{\Delta \nu_D c}{\nu_0} = \sqrt{\frac{2kT}{m}} = 13 \left(\frac{T}{10^4 K}\right)^{1/2} \text{km/s}$$

a rough number worth remembering.

Example

- Show that the line width of hydrogen in the sun (6000K) is dominated by thermal velocities and that it is also wider than the line width of metals (use iron with atomic weight 56).

$$\frac{\Delta \lambda}{\lambda} = \frac{v}{c} = \frac{1}{c}\sqrt{\frac{2kT}{m}} = \frac{1}{3\times 10^8}\sqrt{\frac{2\times 1.38\times 10^{-23}\times 6\times 10^3}{56\times 1.66\times 10^{-27}}} \approx 4\times 10^6$$

- Doppler width for λ=400 nm is then 0.0016 nm
- Larger than the damping width ~ 0.00011 nm for Balmer lines of hydrogen
- Hydrogen's thermal velocity is 10 km/s compared to 1.4 km/s for iron

Voigt profile

- The combination of thermal broadening with the natural or collisionally broadened line profile is called the *Voigt profile*.

- This is the convolution of the Lorentz profile with a Doppler profile. No simple analytic form.
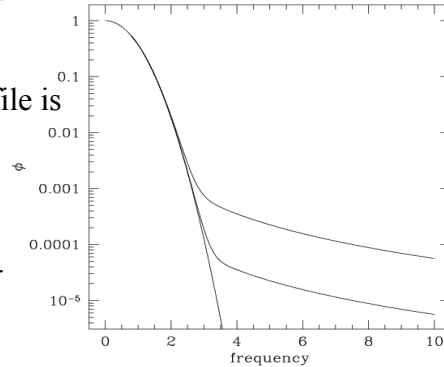
Curves show the profile as the natural (or collisional) linewidth is increased. Lorentz profile falls off slower than Doppler profile, so core remains roughly Gaussian, while the wings look like a Lorentz profile.

Example of use of Voigt profile

- Voigt profiles are fit to observations of absorption towards quasars to measure temperatures and column densities of the gas along the line of sight.

- Deuterium absorbs at slightly different frequencies than hydrogen. Detecting a corresponding deuterium feature provides a limit on the primordial abundance of deuterium, since stellar processes always destroy it (figure from Tytler et al. 1999).
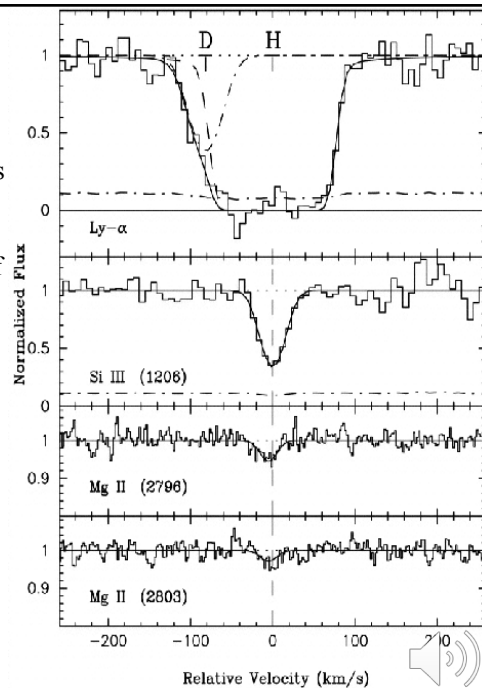
Lecture 8 revision quiz

- Show that $\phi(\nu) \propto \Delta\nu^{-2}$ well away from the centre of a line with a Lorentzian broadening profile.
- Calculate the line-of-sight thermal velocity dispersion Δv_D of line photons emitted from a hydrogen cloud at a temperature of 10^4 K.
- Calculate the natural broadening linewidth in nm of the Lyman α line, given that $A_{ul}=5\times10^8$ s^{-1} . Convert to km/sec via the Doppler formula.
- Compute both profiles and plot them on the same velocity scale.
- Work through the damped harmonic oscillator derivation of the Lorentz function in the textbook.

Lecture 8 revision quiz

- Why are Lorentzian line wings only seen in low density environments?
- Why are spectral lines broader in dwarf stars than giants of the same spectral type?
- How is the damping width for Balmer lines determined?
- What is deuterium?
- How can temperature and density be determined from observations of absorption lines?