Nebular heating and thermal equilibrium

- Assume nebulae are heated by photoionizations only
- Every photoionization creates an electron with energy

$$\frac{1}{2}m\upsilon_p^2 = h(\nu - \nu_0)$$

- Every recombination removes $\frac{1}{2}m\upsilon_r^2$.
- Difference must be lost by radiation if nebula is in thermal equilibrium
- Average energy of photoionized electron is

$$\frac{3}{2}kT_i = \frac{\int_{\nu_0}^{\infty} \frac{J_\nu}{h\nu} h(\nu - \nu_0) d\nu}{\int_{\nu_0}^{\infty} \frac{J_\nu}{h\nu} d\nu}$$

Ionization temperature

- Suppose ionizing spectrum is on Wien part (UV tail) of blackbody curve:

$$J_\nu \propto B_\nu(T_*) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT_*} - 1} \approx \frac{2h\nu^3}{c^2} e^{-h\nu/kT_*}.$$

- After some math, can show that

$$\frac{3}{2}kT_i \approx kT_* \Rightarrow T_i \approx \frac{2}{3}T_*.$$

- So T_i depends on *shape* not strength of ionizing spectrum
  - Energy of ionized electron depends on energy of photon that hit it, not on how many photons there are
- Photoionization x-section decreases as ν^{-3} so T_i may increase with distance from star

Heating rate

- Energy gained by nebula through photoionization is

$$G(\mathrm{H}) = N(\mathrm{H}^0)\int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} h(\nu - \nu_0) a_\nu (\mathrm{H}^0) d\nu$$

Photoionization x-sect [m^2] for H^0

- Use equation for photoionization equilibrium to get G(H):

$$N(H^0)\int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} a_\nu(H^0) d\nu = N_e N_p \alpha_A(H^0, T)$$

$$G(\mathrm{H}) = N_e N_p \alpha_A(\mathrm{H}^0, T) \frac{\int_{\nu_0}^{\infty} \frac{J_\nu}{h\nu} h(\nu - \nu_0) a_\nu(\mathrm{H}^0) d\nu}{\int_{\nu_0}^{\infty} \frac{J_\nu}{h\nu} a_\nu(\mathrm{H}^0) d\nu}$$

$$= N_e N_p \alpha_A(\mathrm{H}^0, T)\frac{3}{2}kT_i \approx N_e N_p \alpha_A(\mathrm{H}^0, T)kT_*.$$

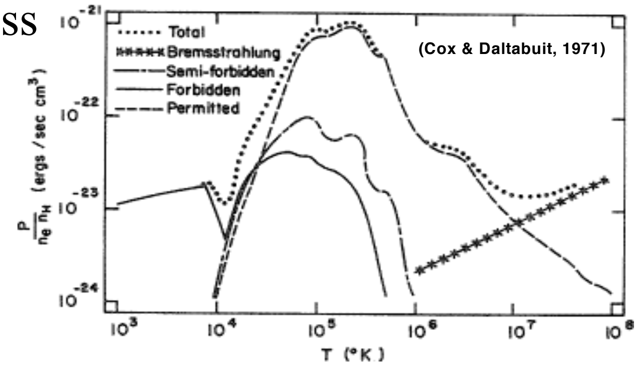
- Similarly for He; hence total $G = G(\mathrm{H}) + G(\mathrm{He}^0) + G(\mathrm{He}^1)$.

Cooling mechanisms

- Free-free (bremsstrahlung)
  - Least important – usually negligible
- Recombination emission
  - Electrons recombine with ions to form excited atoms
  - Atom cascades to lower levels
  - Emitted photons that escape serve to cool the nebula
  - Calculated from kinetic energy-weighted recombination coefficients to each level of atom
- Collisionally excited line cooling
  - Most important mode of nebular cooling
  - Metals have many low-lying energy states
  - These states can be populated by collisions with electrons having $kT \sim$ 1 eV since $kT = 0.86$ eV at $T = 10000$ K
  - If spontaneous de-excitation takes place before another collision de-excites the atom, photon escapes and cools nebula

$$G = L_{ff} + L_R(\mathrm{H}) + L_C.$$

Radiative loss function



- We need to sum radiative losses from several processes to yield the *radiative loss function* = power per unit volume radiated by a plasma with given element abundances in equilibrium
- Plot shows results for n_e=n_H=1 cm^{-3}. Important features:
  - Forbidden transitions dominate for $T \leq 10^4$ K
  - Permitted transitions dominate for 10^4 K $\leq T \leq 10^7$ K
  - Free-free emission (Bremsstrahlung) is most important at high $T \geq 10^7$ K
  - Gas at $T \sim 10^5$ K cools quickly – lots of atomic losses at these temperatures

Cooling by recombination emission

- Kinetic energy lost by electron gas per unit volume per unit time:

$$L_R(H) = N_e N_p kT \beta_A(H^0, T)$$

where $\beta_A(H^0,T) = \sum_{n=1}^{\infty} \sum_{L=0}^{n-1} \beta_{nL}(H^0,T)$

Maxwell-Boltzmann

and $\beta_{nL}(H^0,T) = \frac{1}{kT}\int_0^{\infty} \upsilon \sigma_{nL}(H^0,T) \frac{1}{2} m\upsilon^2 f(\upsilon) d\upsilon.$

- β_{nL}(H^0,T) is effectively a kinetic energy-averaged recombination coefficient
- *n, L* are the shell number and total angular momentum quantum number of the state concerned

Collisional cooling and detailed balance - I

- Common ions of O, N, C, Ne, Ar all have levels that are ~ *1 eV* above ground state (~ kT at T~10^4 K)
  - Efficient for cooling nebulae
- How many collisional excitations $\mathrm{m}^{-3}\ \mathrm{s}^{-1}$?
- Answer involves:
  - Electron, ion densities N_e, N_i
  - Volume swept out by an electron each second which depends on velocity so multiply $N_e N_i$ by a factor $q \sim T_e^{-1/2}$
  - The Boltzmann factor. If $m_e \upsilon^2/2 < \Delta E$, collision won't have enough energy to excite atom/ion from ground state. Larger ΔE implies fewer electrons with enough energy at fixed T. No energy barrier for downward collisional transitions.
  - Statistical weights of levels involved. Collision rate out of a level with statistical weight g is g times smaller than out of a similar level with just 1 state
  - A dimensionless quantum mechanical collision strength: $\Omega(i,j)$

Collisional cooling and detailed balance - II

- Putting it all together, collision rate from level i to j is $N_e N_i q_{ij}$
- If gas gains energy from ion by collisional de-excitation, ion de-excites from i to j ($i > j$), then

$$q_{ij} = \left(\frac{2\pi}{kT_e}\right)^{1/2} \frac{\hbar}{m_e^{3/2}} \frac{\Omega(i,j)}{g_i} = 8.269 \times 10^{-12} \frac{\Omega(i,j)}{g_i T_e^{1/2}} \ \mathrm{m}^3\ \mathrm{s}^{-1}$$

- But if gas loses energy to ion by collisional excitation, ion is excited from i to j ($i < j$), then

$$q_{ij} = 8.269 \times 10^{-12} \frac{\Omega(i,j)}{g_i T_e^{1/2}} e^{-\Delta E/kT} \ \mathrm{m}^3\ \mathrm{s}^{-1}.$$

- Collision strength is symmetrical: $\Omega(i,j) = \Omega(j,i)$

$$\Rightarrow q_{ij} = \frac{g_j}{g_i} e^{-\Delta E/kT}\ q_{ji}.$$

The low-density limit

- If particle density is low, then it's unlikely that a collision will occur before a spontaneous decay occurs
- So every collisional excitation creates a photon that escapes the nebula, and cooling rate is:

$$L_C = \sum_j N_e N_1 q_{1j} h\nu_{1j}$$

All excited levels (typically 4)

Energy diff between state j and ground state

- But we need to consider collisional de-excitation too

Collisional cooling and detailed balance

- To begin, consider a 2-level atom or ion with number density N_1 in ground state and N_2 in excited state
- Detailed balance: $N_e N_1 q_{12} = N_e N_2 q_{21} + N_2 A_{21}$

$$\Rightarrow \frac{N_2}{N_1} = \frac{N_e q_{12}}{N_e q_{21} + A_{21}}.$$

- Collisional cooling rate:

$$L_C = N_2 A_{21} h\nu_{21} = \frac{N_1 N_e q_{12}}{N_e q_{21} + A_{21}} A_{21} h\nu_{21}.$$

- Divide top & bottom by A_{21} :

$$L_C = N_2 A_{21} h\nu_{21} = \frac{N_1 N_e q_{12}}{1 + N_e q_{21}/A_{21}} h\nu_{21}.$$

Low and high density limits

$$L_C = N_2 A_{21} h\nu_{21} = \frac{N_1 N_e q_{12}}{1 + N_e q_{21}/A_{21}} h\nu_{21}.$$

- Low-density limit:

$$\text{As } N_e \to 0,\ L_C = N_e N_1 q_{12} h\nu_{21}$$

  – Every collision produces a photon that escapes

- High-density limit:

$$\text{As } N_e \to \infty,\ L_C \to N_1 \frac{g_2}{g_1} e^{-\Delta E/kT} A_{21} h\nu_{21}$$

  – The thermodynamic-equilibrium cooling rate

Multilevel atoms

- Most atoms have more than 2 levels so we get a set of simultaneous linear rate equations:

$$\sum_{j \neq i} N_e N_j q_{ji} + \sum_{j > i} N_j A_{ji} = \sum_{j \neq i} N_e N_i q_{ij} + \sum_{j < i} N_i A_{ij}$$

Collision rate into level i	Radiative decays into level i from higher levels	Collision rate out of level i	Radiative decays from level i into lower levels

- Total number of this species of atom/ion:

$$\sum_j N_j = N$$

- This makes $n + 1$ equations in n unknowns so we can solve for the relative population in each level

Cooling rate and critical density

- Having solved for level populations N_i , get collisionally excited radiative cooling rate:

$$\text{As } N_e \to 0,\ L_C = \sum_i N_i \sum_{j<i} A_{ij} h\nu_{ij}.$$

- Collisions out of a level become more important than radiative transitions at a critical density when:

$$\sum_{j\neq i} N_e N_i q_{ij} > \sum_{j<i} N_i A_{ij}$$

$$\Rightarrow N_{\text{crit}}(i) > \sum_{j<i} A_{ij} \Big/ \sum_{j\neq i} q_{ij}.$$

- When $N_e < N_{\text{crit}}(i)$ collisional deexcitation out of level i is negligible

Lecture 17 Revision quiz

- Why is the distribution of kinetic energies for electrons liberated by photoionization different from the distribution of kinetic energies removed by recombination?
- Try the challenge of Slide 2 to show that

$$\frac{3}{2}kT_i \approx kT_* \Rightarrow T_i \approx \frac{2}{3}T_*.$$

- Why does the photoionization cross-section appear in the integrals on Slide 3, but not in slide 1?

Lecture 17 revision quiz

- Look at Fig 4 in the paper by Aggarwal et al (1982) MNRAS 201, 923 which I've placed on my website for this course. Why does the emissivity of the 500.7 nm and 495.9 nm lines of [OIII] saturate at electron densities above 10^7 cm^{-3}? Is this a density you would expect to find in an emission-line nebula?
- Why is the plot flat above this density?
- Can you use this plot to estimate the collisional excitation rates *q* and spontaneous emission coefficients *A* for these two lines?