Measuring ionizing flux of photons

- If Case B holds, recombination lines give ionizing flux:

$$\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu = Q(H^0) = \int_0^R N_e N_p \alpha_B dV$$

- Where V is the nebular volume
- In Case B every ionization causes a recombination which creates a Balmer photon that escapes
- Hβ luminosity is

$$L(\mathrm{H}\beta) = \int_0^R 4\pi j_{\mathrm{H}\beta} dV = h\nu_{\mathrm{H}\beta} \int_0^R N_e N_p \alpha_{\mathrm{H}\beta}^{eff} dV.$$

Measuring ionizing flux of photons

- Ratio of Hβ photons to ionizing photons:

$$\frac{L(\mathrm{H}\beta)/h\nu_{\mathrm{H}\beta}}{\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu} = \frac{\int_0^R N_e N_p \alpha_{\mathrm{H}\beta}^{eff} dV}{\int_0^R N_e N_p \alpha_B dV} \approx \frac{\alpha_{\mathrm{H}\beta}^{eff}}{\alpha_B}.$$

Density weighted average values

- Since the recombination coefficients α depend only on temperature, this ratio is easily determined

Temperature of ionizing source

- In Case B, can use nebula's Hβ flux and star's V magnitude to measure temperature of star
- First approximate star's spectrum as a blackbody $B_\nu(T)$
- Ratio of V-band flux to ionizing photons is a function $F(T)$ of temperature:

$$F(T) = \frac{B_V(T)}{\int_{\nu_0}^{\infty} \frac{B_\nu(T)}{h\nu} d\nu} = \frac{L_V}{\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu} = \frac{L_V}{\frac{L(\mathrm{H}\beta)}{h\nu_{\mathrm{H}\beta}} \frac{\alpha_{\mathrm{H}\beta}^{eff}}{\alpha_B}}$$

$$= \frac{L_V}{L(\mathrm{H}\beta)} \frac{\alpha_B}{\alpha_{\mathrm{H}\beta}^{eff}} h\nu_{\mathrm{H}\beta} .$$

Temperature of ionizing source

- Convert luminosities L to observed fluxes f using inverse-square law. Distances cancel so:

$$F(T) = \frac{f_V}{f(\mathrm{H}\beta)} \frac{\alpha_B}{\alpha_{\mathrm{H}\beta}^{eff}} h\nu_{\mathrm{H}\beta} .$$

- If both Hβ flux and V-band flux are visible, just evaluate RHS from observed $f_V/f_{\mathrm{H}\beta}$ and deduce T from the computed function $F(T)$
- Known as the *Zanstra method* for determining stellar temperatures
- Also works for He I 5876 and He II 4686

Estimating extinction to a nebula

- Balmer emission-line ratios are very insensitive to T, N_e
- Can't go far wrong by assuming $\dfrac{I(\mathrm{H}\alpha)}{I(\mathrm{H}\beta)} \approx 2.86.$
- If ratio is larger the discrepancy is most likely due to reddening (differential extinction), since

$$A_\lambda \propto 1/\lambda$$

$$\Rightarrow \log f_\lambda^{\mathrm{obs}} = \log f_\lambda^0 - \frac{a}{\lambda}$$

Constant representing amount of extinction

Observed flux at wavelength λ

Dereddened flux at wavelength λ

Estimating extinction to a nebula

- If we use natural logs, τ = extinction optical depth at Hβ, i.e., $\tau = a/\lambda 4861$
- So:

$$\log f_{\mathrm{H}\beta}^{\mathrm{obs}} = \log f_{\mathrm{H}\beta}^0 - \tau_{\mathrm{H}\beta}$$

- And at any other wavelength λ, $\tau_\lambda = \tau_{\mathrm{H}\beta}\lambda_{\mathrm{H}\beta}/\lambda$.
- Ratio of observed flux at wavelength λ to observed flux at Hβ is

$$\log\left(\frac{f_\lambda^{\mathrm{obs}}}{f_{\mathrm{H}\beta}^{\mathrm{obs}}}\right) = \log\left(\frac{f_\lambda^0}{f_{\mathrm{H}\beta}^0}\right) - \tau_{\mathrm{H}\beta}\left(\frac{\lambda_{H\beta}}{\lambda} - 1\right).$$

- Since we know the intrinsic Hα/Hβ ratio, we can solve for $\tau_{\mathrm{H}\beta}$, then get the dereddened Hβ flux and hence get the dereddened flux at any wavelength

Deriving nebular abundances

- First, use nebular diagnostics to fix the electron temperatures and densities
- Abundances follow from observed fluxes
- For recombination lines of H, He I, He II:

$$I(\mathrm{H}\beta) = N_e N_p \alpha^{\mathrm{eff}}_{\mathrm{H}\beta}$$

$$I(\lambda 5876) = N_e N_{\mathrm{He}^+} \alpha^{\mathrm{eff}}_{\lambda 5876}$$

$$I(\lambda 4686) = N_e N_{\mathrm{He}^{++}} \alpha^{\mathrm{eff}}_{\lambda 4686},$$

- While for collisionally excited ions: $I_{ji} = N_e N_i q_{ij}$.
- The α and q_{ij} are functions of temperature only. Once they are known, the line strengths are directly proportional to abundance.

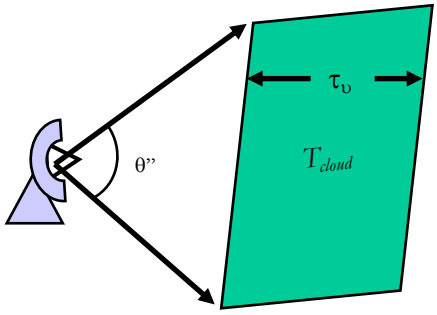
Deriving nebular abundances

- To get total abundance of, e.g. oxygen, need to add up O, O^+, O^{++} etc
- Fortunately the recombination-line strengths of H, He I and He II, give the underlying distribution of ionizing photons
  - Hence can estimate relative abundance of, say, O^{+++} to O^{++} even if no lines of O^{+++} are observable
- There is a fair bit of redundancy in the information provided by emission lines
- Sometimes get inconsistent N_e and/or T from different diagnostics
  - e.g., [OII] lines often give N_e greater than found using radio observations of nebular continuum
  - Can explain if nebula has clumps at N_e separated by regions of zero density, defining a *filling factor f*
  - This affects many equations, e.g., ionization balance becomes

$$Q(\mathrm{H}^0) = \frac{4}{3}\pi R^3 N_e N_p \alpha_B . f .$$

Radio-line emission and absorption (Tutorial sheet2, Q2)

Observe on and off a hot continuum source behind a cool foreground cloud, on and off the HI 21 cm radio line. Get temperatures of both.



$T_{back} >> T_{cloud}$

Example: HI 21 cm radio absorption/emission

Off-source emission

On-source absorption

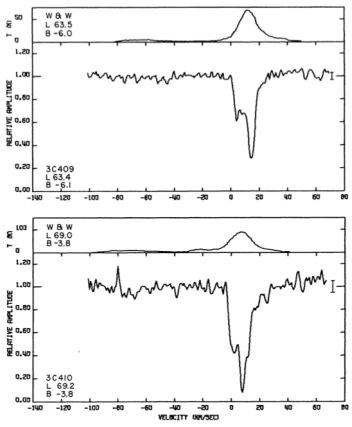


FIG. 1.—*Continued*

Hughes et al 1971, ApJS 23, 323

Temperatures from radio observations

- Recall equation of radiative transfer in terms of brightness temperature in Rayleigh-Jeans limit, where $I_\nu \sim T_b$ and $S_\nu \sim T$ for a thermal source function.

$$\frac{dT_b}{d\tau_\nu} = -T_b + T$$

- With formal solution:

$$T_b(\tau_\nu) = T_b(0)e^{-\tau_\nu} + T(1 - e^{-\tau_\nu})$$

Temperatures from radio observations

- Suppose we observe a background continuum source of brightness temperature T_b through a nebula
- At frequencies where cloud is optically thin (high frequency continuum):

  $\tau_C \to 0 \Rightarrow T_b(\tau_C) \to T_b(0)$
- At frequencies where cloud is optically thick (radio line or low-frequency continuum):

  $T_b(\tau_L) \to T_b(0)e^{-\tau_L} + T(1 - e^{-\tau_L})$
- Now observe off the background source, so:

  $T_b(\tau_C) \to 0$ and $T_b(\tau_L) = T(1 - e^{-\tau_L})$.

Temperatures from radio observations

- Difference between brightness temperatures on-source:
  $\Delta T_{on} = T_b(0)(e^{-\tau_L} - 1) + T(1 - e^{-\tau_L})$.
- Difference between brightness temperatures off-source:

  $\Delta T_{off} = T(1 - e^{-\tau_L})$.
- So $\Delta T_{off} - \Delta T_{on} = T_b(0)(1 - e^{-\tau_L})$

  $\Rightarrow \tau_L = -\ln\left[1 - \left(\Delta T_{off} - \Delta T_{on}\right)/T_b\right]$.
- Now that we know τ_L, get T from ΔT_{off}

Lecture 20 revision quiz

- Light emerging from an absorbing cloud of optical thickness τ_ν has its intensity diminished by a factor $\exp(-\tau_\nu)$. Observers usually convert the ratio of the observed flux to the original flux to magnitudes, and call it interstellar extinction, A. What's the relationship between τ_ν and A?
- Derive the equation relating observed and intrinsic flux from an HII region.
- Sketch the physical situation of the *filling factor* approach for the structure of an HII region.