Local Thermodynamic Equilibrium

- LTE assumptions
- LTE validity
- Saha, Boltzmann equations

Aim to determine:

- Gas pressure, density, absorption coefficients
- Wavelength dependence of emergent radiation field

Need to know:

- Opacity sources and wavelength dependences
- Level populations: how many atoms can absorb/emit radiation of a given wavelength
- Intensity of local radiation field

But:

- Level populations determined by radiation field & pressure
- Make simplifying assumption that levels determined only by temperature…

LTE

All atomic, ionic, and molecular level populations given by Maxwellian-like Saha-Boltzmann statistics defined by local temperature:

$$N_i / N \sim \exp(-E_i / kT).$$

Two variables uniquely define state of gas: temperature and density, via equilibrium relations of statistical mechanics

T describes: velocity distribution
energy level populations
intensity of black body radiation

LTE Validity

- Detailed Balance:
  Rate process occurs = rate of inverse process
- Transitions: radiative & collisional
- Collisional processes in detailed balance when velocity is Maxwellian
- Radiative processes in detailed balance only if radiation field is isotropic & Planck distribution, $B_\nu(T)$
- LTE OK in deepest atmosphere layers: densities high, collision rates large, optical depths large, photons trapped, and radiation field approaches $B_\nu(T)$
- Clearly not the case in observable layers
- But LTE makes calculations simple…

Basic Relationships: General & LTE

- Perfect gas law: low densities
- Level populations: Boltzmann
- Ionization stage: Saha
- Velocity distribution: Maxwellian
- Radiation field: Planck (black body)

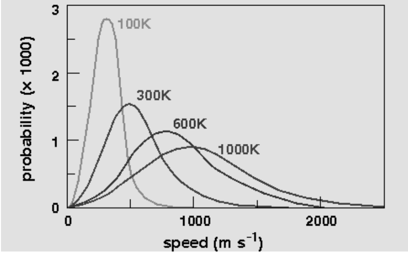
Matter in LTE

Maxwell Distribution:

$$n(v) / N \, \mathrm{d}v = (m / 2\pi kT)^{3/2} 4\pi v^2 \exp(-mv^2 / 2kT) \, \mathrm{d}v$$

Mass = m, total velocity = v, number density = N

High velocity tail due to v^2 factor
Peak => most probable speed, $v_p = (2kT/m)^{1/2}$
Average speed, $v_{av} = (3kT/m)^{1/2}$



Boltzmann Distribution:

$$\frac{n_{r,s}}{n_{r,t}} = \frac{g_{r,s}}{g_{r,t}} \exp[-(\chi_{r,s} - \chi_{r,t})/kT]$$

$n_{r,s}$ = number density of level s of ionization stage r
$g_{r,s}$ = statistical weight of level s in stage r
$\chi_{r,s}$ = excitation energy of (r, s) measured from ground (r, 1)
$\chi_{r,s} - \chi_{r,t} = h\nu$ for a radiative transition (r, s) to (r, t)

$n_{r,s}$ relative to total n_r :

$$\frac{n_{r,s}}{n_r} = \frac{g_{r,s}}{U_r(T)} \exp[-\chi_{r,s}/kT]$$

Partition function:

$$U_r \equiv \sum_s g_{r,s} \exp(-\chi_{r,s}/kT)$$

Convenient logarithmic form (base 10):

$$\log\left[\frac{n_{r,s}}{n_r}\right] = \log\left[\frac{g_{r,s}}{U_r(T)}\right] - \theta\chi_{r,s}$$

Excitation potential $\chi_{r,s}$ in eV
θ = inverse temperature = 5040 / T , when $\chi_{r,s}$ in eV
1 eV = 1.60218 E –19 J

Partition Functions

- Appear to require total knowledge of energy level structure
- In many cases most atoms/ions are in or near ground state and $g_{r,1}$ dominates U
- At higher T: more excited atoms, approximations exist
- Detailed calculations are tabulated

Saha Ionization Distribution

Population ratio between successive ionization stages is:

$$\frac{N_{r+1}}{N_r} = \frac{1}{N_\mathrm{e}} \frac{2U_{r+1}}{U_r} (2\pi\, m_\mathrm{e}\, k\, T / h^2)^{3/2} \exp(-\chi_r / kT)$$

N_e , m_e: electron number density and mass

N_{r+1} , N_r: population densities of ionization stages r , r+1

$\chi_\mathrm{r} = h\nu_\mathrm{edge}$: ionization potential of stage r

U_{r+1} , U_r: partition functions

Using perfect gas law, $P_e = N_e\, k\, T$:

$$\frac{N_{r+1}\, P_e}{N_r} = \frac{2U_{r+1}}{U_r} \frac{(2\pi\, m_e)^{3/2} (kT)^{5/2}}{h^3} \exp(-\chi_r / kT)$$

For computations use:

$$\log \frac{N_{r+1}\, P_e}{N_r} = \log \frac{2U_{r+1}}{U_r} + 2.5 \log T - \theta\, \chi_r - 1.48$$

P_e in Pa, χ_r in eV

Radiation in LTE

Planck Function

LTE line source function simplifies to the Planck Function:

$$S_\nu^l = \frac{2h\nu^3}{c^2} \frac{1}{\left[\frac{g_u n_l}{g_l n_u}\right]_{\text{LTE}} - 1} = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu / kT) - 1} \equiv B_\nu(T)$$

Equality $S_\nu = B_\nu$ is formally derived via Einstein coefficients for bound-bound process, but holds for all LTE or “thermal” photon processes

Wien Approximation

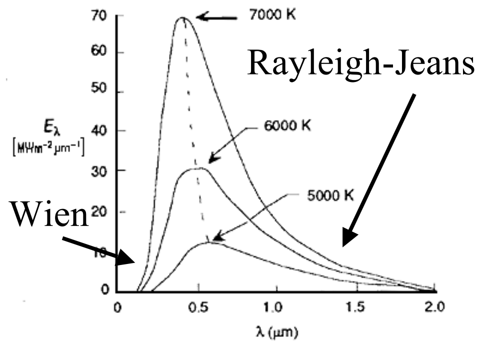
Large ν / T : $$B_\nu(T) \approx \frac{2h\nu^3}{c^2}\exp(h\nu / kT)$$

Particle-like behaviour of high energy photons, similar to Boltzmann distribution

Rayleigh-Jeans

Small ν / T :

$$B_\nu(T) \approx \frac{2h\nu^2 kT}{c^2}$$



Stefan-Boltzman

Integrate B_ν : $$B(T) = \int_0^\infty B_\nu \, \mathrm{d}\nu = \frac{\sigma T^4}{\pi}$$

Stefan-Boltzmann constant:

$$\sigma = \frac{2\pi^5 k^4}{15h^3c^2} = 5.67 \times 10^{-5} \text{ J m}^{-2} \text{ K}^{-4} \text{ s}^{-1}$$

Fraction of Ionized H in Solar Photosphere

- $T \sim 6000$K, $\theta = 0.84$, $P_e \sim 3$ Pa, $\log P_e \sim 0.5$
- $U_1 = 1$(only one state for proton)
- $U_0 = 2$, $g_{0,1} = 2$ (spin up/down)
- χ for higher levels so high that populations negligible

$$\log \frac{N_1}{N_0} = -4$$

- Only 1 in 10^4 atoms is ionized

How much H^- in Solar Photosphere?

- Binding energy $\chi_{-1} = 0.74$ eV. Photons with $\lambda < 1.6\mu$m can ionize H^- back to H^0 and free electron
- Both electrons in level 1 so have opposite spins => $g_{-1,1} = U_{-1} = 1$
- Saha: $$\log \frac{N_0}{N_{-1}} \Rightarrow N(H^-)/N(H^0) \approx 3 \times 10^{-8}$$
- So less than 1 in 10^7 H atoms is H^-
- But there's LOTS of H atoms!

Electrons in Stellar Atmospheres

- Solar stars (T~ 6000 K): H mostly neutral, e^- from metals (Na, Mg, Al, Si, Ca, Fe) with low ionization potential (5-7eV)
- A stars (T ~ 10 000 K): H ionized, dominant source of e^-
- O & B stars (T > 20 000 K): He ionized, contributes to e^-