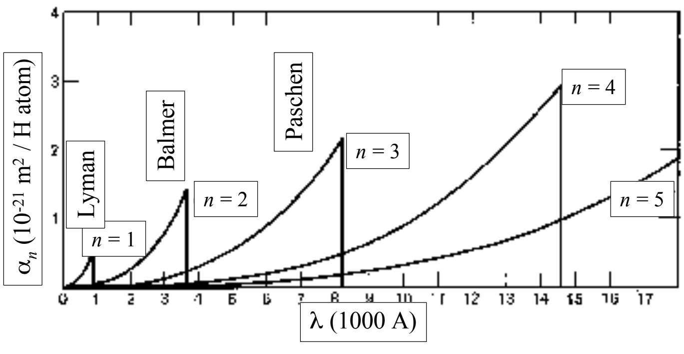
Producing a Theoretical Spectrum

- Pure hydrogen atmosphere, $T_{\text{eff}} = 5600$ K
- Assume opacity independent of depth
- Determine wavelength dependent opacity
- Determine temperature structure
- Determine emergent flux in optical range $3000 \text{ A} < \lambda < 8600 \text{ A}$

HI Continuous Extinction



For $912 < \lambda < 3646$ can ionize out of $n > 1$: need $N_2, N_3, N_4, \ldots$

For $3646 < \lambda < 8206$ can ionize out of $n > 2$: need $N_3, N_4, \ldots$

For $\lambda > 8206$ can ionize out of $n > 3$: need $N_4, \ldots$

Hydrogen Bound-Free Opacity

- Level populations for $T_{\text{eff}} = 5600$ K
- Use Boltzmann equation to get populations of H levels $n = 2$, 3, and 4
- Use Kramer's opacity formula to get wavelength dependence

Calculate Level Populations

Use Boltzmann distribution, $g_n = n^2$:

$$\frac{N_n}{N_1} = \frac{g_n}{g_1} \exp[-(\chi_1 - \chi_n)/kT]$$
$$= n^2 \exp[-\Delta E_{1n}/kT]$$
$$= n^2\ 10^{[-\Delta E_{1n}(\text{eV})5040/T]}$$

Use Rydberg formula: $\Delta E_{1n} = \chi\,(1 - 1/n^2)$, $\chi = 13.6$ eV

$$N_2/N_1 = 4 \times 10^{[-10.2\times0.9]} = 2.6 \times 10^{-9}$$
$$N_3/N_1 = 9 \times 10^{[-11.9\times0.9]} = 1.2 \times 10^{-10}$$
$$N_4/N_1 = 16 \times 10^{[-12.75\times0.9]} = 5.4 \times 10^{-11}$$

Calculate Opacity

Kramer's opacity for hydrogen b-f cross-section (m^2)

$$\sigma_\nu^{\rm bf} = 2.815\times10^{25}\frac{Z^4}{n^5\nu^3}g_{\rm bf} \quad \text{for } \nu \ge \nu_0$$
$$= 1.044\times10^{-30}\,\lambda^3/n^5 = \sigma_0\,\lambda^3/n^5$$

with λ in A. Total opacity = sum of absorption coefficients from all levels that can be ionized by photon at given λ, times population of level:

$$\begin{aligned}\alpha_\nu^{\rm bf} &= N_2\sigma_2 + N_3\sigma_3 + N_4\sigma_4 && \text{for } \lambda \le 3647\text{A}\\ &= N_3\sigma_3 + N_4\sigma_4 && \text{for } 3648\text{ A} \le \lambda \le 8206\text{A}\\ &= N_4\sigma_4 && \text{for } \lambda \ge 8207\text{A}\end{aligned}$$

Ignore contributions from levels $n > 4$

Calculate Opacity

For λ < 3646 A, enough energy to ionize out of $n > 1$:

$$\begin{aligned}\alpha_\nu^{\rm bf} &= N_2\sigma_2 + N_3\sigma_3 + N_4\sigma_4\\ &= N_1\sigma_0\lambda^3(1/32\,N_2/N_1 + 1/243\,N_3/N_1 + 1/1024\,N_4/N_1)\\ &= 8.2\times10^{-11}N_1\sigma_0\lambda^3\end{aligned}$$

For 3647 A < λ < 8206 A, can ionize out of $n > 2$

$$\begin{aligned}\alpha_\nu^{\rm bf} &= N_1\sigma_0\lambda^3(1/243\,N_3/N_1 + 1/1024\,N_4/N_1)\\ &= 5.5\times10^{-13}N_1\sigma_0\lambda^3\end{aligned}$$

For λ > 8207 A, can ionize out of $n > 3$

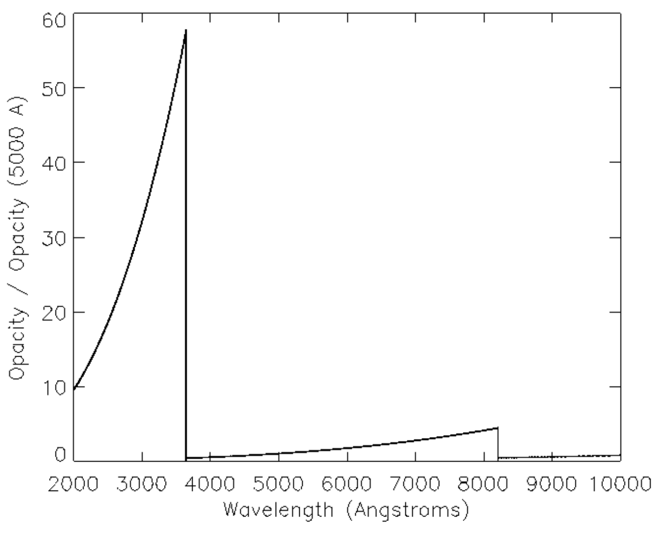
$$\alpha_\nu^{\rm bf} = N_1\sigma_0\lambda^3(1/1024\,N_4/N_1) = 5.3\times10^{-14}N_1\sigma_0\lambda^3$$

Calculate Opacity

- *Gray Atmosphere* gives solution for $T\,[\tau(\lambda)]$, so we'll need some reference λ for τ: normalize opacity to α(5000 A):

$$\alpha_\nu^{\mathrm{bf}}(5000A) = 5.5\times10^{-13} N_1\,\sigma_0\,(5000)^3$$

$$\begin{aligned}
\alpha_\nu^{\mathrm{bf}} / \alpha_\nu^{\mathrm{bf}}(5000A) &= 149\,(\lambda/5000)^3 && \lambda \le 3646\,\mathrm{A} \\
&= (\lambda/5000)^3 && 3646\,\mathrm{A} \le \lambda \le 8206\,\mathrm{A} \\
&= 0.096\,(\lambda/5000)^3 && \lambda \ge 8206\,\mathrm{A}
\end{aligned}$$



Have opacity, now need to solve ERT to get temperature structure and emergent intensity…

Theoretical Spectrum

- Calculate wavelength dependent opacity: hydrogen bound-free opacity
- Temperature structure: Gray atmosphere: $T^4 = T^4_{eff}(3\tau/4+1/2)$
- Emergent flux: LTE + Eddington-Barbier:
  $F = S(\tau = 2/3) = B(T[\tau = 2/3])$

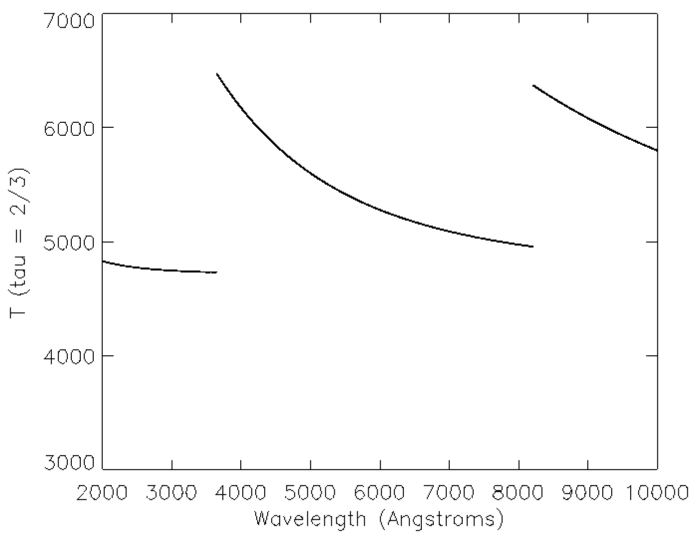
Temperature Structure

Assume gray temperature structure: Opacity isn't really gray, so pick 5000A as a representative wavelength: $\tau_{5000} = \tau(\lambda = 5000\ \text{A})$, and assume temperature structure is

$$T^4(\tau) = T^4_{\text{eff}}\left(\frac{3}{4}\tau_{5000} + \frac{1}{2}\right)$$

Eddington-Barbier gives $F(\lambda) = S(\tau_\lambda = 2/3) = B(T[\tau_\lambda = 2/3])$
Radiation comes from $\tau_\lambda = 2/3$, so want temperature at this depth. If $\tau_\lambda = 2/3$ then $\tau_{5000} = \tau_\lambda /(\kappa_\lambda/ \kappa_{5000})$.
So temperature at $\tau_\lambda = 2/3$ is

$$T^4(\tau_\lambda) = T^4_{\text{eff}}\left(\frac{3}{4} \times \frac{2}{3}[\kappa_\lambda / \kappa_{5000}] + \frac{1}{2}\right)$$



Have temperature structure, can now get emergent flux…

Emergent Flux

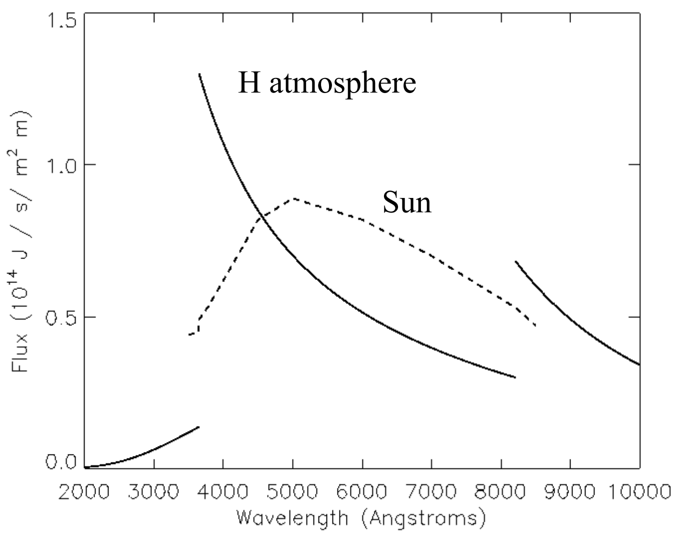
From Eddington-Barbier surface relation:

$$F_\lambda(0) = \pi B_\lambda(T[\tau_\lambda = 2/3])$$

Remember $\nu B_\nu = \lambda B_\lambda$ so

$$F_\lambda(0) = \frac{2\pi h c^2}{\lambda^5} \frac{1}{\exp(hc/\lambda k T[\tau_\lambda]) - 1}$$

Where we use the $T(\tau_\lambda)$ relation derived above to finally give the emergent flux spectrum…



Flux from pure hydrogen atmosphere different from Sun
Solar opacity dominated by H^- opacity