

1.1 Motivation

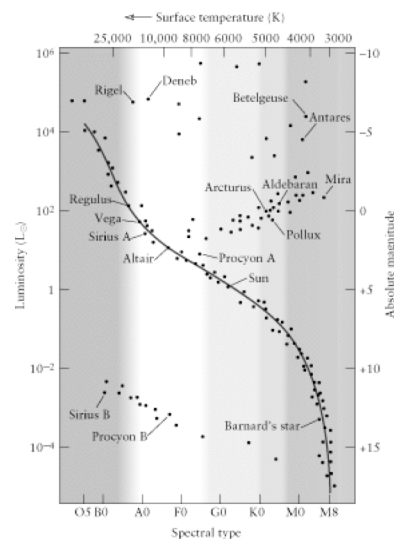
- **Observational:** How do we explain stellar properties as demonstrated, e.g. by the H-R diagram?
- **Theoretical:** How does an isolated, self-gravitating body of gas behave?
- **Aims:**
 - Identify and understand physical processes
 - Make simple models
 - Understand fundamental properties of stars
 - Learn how stars form and evolve

AS 3003

Stellar Physics

1.2 The H-R diagram

- **Solar neighbourhood**
 - stars of known distance.
 - Main sequence, giants, supergiants, white dwarfs
 - All masses and evolutionary stages present



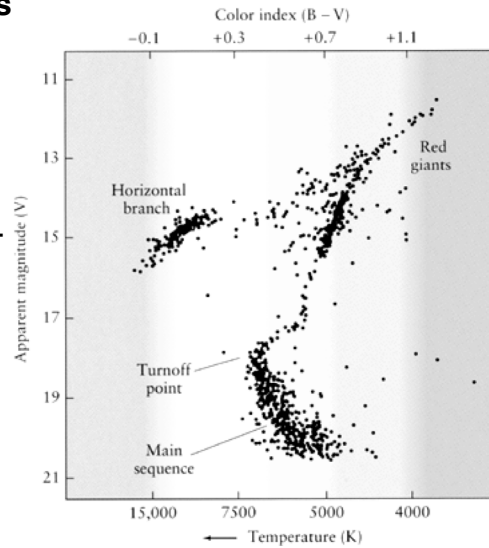
AS 3003

Stellar Physics

Cluster H-R diagrams

- **Open and globular clusters**

- “Snapshots” at a single age
- Assume all stars formed at same time
- All stars at same distance
- Open clusters: disc population, young, solar-like metallicity
- Globulars: Halo pop, old, metal-poor



Typical globular-cluster HR diagram.

AS 3003

Stellar Physics

1.3 Temperature-luminosity relation

- Most important empirical relation between stellar properties.
- Theoretical H-R diagram: plot

$$\log\left(\frac{L}{L_{\text{Sun}}}\right) \text{ against } \log\left(\frac{T_{\text{eff}}}{T_{\text{eff, Sun}}}\right)$$

- Main sequence shows luminosity correlated with effective temperature:

$$\frac{L}{L_{\text{Sun}}} = \left(\frac{T_{\text{eff}}}{T_{\text{eff, Sun}}}\right)^{\alpha} \text{ where on average } \alpha = 6.67.$$

AS 3003

Stellar Physics

1. 4 Mass - luminosity relation

- From masses of many binary stars, find for m-s stars:

$$\frac{L}{L_{\text{Sun}}} = \left(\frac{M}{M_{\text{Sun}}} \right)^{4.0 \pm 0.02} \quad \text{for } 0.4 < M < 10M_{\text{Sun}}$$

$$\frac{L}{L_{\text{Sun}}} = \left(\frac{M}{M_{\text{Sun}}} \right)^{3.6 \pm 0.1} \quad \text{for } 5 \leq M \leq 40M_{\text{Sun}}$$

- Theory of stellar structure must reproduce this!

AS 3003

Stellar Physics

1.6 Definitions

- Symbols & units commonly used for stellar parameters :
- Stellar mass (M) -- solar units M/M_{Sun}
- Stellar radius (R) -- solar units R/R_{Sun}
- Surface gravity ($g=GM/R^2$) -- ms^{-2}
- Effective temperature (T_{eff}) -- K
- Stellar luminosity (L) -- solar units L/L_{Sun}
- Composition (X, Y, Z) -- mass fractions of H (X), He (Y) and other elements (Z).
- Stellar age (t) -- years

AS 3003

Stellar Physics

Example: The Sun

$$M = 1M_{\text{Sun}} = 1.99 \times 10^{30} \text{ kg}$$

$$R = 1R_{\text{Sun}} = 6.96 \times 10^8 \text{ m}$$

$$g = 2.74 \times 10^2 \text{ m s}^{-2}$$

$$T_{\text{eff}} = 5780 \text{ K}$$

$$L = 1L_{\text{Sun}} = 3.86 \times 10^{26} \text{ W}$$

$$X = 0.71, Y = 0.265, Z = 0.025$$

$$t \cong 4.6 \times 10^9 \text{ y}$$

AS 3003

Stellar Physics

1.7a Dynamical (free-fall) timescale

- **Ball of gas supported by pressure.**
- **How long to collapse if support removed?**
- **Spherical shell, initially at rest with radius R_0 encloses mass M .**
- **After collapse to radius r , inward velocity conserves PE+KE:**

$$\frac{1}{2} \left(\frac{dr}{dt} \right)^2 = \frac{GM}{r} - \frac{GM}{R_0}$$

• **Hence**

$$t_{\text{ff}} = \int_{R_0}^0 \frac{dt}{dr} dr = - \int_{R_0}^0 \left(\frac{2GM}{r} - \frac{2GM}{R_0} \right)^{-1/2} dr$$

Choose negative root
to ensure positive answer!

AS 3003

Stellar Physics

Example: Solar free-fall time

Define $x = r / R_0$:

$$t_{\text{ff}} = \left(\frac{R_0^3}{2GM} \right)^{1/2} \int_0^1 \left(\frac{x}{1-x} \right)^{1/2} dx = \frac{\pi}{2} \left(\frac{R_0^3}{2GM} \right)^{1/2},$$

(use substitution $x = \sin^2 \theta$ to evaluate integral).

- cf. period of particle orbiting at R_0 .
- Numerical value:

$$t_{\text{ff}} = 1770 \left(\frac{R_0}{R_{\text{Sun}}} \right)^{3/2} \left(\frac{M_{\text{Sun}}}{M} \right)^{1/2} \text{ s.}$$

AS 3003

Stellar Physics

1.7b Thermal (Kelvin) timescale

- Treat star as a reservoir of heat energy U .
- Length of time to radiate this energy away:

$$\tau_{\text{th}} \approx U / L$$

- Virial theorem relates U to star's gravitational binding energy Ω :

$$2U + \Omega = 0 \Rightarrow -\Omega = 2U$$

- We'll see later that for a star:

$$\Omega = -q \frac{GM^2}{R}$$

- where q is a dimensionless constant of order 1,

$$\text{so: } \tau_{\text{th}} = \frac{q GM^2}{2 LR} \approx 3 \times 10^7 \left(\frac{M}{M_{\text{Sun}}} \right)^2 \left(\frac{L_{\text{Sun}} R_{\text{Sun}}}{LR} \right) \text{ y.}$$

AS 3003

Stellar Physics

1.7c Nuclear timescale

- Radioisotope dating => Earth rocks much older than K-H “age” for Sun (oops...!)
- Energy available from thermonuclear fusion:

$$E = mc^2 = 0.007 q_{sc} X M c^2$$

Mass fraction converted

Schönberg-Chandrasekhar

- Nuclear timescale = time to radiate this amount of energy at main-sequence luminosity L :

$$t_{\text{MS}} = E/L = 0.007 q_{sc} X c^2 M/L$$

$$\approx 7 \times 10^9 \frac{q_{sc} M}{(q_{sc} M)_{\text{Sun}}} \frac{L}{L_{\text{Sun}}} \text{ y.}$$

AS 3003

Stellar Physics

Diffusion time

- If Sun were transparent, γ -rays generated in core would emerge in 2.3s with no energy loss.
- Actual escape time is much longer due to multiple absorptions & re-emissions.
- Total radiant energy content:

$$U_r = u_r V = a \bar{T}^4 \cdot \frac{4}{3} \pi R^3$$

- Crude approximation: $\bar{T}_{\text{Sun}} \approx T_{c,\text{Sun}} / 2 = 7 \times 10^6 \text{ K.}$
- So time for a photon to diffuse out is roughly:

$$t_{\text{diff}} = U_r / L \approx 2 \times 10^5 \left(\frac{R}{R_{\text{Sun}}} \right)^3 \left(\frac{T_c}{T_{c,\text{Sun}}} \right)^4 \left(\frac{L_{\text{Sun}}}{L} \right) \text{ y.}$$

AS 3003

Stellar Physics

2. Equations of stellar structure

- **Object: find a set of n equations in n variables that describe the structure of a star.**
- **Augment them with n boundary conditions.**
- **We'll start with the basics like making sure that mass, momentum and energy are conserved.**

AS 3003

Stellar Physics

2.1 Mass continuity

- **Gotta conserve mass!**
- **By considering a shell of radius r in a region of local density $\rho(r)$, we found in AS2001 that:**

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

- **This introduces 2 unknowns:**
 - $M(r)$ = mass enclosed within sphere radius r
 - $\rho(r)$ = local gas density at radius r .
- **So we need at least 1 more equation!**

AS 3003

Stellar Physics

2.2 Hydrostatic equilibrium

- Otherwise known as force balance or conservation of momentum!
- Main forces operating are self-gravity and internal pressure.
- In AS 2001, considered force balance across a thin shell to get:

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

- So now we have a third unknown:
 - $P(r)$ = local pressure at radius r .
- But still only 2 equations so we need another!

AS 3003

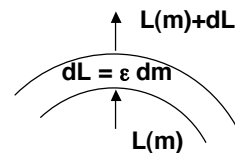
Stellar Physics

2.3 Energy Conservation I: sources & sinks

- Energy moves through the star.

- Consider a shell of mass:

$$dm = \rho dV = 4\pi r^2 \rho dr$$



- Energy flows into bottom of shell at rate $L(m)$
- Energy leaves top of shell at rate $L(m)+dL$
- Denote rate at which energy is produced or absorbed within shell (per unit mass) by:

$$\frac{dL}{dm} = \varepsilon, \text{ and so } \frac{dL}{dr} = 4\pi r^2 \rho \varepsilon$$

- That's one more equation but 2 extra unknowns:
 - $L(r)$ = local luminosity at radius r .
 - $\varepsilon(r)$ = energy generation per unit mass at radius r .

AS 3003

Stellar Physics

Energy sources and sinks

- We'll consider these later. They include:

- Nuclear, e.g. $4p \rightarrow \alpha + 26.72 \text{ MeV}$

- Neutrino, e.g. $\gamma + e^- \rightarrow e^- + \gamma$
or $\gamma + e^- \rightarrow e^- + \nu + \bar{\nu}$

- Nonadiabatic expansion, i.e.

$$dQ = TdS = dE + PdV$$

- Note that these imply a relationship between T, P and energy generation rate, i.e. another equation.