Thermonuclear Physics

- **Four fundamental forces:**

Force	Source	Range	Nuclear reactions?
gravitational	mass	$1/r^2$	No
electrostatic	charge	$1/r^2$	Yes
weak nuclear	baryon-lepton	$1/r^w$: w>>2	Some
strong nuclear	baryon-baryon	$1/r^s$: s>>2	Yes

- **Electrostatic repulsion between ⁺vely charged nuclei gives potential:**

$$E_{pot} = \frac{q_1 q_2}{4\pi\varepsilon_0 r} \approx 2.3 \times 10^{-13} \text{ J for protons.}$$

- **Average KE of proton at 2x10⁷ K is:**

$$E_{kin} = \frac{3}{2}kT \approx 4 \times 10^{-16} \text{ J .}$$

> **Given a Maxwellian velocity distribution, only ~ 1 proton in 10⁸ has enough KE to get over this Coulomb barrier.**

Quantum mechanical description

- **Schrödinger equation for proton wavefunction:**

$$\frac{d^2\psi}{dr^2} + \frac{2m}{\hbar^2}(E_{kin} - E_{pot})\psi = 0.$$

For $r > r_1$ and $r < r_2$,
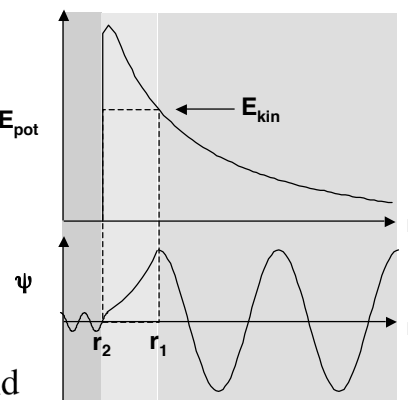
$(E_{kin} - E_{pot}) > 0 \Rightarrow \psi$ is real.

$r > r_1$:	$\psi \sim \sin kr$
$r_2 < r < r_1$:	$\psi \sim e^{-kr}$
$r < r_2$:	$\psi \sim \sigma \sin kr$

where $k = \frac{2m}{h^2}\left(E_{kin} - E_{pot}\right)$, and

$\sigma = $ probability of barrier penetration.

Two things we need to know

- **Reaction rate:**
    - Related to tunnelling probability and energy distribution of particles.
    - Can calculate penetration probability and hence reaction cross-section from Schrödinger eq.
- **Energy released:**
    - approximately, the binding energy per nucleon.
    - All nuclei consist of Z protons and N neutrons.
    - Total rest mass energy of individual particles > rest mass energy of nucleus.
    - Deficit represents binding energy of nucleus:

$$Q(Z,N) = [Zm_{\mathrm{p}} + Nm_{\mathrm{n}} - m(Z,N)]c^2$$

- (=net energy released during construction of nuclide.)
- **Also need to know how much of this energy is "lost" in the form of neutrinos.**

Thermonuclear cross-sections

- **Calculated from**

$$\sigma(E) = \frac{S(E)}{E} \, e^{-(E_{\mathrm{G}}/E)^{1/2}}$$

where E_{G} is the Gamow energy :

$$E_{\mathrm{G}} = (\pi\alpha Z_i Z_j)^2 \, 2\mu c^2$$

Fine structure constant

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 hc} \approx \frac{1}{137}$$

Reduced mass

$$\mu = \frac{m_i m_j}{m_i + m_j}$$

- **S(E) depends on nuclear physics of the reaction and is in practice very difficult to measure.**

Thermonuclear rate coefficients

- **To convert tunnelling probability to reaction rate r_{ij} (reactions $s^{-1}\,kg^{-1}$):**
  - average over particle velocities to get temperature dependence:

$$\langle \sigma v \rangle = \int_0^\infty \sigma v N(v)\, dv \Big/ \int_0^\infty N(v)\, dv\,,$$

  e.g. Maxwellian distribution at temperature T

  - then multiply by number densities:

$$r_{ij} = n_i n_j \langle \sigma v \rangle / \rho\,.$$

- **If Q_{ij} is the energy released per reaction ij, total energy released is:**

$$\varepsilon_{ij}(\rho, T, n_i, n_j) = r_{ij} Q_{ij}\,.$$
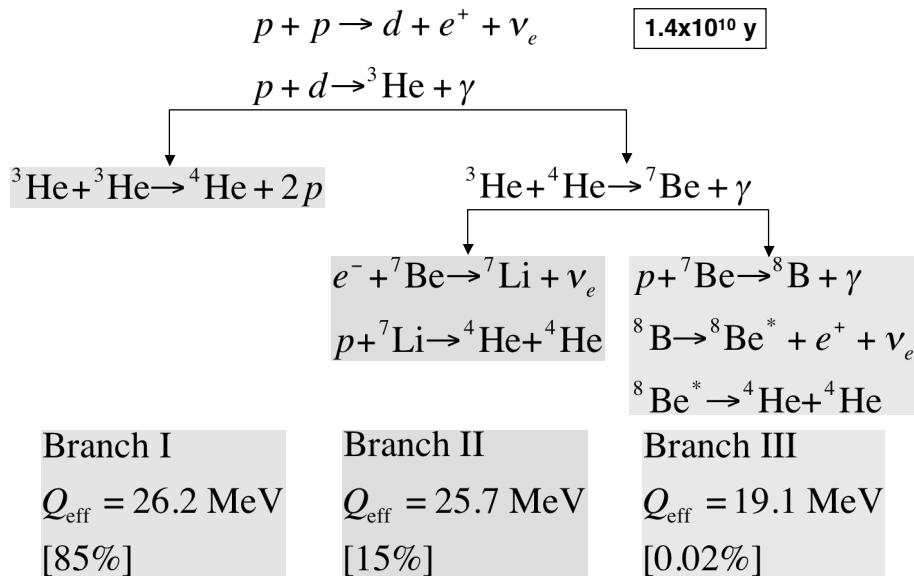

Thermonuclear networks

- **Common networks: pp chain, CN(O) cycle.**
- **Reaction rate of a thermonuclear network depends on slowest reaction in network.**
- **Energy released per product nucleon is that for entire cycle:**

$$\varepsilon_{cycle} = r_{slowest} Q_{cycle}\,.$$

Reactions of the pp chains

$$p + p \rightarrow d + e^+ + \nu_e \qquad \boxed{\textbf{1.4x10}^{\textbf{10}}\textbf{ y}}$$

$$p + d \rightarrow {}^3\text{He} + \gamma$$

$${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + 2p \qquad\qquad {}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be} + \gamma$$

$$e^- + {}^7\text{Be} \rightarrow {}^7\text{Li} + \nu_e \qquad p + {}^7\text{Be} \rightarrow {}^8\text{B} + \gamma$$

$$p + {}^7\text{Li} \rightarrow {}^4\text{He} + {}^4\text{He} \qquad {}^8\text{B} \rightarrow {}^8\text{Be}^* + e^+ + \nu_e$$

$${}^8\text{Be}^* \rightarrow {}^4\text{He} + {}^4\text{He}$$

Branch I	Branch II	Branch III
$Q_{\text{eff}} = 26.2$ MeV	$Q_{\text{eff}} = 25.7$ MeV	$Q_{\text{eff}} = 19.1$ MeV
[85%]	[15%]	[0.02%]

Reaction rates for pp chains

- **e.g. slowest reaction in any of the pp chains is the pp reaction itself:**

$$r_{\text{pp}} \sim \frac{\rho}{T_6^{2/3}} X_{\text{H}}^2 e^{25.44 - 33.81/T_6^{1/3}}. \qquad T_6 \equiv T/10^6 \text{ K}$$
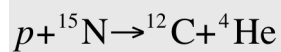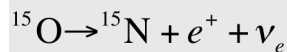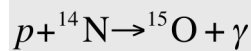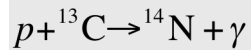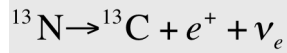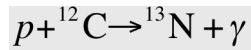
- **Energy released depends how far chain goes.**
- **Choice of pp-chain governed mainly by relative abundances of individual species:**
  - pp II and III dominate when ^{4}He is abundant; deplete ^{7}Li.
  - PPIII is rare but produces the most energetic neutrinos.
- **Approximate reaction rate for pp chain:**

$$\varepsilon_{\text{pp}} \approx 120 \left(\frac{X_{\text{H}}}{0.5} \right)^2 \left(\frac{\rho}{10^5 \text{ m}^{-3}} \right) \left(\frac{T}{15 \times 10^6 \text{ K}} \right)^4 \text{ Wm}^{-3}.$$

Reactions of the CN cycles
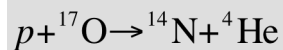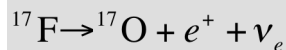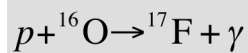
$$p + {}^{12}\mathrm{C} \rightarrow {}^{13}\mathrm{N} + \gamma$$

$${}^{13}\mathrm{N} \rightarrow {}^{13}\mathrm{C} + e^+ + \nu_e$$

**CNO cycle
(less important)**

$$p + {}^{13}\mathrm{C} \rightarrow {}^{14}\mathrm{N} + \gamma$$

$$p + {}^{14}\mathrm{N} \rightarrow {}^{15}\mathrm{O} + \gamma \qquad \boxed{\text{3x10}^8 \text{ y}} \qquad p + {}^{14}\mathrm{N} \rightarrow {}^{15}\mathrm{O} + \gamma$$

$${}^{15}\mathrm{O} \rightarrow {}^{15}\mathrm{N} + e^+ + \nu_e \qquad\qquad {}^{15}\mathrm{O} \rightarrow {}^{15}\mathrm{N} + e^+ + \nu_e$$

$$p + {}^{15}\mathrm{N} \rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He} \qquad\qquad p + {}^{15}\mathrm{N} \rightarrow {}^{16}\mathrm{O} + \gamma$$

**CN cycle
(most important)**

$$p + {}^{16}\mathrm{O} \rightarrow {}^{17}\mathrm{F} + \gamma$$

$${}^{17}\mathrm{F} \rightarrow {}^{17}\mathrm{O} + e^+ + \nu_e$$

$$p + {}^{17}\mathrm{O} \rightarrow {}^{14}\mathrm{N} + {}^{4}\mathrm{He}$$
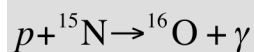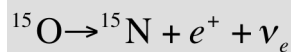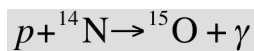
Reaction rates for CN cycles

- **Reaction rate is controlled by p+^{14}N → ^{15}O +γ:**

$$r_{\mathrm{CN}} \sim \frac{\rho}{T_6^{2/3}} X_{\mathrm{H}} X_{\mathrm{N14}} e^{74.4 - 152.3/T_6^{1/3}} \,.$$

- **Rate is cycle dependent since conversion of ^{12}C and ^{16}O to ^{14}N modifies ^{14}N abundance.**
- **High Coulomb barriers: strong T dependence.**
- **Energy produced ~25.02 MeV, independent of cycle.**
- **Abundance of ^{14}N in Sun is 0.6%**

  **anywhere
  from 13 to 18**

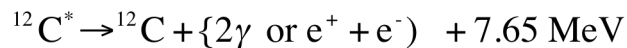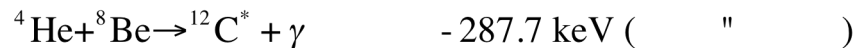- **Approximate reaction rate for CN cycle:**

$$\varepsilon_{\mathrm{CN}} \approx 2.0 \left(\frac{X_{\mathrm{H}}}{0.5} \right) \left(\frac{X_{\mathrm{N14}}}{0.006} \right) \left(\frac{\rho}{10^5 \ \mathrm{m^{-3}}} \right) \left(\frac{T}{15 \times 10^6 \ \mathrm{K}} \right)^{16} \mathrm{Wm^{-3}} \,.$$

Triple-α reactions

- **Once H has been completely converted to He, next most favourable reaction is:**

$$^4\text{He} + {}^4\text{He} \rightarrow {}^8\text{Be} + \gamma \qquad \text{- 91.8 keV (endothermic!)}$$

$$^4\text{He} + {}^8\text{Be} \rightarrow {}^{12}\text{C}^* + \gamma \qquad \text{- 287.7 keV (\quad '' \quad)}$$

$$^{12}\text{C}^* \rightarrow {}^{12}\text{C} + \{2\gamma \text{ or } e^+ + e^-) \quad \text{+ 7.65 MeV}$$

- **High threshold: ^{4}He+^{8}Be fusion needs T>10^8 K.**
- **Need high energy ^{4}He to collide with ^{8}Be within 10^{-16} s to prevent ^{8}Be decaying again to 2α.**
- **Once formed, most ^{12}C* decay back to ^{8}Be+ ^{4}He, but a few survive to decay via photon cascade or positron-electron pair emission.**
- **To estimate ε, see Phillips pp125-126.**

Neutrinos

- **Produced as electron (or positron) decay capture products.**
- **Capture cross section:** $\quad \sigma_\nu \approx 10^{-48} (\varepsilon_\nu / \text{MeV})^2 \text{ m}^2$
- **Hence mean free path:** $\quad \lambda_\nu \approx 10^{21} (\varepsilon_\nu / \text{MeV})^{-2} / \rho \text{ m}$
- **i.e. neutrinos escape more or less unimpeded.**
- **Neutrino losses account for only a small fraction of energy produced in pp, CN and 3α.**
- **i.e. little effect on main-sequence or red-giant structure & evolution.**
- **Important in stellar collapse: neutrino flux ~ photon flux during final stages of collapse in SN explosion.**
- **Can get λ_ν ~ 25m in supernova core!**