AS4023 STARS

STELLAR STRUCTURE

Problem Sheet 1

1. Calculate the dynamical, Kelvin and nuclear timescales for main-sequence stars of 1 and 20 $M_\odot$ (you may suppose that $L \propto M^{3.8}$ and $R \propto M^{0.8}$). Sketch the nuclear timescale as a function of main-sequence mass. [1.77×10^3 s; 3×10^7 y; 7×10^9y; 1.44×10^4 s; 1.24×10^4 y; 1.6×10^6 y]

2. A shell of matter enclosing a mass m_0 is initially at rest, with radius r_0. The energy conservation equation may be written:

$$\frac{1}{2}\left[\frac{dr}{dt}\right]^2 = \frac{Gm_0}{r} - \frac{Gm_0}{r_0}.$$

   Integrating and using substitutions $x = r/r_0$, $x = sin^2\theta$, or otherwise, show that the timescale for gravitational collapse can be written as

$$t_{FF} = \left(\frac{3\pi}{32G\rho}\right)^{1/2}$$

3. Derive the equations for convective instability:

$$\frac{P}{\rho}\frac{d\rho}{dP} > \frac{1}{\gamma}$$

   and for the temperature gradient under convective equilibrium:

$$\frac{d\ln T}{d\ln P} = \frac{\gamma - 1}{\gamma}.$$

4. Transform the four ordinary differential equations of stellar structure and their boundary conditions from Eulerian to Lagrangian coordinates.

5. Consider a solar-mass protostar with radius 10^{11} m and average internal temperature 30,000 K. What would its average internal temperature be after contracting to a radius of 10^9 m?

6. The Virial theorem provides an estimate for the average internal temperature of a contracting star, $kT \simeq G\mu m_H M^{2/3}\rho^{1/3}$. Electrons become degenerate at a point where $\rho \simeq \mu m_H(m_e kT)^{3/2}/h^3$. Derive expressions for the radius, density, and temperature of a star at this point, assuming that nuclear reactions are not ignited. What values do these take for a solar-mass star?

7. If the internal density distribution of a sequence of chemically homogeneous stars is given by $\rho = (M/R^3)F_\rho(x)$, where M and R are the stellar mass and radius and $F_\rho(x)$ is a function of radius $x = r/R$, and if the equation of state is given by $P \propto \rho T$, show that: $m = M.F_M(x)$, $P = (M^2/R^4)F_P(x)$ and $T = (M/R)F_T(x)$, where $F_M(x)$, $F_P(x)$ and $F_T(x)$ are also functions of radius only. If the opacity κ and nuclear energy generation rate ϵ are given respectively by $\kappa \propto \rho T^{-3.5}$ and $\epsilon \propto \rho T^\nu$, show that

$$L_{rad} = \frac{M^{5.5}}{R^{0.5}} F_{rad}(x) \qquad \text{and} \qquad L_{nuc} = \frac{M^{2+\nu}}{R^{3+\nu}} F_{nuc}(x).$$

Hence derive the mass-radius and mass-luminosity relations for solar-type ($\nu = 4$) and massive ($\nu = 15$) stars.