Diagnostic radiative transfer in astrophysics with RADMC-3D

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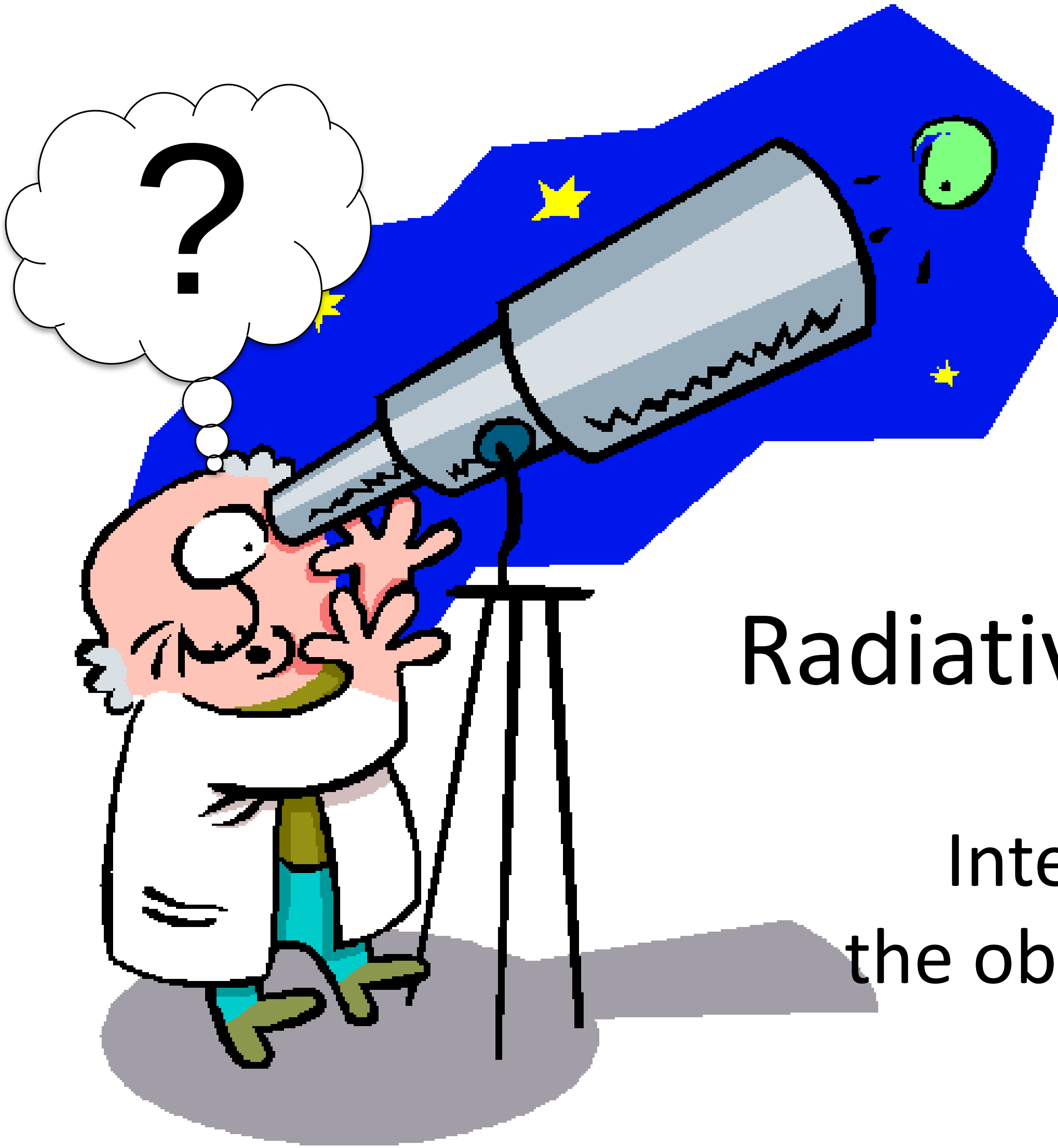


Radiative Transfer:

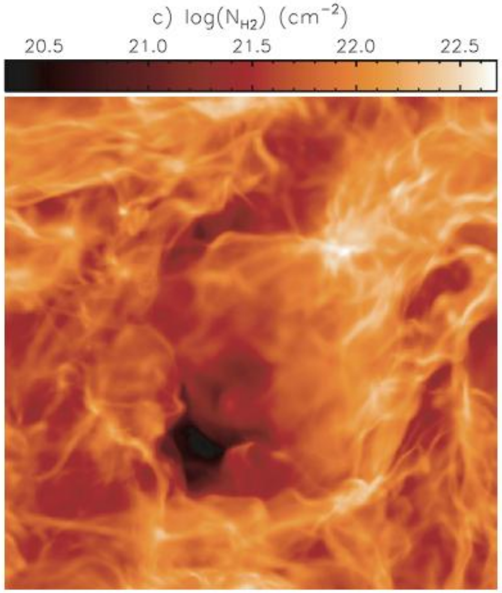
Interpreting
the observed light

Radiative transfer: Diagnostic tool

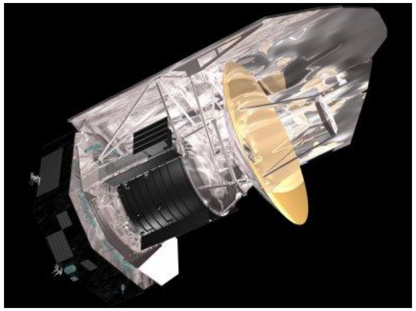
Model

Diagnostic
radiative transfer

Observation



Forward modeling



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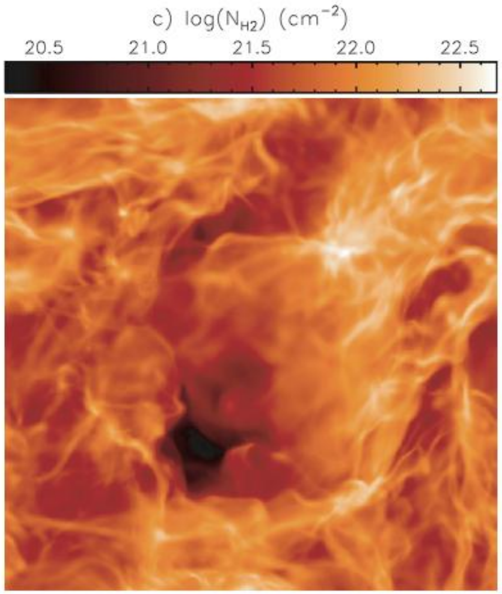

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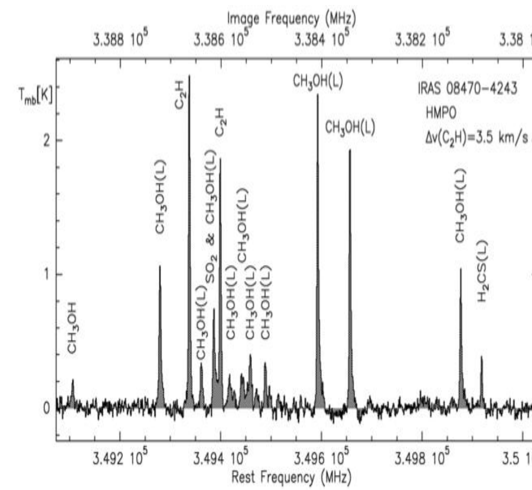

Radiative transfer: Diagnostic tool

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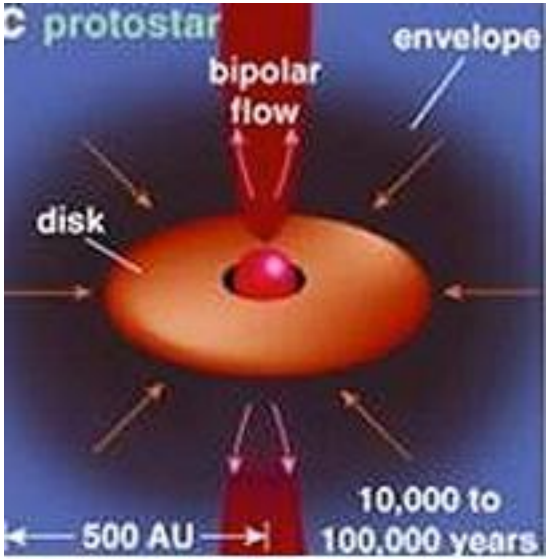
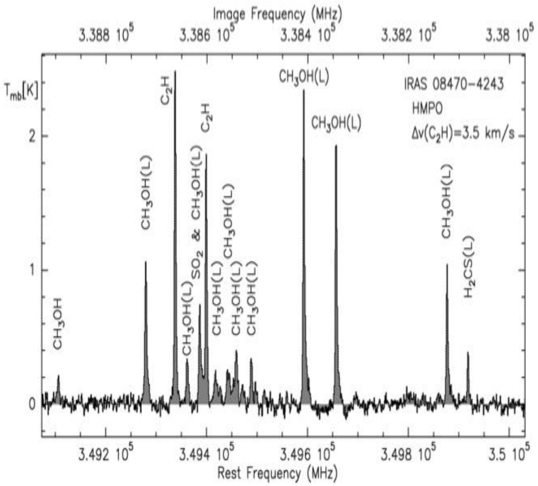


Forward modeling



Radiative transfer: Diagnostic tool

Model

Diagnostic
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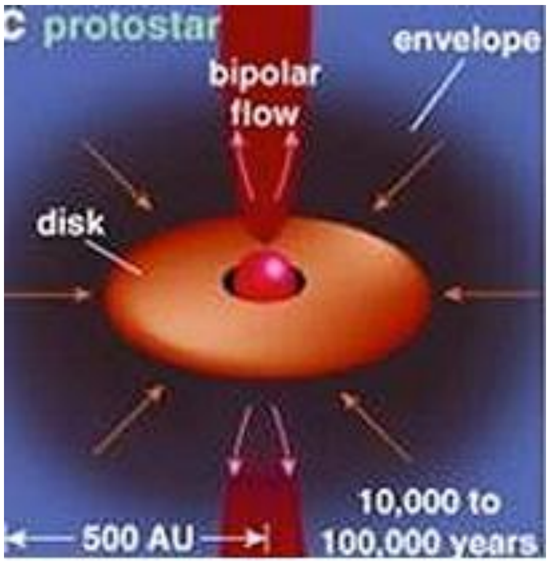
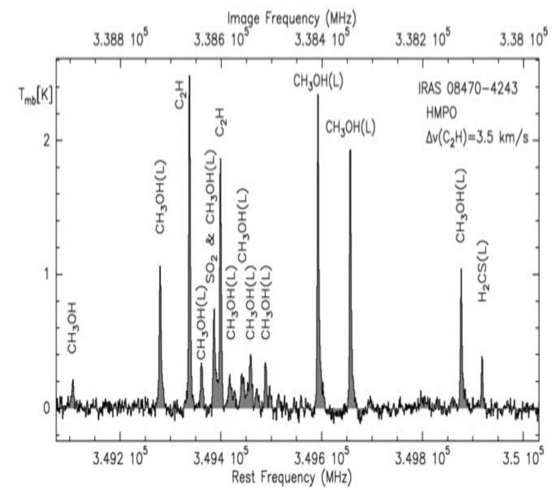


Forward modeling



Radiative transfer: Heating, cooling and energy transport

- Astrophysical objects cool by emitting radiation
- That same radiation is the radiation we observe with our telescopes
- Inside the object: Radiation can transport energy from one place to another
- Often linked to hydrodynamics: „Radiation hydrodynamics“

Radiative transfer: Driving photochemistry

- Energetic photons can:
  - photoionize atoms, molecules
  - photodissociate molecules
  - charge dust grains
- This powers a complex photochemical network

In summary:

- Radiative transfer is BOTH about:
  - How radiation affects the object *and*
  - how we can interpret our observations
- In many cases these two are *linked*, so that we cannot interpret our observations without computing how the radiation affects the object.

A short review of radiative transfer

(See also Kenny Wood's lecture)

Radiative transfer: A short review

Radiative Transfer is a 7-dimensional problem
(that's *one* of the reasons it is so hard and expensive to solve):

$$I(x,y,z,\theta,\phi,\nu,t) \qquad [\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,ster^{-1}}]$$

Usually: semi-steady-state:

$$I(x,y,z,\theta,\phi,\nu) \qquad [\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,ster^{-1}}]$$

If the emission and extinction coefficients are known, you can reduce this to the Formal Transfer Equation along a single ray:

$$I(s,\nu) \qquad [\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,ster^{-1}}]$$

Radiative transfer: A short review

Formal Transfer Equation along a ray:

$$\frac{dI_n}{ds} = r\,k_n\,(S_n - I_n)$$

Over length scales larger than $1/\rho\kappa_\nu$ intensity I tends to approach source function S.

Photon mean free path:

$$l_{\mathrm{free},n} = \frac{1}{r\,k_n}$$

Optical depth of a cloud of size L:

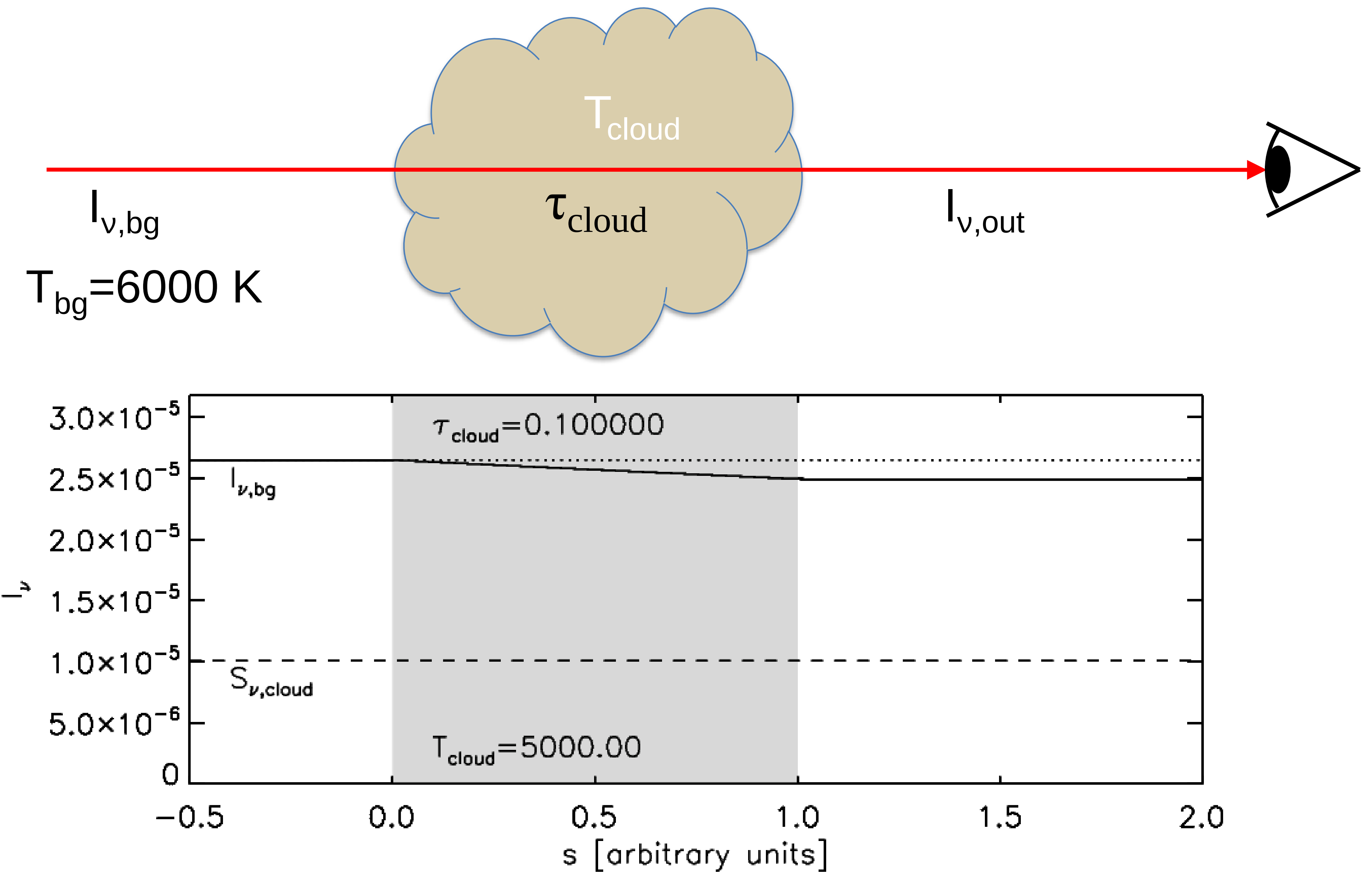
$$t_n = \frac{L}{l_{\mathrm{free},n}} = L\,r\,k_n$$

In case of local thermodynamic equilibrium: S is Planck function:

$$S_n = B_n(T)$$

Kirchhoff's law

Rad. trans. through a cloud of fixed T



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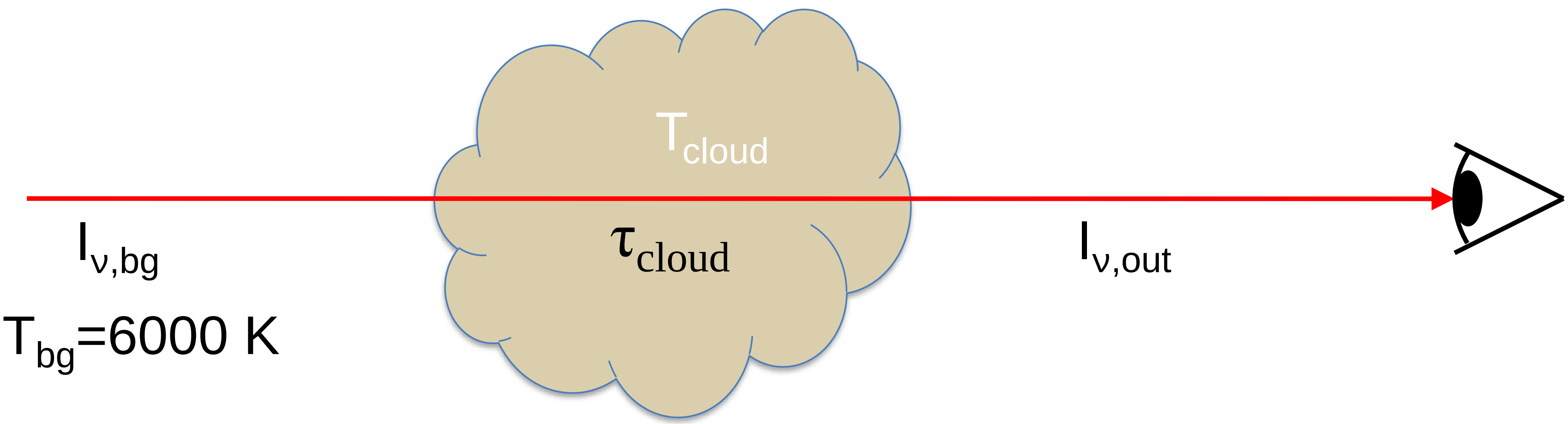
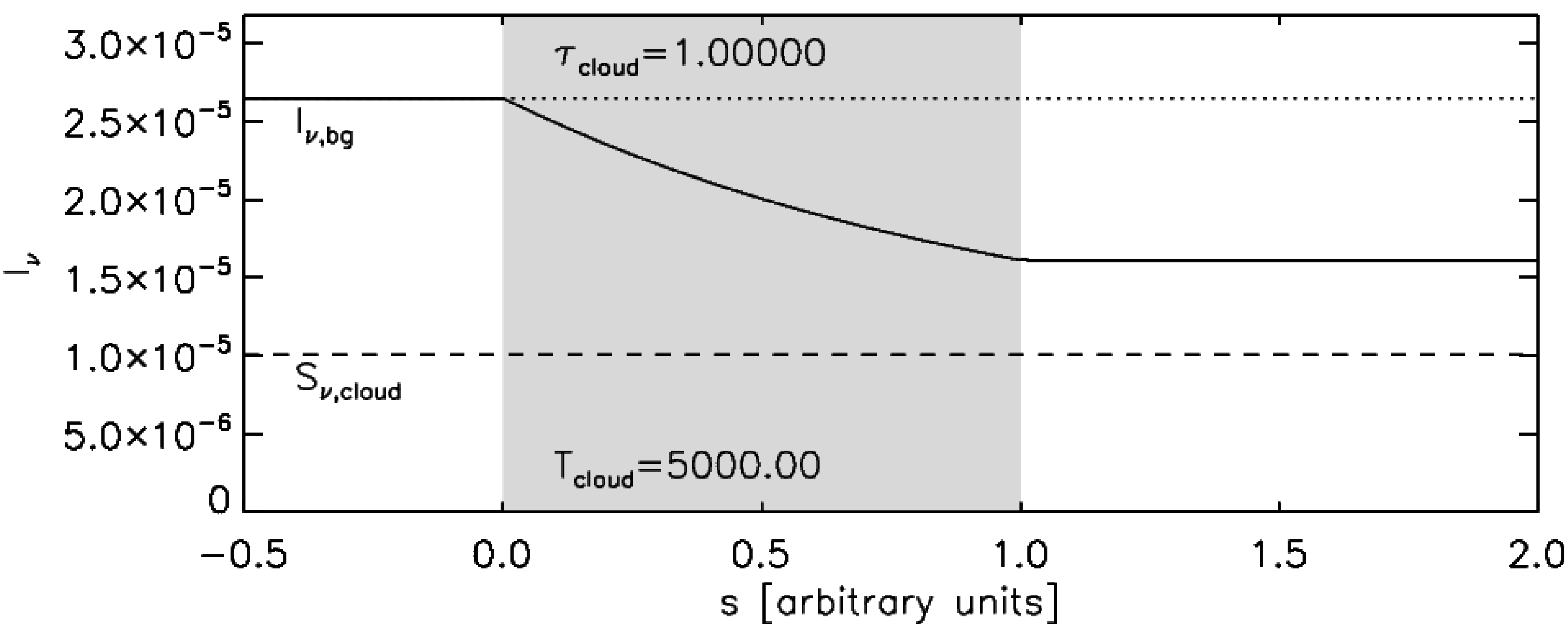
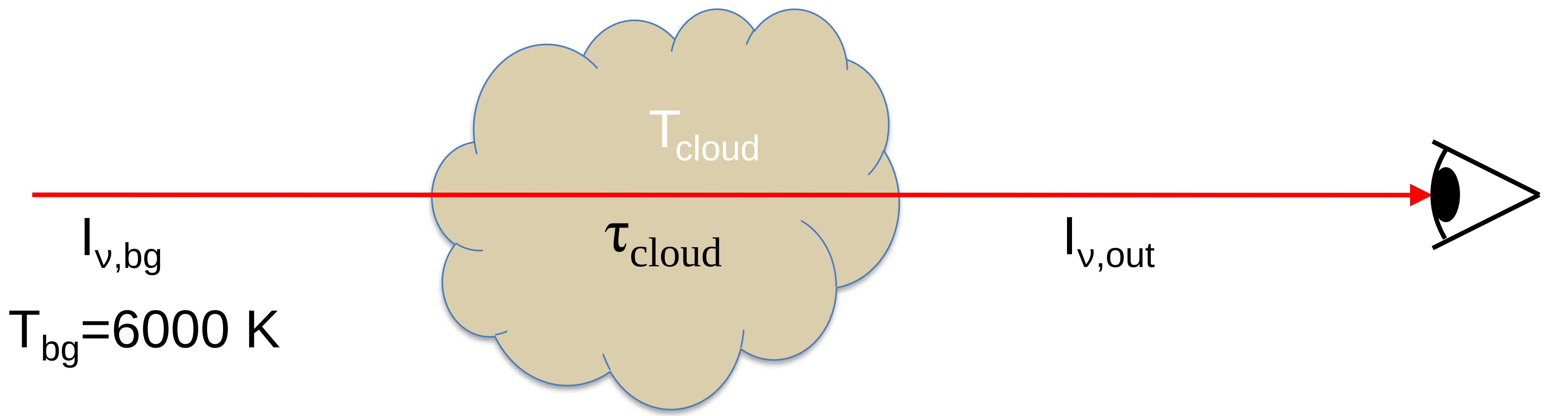
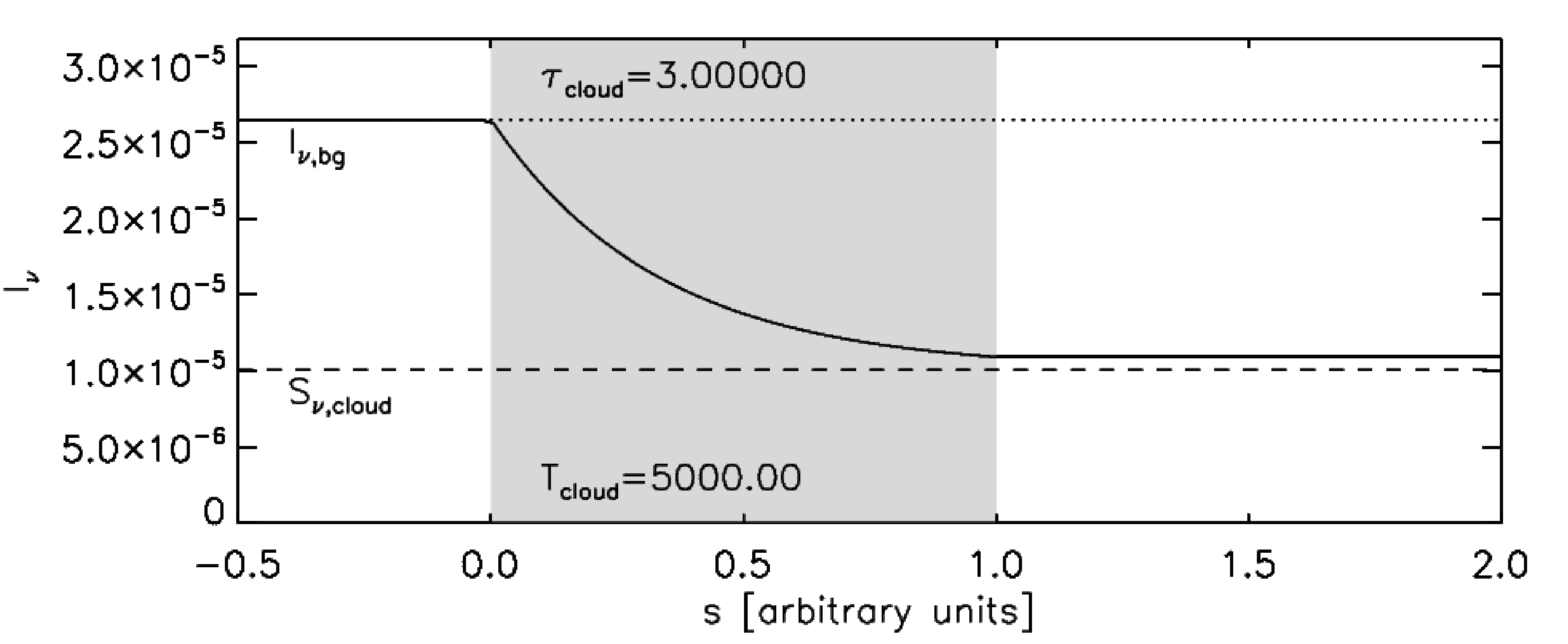




Rad. trans. through a cloud of fixed T





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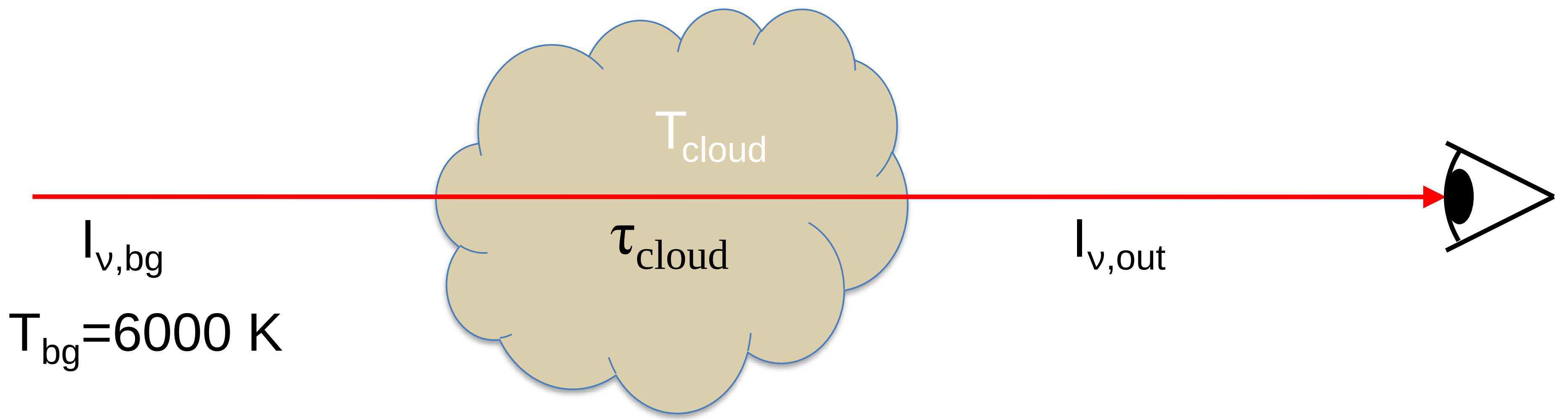

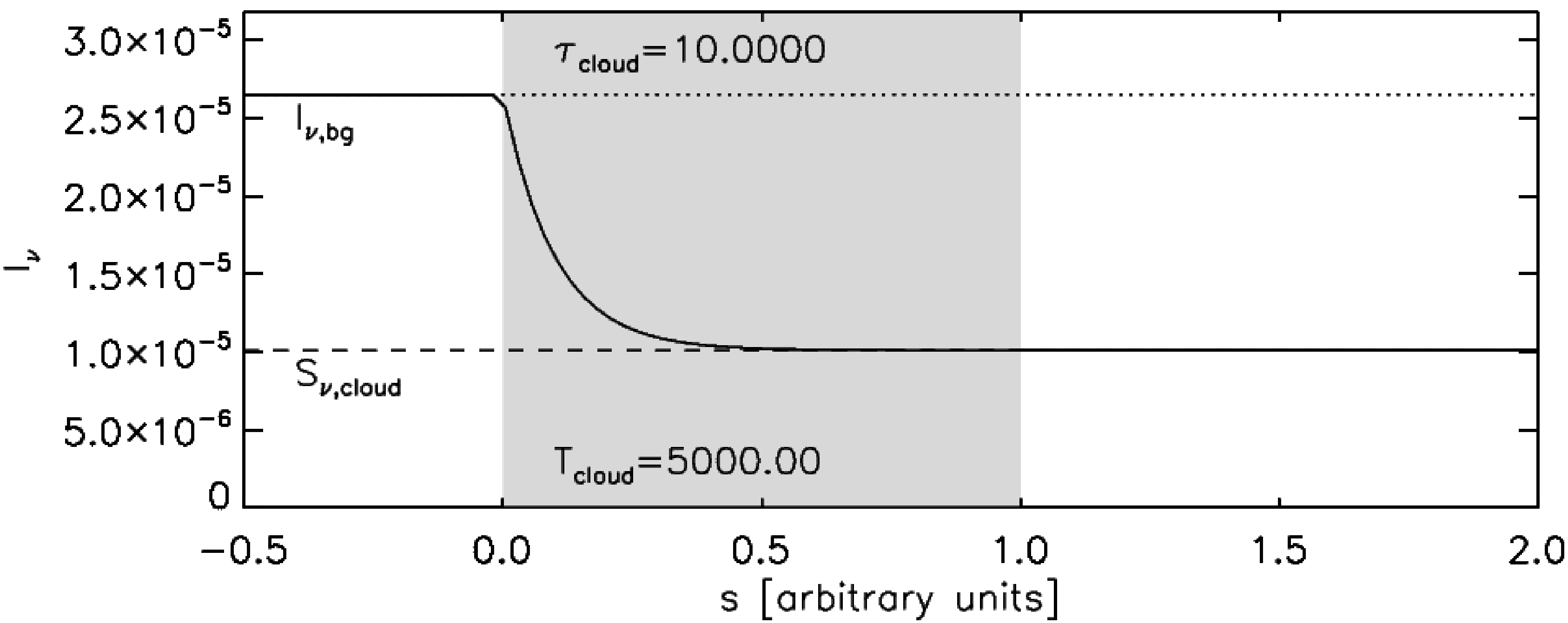

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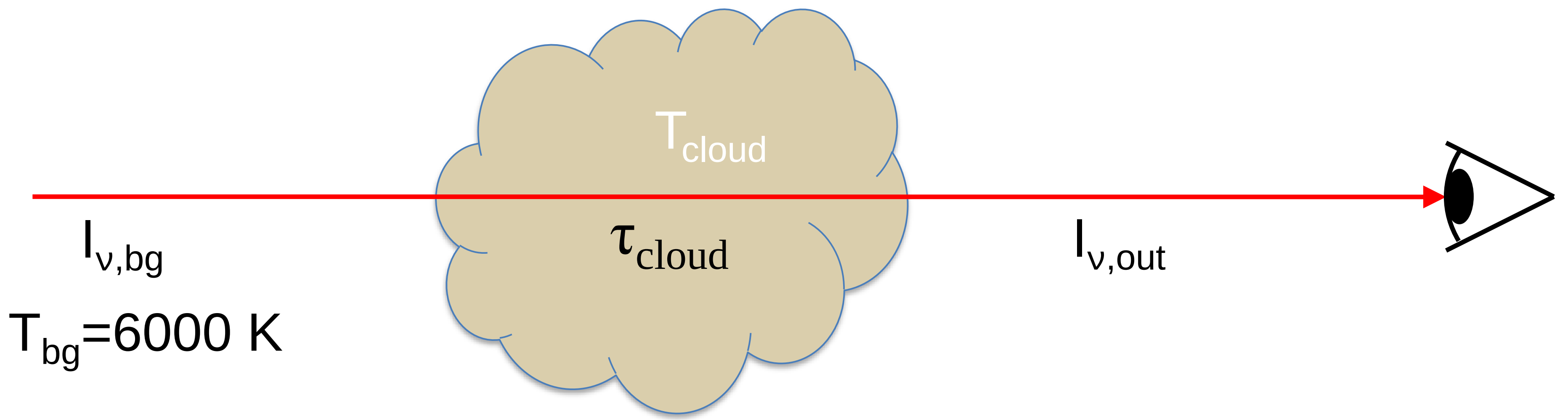

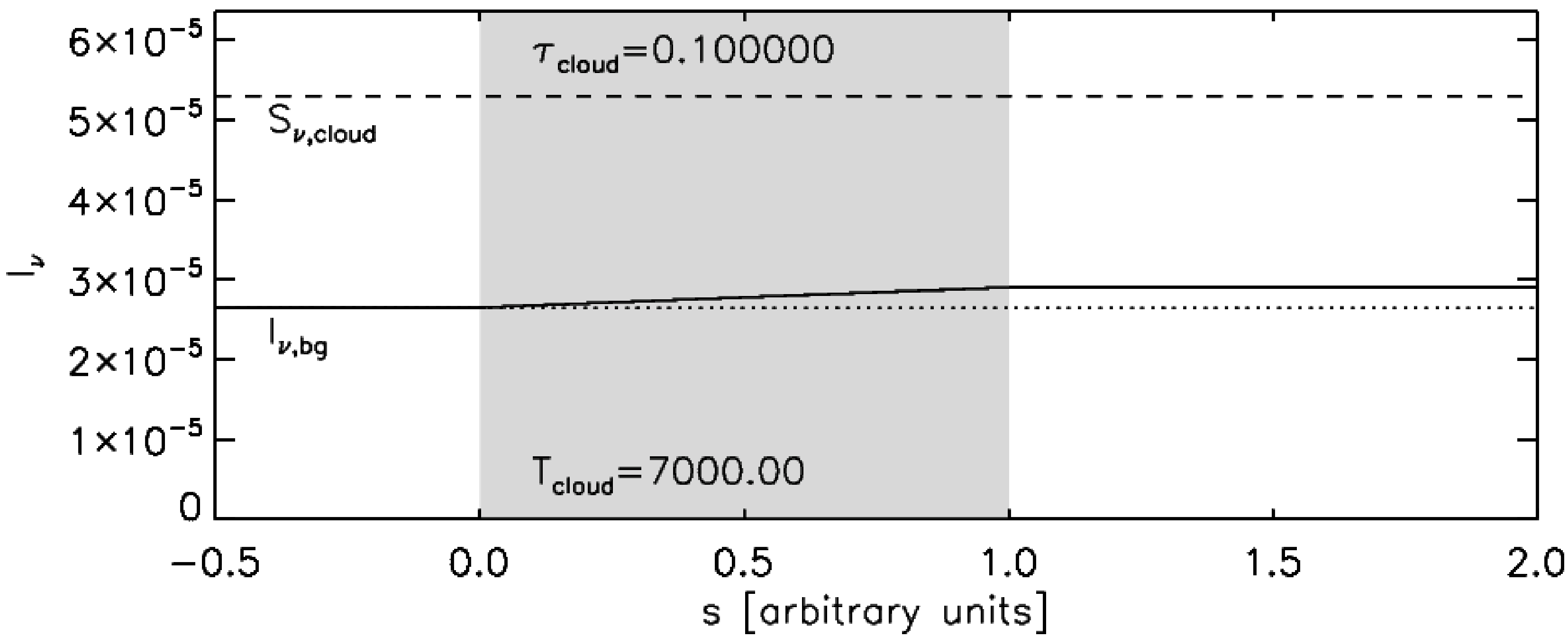

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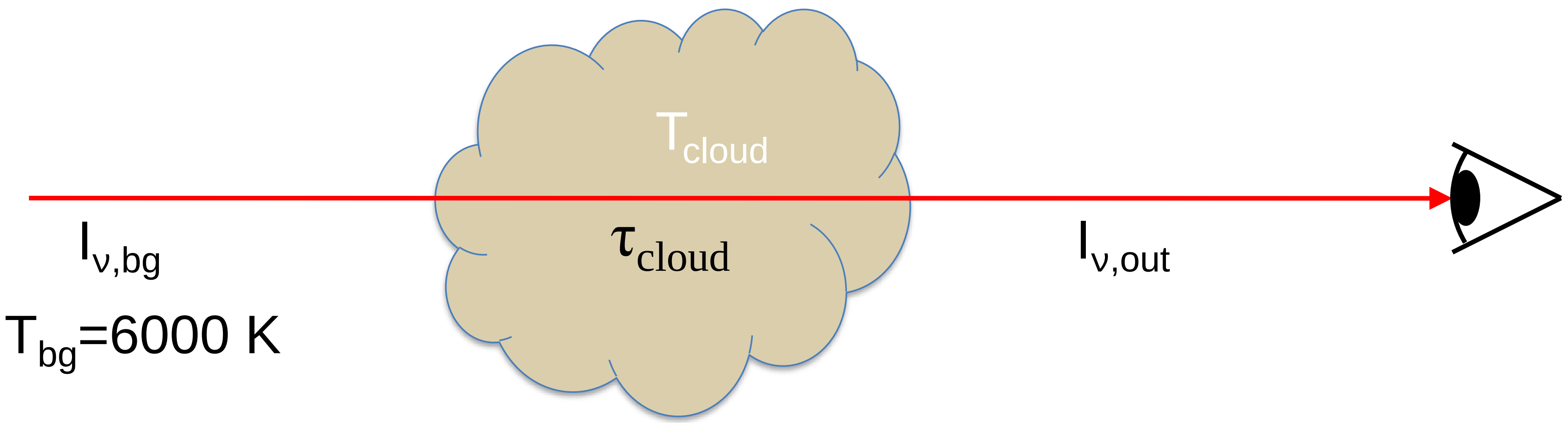

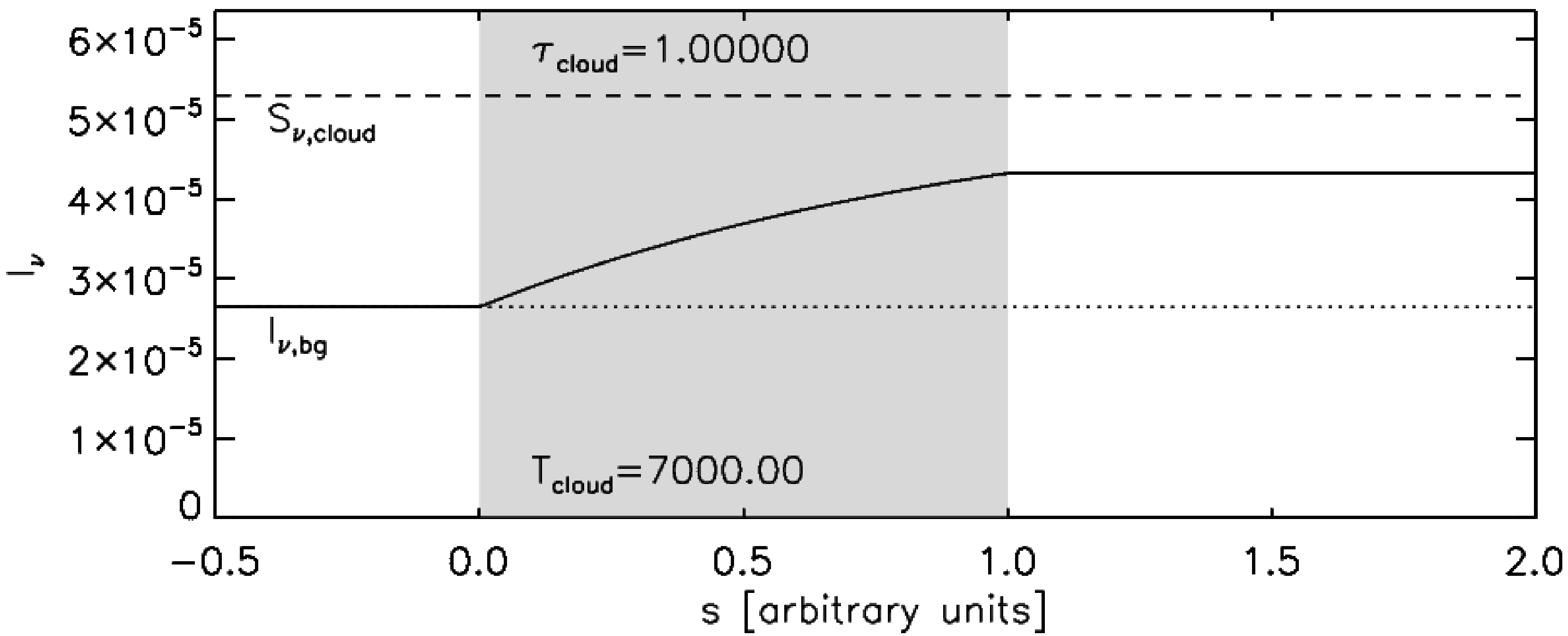

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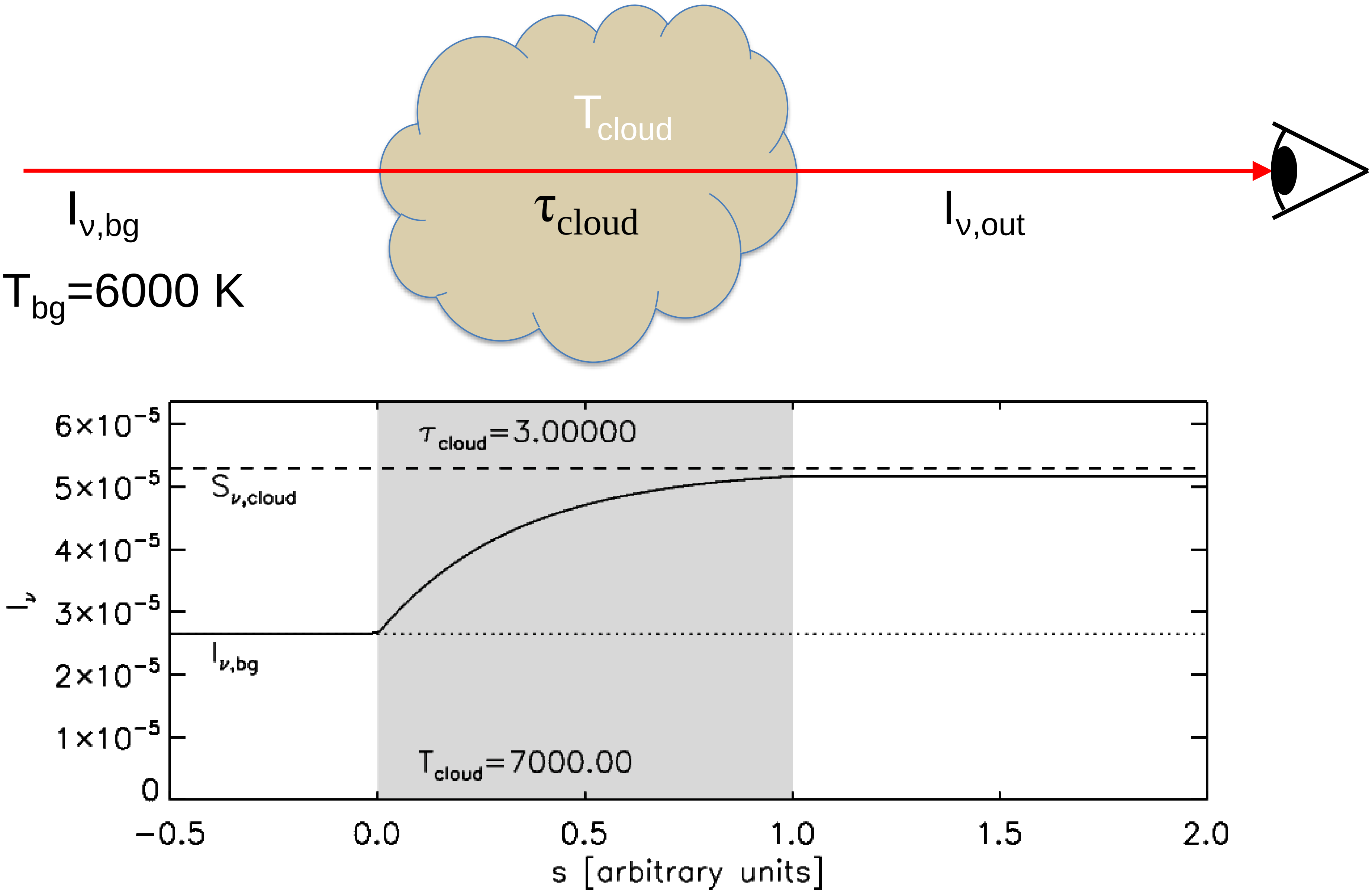

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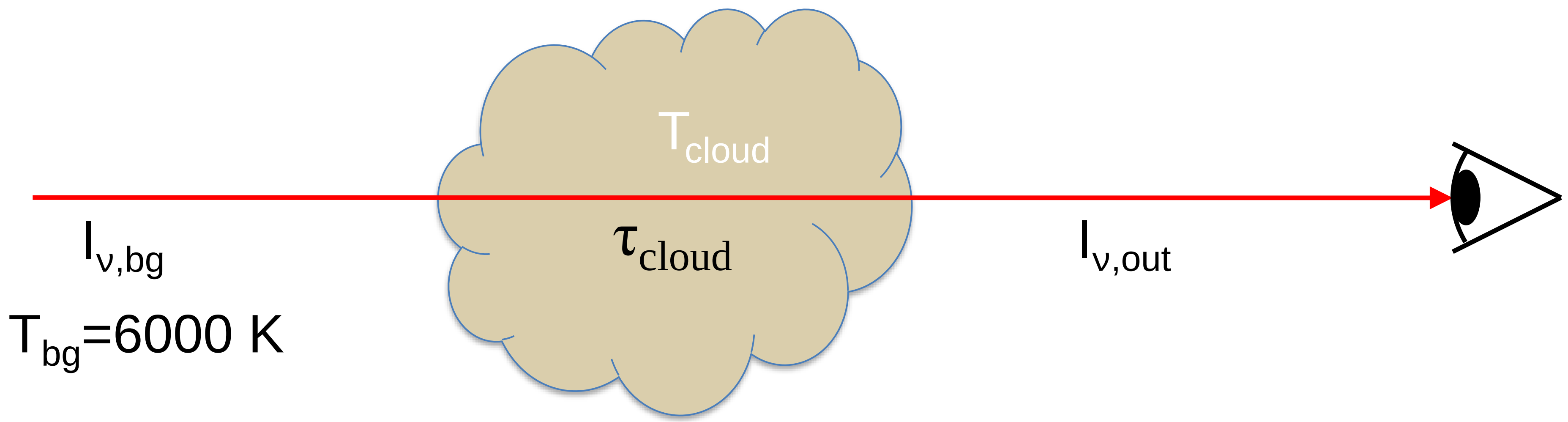
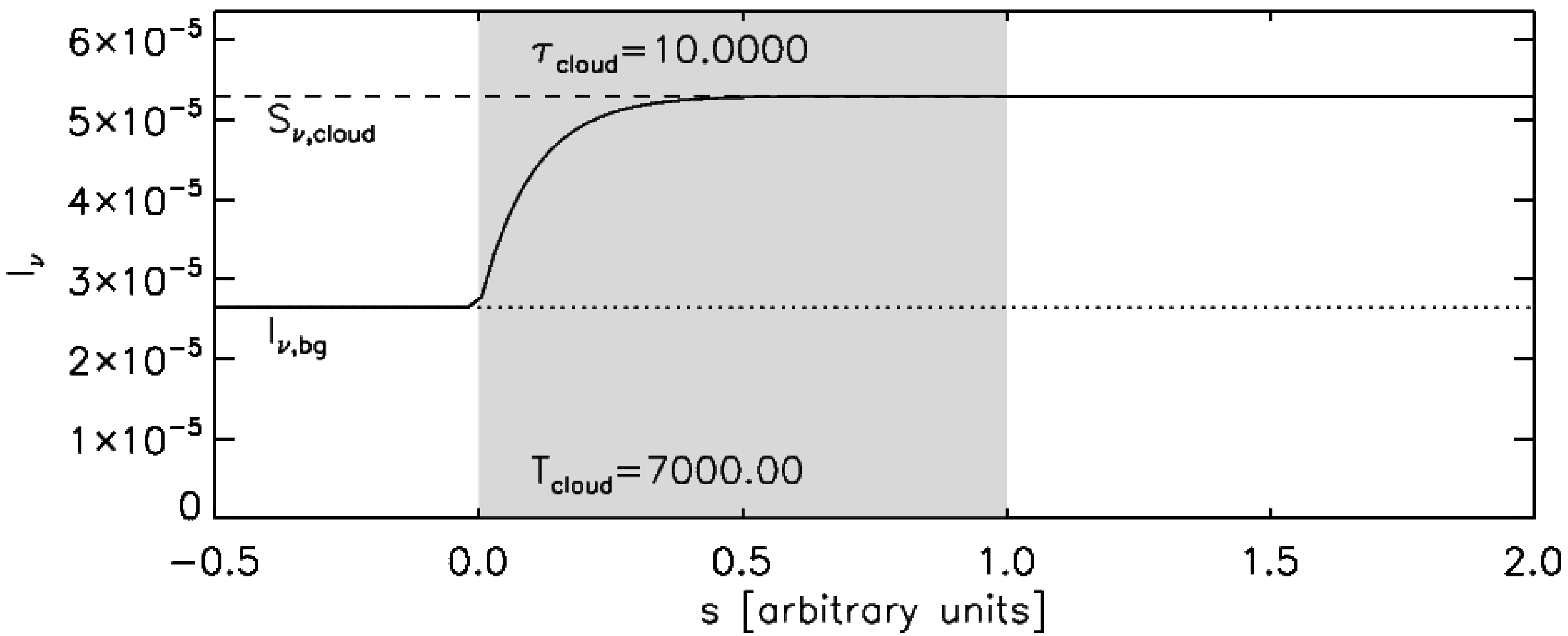




Formal radiative transfer solution

Radiative transfer equation again:

$$\frac{dI_n}{ds} = r\,k_n\,(S_n - I_n)$$

Observed flux from single-temperature slab:

$$I_n^{\mathrm{obs}} = I_n^0 e^{-t_n} + (1 - e^{-t_n})\,B_n(T) \qquad t_n = L\,r\,k_n$$

$$» \; t_n B_n(T)$$

for $t_n << 1$ and $I_n^0 = 0$

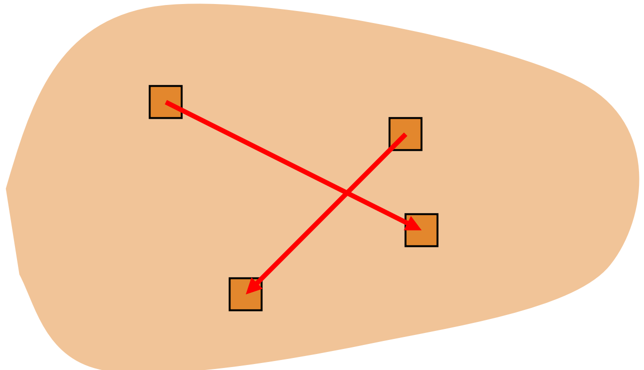
Dust continuum radiative transfer

(See also Tom Robitaille lecture)

Difficulty of dust radiative transfer

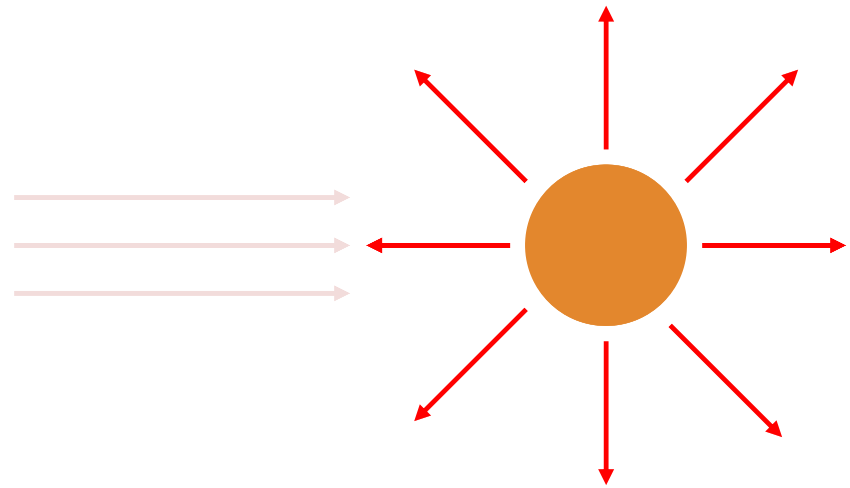
I. The thermal equilibrium problem

- If temperature of dust is given (ignoring scattering for the moment), then radiative transfer is a mere integral along a ray: i.e. easy.
- Problem: dust temperature is affected by radiation, even the radiation it emits itself.
- Therefore: must solve radiative transfer and thermal balance simultaneously.
- Difficulty: each point in cloud can heat (and receive heat from) each other point.



Thermal balance of dust grains

<u>Optically thin case:</u>



Heating:

$$Q_+ = \pi a^2 \int F_\nu \, \varepsilon_\nu \, d\nu$$

a = radius of grain

ε_ν = absorption efficiency (=1 for perfect black sphere)

Cooling:

$$Q_- = 4\pi a^2 \int \pi B_\nu(T) \, \varepsilon_\nu \, d\nu$$

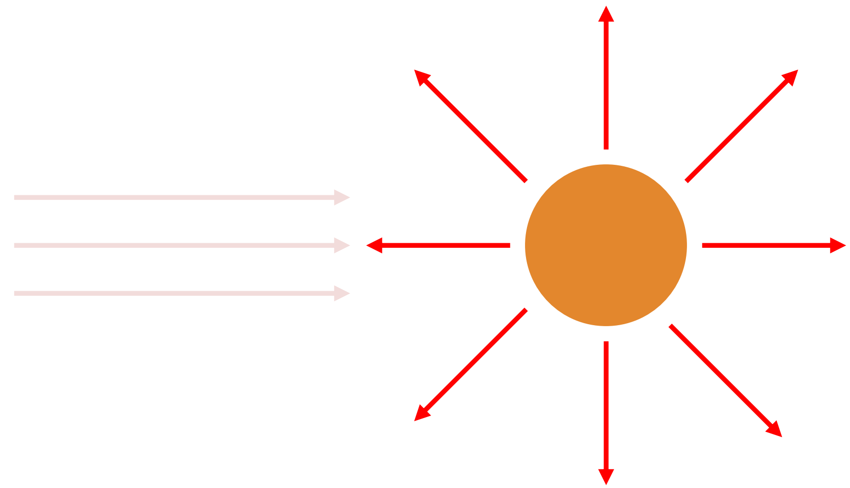
$$\kappa_\nu = \frac{\pi a^2 \varepsilon_\nu}{m}$$

Thermal balance:

$$4\pi a^2 \int \pi B_\nu(T) \, \varepsilon_\nu \, d\nu = \pi a^2 \int F_\nu \, \varepsilon_\nu \, d\nu$$

Thermal balance of dust grains

<u>Optically thin case:</u>



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$$Q_- = 4\pi a^2 \int \pi B_\nu(T) \epsilon_\nu \, d\nu$$

$$\kappa_\nu = \frac{\pi a^2 \epsilon_\nu}{m}$$

Thermal balance:

$$\int B_\nu(T) \kappa_\nu \, d\nu = \frac{1}{\pi} \int F_\nu \kappa_\nu \, d\nu$$

Optically thick case

Additional radiation field: diffuse infrared radiation from the grains

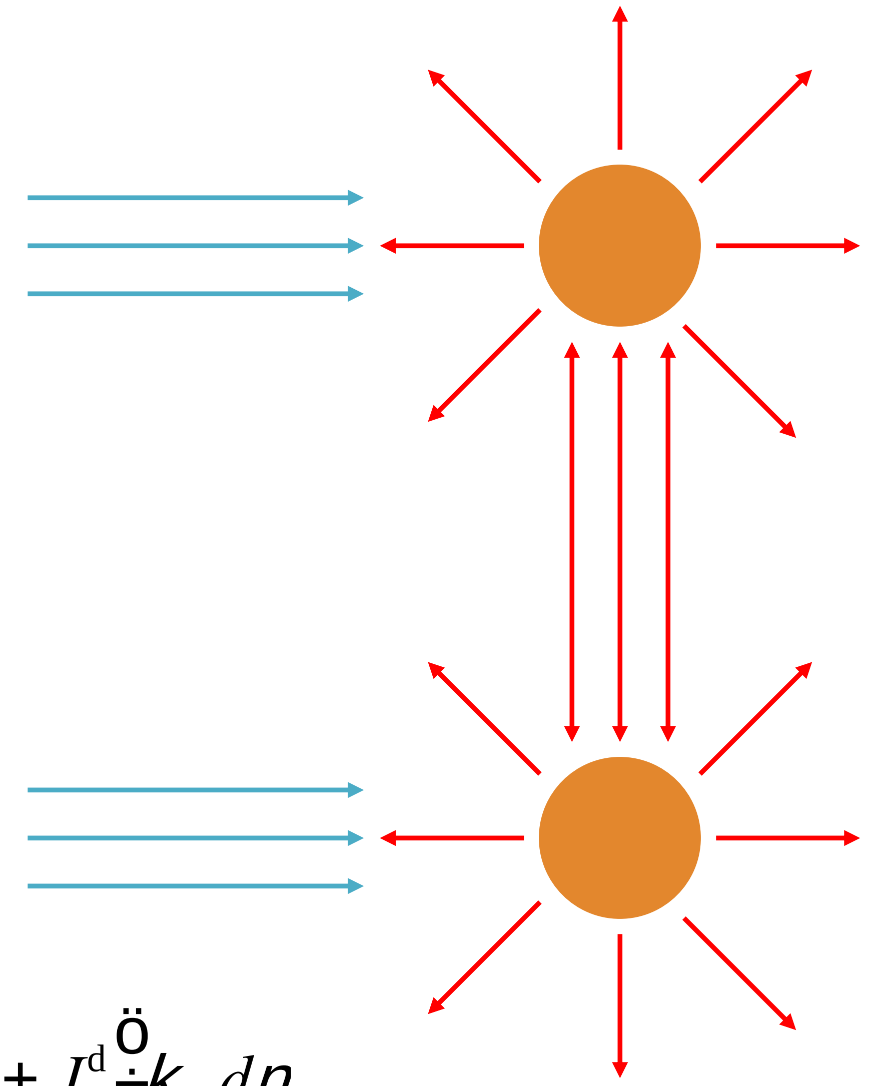
$$J_\nu^{\mathrm{d}} = \frac{1}{4\pi} \oint I_\nu^{\mathrm{d}} \, d\Omega$$

Intensity obeys tranfer equation along all possible rays:

$$\frac{dI_\nu^{\mathrm{d}}}{ds} = \rho \kappa_\nu \left(B_\nu(T) - I_\nu^{\mathrm{d}} \right)$$

Thermal balance:

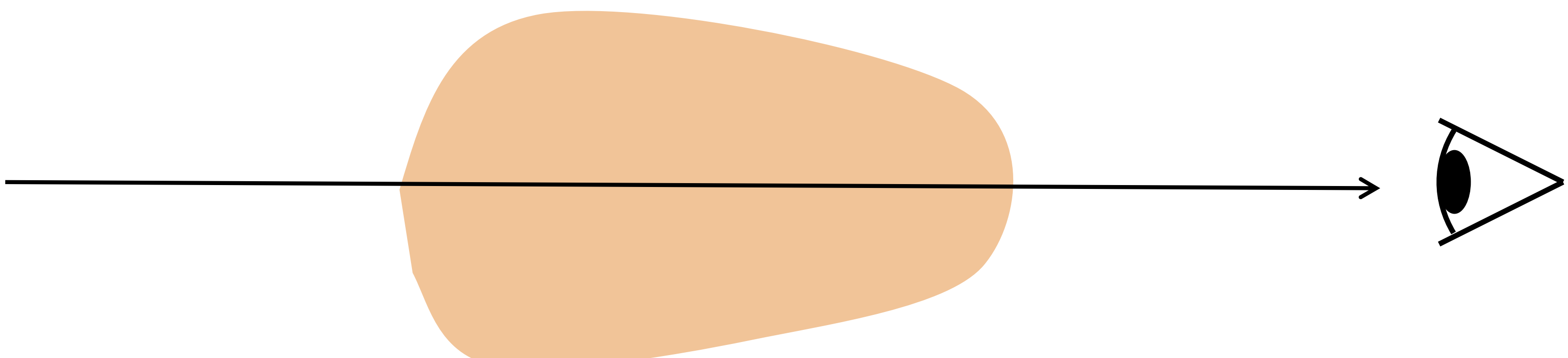
$$\int B_\nu(T) \kappa_\nu \, d\nu = \int \left(\frac{1}{\pi} F_\nu e^{-\tau_\nu} + J_\nu^{\mathrm{d}} \right) \kappa_\nu \, d\nu$$



Once we have the Temperature...

Simply integrate the Formal Transfer Equation

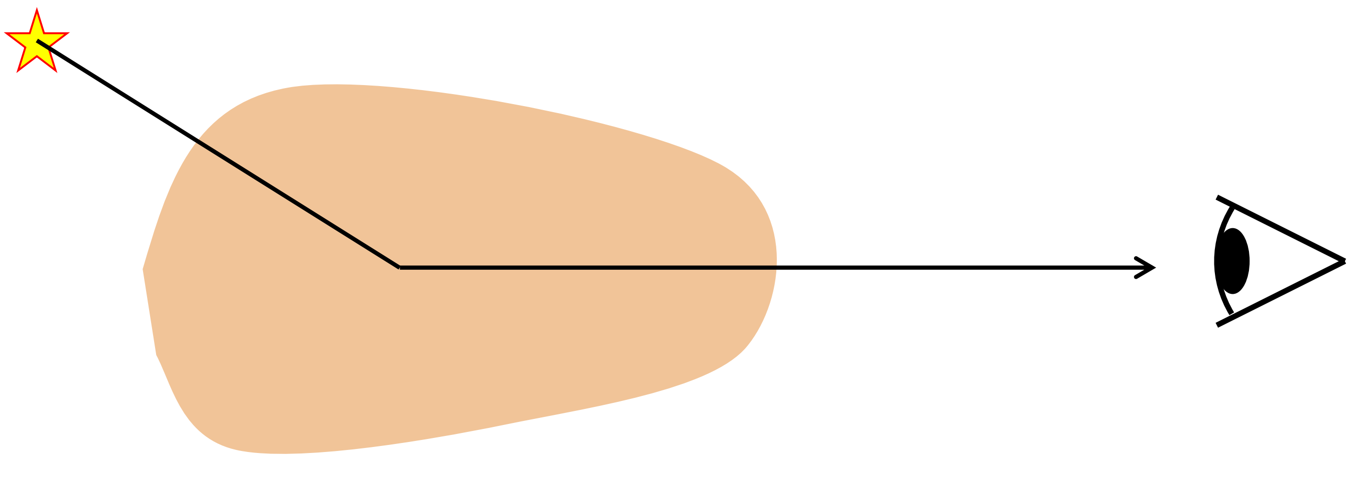
$$\frac{dI_n^{\mathrm{d}}}{ds} = r\,k_n\left(B_n(T) - I_n^{\mathrm{d}}\right)$$



Difficulty of dust radiative transfer

II. The scattering problem

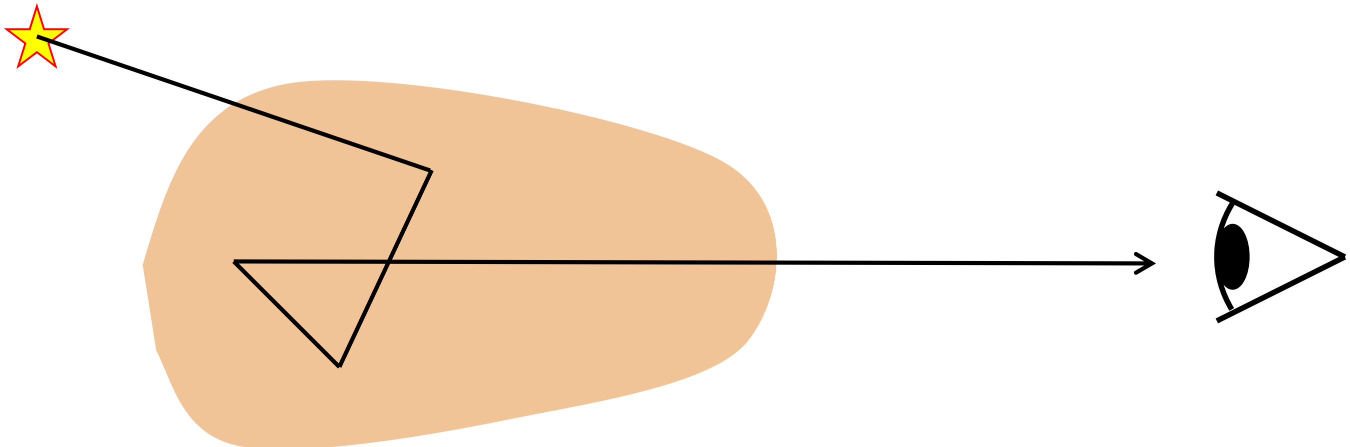
- Light from a star, or even from other regions of the cloud can scatter into the line of sight:



Difficulty of dust radiative transfer

II. The scattering problem

- Light from a star, or even from other regions of the cloud can scatter into the line of sight.
- Multiple scattering can happen:



Scattering source function

$$\frac{dI_\nu(s)}{ds} = j_\nu^{\mathrm{emis}}(s) + j_\nu^{\mathrm{scat}}(s) - \rho(s)(\kappa_\nu^{\mathrm{abs}} + \kappa_\nu^{\mathrm{scat}})I_\nu(s)$$

Scattering source function

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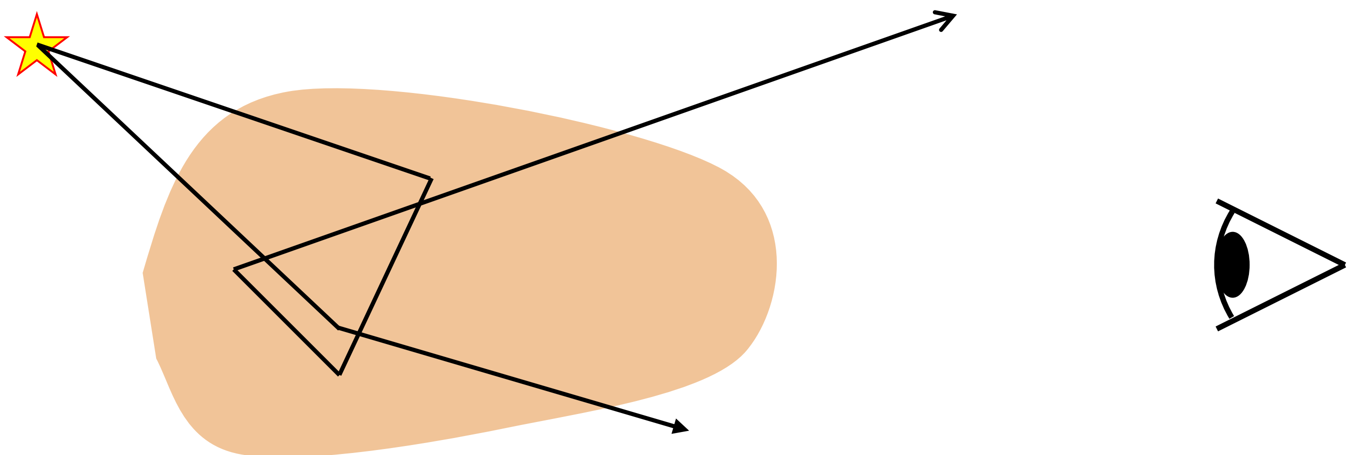
So now use Monte Carlo to compute S^{scat}



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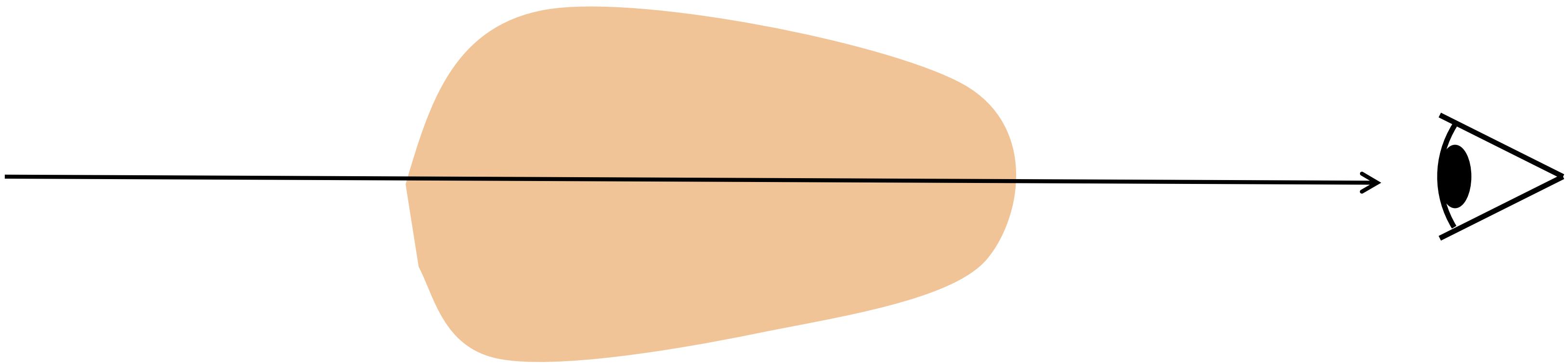
So now use Monte Carlo to compute $S^{\rm scat}$

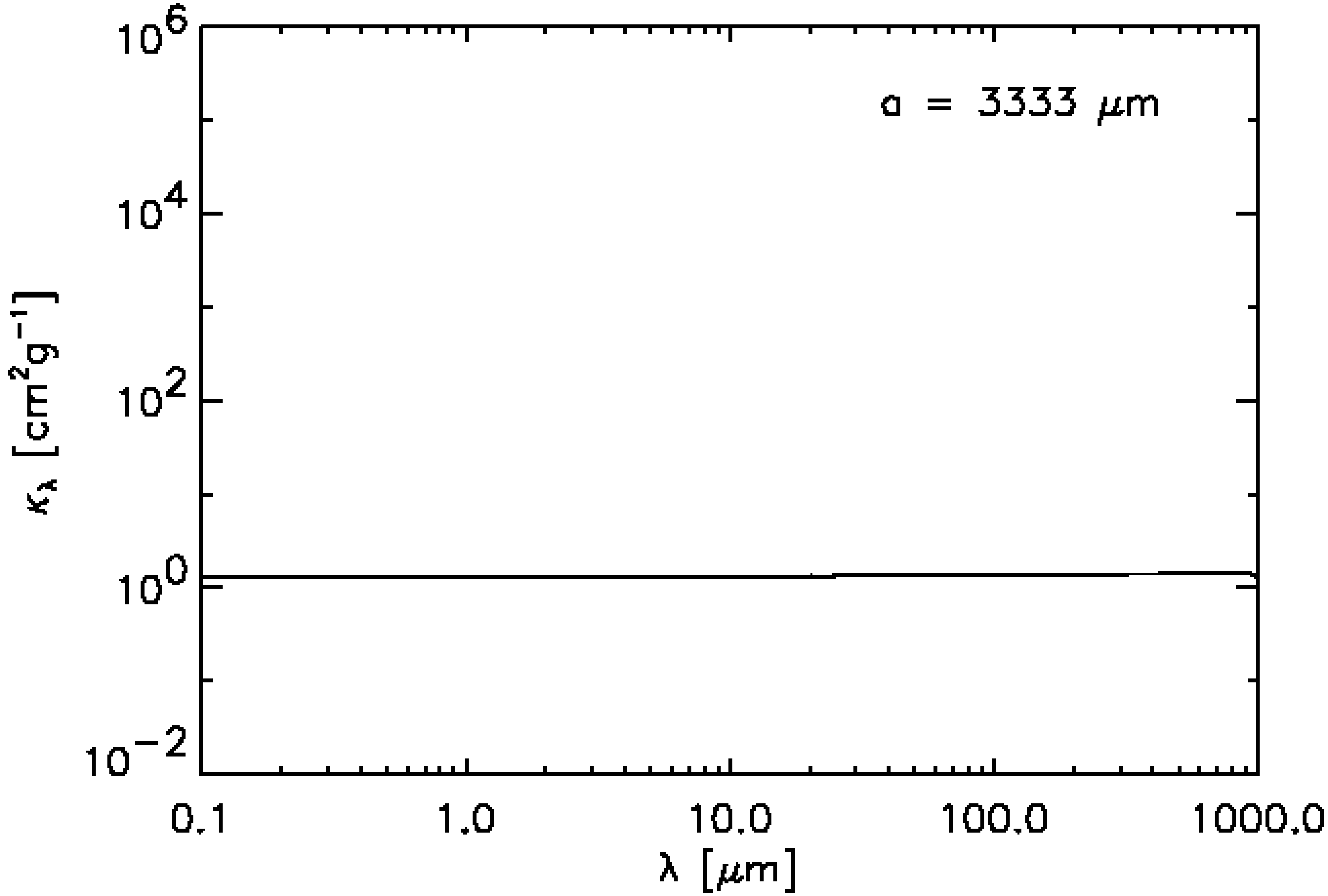


Dust opacities

- Dust opacities depend on:
  - Material properties (silicate, carbon, water ice, you name it!)
  - Grain size
  - Grain shape (spherical, compact, porous, fluffy)

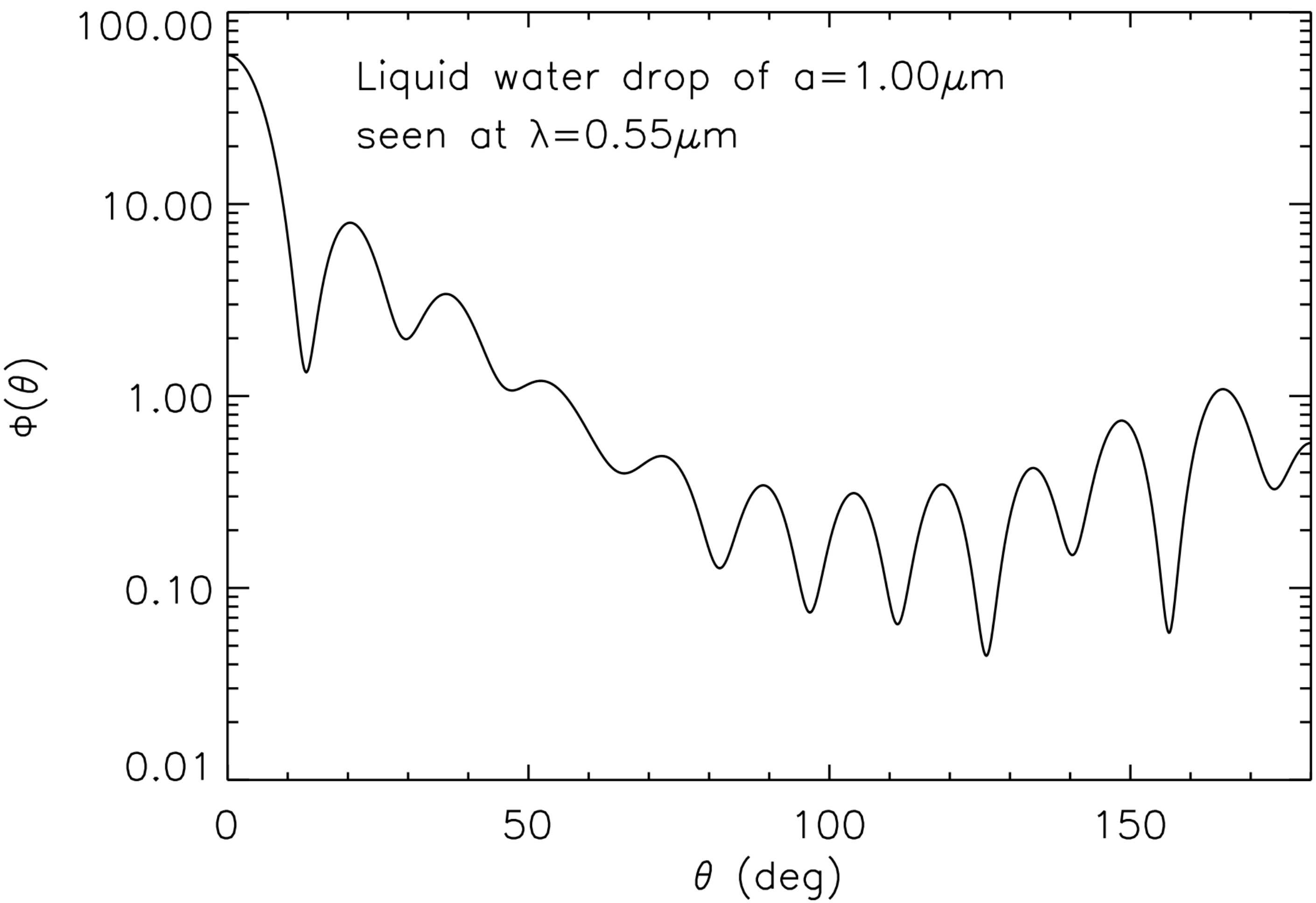
Dust opacities

Example: Silicate dust opacity for different grain sizes



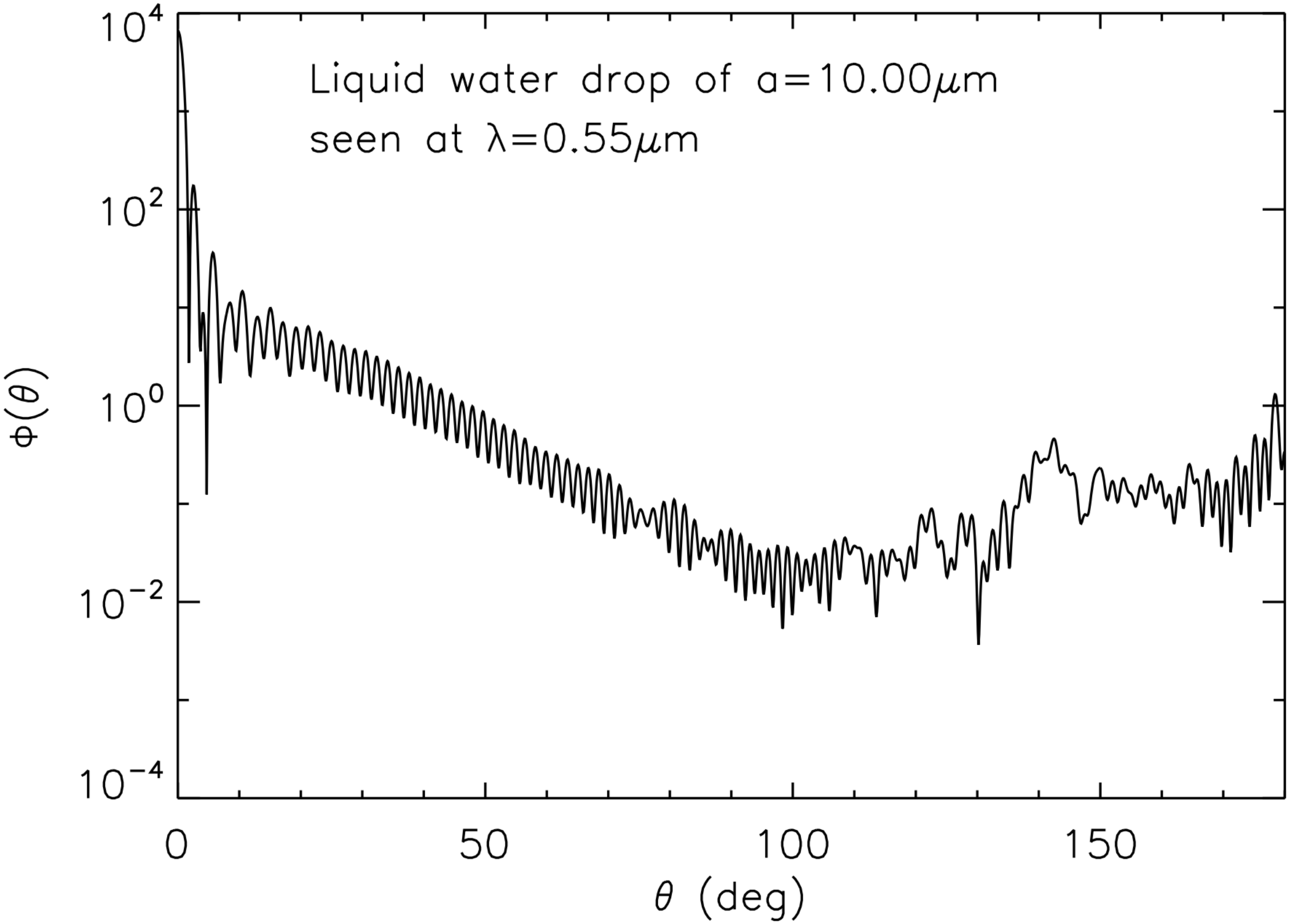
Dust opacities

Example: Phase functions for non-isotropic scattering



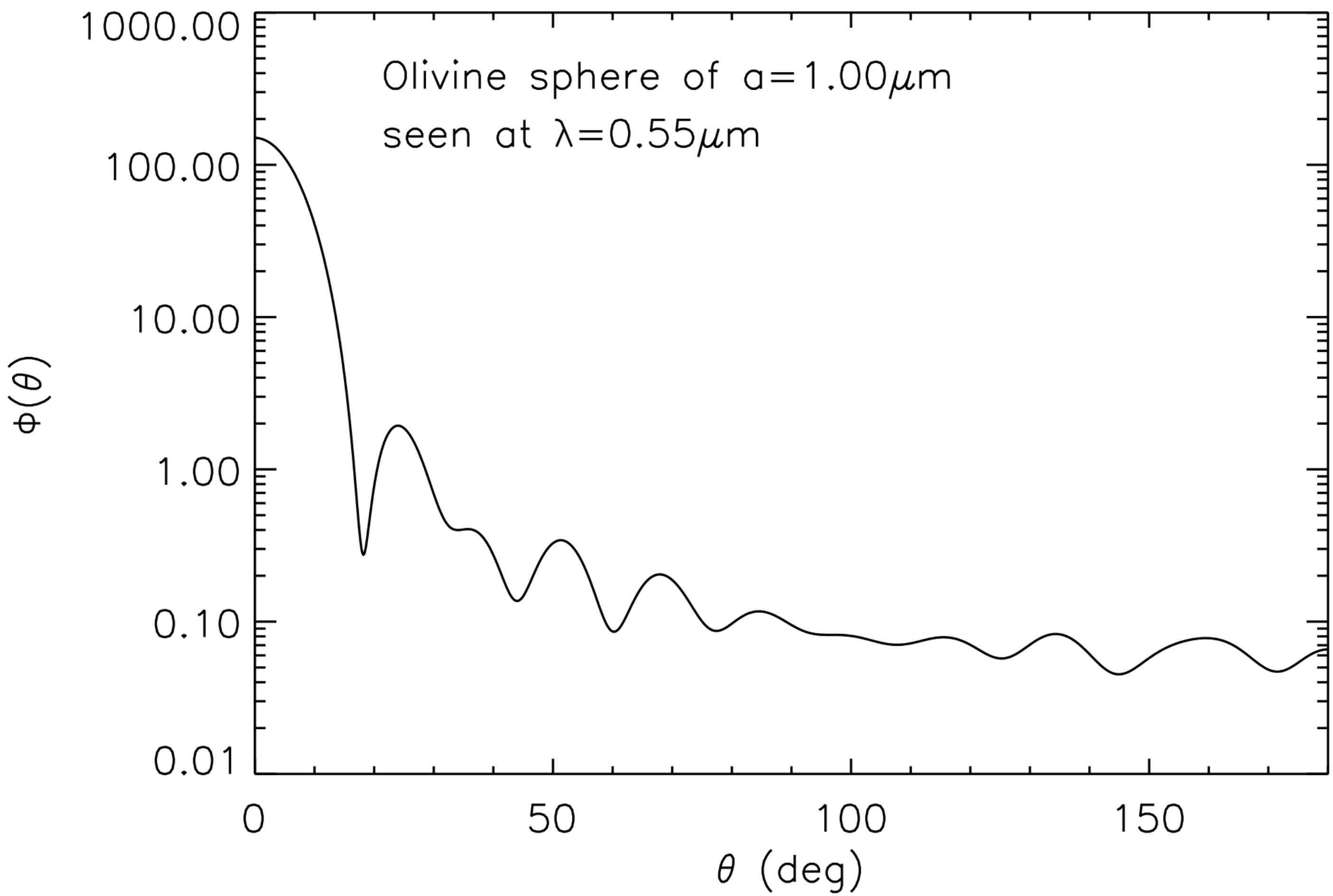
Dust opacities

Example: Phase functions for non-isotropic scattering



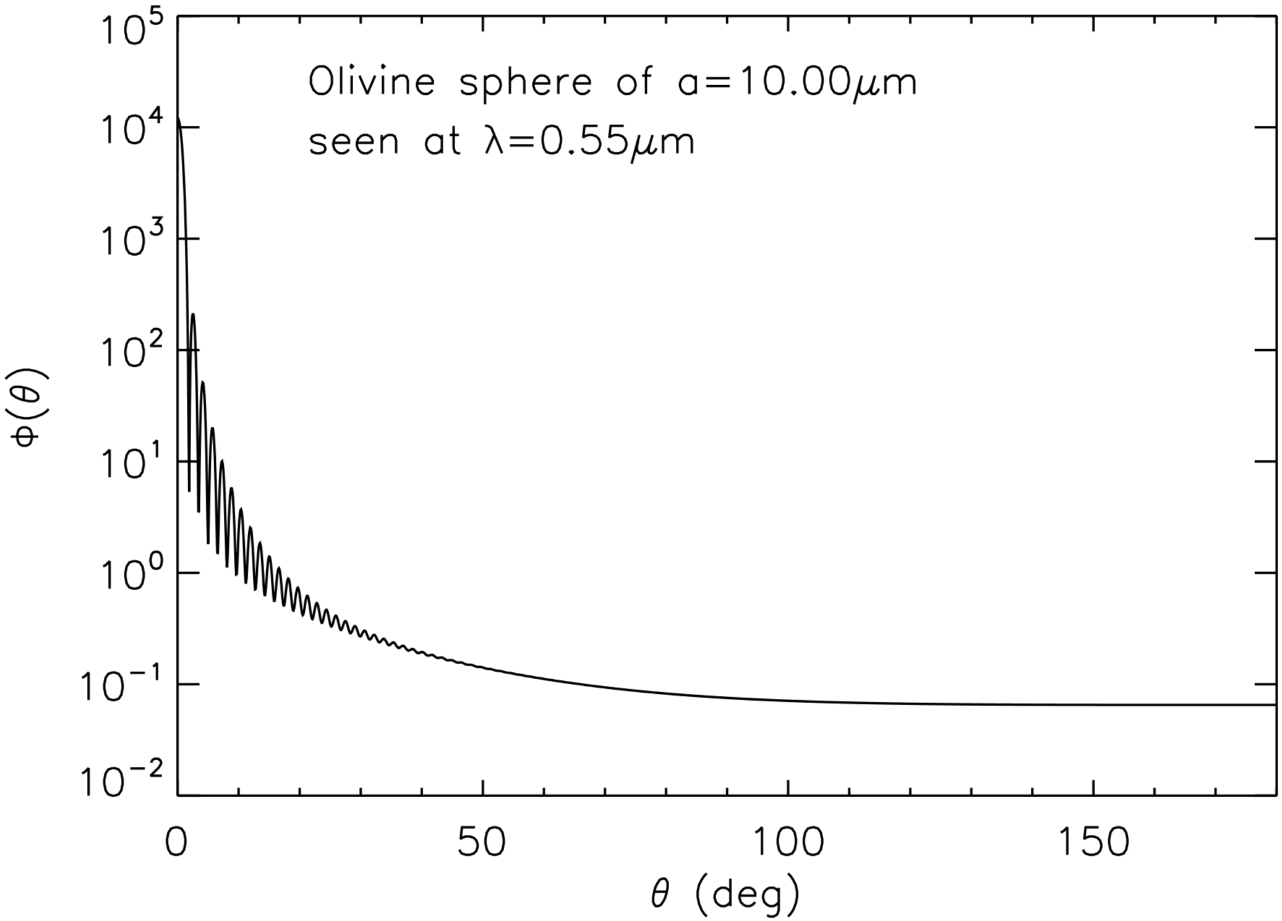
Dust opacities

Example: Phase functions for non-isotropic scattering



Dust opacities

Example: Phase functions for non-isotropic scattering



Compute your own dust opacities

- Different shapes need different opacity codes:
  - Spheres: *Mie Code* (= simplest!)
  - Polygons: *T-Matrix Code* (= moderately complex)
  - Complex shapes: DDA Code (= very complex/heavy)

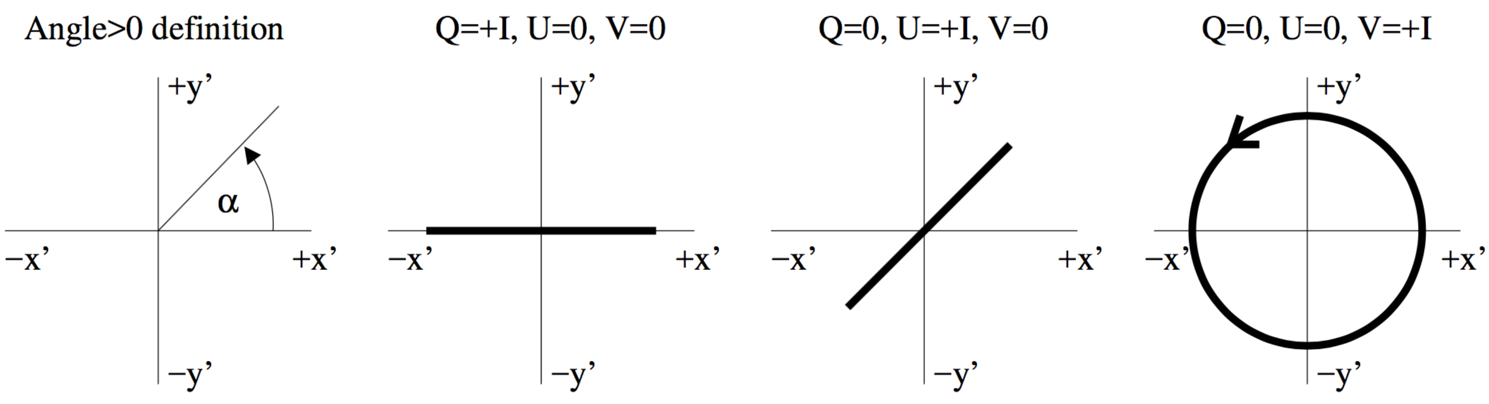
Polarized radiation

- So far we have only talked about "the" intensity:

$$I(s,\nu) \qquad [\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,ster^{-1}}]$$

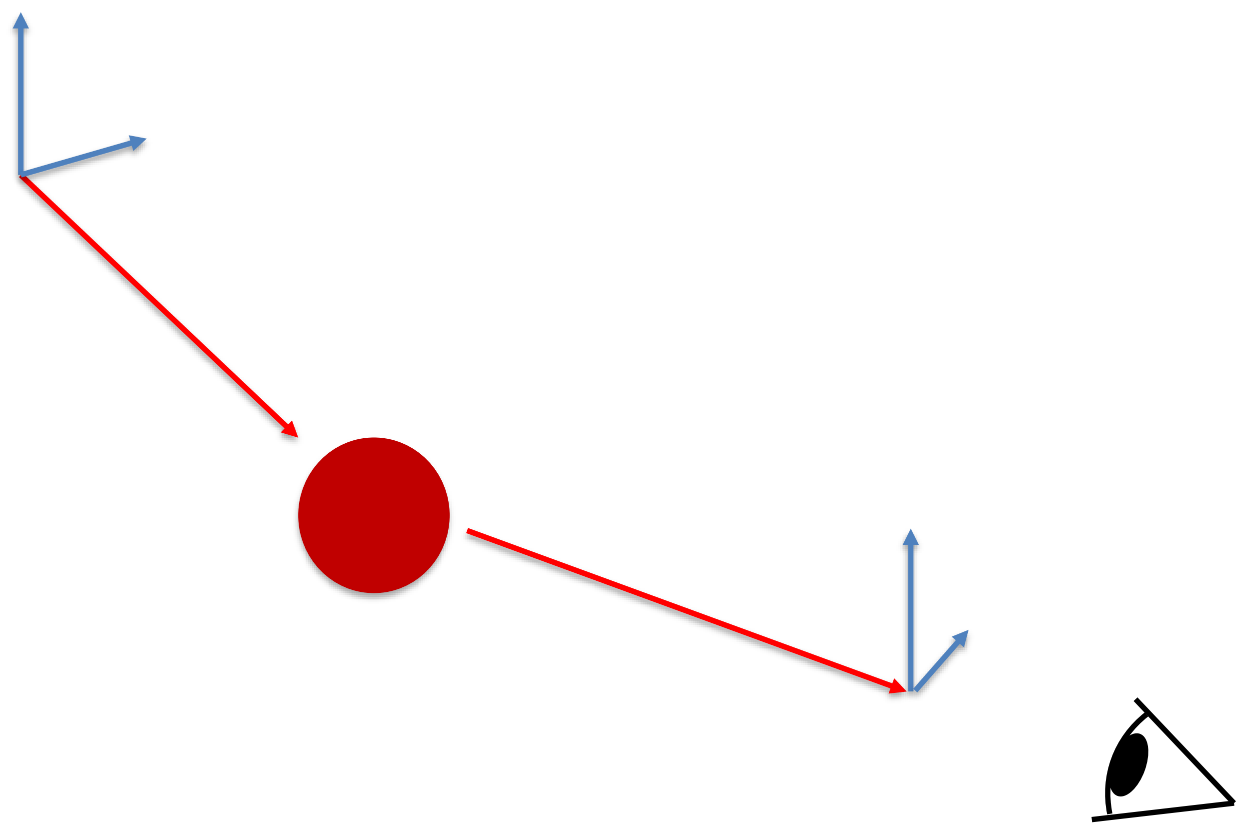
- But in reality radiation can be polarized and should be described by a Stokes vector:

$$\mathcal{I}_\nu = \begin{pmatrix} I_\nu \\ Q_\nu \\ U_\nu \\ V_\nu \end{pmatrix}$$

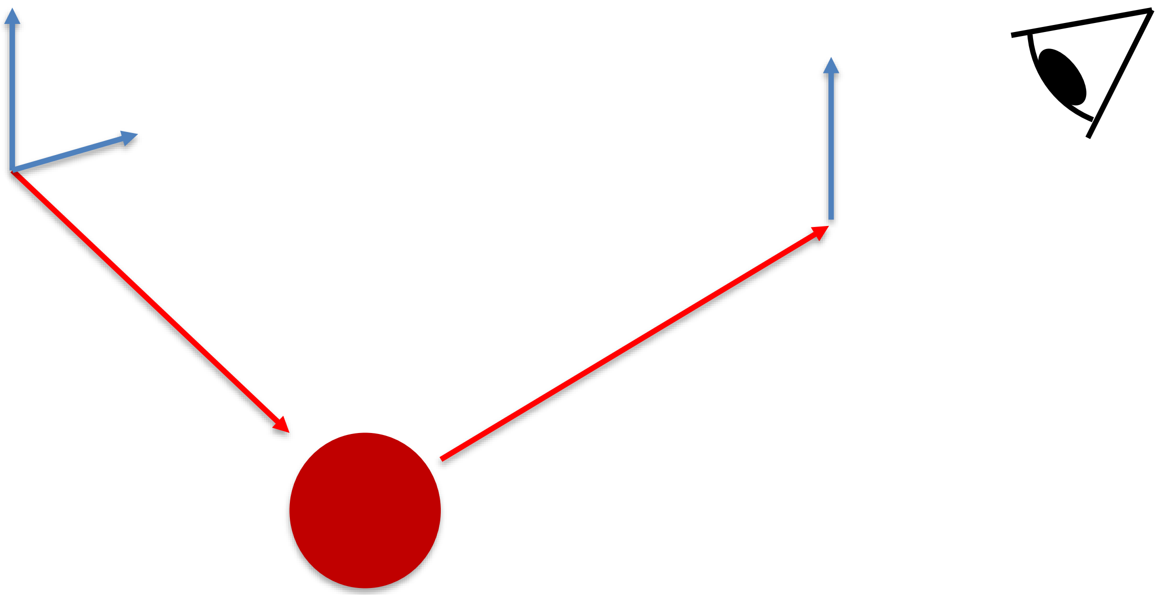


<u>Note:</u> Depends on convention and on a choice of Q-direction!

Scattering induces polarization

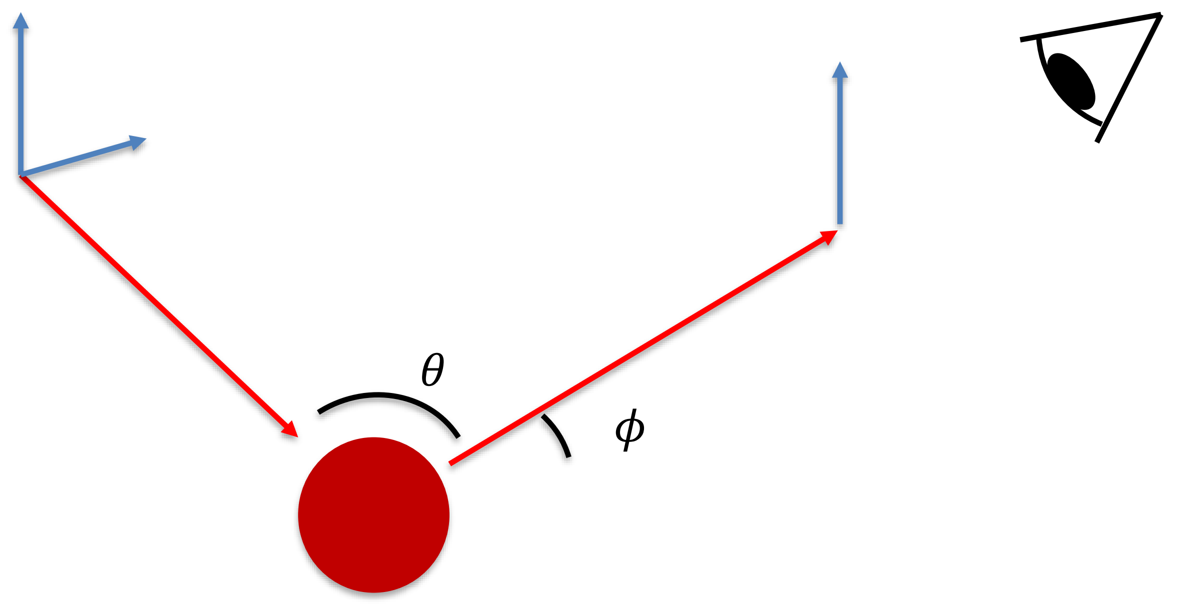


Scattering induces polarization



$$j_\nu^{\rm scat} = \begin{pmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{pmatrix} \begin{pmatrix} I_\nu \\ Q_\nu \\ U_\nu \\ V_\nu \end{pmatrix}$$

Scattering induces polarization



$$j_\nu^{\mathrm{scat}} = \begin{pmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{pmatrix}_{\theta,\phi} \begin{pmatrix} I_\nu \\ Q_\nu \\ U_\nu \\ V_\nu \end{pmatrix}$$

Line radiative transfer

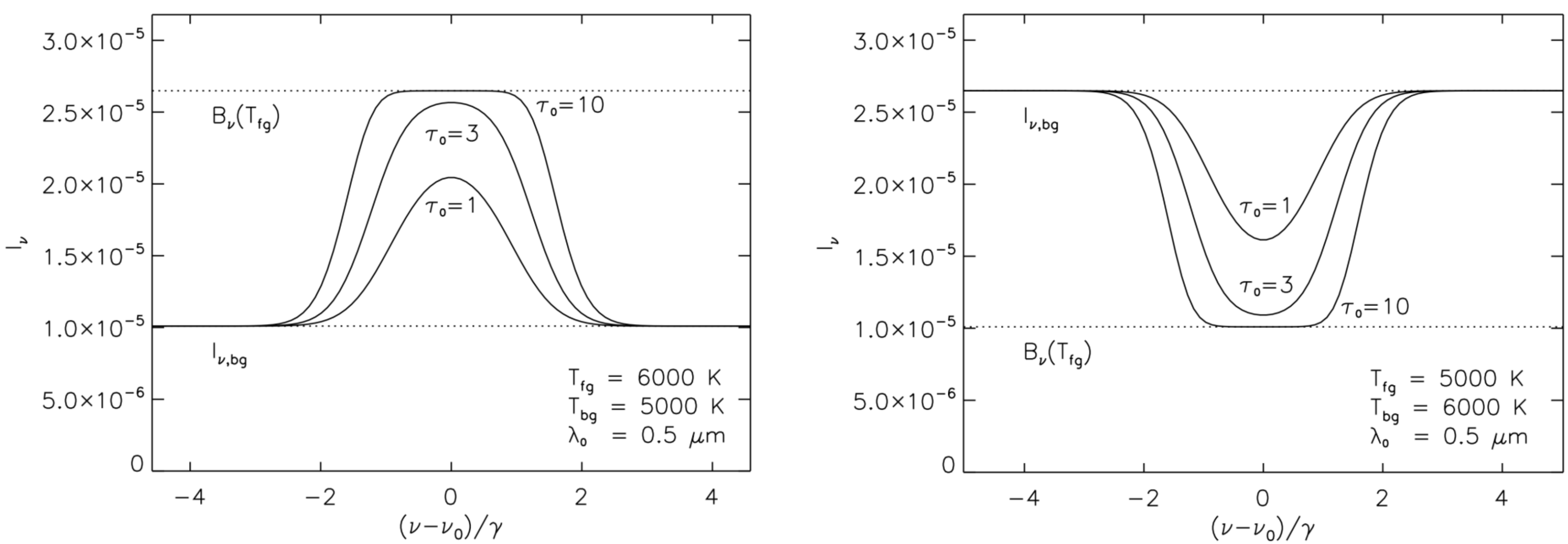
(See also Christian Brinch's lecture)

RT Equation for lines in LTE

For LTE the Formal Transfer equation is the same as for dust:

$$\frac{dI_\nu}{ds} = \rho(s)\kappa_\nu B_\nu(T(s)) - \rho(s)\kappa_\nu I_\nu(s)$$

Just replace dust continuum opacity with line opacity.
(works only in LTE!)



RT Equation for lines in non-LTE

For non-LTE the Formal Transfer equation is:

$$\frac{dI_\nu}{ds} = j_\nu(s) - \alpha_\nu(s) I_\nu(s)$$

$$j_{ij,\nu} = \frac{h\nu_{ij}}{4\pi} N_i A_{ij} \phi_{ij}(\nu)$$

$$\alpha_{ij,\nu} = \frac{h\nu_{ij}}{4\pi} (N_j B_{ji} - N_i B_{ij}) \phi_{ij}(\nu)$$

Methods for solving the populations:

- Optically thin populations
- Escape probability
- Large Velocity Gradient
- Full non-LTE (not included in RADMC-3D)

Literature:

- A standard book on radiative processes in astrophysics is: Rybicki & Lightman "Radiative Processes in Astrophysics" Wiley-Interscience
- For radiative transfer in particular there are some excellent lecture notes on-line by Rob Rutten "Radiative transfer in stellar atmospheres" http://www.staff.science.uu.nl/~rutte101/
- For stellar atmospheres: pleasantly written book by Böhm-Vitense „Stellar Astrophysics Vol. 2: Stellar atmospheres“

Literature:

- In-depth reference work by Mihalas „Stellar atmospheres“
- Allround bible on radiation hydrodynamics by Mihalas & Mihalas „Radiation Hydrodynamics“
- Book on Exoplanetary atmospheres by Seager „Exoplanet Atmospheres“
- Book on radiative transfer in Earth‘s atmosphere (application to e.g. climate research): Wendisch & Yang „Theory of Atmospheric Radiative Transfer“

Literature:

- My own set of lecture notes:
  http://www.ita.uni-heidelberg.de/~dullemond/teaching.shtml