Monte Carlo Radiation Transfer

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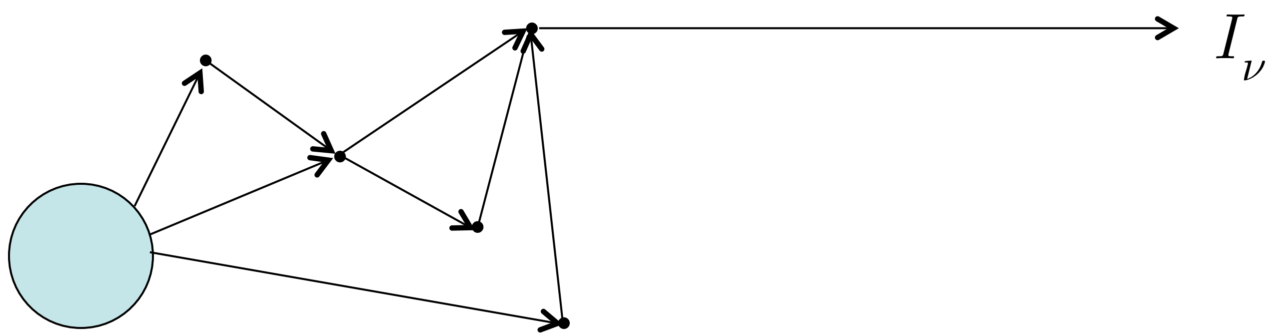
3-D Radiation Transfer

- Transfer Equation

$$\hat{\mathbf{n}} \cdot \nabla I_\nu = -\chi_\nu \rho I_\nu + j_\nu + \sum_i n_i \sigma_\nu^i \int \left(\frac{1}{\sigma_\nu^i} \frac{d\sigma_\nu^i}{d\Omega} \right) I_\nu(\hat{\mathbf{n}}') d\Omega'$$

  – Ray-tracing (requires lambda-iteration)



  – Monte Carlo (exact integration using random paths)
    - *May* avoid lambda-iteration
    - automatically an adaptive mesh method
      – Paths sampled according to their importance

Monte Carlo Radiation Transfer

- Transfer equation traces flow of energy
- Divide luminosity into equal energy packets ("photons")

$$E_\gamma = L\Delta t \,/\, N_\gamma$$

  - Number of physical photons

$$n = E_\gamma \,/\, h\nu$$

  - Packet may be partially polarized

$$\begin{aligned} I &= 1 \\ Q &= (E_{\updownarrow} - E_{\leftrightarrow}) \,/\, E_\gamma \\ U &= (E_{\swarrow\!\!\!\nearrow} - E_{\nwarrow\!\!\!\searrow}) \,/\, E_\gamma \\ V &= (E_{\circlearrowright} - E_{\circlearrowleft}) \,/\, E_\gamma \end{aligned}$$

Monte Carlo Radiation Transfer

- Split luminosity between star and envelope

$$L = L_* + L_{\text{env}}$$

Star

$$\mu I_\nu = \frac{dE / dt}{dA\, d\nu\, d\Omega}$$

$$\frac{dP}{dA\, d\nu\, d\Omega} \propto \mu I_\nu$$

$$\frac{dP}{dA\, d\nu} \propto H_\nu$$

$$\frac{dP}{dA} \propto H$$

Envelope

$$j_\nu = \frac{dE / dt}{dV\, d\nu\, d\Omega}$$

$$\frac{dP}{dV\, d\nu\, d\Omega} \propto j_\nu$$

$$\frac{dP}{dV\, d\nu} \propto 4\pi j_\nu$$

$$\frac{dP}{dV} \propto 4\pi \int_0^\infty j_\nu \, d\nu$$

Monte Carlo Radiation Transfer

- Pick random starting location, frequency, and direction
  - Sample from appropriate probability distributions:
    - Location on star: $$\frac{dP}{dA} \propto H \qquad \text{(flux)}$$
    - Frequency: $$\frac{dP}{d\nu} \propto H_\nu \qquad \text{(stellar spectrum)}$$
    - Direction: $$\frac{dP}{d\Omega} \propto \mu I_\nu \qquad \text{(intensity)}$$

Monte Carlo Radiation Transfer

- Doppler Shift photon packet as necessary
  - packet energy is frame-dependent

$$E_\gamma \to wE_\gamma \qquad w \text{ is photon "weight"}$$

- Transport packet to random interaction location

$$dP = d\tau = \chi_\nu \rho ds \qquad \text{(Poisson Distribution)}$$

$$dN = -N d\tau$$

$$P = 1 - e^{-\tau} \qquad \text{(Cumulative Probability)}$$

$$\tau = -\ln \xi \qquad (\xi \text{ is uniform random number})$$

$$\tau = \int_0^s \chi_\nu \rho ds \qquad \text{(find distance, } s\text{)} \qquad \text{most CPU time}$$

$$\mathbf{x} = \mathbf{x}_0 + s\,\hat{\mathbf{n}} \qquad \text{(move photon)}$$

Monte Carlo Radiation Transfer

- Randomly scatter or absorb photon packet

$$a = \frac{\sigma_\nu}{\sigma_\nu + \kappa_\nu} \qquad \text{(albedo)}$$

$$\begin{cases} \xi > a & \text{(absorb)} \\ \xi < a & \text{(scatter)} \end{cases}$$

$$\frac{dP}{d\Omega} = \frac{1}{\sigma_\nu}\frac{d\sigma_\nu}{d\Omega} \qquad \text{(phase function)}$$

- If photon hits star, reemit it locally
- When photon escapes, place in observation bin (direction, frequency, and location)

REPEAT 10^6-10^9 times

Monte Carlo Maxims

- Monte Carlo is EASY
  - to do wrong (G.W. Collins III)
  - code must be tested *quantitatively*
  - being clever is dangerous
  - try to avoid discretization
- The Improbable event WILL happen
  - code must be bullet proof
  - and error tolerant

Sampling and Measurements

- MC simulation produces random events
  - Photon escapes
  - Photon interactions
  - Cell wall crossings
  - Photon motion
- Events are sampled/counted
  - Cumulative energy => measurements (flux)
  - Histogram => distribution function (spectrum)

SEDs and Images

- Sampling Photon Escapes

$$\frac{F_\nu}{F_*} = \frac{4\pi d^2}{L}\frac{dE}{dt\,dA\,d\nu} = \frac{4\pi d^2}{L}\frac{N_{ij}E_\gamma \,/\, \Delta t}{d^2 d\Omega_i \Delta\nu_j} = \frac{4\pi N_{ij}}{N_\gamma d\Omega_i \Delta\nu_j}$$

where $N_{ij} = \sum w_{ij}$

$$\frac{I_\nu}{F_*} = \frac{4\pi N_{ijkl}}{N_\gamma d\Omega_{ij} d\Omega_k \Delta\nu_l}$$

SEDs and Images

- Advanced Sampling
  - Photon interactions: scattering, emission (source function samping)

$$dN_i = \begin{cases} w\left(\frac{1}{\sigma}\frac{d\sigma}{d\Omega}\right)e^{-\tau_{\mathrm{esc}}}\Delta\Omega_i & \text{(scattered)} \\ \frac{w}{4\pi}e^{-\tau_{\mathrm{esc}}}\Delta\Omega_i & \text{(emitted)} \end{cases}$$

  - Photon motion (Lucy path length sampling)

$$dN_{\mathrm{abs}} = w d\tau_{\mathrm{abs}} = w\kappa_\nu \rho ds$$

$$dN_i = w d\tau_{\mathrm{sc}}\left(\frac{1}{\sigma}\frac{d\sigma}{d\Omega}\right)e^{-\tau_{\mathrm{esc}}}\Delta\Omega_i$$

Error Estimation

- Unweighted Photons
  - Number in bin has a binomial distribution

$$\frac{\delta E}{E} = \sqrt{(1 - N_i / N_\gamma) / N_i}$$

$$\approx 1 / \sqrt{N_i}$$

- Weighted Photons
  - Each photon track is statistically independent

$$w_i = \sum_{\text{track}} w_{\text{obs}}$$

$$\frac{\delta E}{E} = \sqrt{\sum_i w_i^2} / \sum_i w_i$$

Parallel Monte Carlo

- Photon paths are independent
  - Divide total among different CPUs
  - Each CPU independently runs its batch of photons
  - Co-add results at end
  - Embarrassingly parallel

Parallel Implementation

- Master/Slave:
  - Master sends messages to slaves:
    - Initialize (includes simulation parameters)
    - Run batch of N photons
    - Retrieve results
    - Reinitialize (zero all counters)
    - Die
  - Each slave reports back to master when done
  - Master gives slave new batch of photons
    - Automatic CPU load balancing
  - Results collected when all slaves are finished
    - Minimizes network load

Monte Carlo Assessment

- Advantages
  - Inherently 3-D
  - Microphysics easily added (little increase in CPU time)
  - Modifications do not require large recoding effort
  - Embarrassingly parallelizable
- Disadvantages
  - High S/N requires large number of photons
  - Achilles heel = no photon escape paths; i.e., large optical depth

Improving Run Time

- Photon paths are random
  - Can reorder calculation to improve efficiency
- Adaptive Monte Carlo
  - Modify execution as program runs
- High Optical Depth
  - Use analytic solutions in "interior" + MC "atmosphere"
    - Diffusion approximation (static media)
    - Sobolev approximation (for lines in expanding media)
  - Match boundary conditions

MC Radiative Equilibrium

- Sum energy absorbed by each cell
- Radiative equilibrium gives temperature

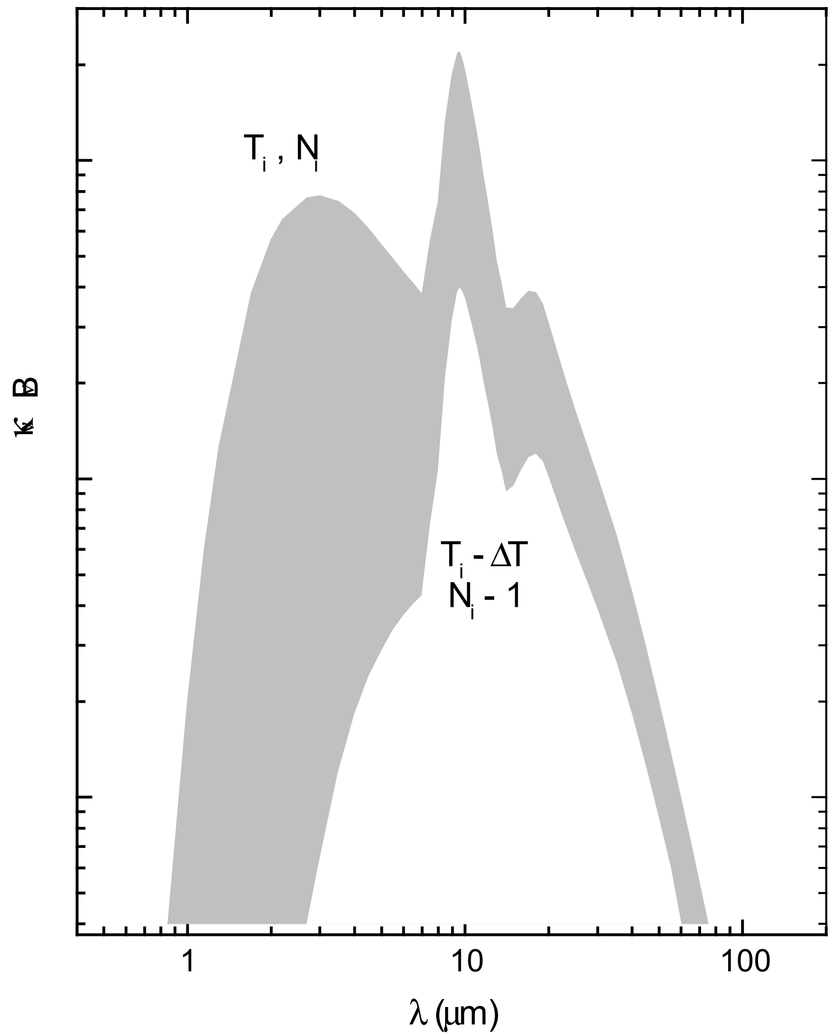
$$E_{\text{abs}} = E_{\text{emit}}$$

$$n_{\text{abs}} E_{\gamma} = 4\pi m_i \kappa_{\text{P}} B(T_i)$$

- When photon is absorbed, reemit at new frequency, depending on T
  - Energy conserved automatically
- Problem: Don't know T *a priori*
- Solution: Change T each time a photon is absorbed and correct previous frequency distribution

avoids iteration

Temperature Correction



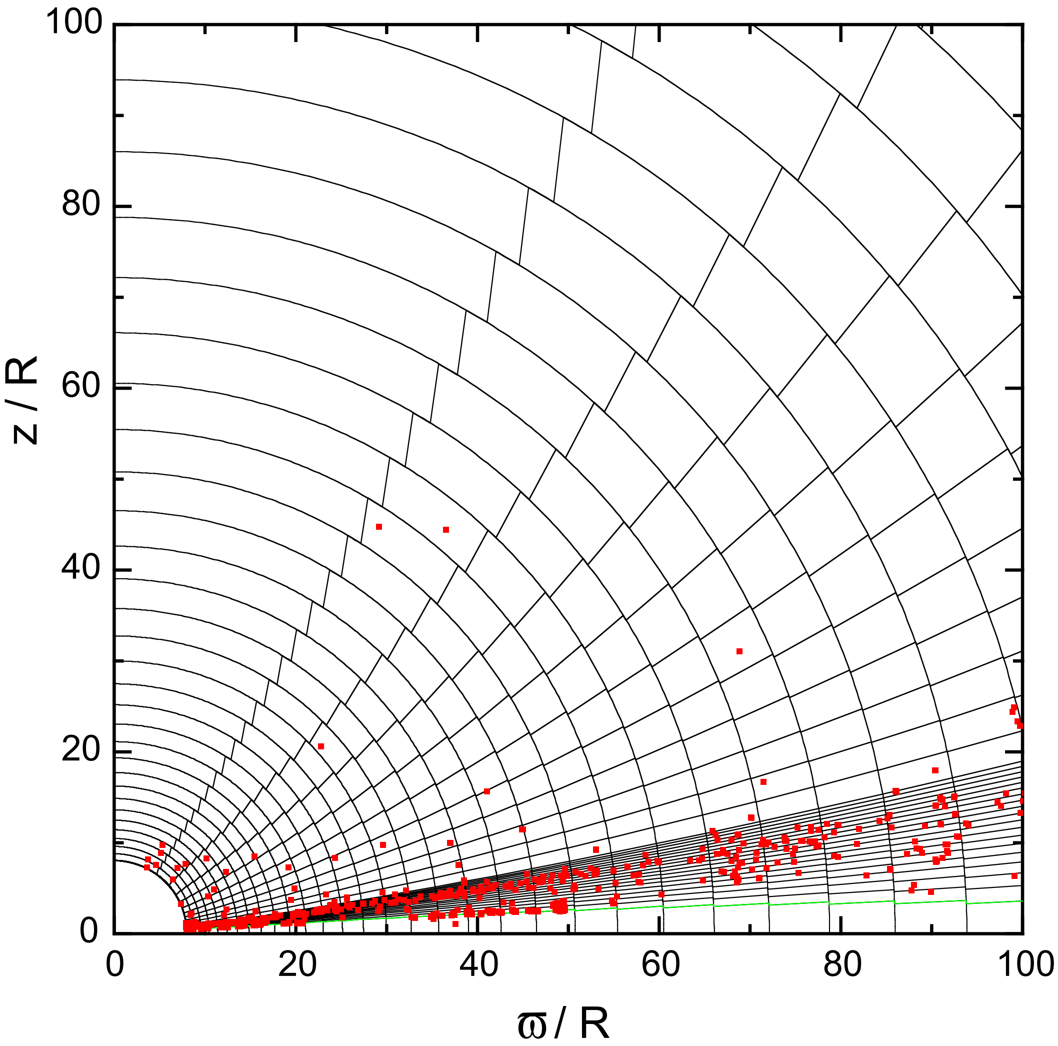
Frequency Distribution:

$$\frac{dP}{d\nu} = j_\nu(T + \Delta T) - j_\nu(T)$$

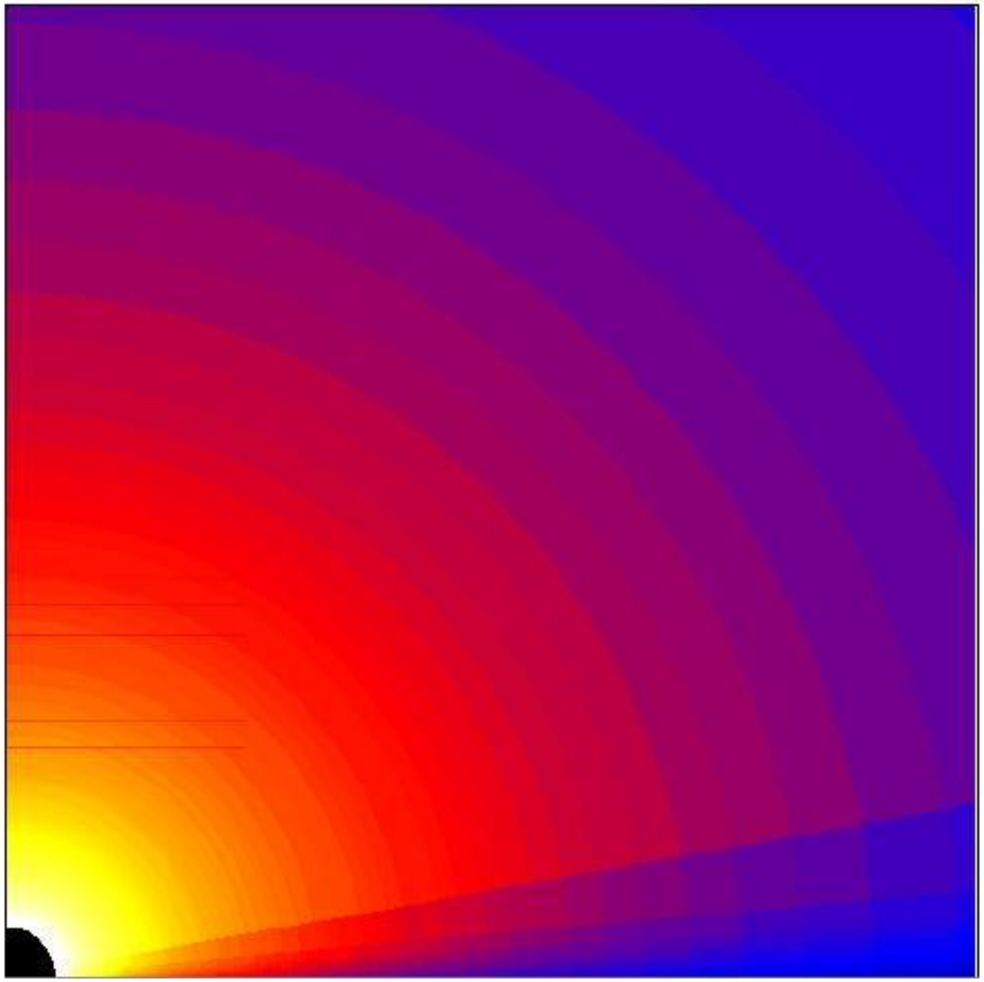
$$= \kappa_\nu \Delta T \frac{dB_\nu}{dT}$$

Bjorkman & Wood 2001

T Tauri Envelope Absorption

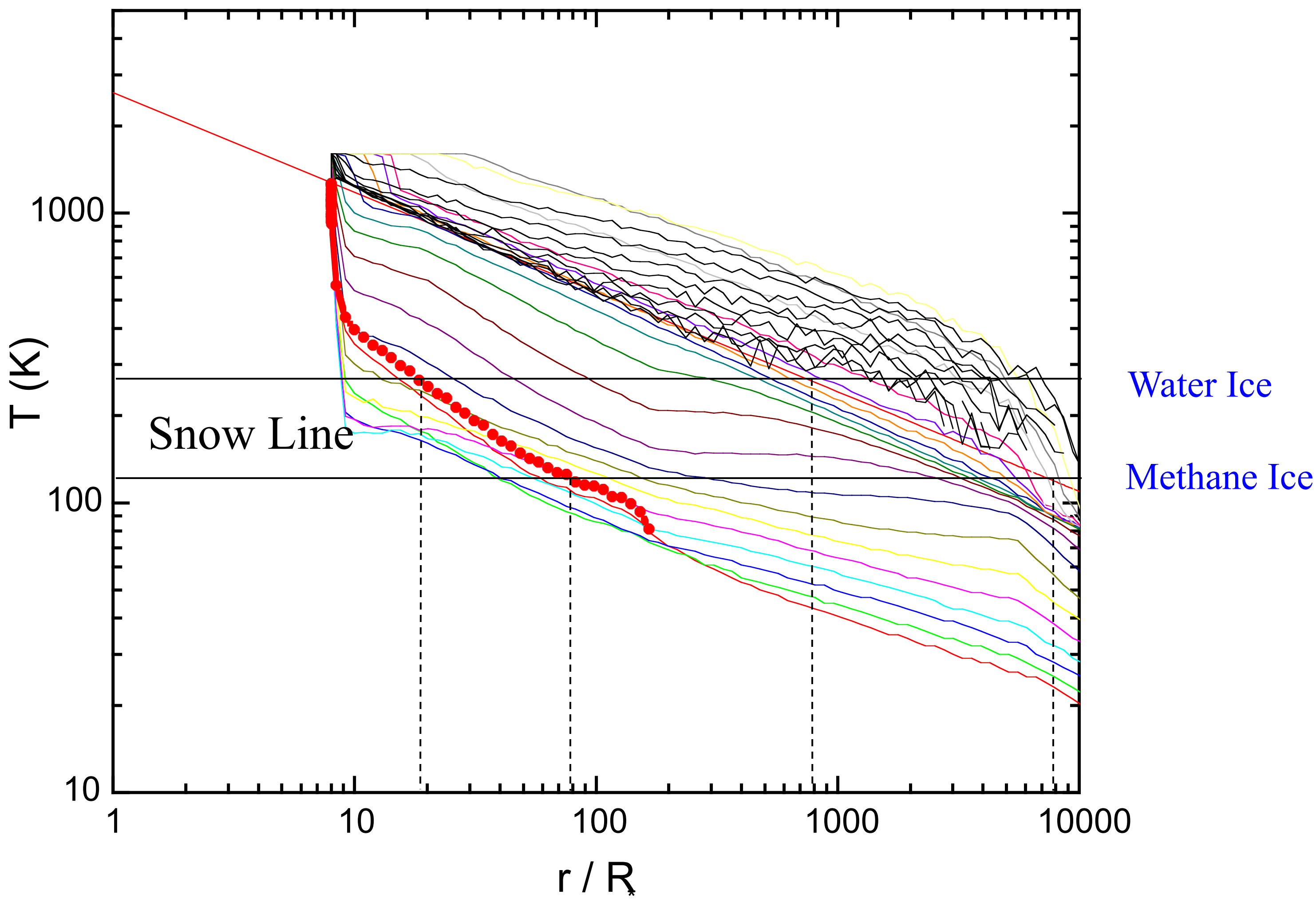


Disk Temperature



Bjorkman 1998

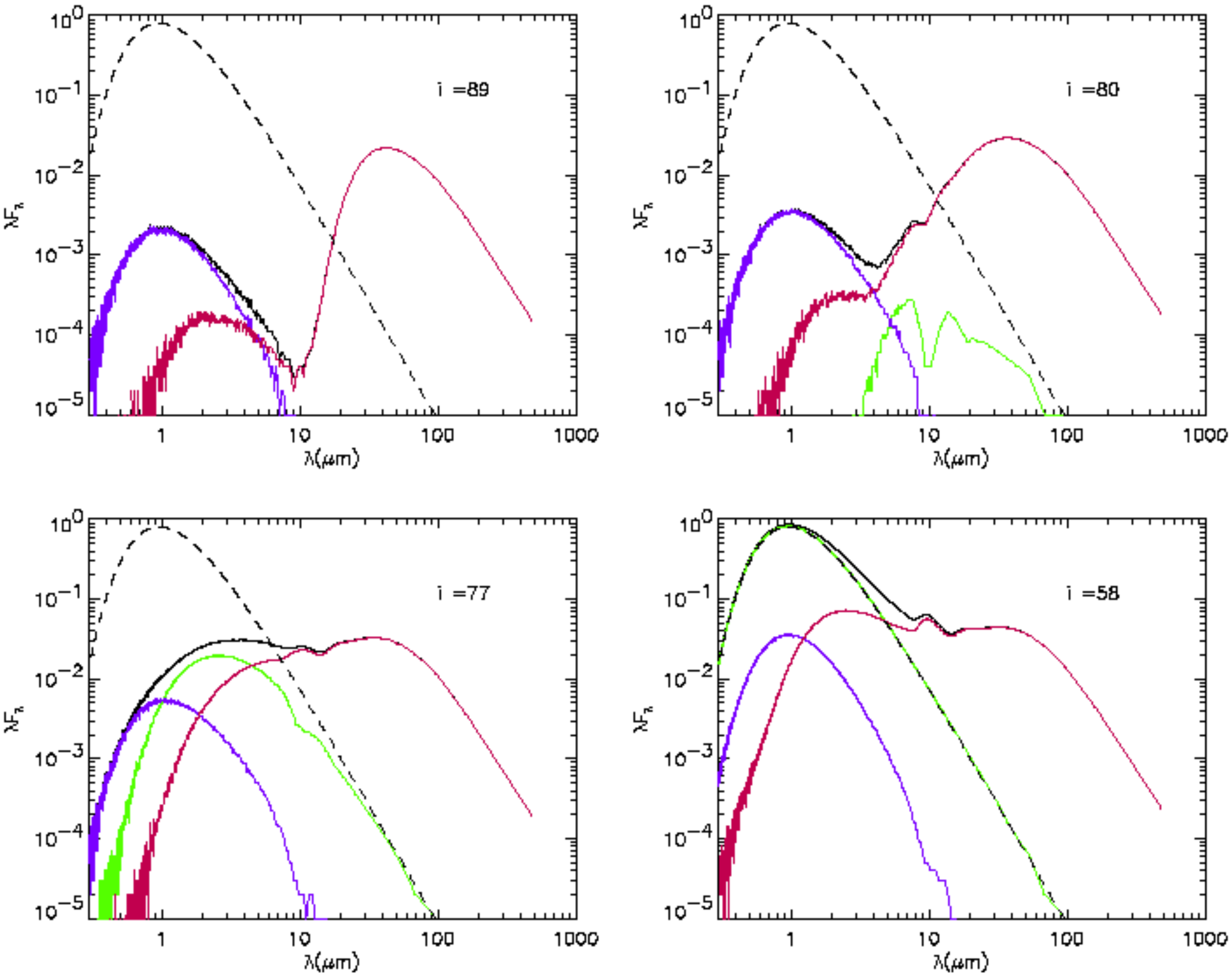
Disk Temperature



Effect of Disk on Temperature

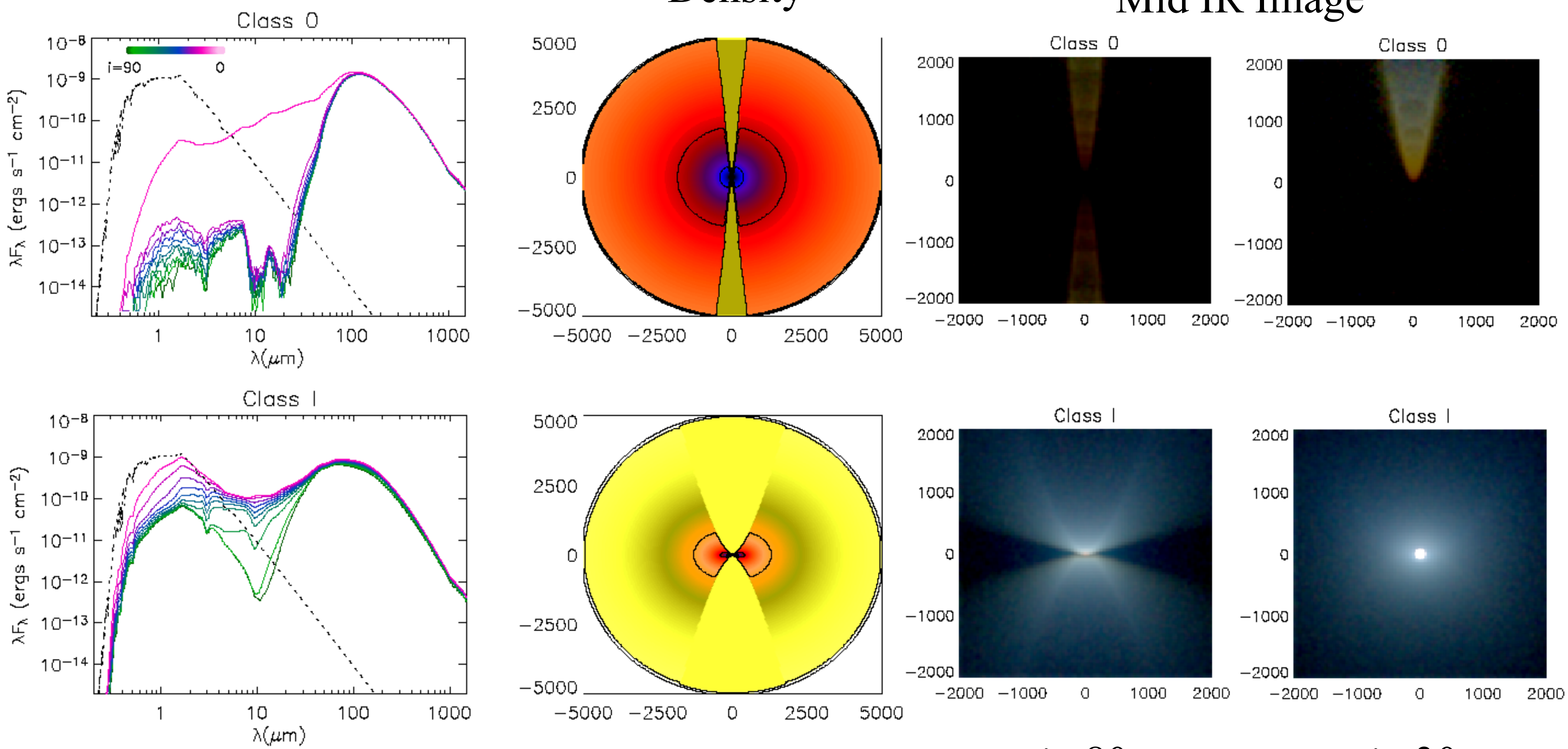
- Inner edge of disk
  - heats up to optically thin radiative equilibrium temperature
- At large radii
  - outer disk is shielded by inner disk
  - temperatures lowered at disk mid-plane

CTTS Model SED



Protostar Evolutionary Sequence

SED | Density | Mid IR Image

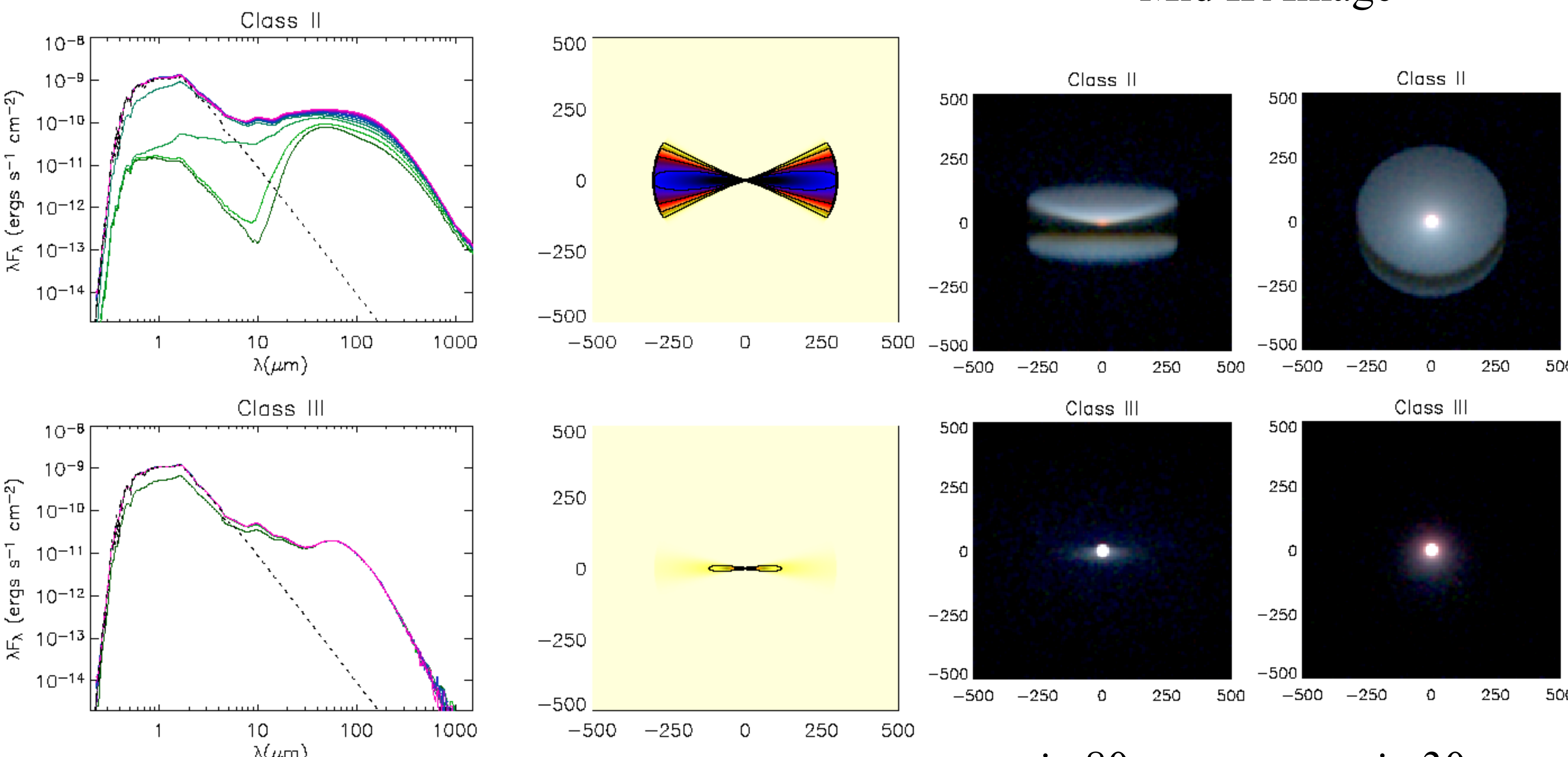


i =80 | i =30

Whitney, Wood, Bjorkman, & Cohen 2003

Protostar Evolutionary Sequence

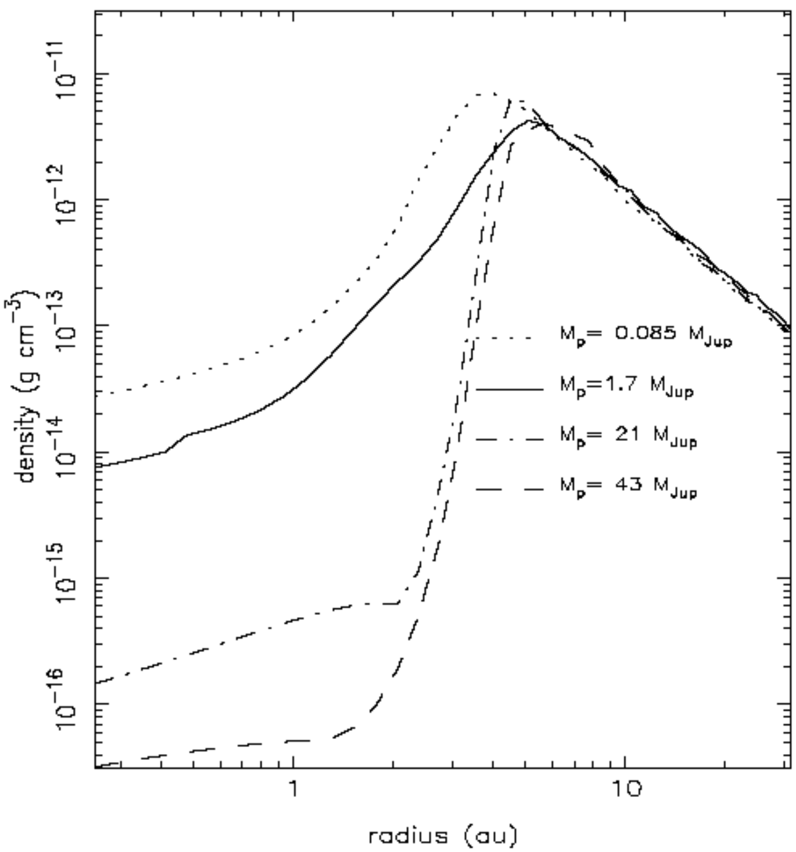
SED | Density | Mid IR Image

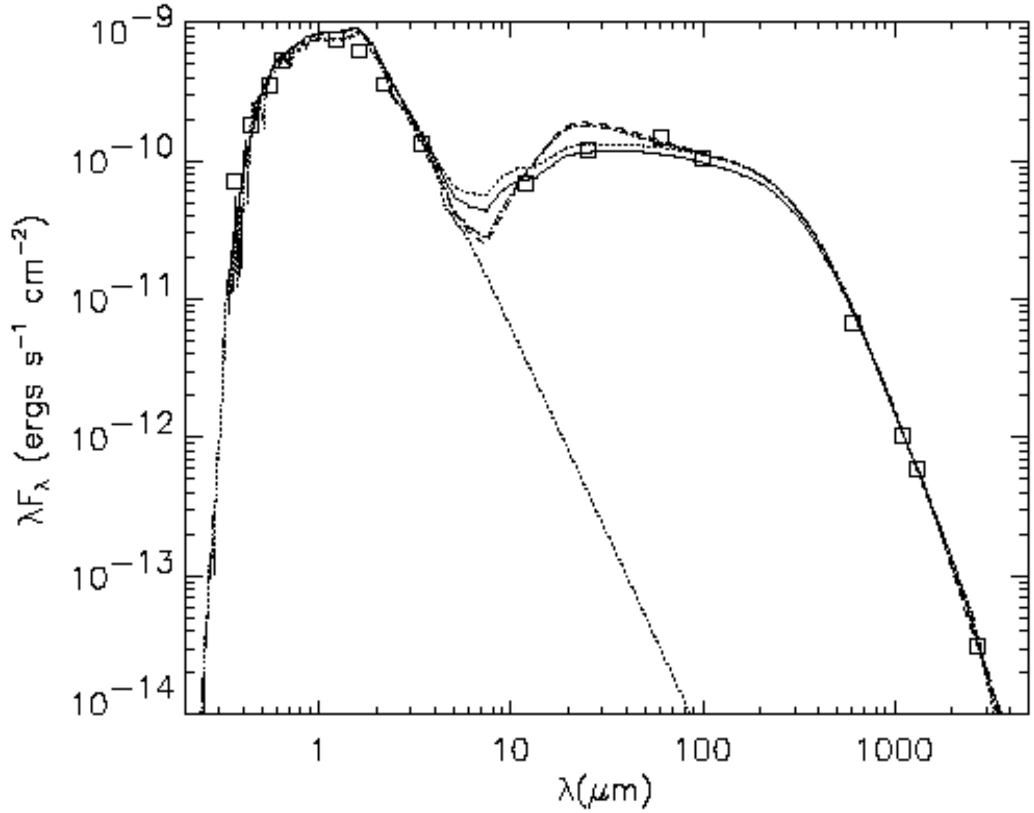


i =80 | i =30

Whitney, Wood, Bjorkman, & Cohen 2003

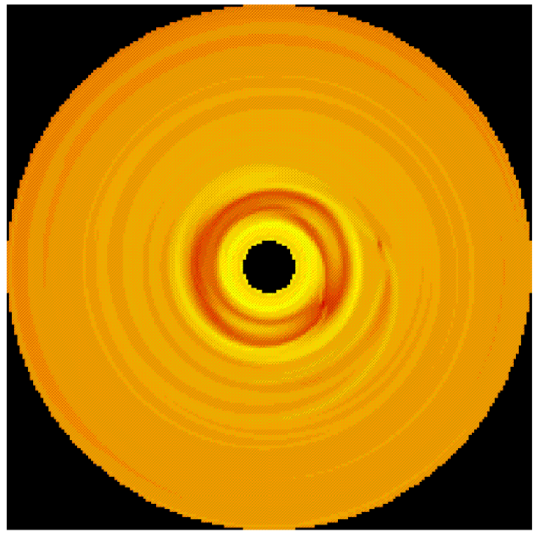
Planet Gap-Clearing Model



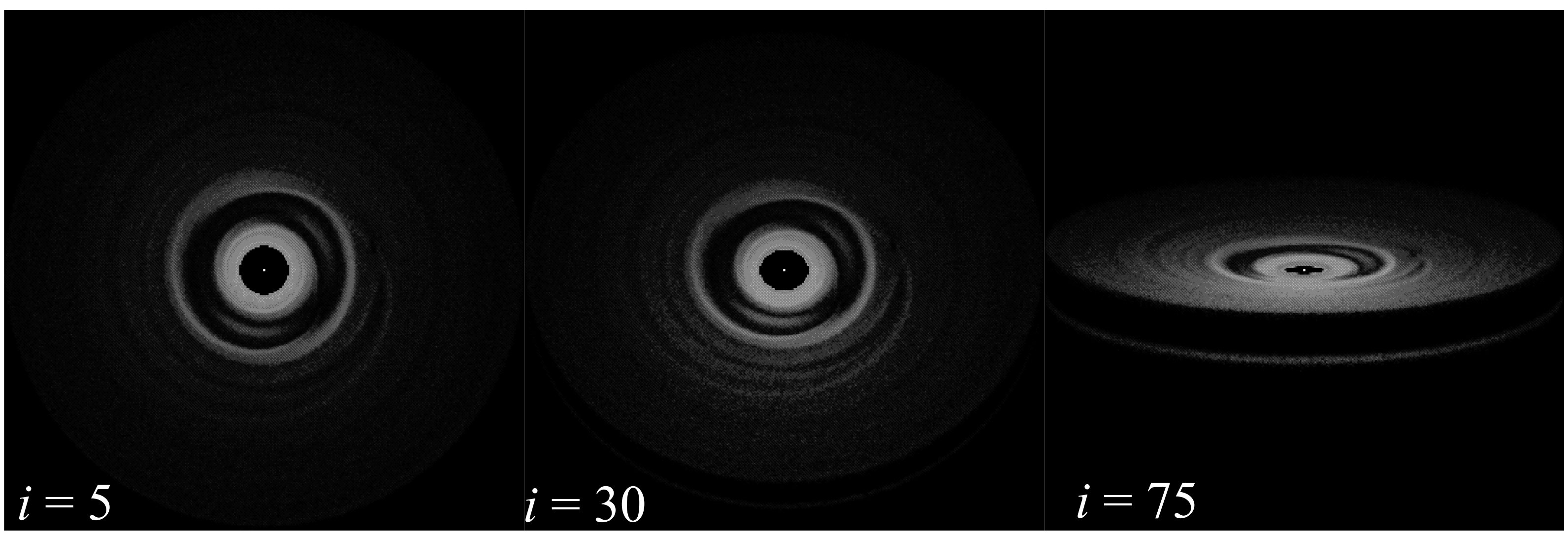


Rice et al. 2003

Protoplanetary Disks



Surface Density



Spectral Lines

- Lines very optically thick
  - Cannot track millions of scatterings
- Use Sobolev Approximation (moving gas)
  - Sobolev length

$$l(\hat{\mathbf{n}}) = \frac{v_D}{\left| dv \,/\, dl \right|} \qquad \frac{dv}{dl} = n^i e_{ij} n^j \qquad e_{ij} = (v_{i;j} + v_{j;i}) \,/\, 2$$

  - Sobolev optical depth

$$\tau_{\mathrm{sob}} = \frac{k_L c}{\nu_0 \left| dv \,/\, dl \right|} \qquad k_L = \frac{\pi e^2}{m_e c} gf \left(\frac{n_l}{g_l} - \frac{n_u}{g_u} \right)$$

  - Assume *S*, rho, etc. constant (within *l*)

Spectral Lines

- Split Mean Intensity

$$J = J_{\text{local}} + J_{\text{diffuse}}$$

- Solve analytically for J_{local}
- Effective Rate Equations

$$\beta_e n_u A_{ul} + \beta_p n_u B_{ul} \bar{J}_{\text{diff}} - \beta_p n_l B_{lu} \bar{J}_{\text{diff}} + \ldots = 0$$

$$\beta_e = \frac{1}{4\pi} \int \frac{1 - e^{-\tau_{\text{sob}}}}{\tau_{\text{sob}}} d\Omega \qquad \text{(escape probability)}$$

$$\beta_p = \frac{1}{4\pi} \int \frac{1 - e^{-\tau_{\text{sob}}}}{\tau_{\text{sob}}} \frac{\bar{I}_{\text{diff}}}{\bar{J}_{\text{diff}}} d\Omega \qquad \text{(penetration probability)}$$

$$\frac{dP}{d\Omega} \propto j_{\text{esc}} = \frac{h\nu n_u A_{ul}}{4\pi} \left(\frac{1 - e^{-\tau_{\text{sob}}}}{\tau_{\text{sob}}} \right) \qquad \text{(effective line emissivity)}$$

Resonance Line Approximation

- Two-level atom => pure scattering
- Find resonance location

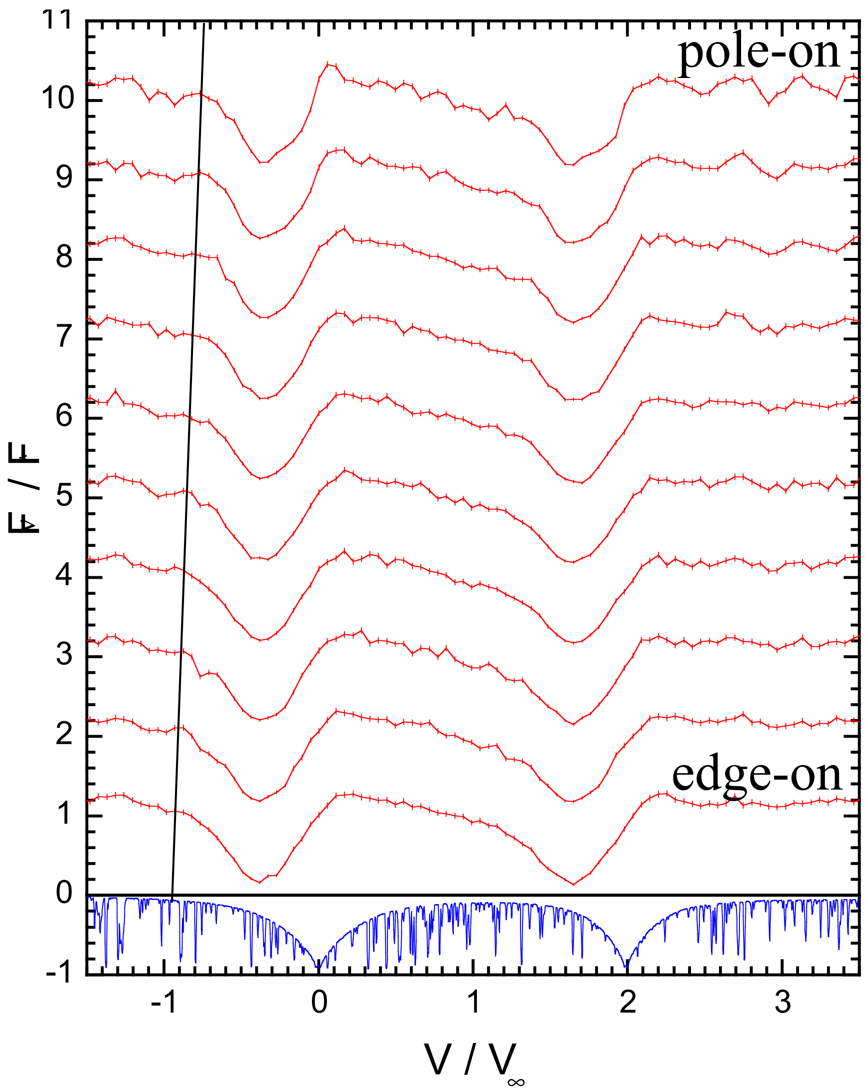
$$\nu_0 = \nu(1 - \mathbf{v} \cdot \hat{\mathbf{n}} / c)$$

- If photon interacts
  - Reemit according to escape probability

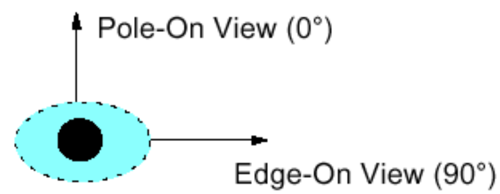
$$\frac{dP}{d\Omega} \propto \frac{1 - e^{-\tau_{\mathrm{sob}}}}{\tau_{\mathrm{sob}}}$$

  - Doppler shift photon; adjust weight

Wind Line Profiles



(b)



Bjorkman 1998

NLTE Monte Carlo RT

- Gas opacity depends on:
  - temperature
  - degree of ionization
  - level populations

  } determined by radiation field

- During Monte Carlo simulation:
  - sample radiative rates
- Radiative Equilibrium
  - Whenever photon is absorbed, re-emit it
- After Monte Carlo simulation:
  - solve rate equations
  - update level populations and gas temperature
  - update disk density (integrate HSEQ)

$$dN = n_\gamma d\tau$$

$$\frac{dP}{d\nu d\Omega} \propto j_\nu^{\mathrm{eff}}$$

What are Be stars?

B stars (T ~ 20000 K) with hydrogen emission lines

Detected by:

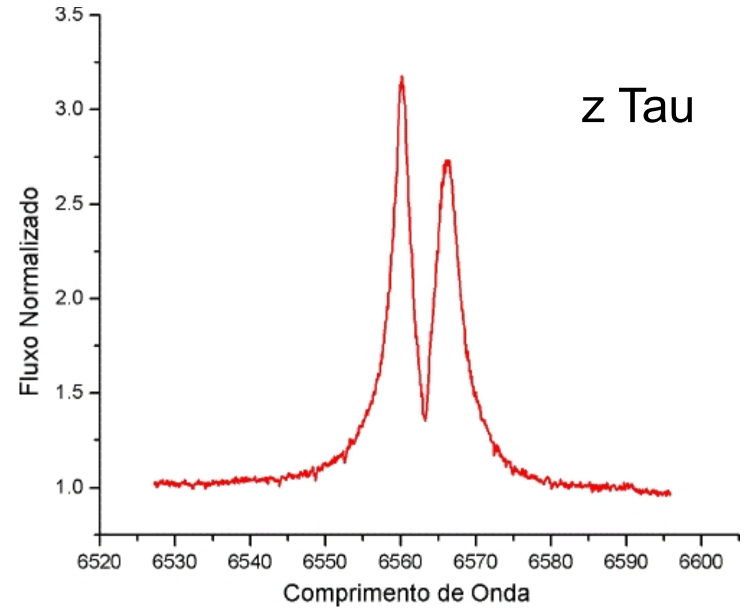
➔ Spectral lines in emission (frequently double peaked)

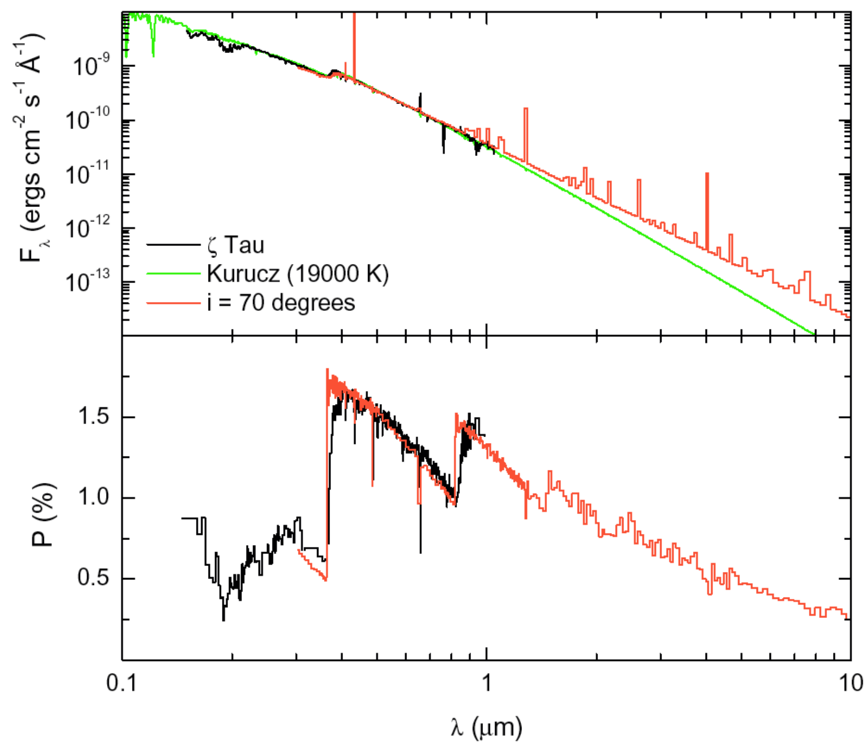
➔ IR excess (ff+bf emission)

➔ Linear polarization (scattering in the disk)

Rapidly rotating (non-supergiants) with a circumstellar disk

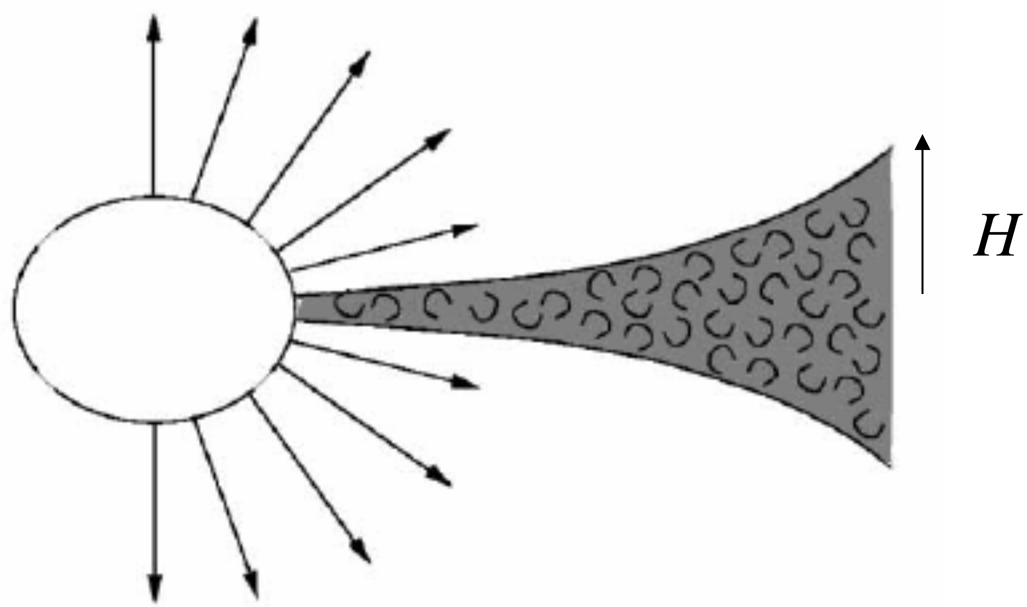
Disk is geometrically thin





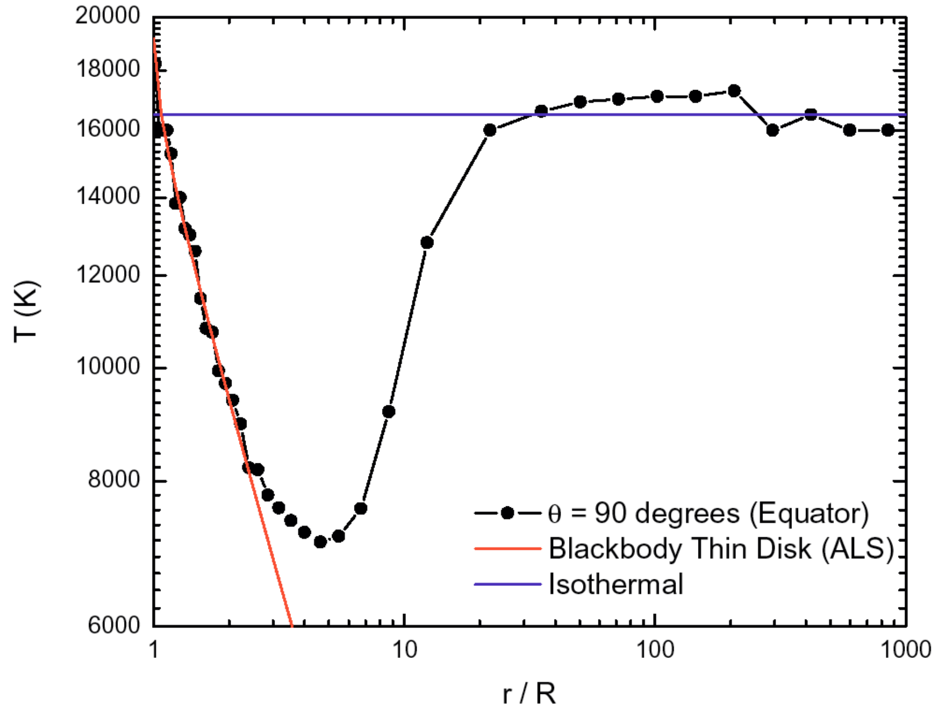
Viscous Decretion Disk

- Lee, Saio, Osaki 1991

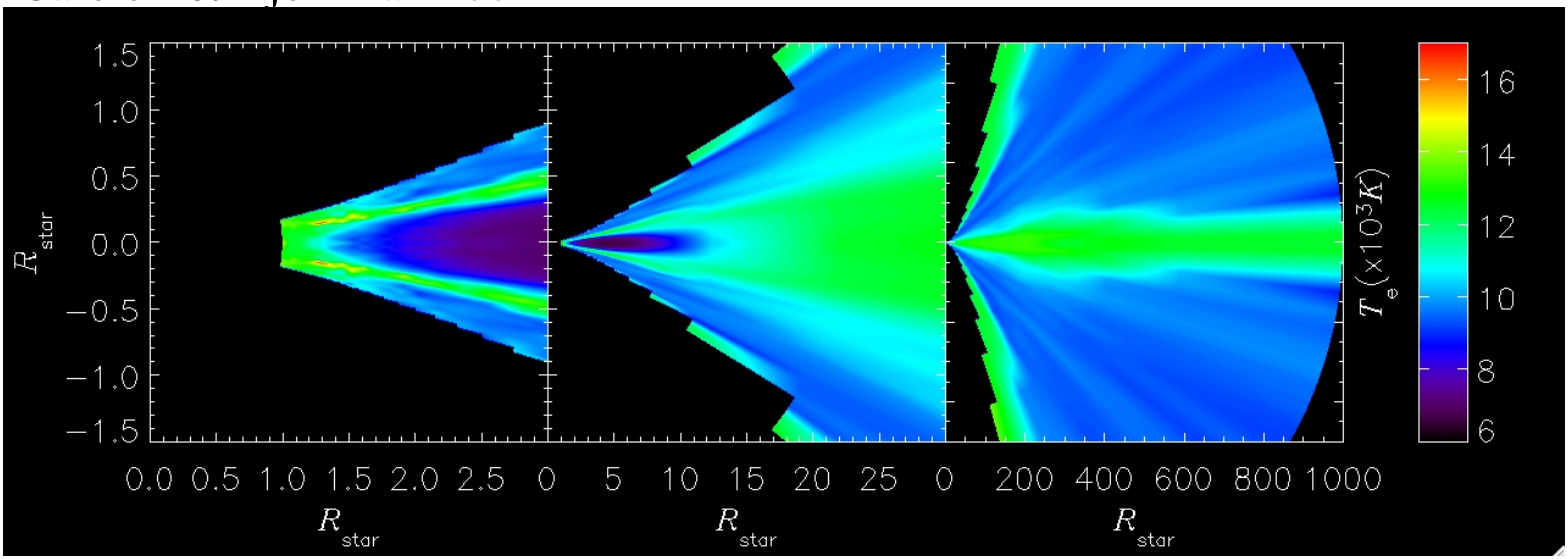


viscous disc

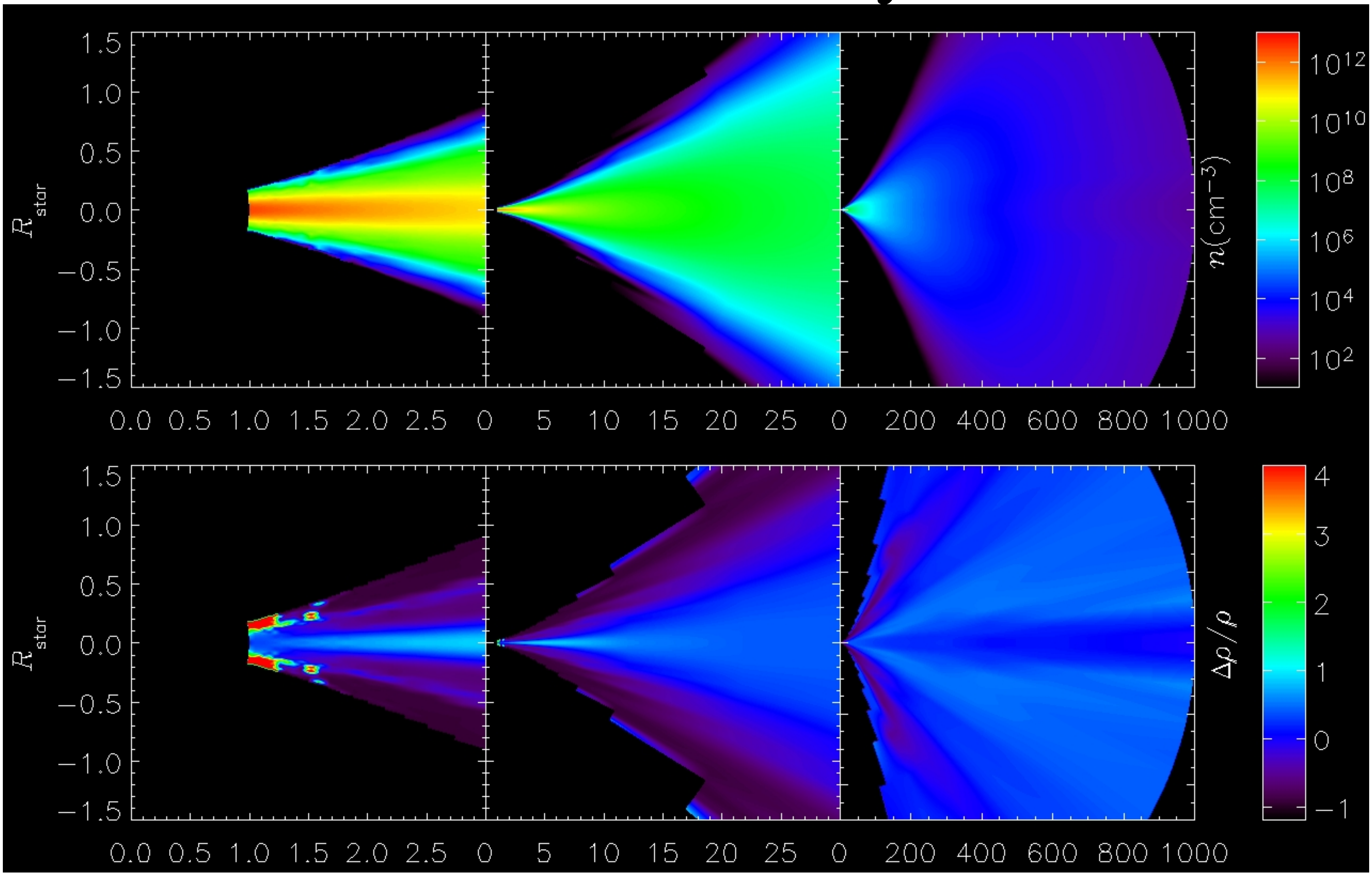
Be Star Disk Temperature



Carciofi & Bjorkman 2004

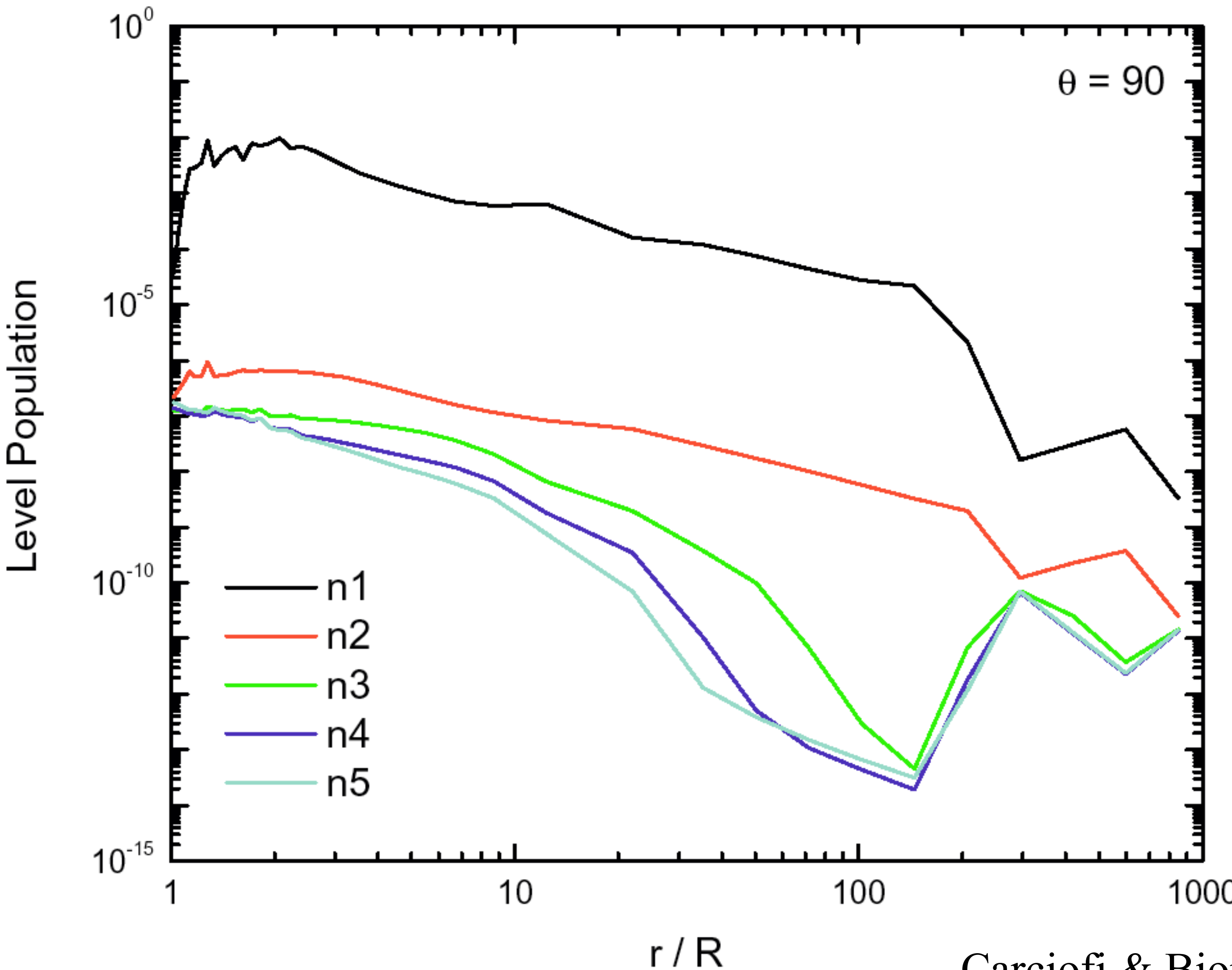


Disk Density



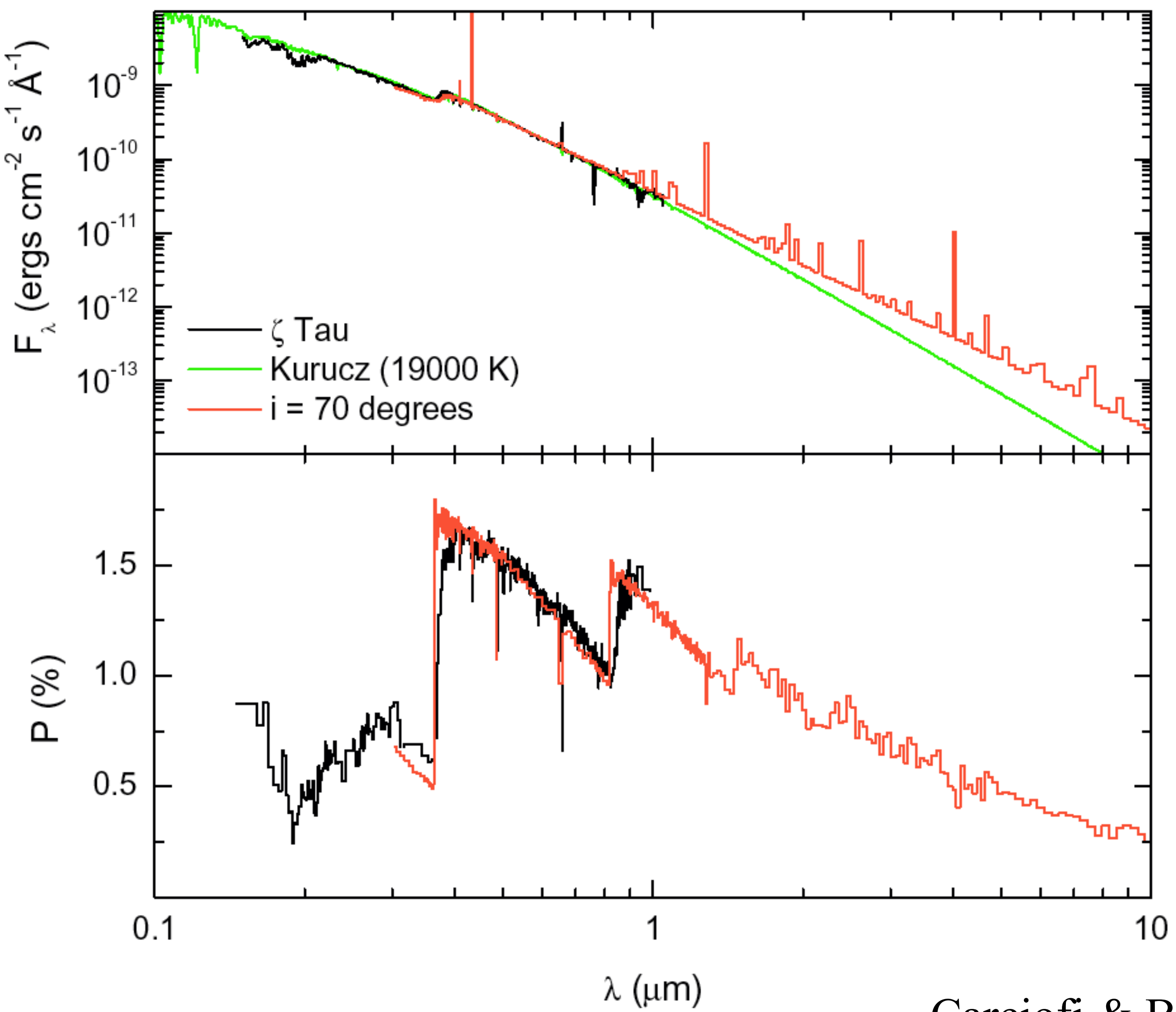
Carciofi & Bjorkman 2004

NLTE Level Populations



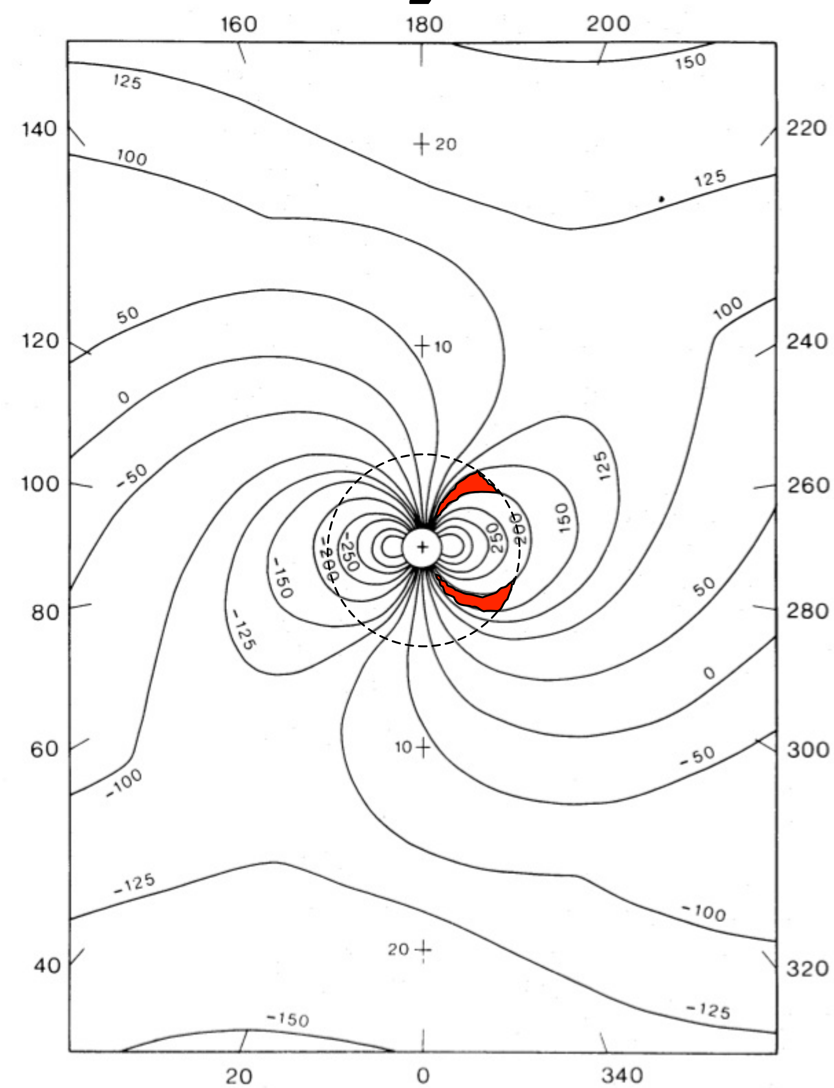
Carciofi & Bjorkman 2004

SED and Polarization



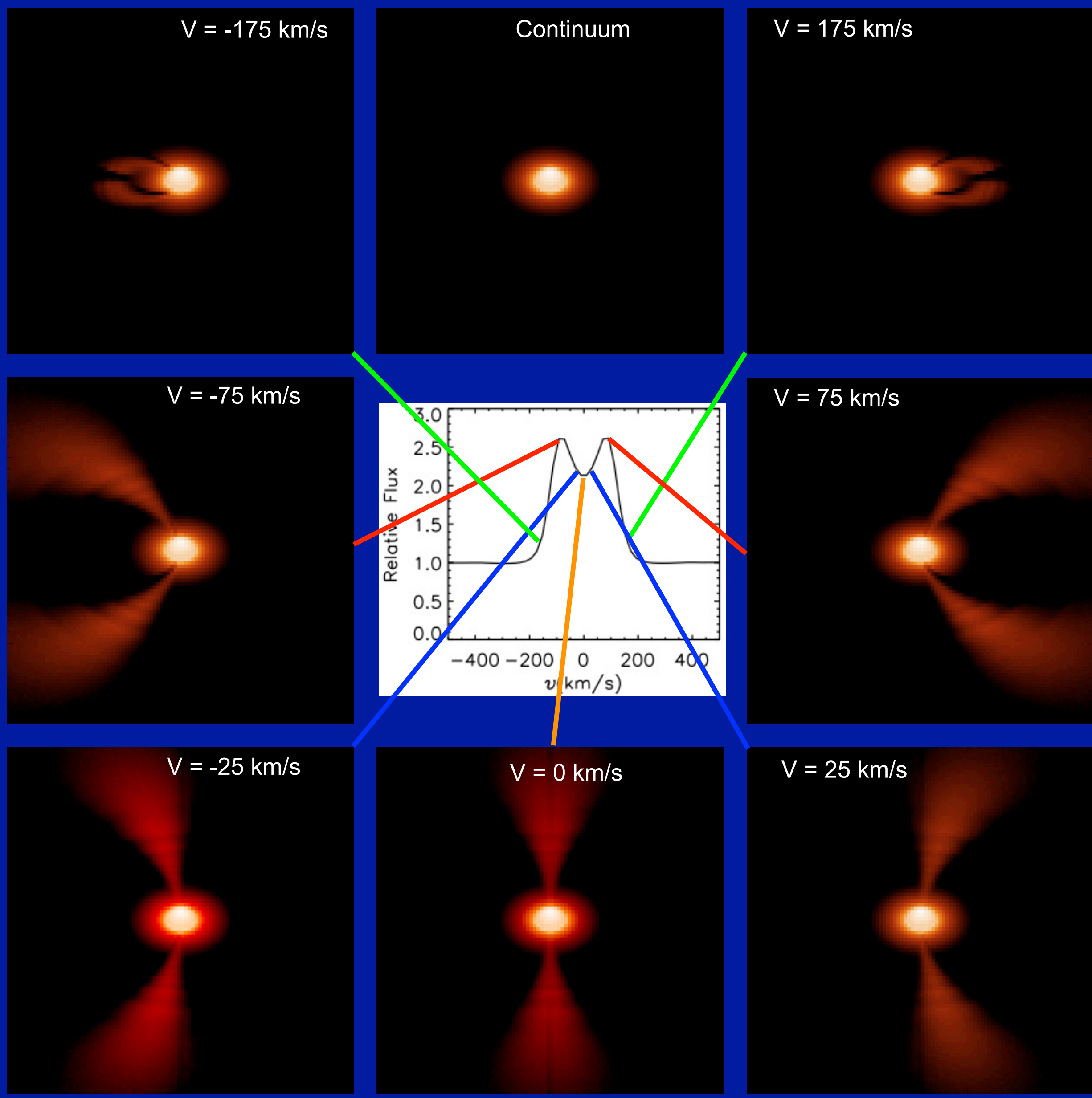
Carciofi & Bjorkman 2004

Emission Line Formation: Iso-Velocity Contours

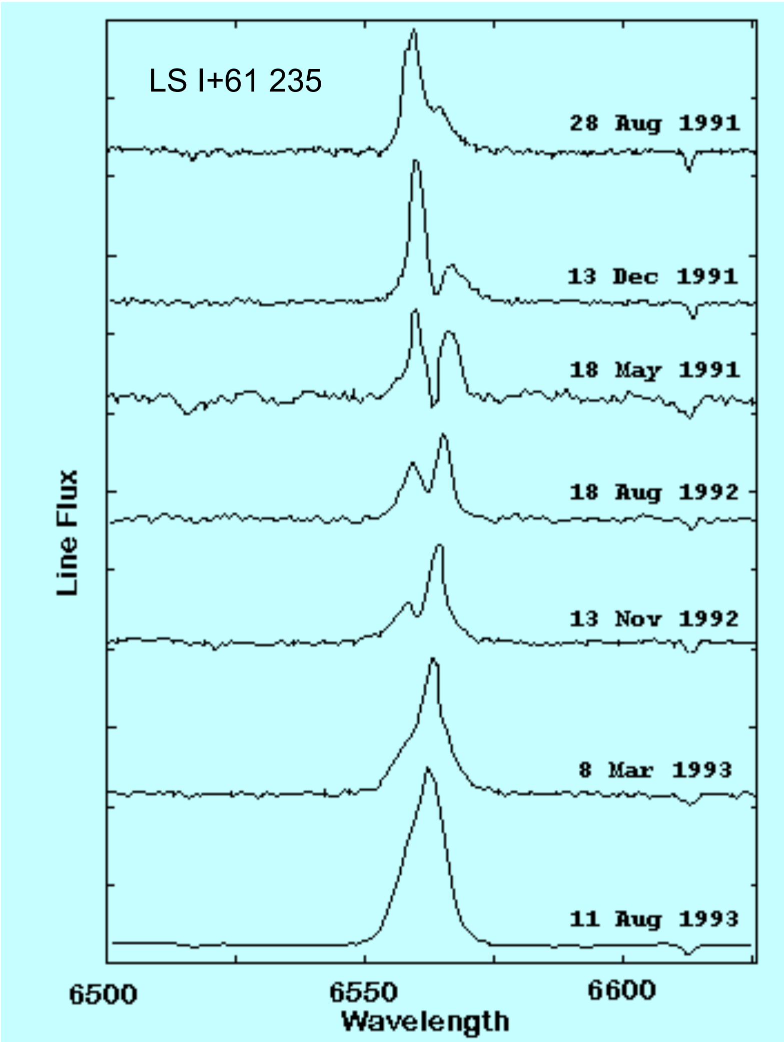


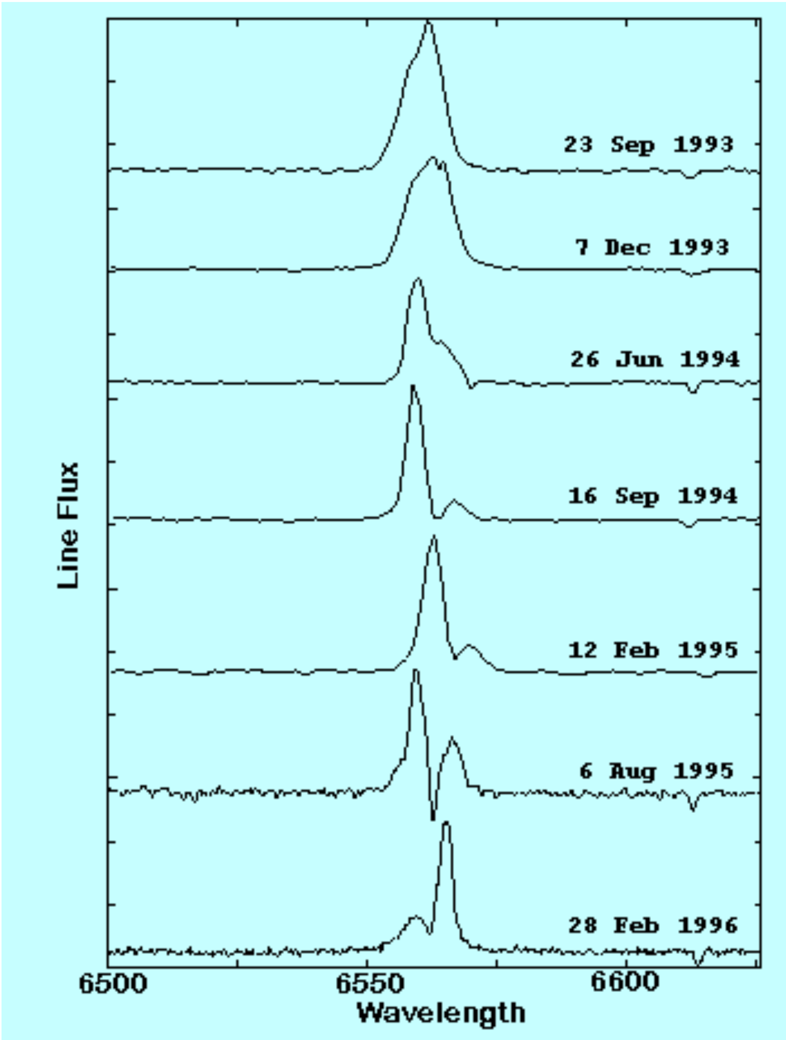
Poeckert & Marlborough 78

Emission Line Formation



V/R Variations

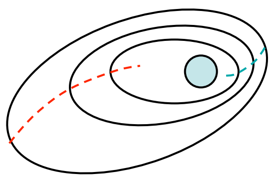




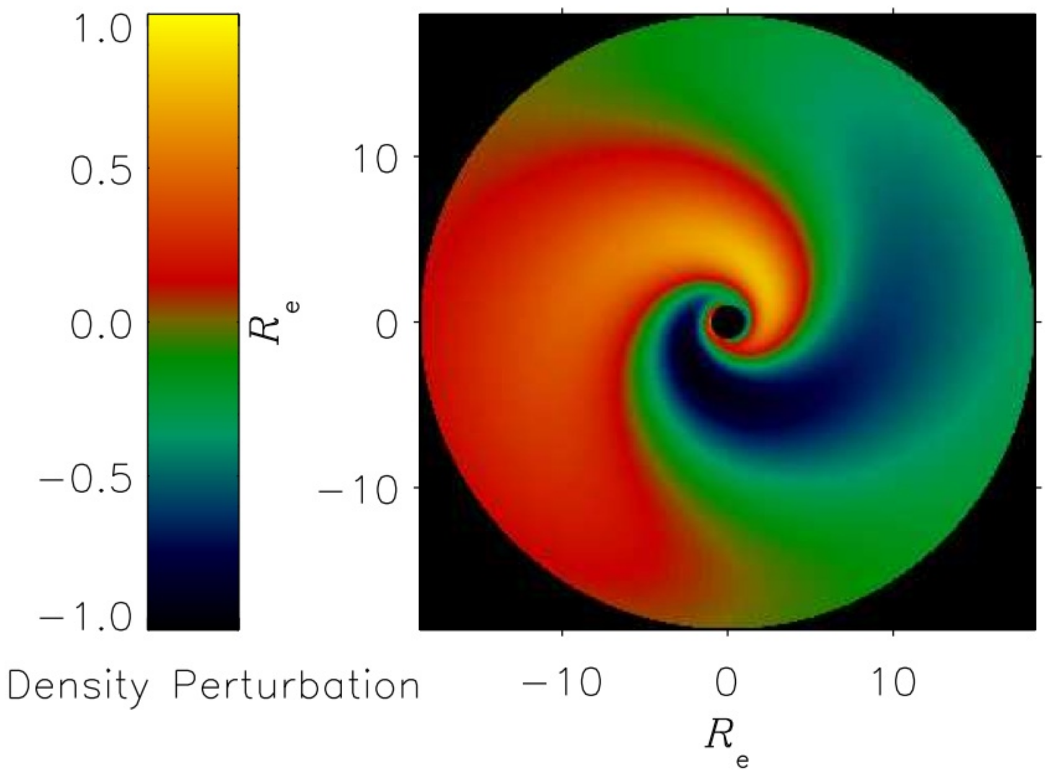
Coe

Global Disk Oscillations

- Kato 1983, Okazaki 1991
- Elliptical Orbits in Disk
  - Periastron
    speed high => low density
  - Apastron
    speed low => high density

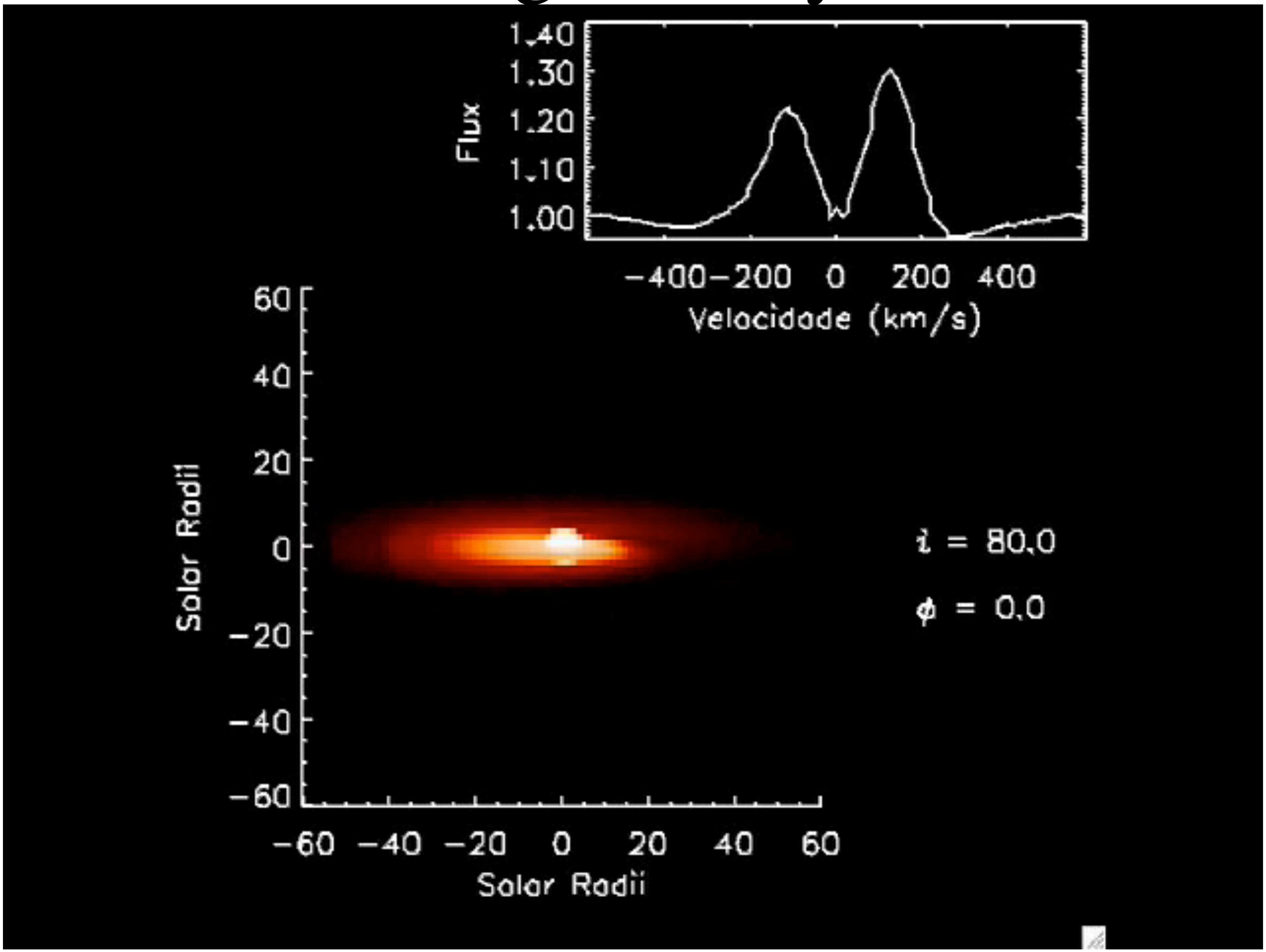


- Orbits can precess (Papaloizou, Savonije & Henrichs 1992)
  - Density wave rotates
    at precession period



Okazaki

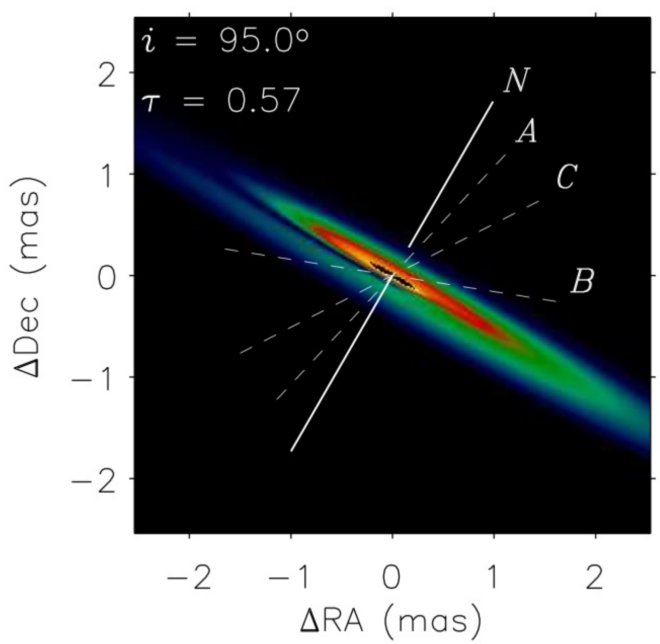
Precessing Density Wave

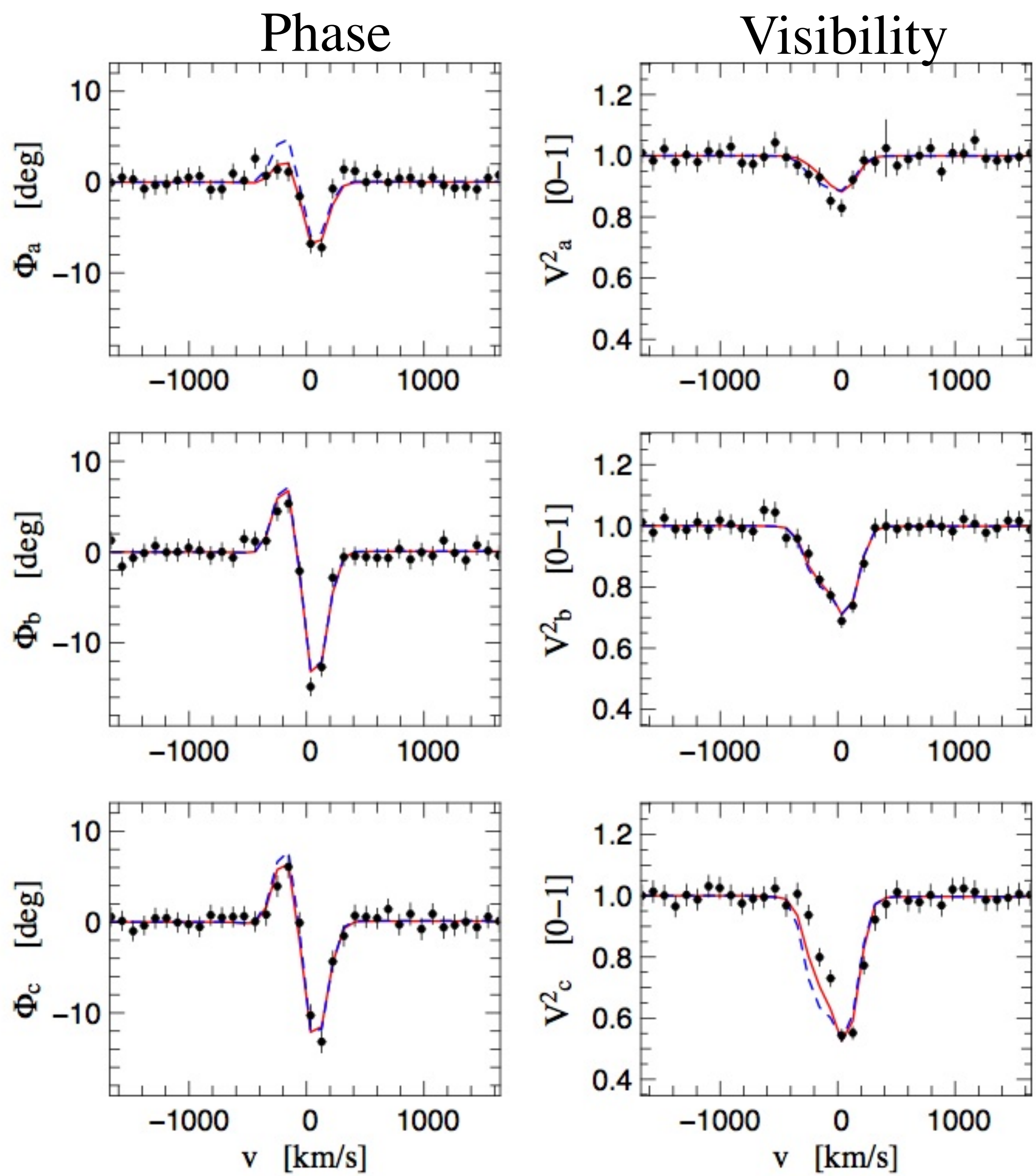


Zeta Tau: Precessing Density Wave

Amber/VLTI Brg Observations

(Stefl et al. 2009)





Carciofi et al. (2009)

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