MCRT for astrophysical explosions:
light curves and spectra

Stuart Sim (QUB)

SAMCSS 2015

Overview:

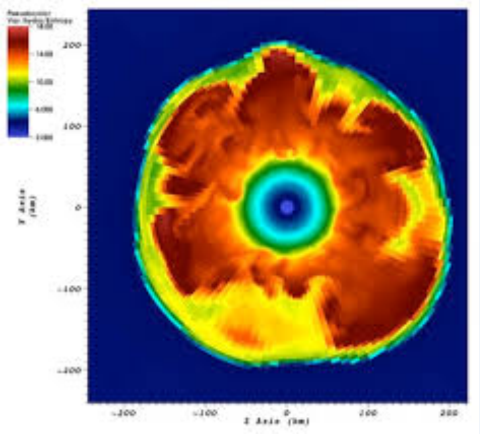
- Brief introduction to supernovae
  - Observations to study
- Considerations for modelling radiation transport for supernovae
  - Suitability of MCRT techniques
- Light curve calculations
  - Simple 1D example
- Spectrum calculation
  - TARDIS code

Variety of bright astronomical transients that *may* be associated with explosions…

Supernovae and related transients

Variety of bright astronomical transients that may be associated with explosions:

- Stellar core collapse
- Thermonuclear runaway (accretion, mergers, collisions)
- Tidal disruption events
- …



Radiation could be powered in many ways:

- Heat deposited in explosion
- Radioactivity (decay of products of explosive nucleosynthesis)
- Central engine (accreting compact object? magnetar?)
- Interaction with environment (rad. hydro)
- …

Supernovae and related transients

Variety of bright astronomical transients that may be associated with explosions:

- Stellar core collapse
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- Tidal disruption events
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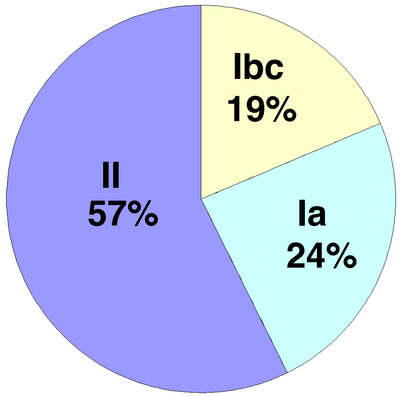
Type Ia supernovae (Li et al. 2011)



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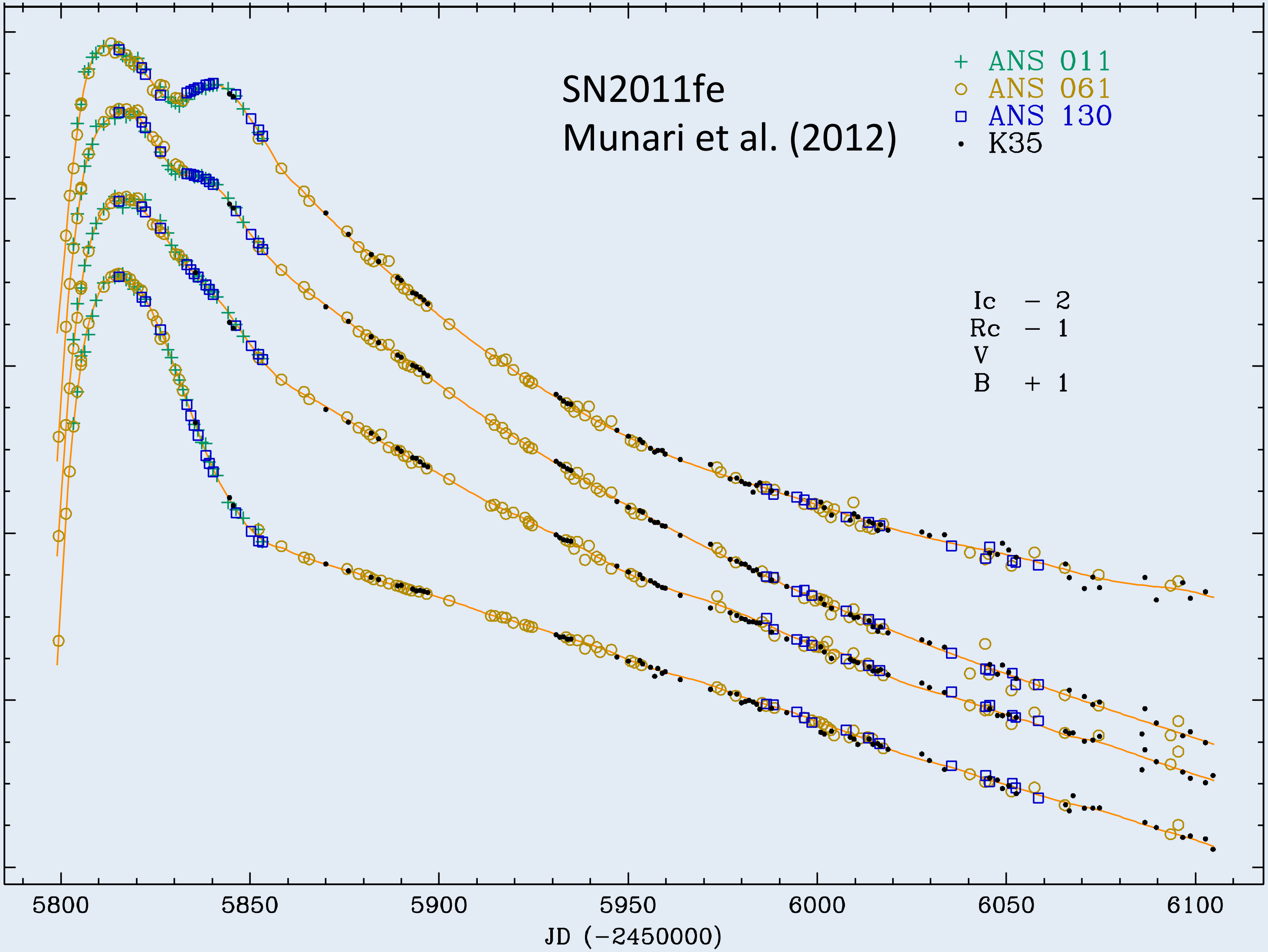
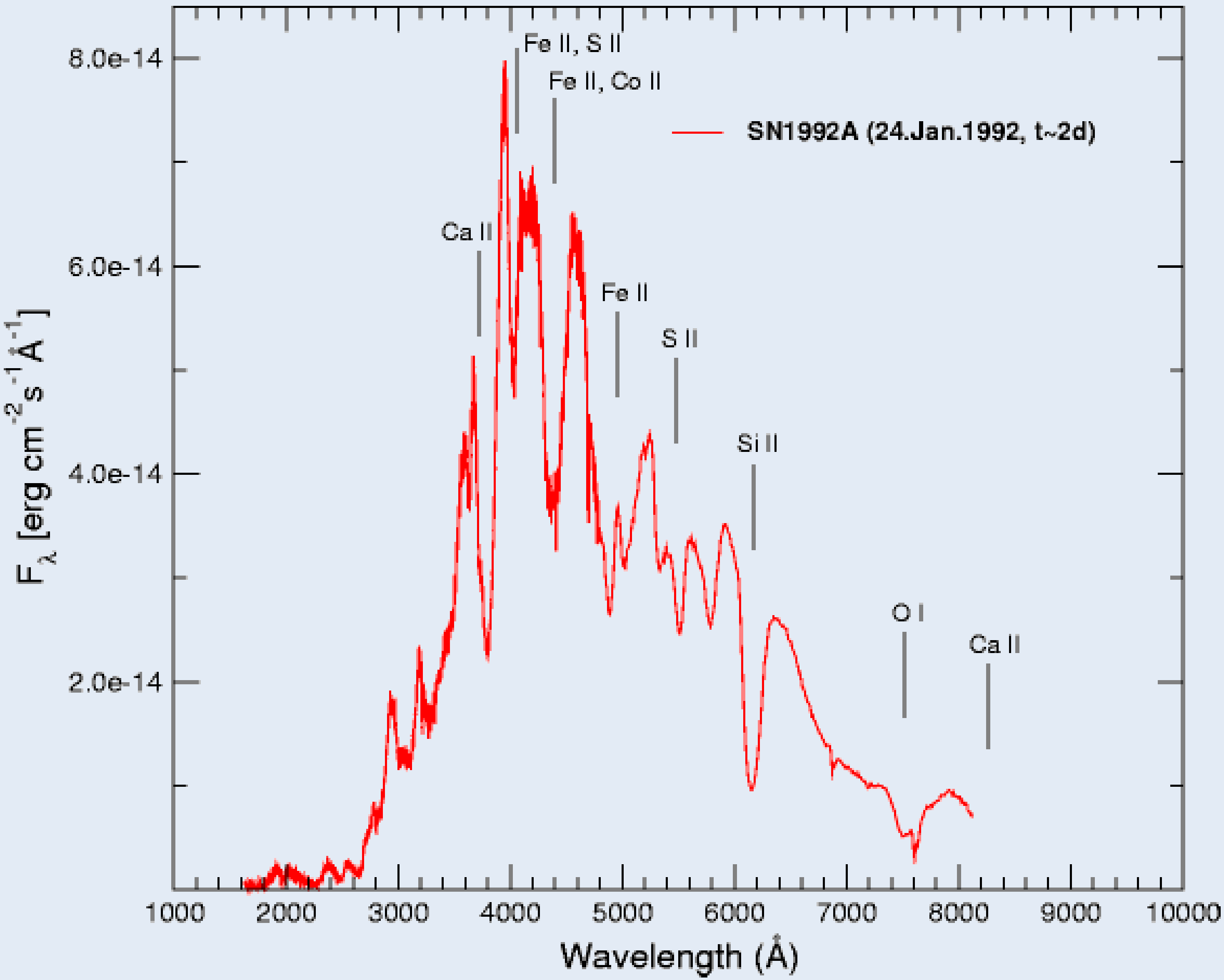
Observing Supernovae

SN1994D in NGC 4526
NASA/HST

Supernova (Type Ia) lightcurves



Supernova (Type Ia) spectrum



Supernova modelling

Considerations:

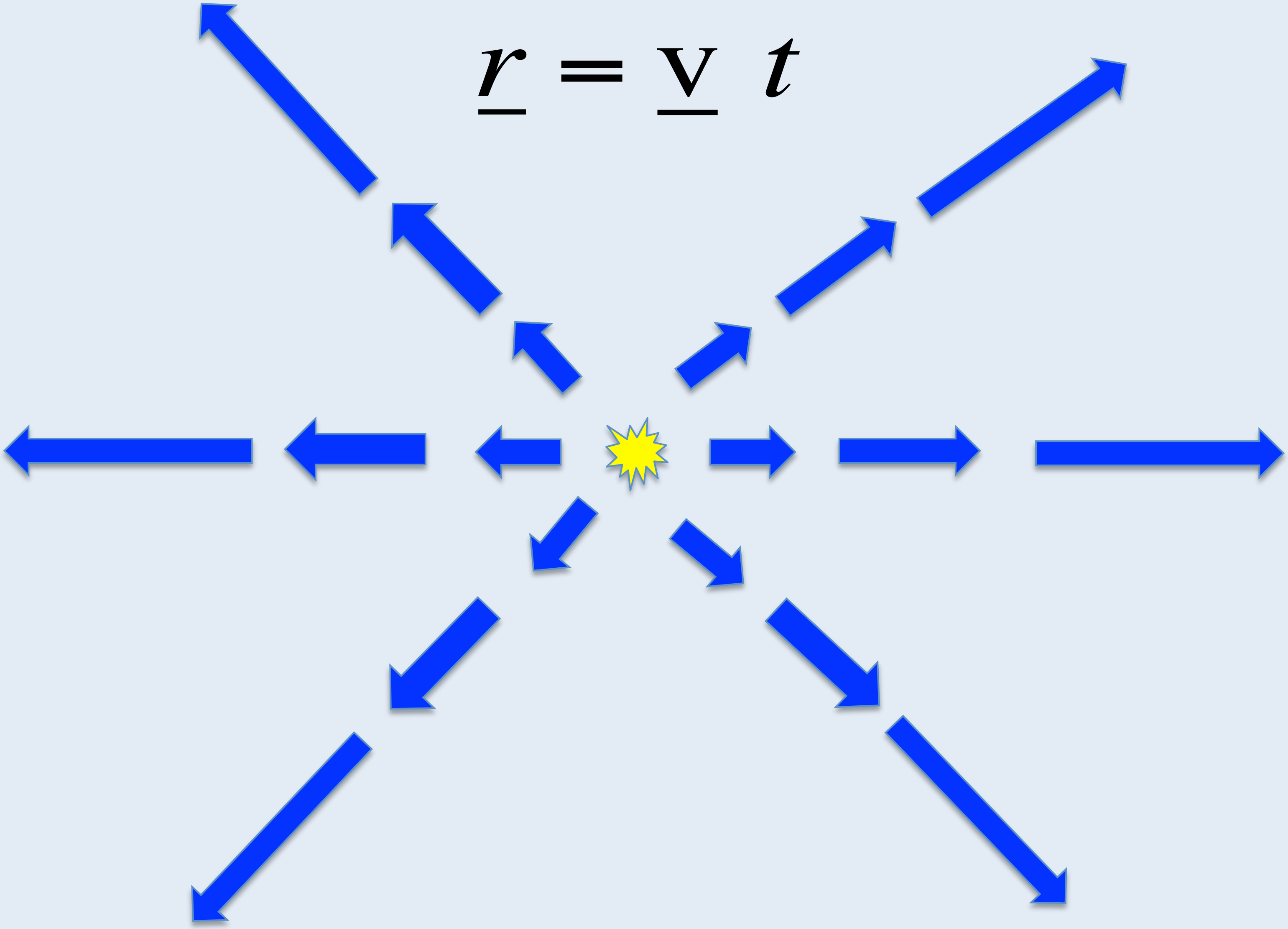
- Time-dependence
- Large velocities
- Homologous flow (i.e. not dynamics)

Monte Carlo RT:

- ➢ Easy to track time on trajectories
- ➢ Mixed frame approach makes easy; line blending
- ➢ Pure radiation (radiative equilibrium)

Homologous flow

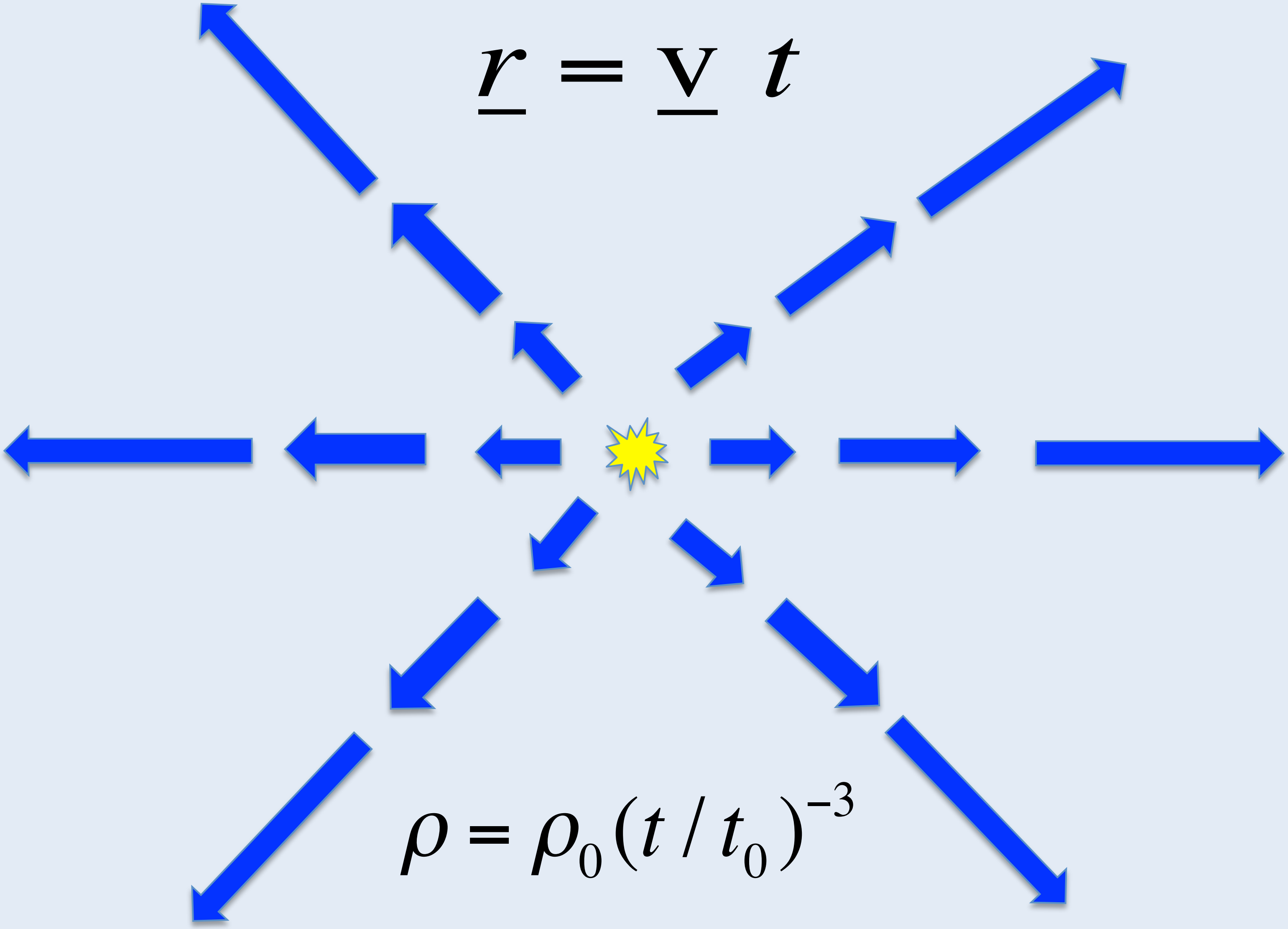
normally established within seconds to hours



$$\underline{r} = \underline{\mathrm{v}}\; t$$

Homologous flow

normally established within seconds to hours



$$\underline{r} = \underline{\mathrm{v}}\ t$$

$$\rho = \rho_0 (t / t_0)^{-3}$$

Supernova modelling

Considerations:

- Time-dependence
- Large velocities
- Homologous flow (i.e. not dynamics)
- Multi-dimensional

Monte Carlo RT:

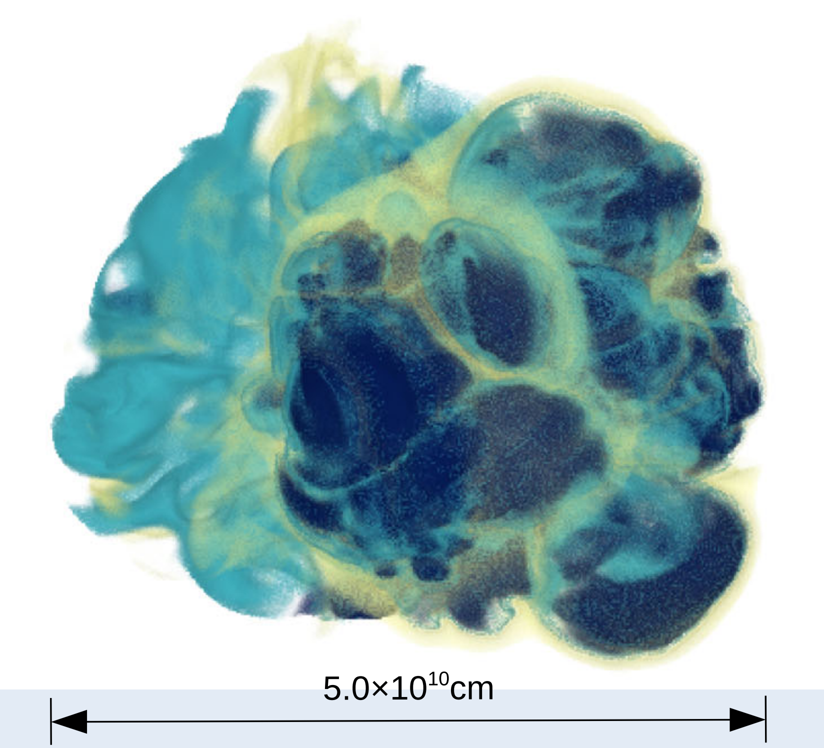
- ➢ Easy to track time on trajectories
- ➢ Mixed frame approach makes easy; line blending
- ➢ Pure radiation (radiative equilibrium)
- ➢ 3D grid easy

Multi-dimensional

departures from sphericity large in some models

Flash Centre simulation (early phase)

Kromer et al. 2015
(composition rendering at 10s)



Supernova modelling

Considerations:

- Time-dependence
- Large velocities
- Homologous flow (i.e. not dynamics)
- Multi-dimensional
- NLTE
- Metal-rich

Monte Carlo RT:

- ➢ Easy to track time on trajectories
- ➢ Mixed frame approach makes easy; line blending
- ➢ Pure radiation (radiative equilibrium)
- ➢ 3D grid easy
- ➢ Most serious challenge (estimators for rates)

Light curve modelling

Lucy (2005), SEDONA/ARTIS codes

Procedure:

- Define homologous model, including energy source (e.g. based on hydro simulation), grid zones and time steps

Light curve modelling

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- Inject energy packets to make a ensemble that represents the injection process.

Light curve modelling

Lucy (2005), SEDONA/ARTIS codes

Procedure:

- Define homologous model, including energy source (e.g. based on hydro simulation), grid zones and time steps
- Inject energy packets to make a ensemble that represents the injection process.
- Use radiative equilibrium MC algorithm to simulate the propagation of an ensemble of energy packets ("photon bundles")
    - Indivisible energy packet algorithm (e.g. Abbott & Lucy 1985 …Lucy 2005)

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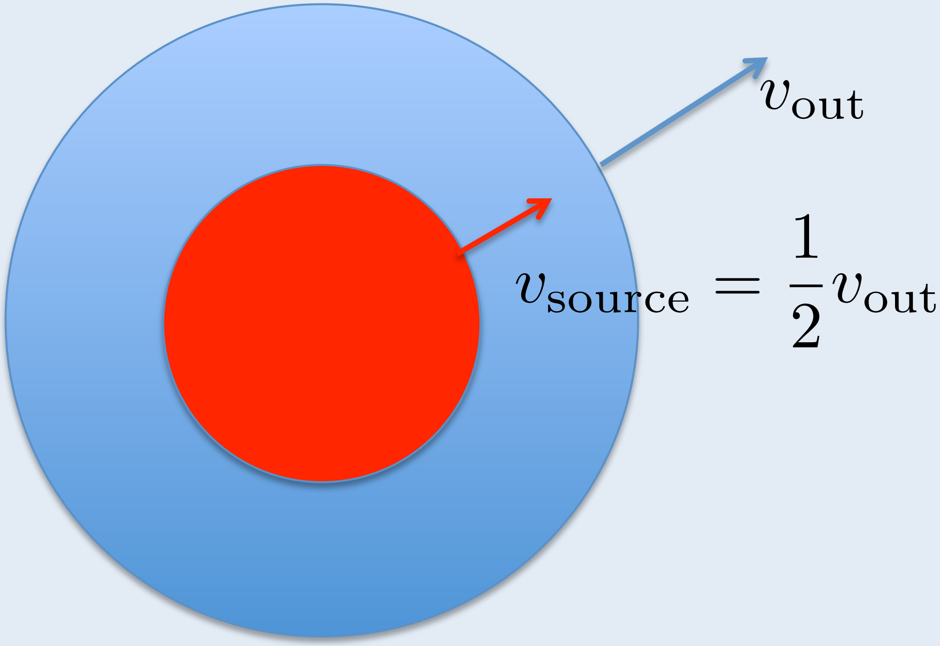
Light curve modelling

Simple example; suppose:

- **Spherical, uniform density ejecta with constant opacity coefficient**
- **Energy source from (single isotope) radioactive decay (inner half of ejecta)**
- **Compute light curve for 2 – 100 days after explosion**



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Light curve modelling

Injecting packets - use random numbers to select

- time of injection
- Location of injection
- Initial direction

Light curve modelling

Time of injection:

- **E.g. for radioactive decays**

$$\frac{\mathrm{d}N(t)}{\mathrm{dt}} = -\frac{N_0}{t_0} \exp\left(-t/t_0\right)$$

- **Which is easy to sample a time using a random number**

$$t = -t_0 \ln z$$

- **Convenient to restrict decay times to duration of simulation**

Light curve modelling

Location of injection:

- **For homologous expansion the outer boundary is located at:**

$$R_{\rm out} = v_{\rm out} t$$

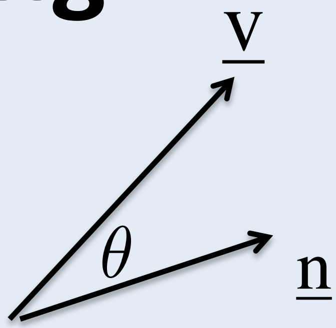
- **Similarly,**

$$R_{\rm source} = v_{\rm source} t$$

- **So select random starting radius for packet inside source region by sampling volume at the time of injection:**

$$r = z^{1/3} \, v_{\rm source} \, t$$

Light curve modelling



Direction of injection:

- For our simple problem, sufficient to specify direction cosine:

$$\mu = \cos\theta$$

- Assuming that emission is isotropic in **fluid frame**, can randomly select

$$\mu_{\mathrm{ff}} = -1 + 2z$$

- Then in **observer frame**, angle aberration gives

$$\mu_{\mathrm{obs}} = \frac{\mu_{\mathrm{ff}} + \beta}{1 + \mu_{\mathrm{ff}}\beta}$$

$$\beta = \frac{v}{c} = \frac{r}{ct}$$

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Light curve modelling

Propagation:

- Once a packet in injected into the simulation box its subsequent propagation can be followed using a standard MCRT "random walk" algorithm:

1. **Compute three time intervals**:
   - To reach grid zone boundary
   - To reach end of current time step
   - To reach (randomly selected) interaction point

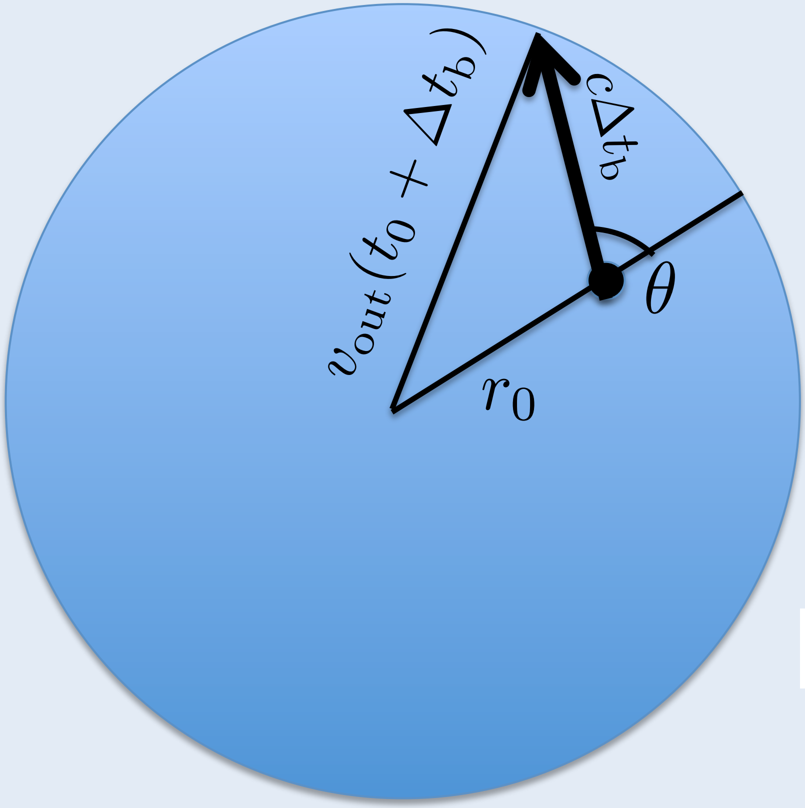
2. **Select shortest of these three distances and accordingly**:
   - Move into next grid zone
   - Move on to next time step [or store and come back]
   - Simulate the interaction

3. **Rinse and repeat…until packet leaves simulation** (or reach final time step)

Light curve modelling

Time intervals for our simple example:

- **To reach (the only!) grid zone boundary, starting from** r_0 , t_0



$$v_{\rm out}^2(t_0 + \Delta t_{\rm b})^2 = r_0^2 + c^2\Delta t_{\rm b}^2 + 2r_0 c\mu\Delta t_{\rm b}$$

…solve for the time interval needed to reach boundary, $\Delta t_{\rm b}$

Light curve modelling

Time intervals for our simple example:

- **To reach end of time step is even simpler:**

$$\Delta t_{\rm t} = T_{\rm next} - t_0$$

Light curve modelling

Time intervals until interaction:

- **Use random number to draw optical depth to interaction point:**

$$\tau = -\ln z$$

- **Convert this to a photon travel time to interaction point:**

$$\Delta t_{\mathrm{i}} = \frac{\tau}{c \rho \kappa}$$

Light curve modelling

Time intervals until interaction:

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In observer frame
(Doppler factor)

Light curve modelling

Decision:

- **Knowing:**

$$\Delta t_{\rm b} \;,\; \Delta t_{\rm t} \;,\; \Delta t_{\rm i}$$

…select the shortest:

$\Delta t_{\rm b}$ → Move packet to boundary and cross into new zone (or flag it as having escaped)

$\Delta t_{\rm t}$ → Move packet to position for end of time step and then proceed to next time step: fluid properties (e.g. density $\rho \propto t^{-3}$) will change;

Light curve modelling

Δt_{i} → Move packet to interaction point and undergo fluid interaction

Complex part, where codes differ in detail, but same general idea:

Radiative equilibrium → **"effective" scattering (Lucy 1999, 2005); conserve radiative energy in fluid frame**

$$\epsilon_{ff}^{\mathrm{after}} = \epsilon_{ff}^{\mathrm{before}}$$

E.g. for grey opacity with isotropic emission, just need Lorentz transformations of packet energy:

$$\epsilon_{ff} = \gamma \epsilon_{obs} (1 - \mu_{obs} \beta)$$

So that in interaction:

$$\epsilon^{\mathrm{after}} = \epsilon^{\mathrm{before}} \frac{(1 - \mu_{obs}^{\mathrm{before}} \beta)}{(1 - \mu_{obs}^{\mathrm{after}} \beta)}$$

Aberration of angles

Δt_{i} → Move packet to interaction point and undergo fluid interaction

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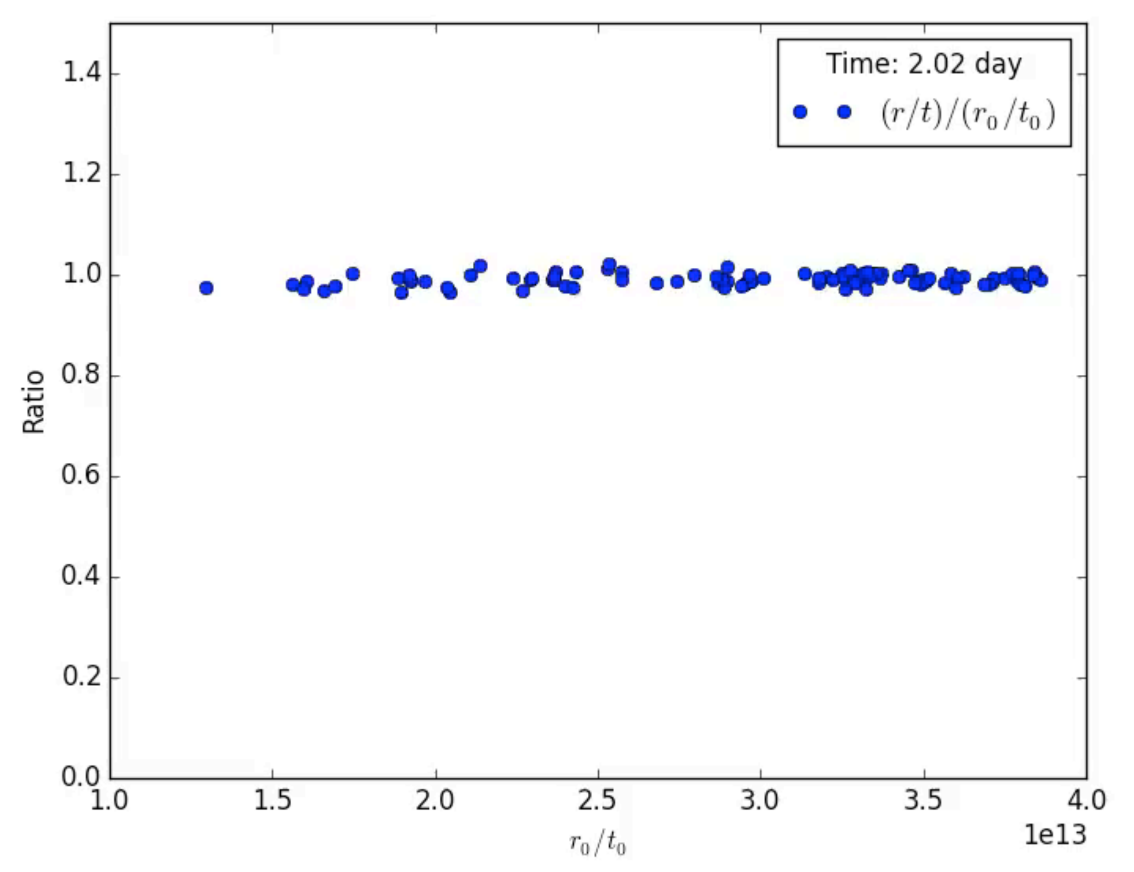
Isotropic
(in fluid frame)

Light curve modelling

Note: correct application of frame transformations is needed for:

- Advection in optically-thick regime
- Work done on ejecta by radiation



Light curve modelling

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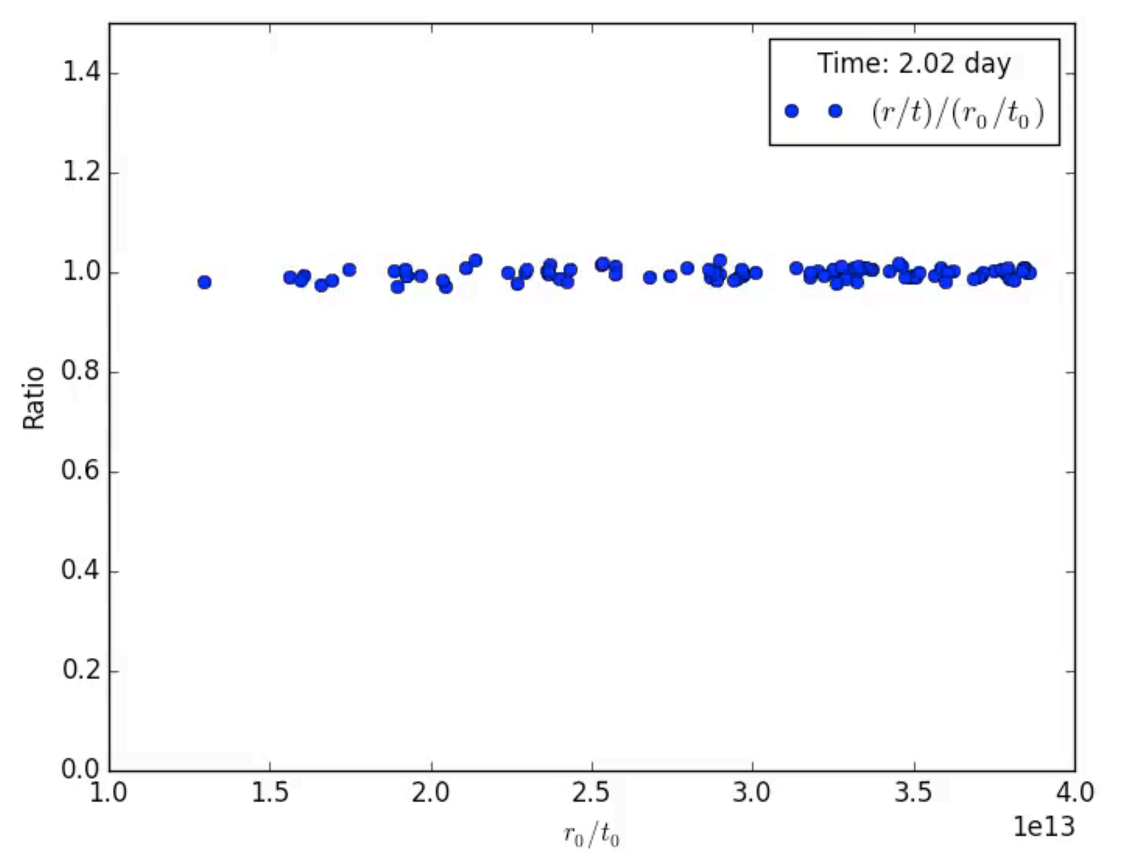

Light curve modelling

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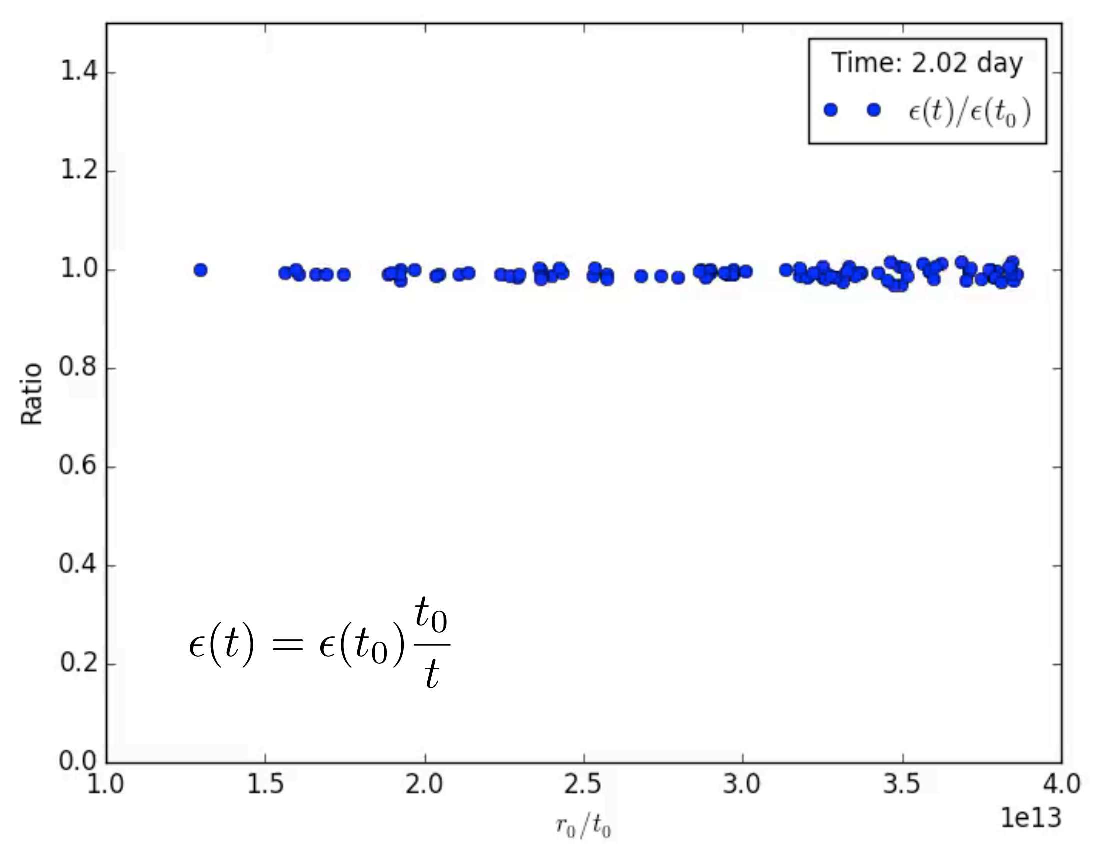

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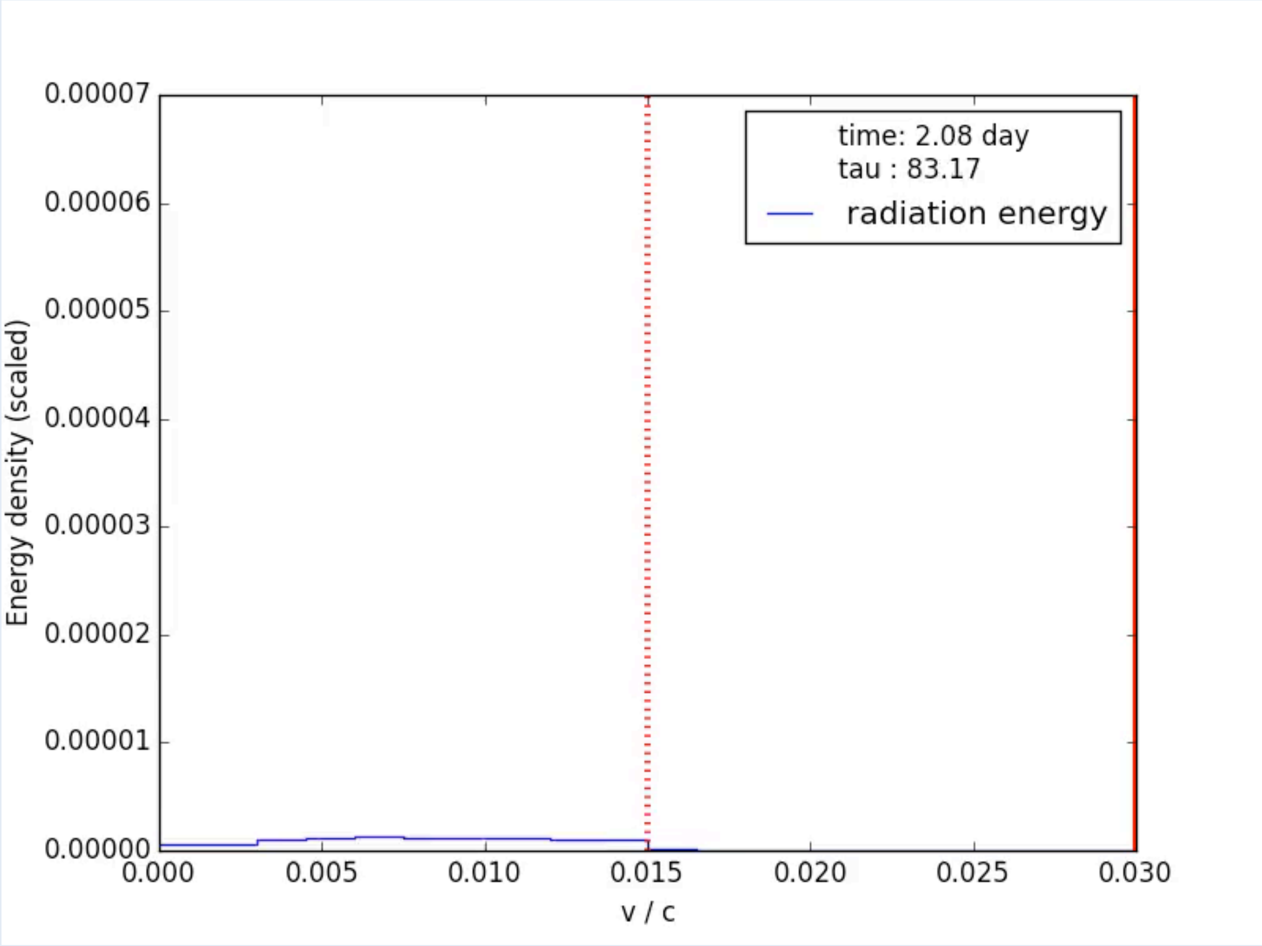
    - Simulate the interaction

3. **Rinse and repeat…until packet leaves simulation** (or reach final time step)
    - Record properties of escaping packets: bin to make light curve
    - Can also use more sophisticated estimators to make light curve…

Light curve modelling

Example toy code: result of code



Light curve modelling

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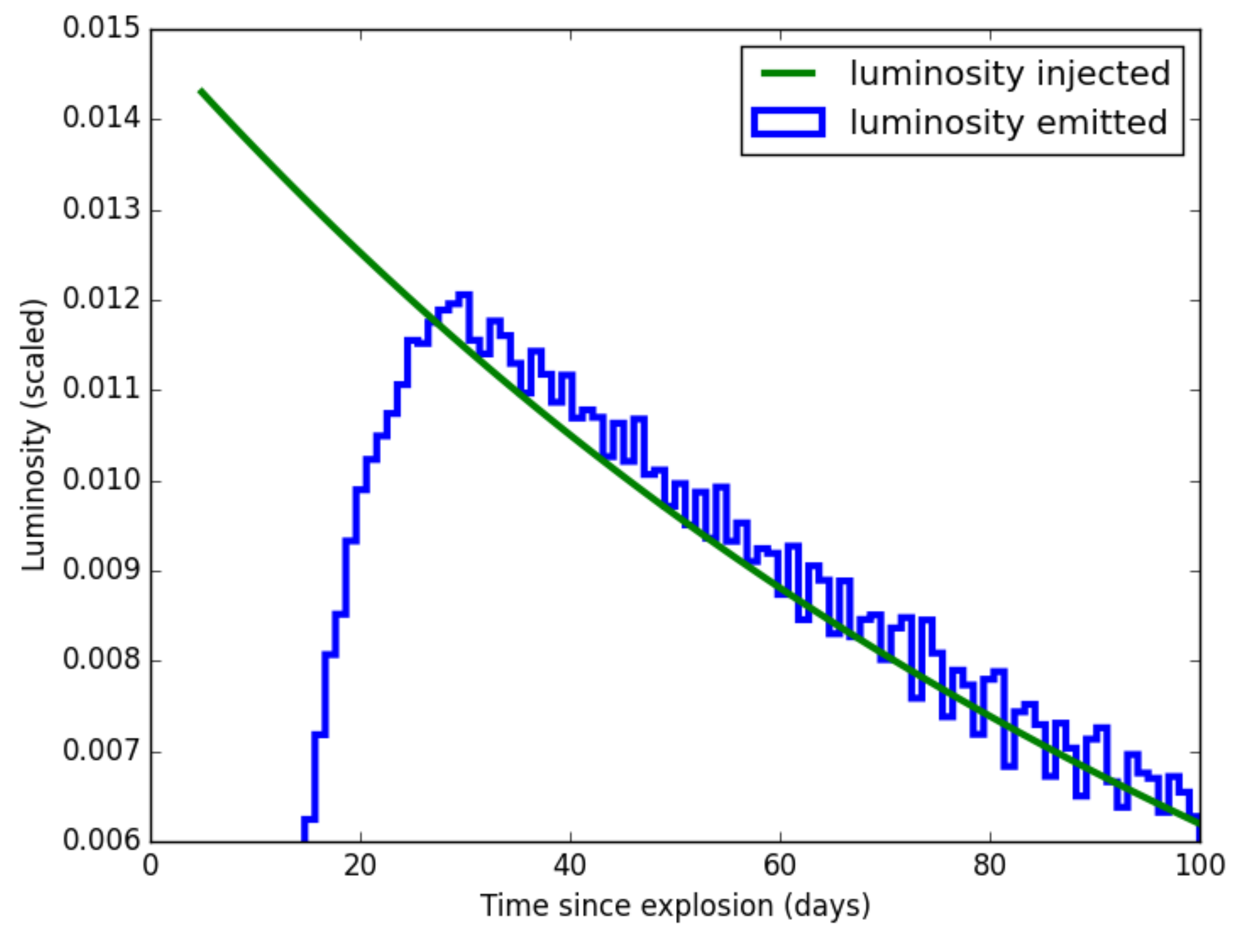


Light curve modelling

Simplifications made today:

- (Minor) Energy source: can be easily generalized for other internal energy sources
- (Minor) Uniform density / spherical: just need to identify boundaries
- (Major) Realistic calculations need non-grey opacities: commonly will involve use of Sobolev line-opacities and continuum; ideally non-LTE
- (Major) Need detailed interaction microphysics: frequency redistribution, thermalisation etc.

Spectral modelling

Context:

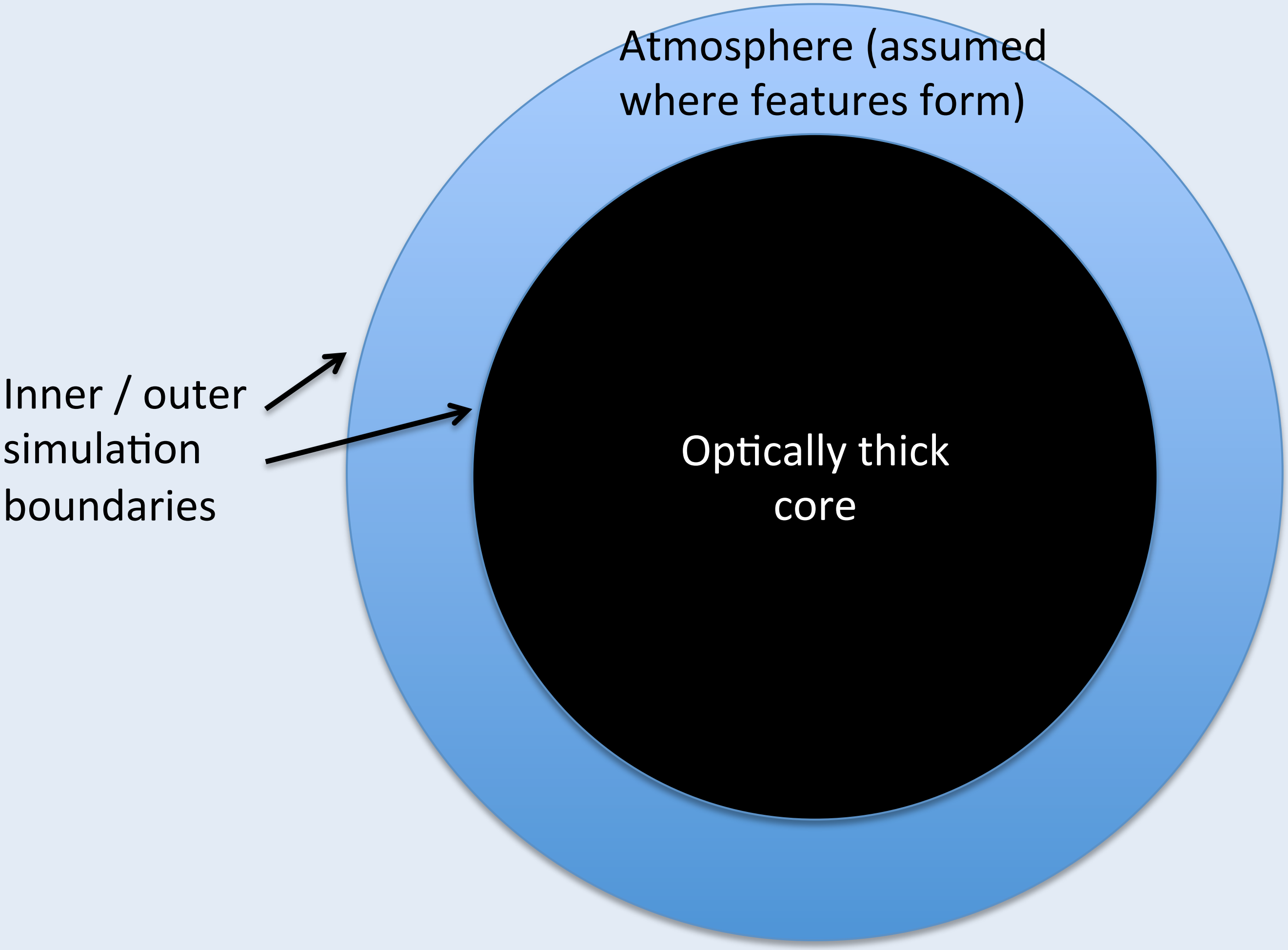
- Wider variety of approaches
- State-of-the-art codes will couple to a light curve approach; simpler impose luminosity

Still need to consider:

- Homologous flow (i.e. not dynamics)
- Realistic opacity
- NLTE

Snap-shot spectral modelling

Mazzali & Lucy (1993), various later (e.g. TARDIS)



Atmosphere (assumed where features form)

Inner / outer simulation boundaries

Optically thick core

Snap-shot spectral modelling

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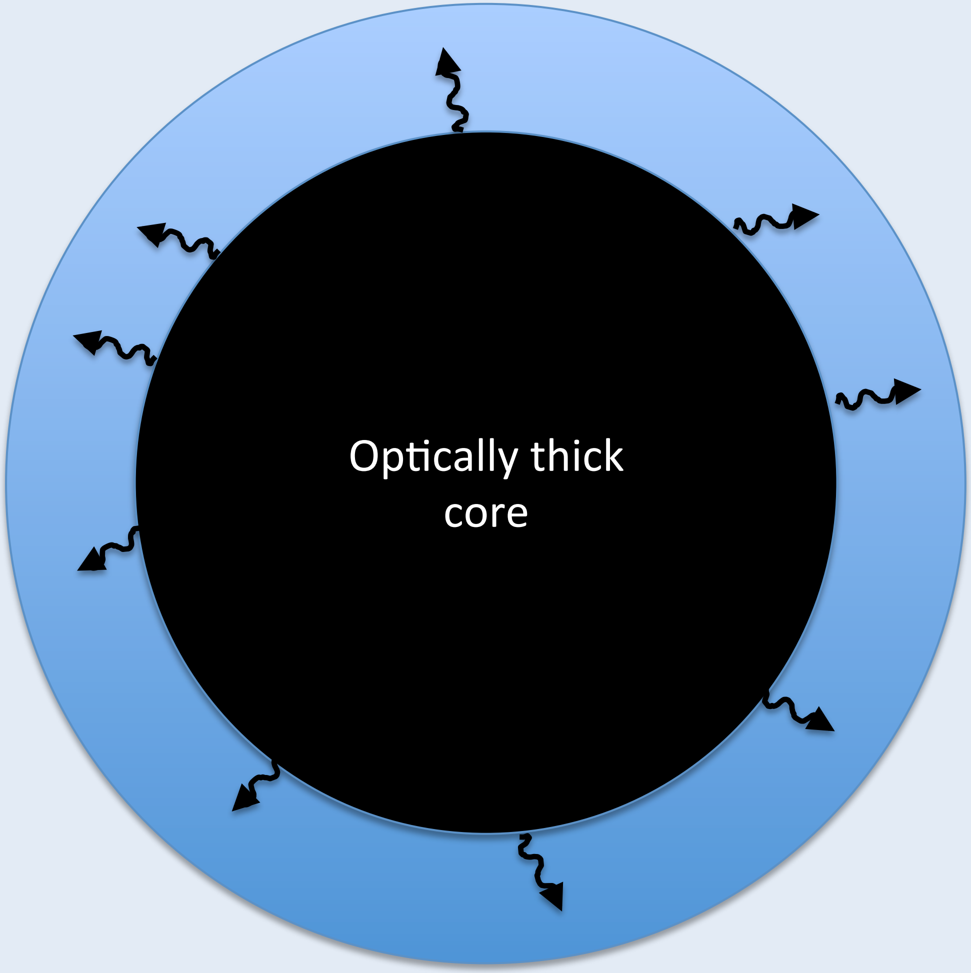

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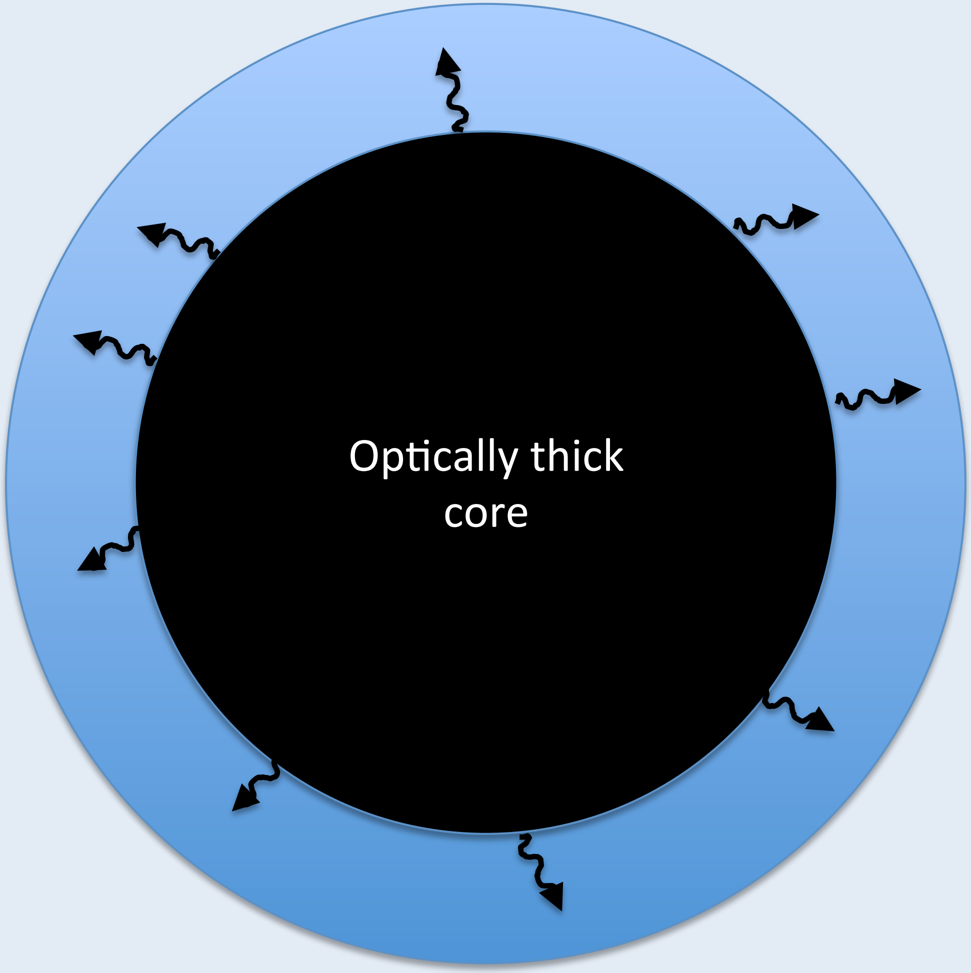

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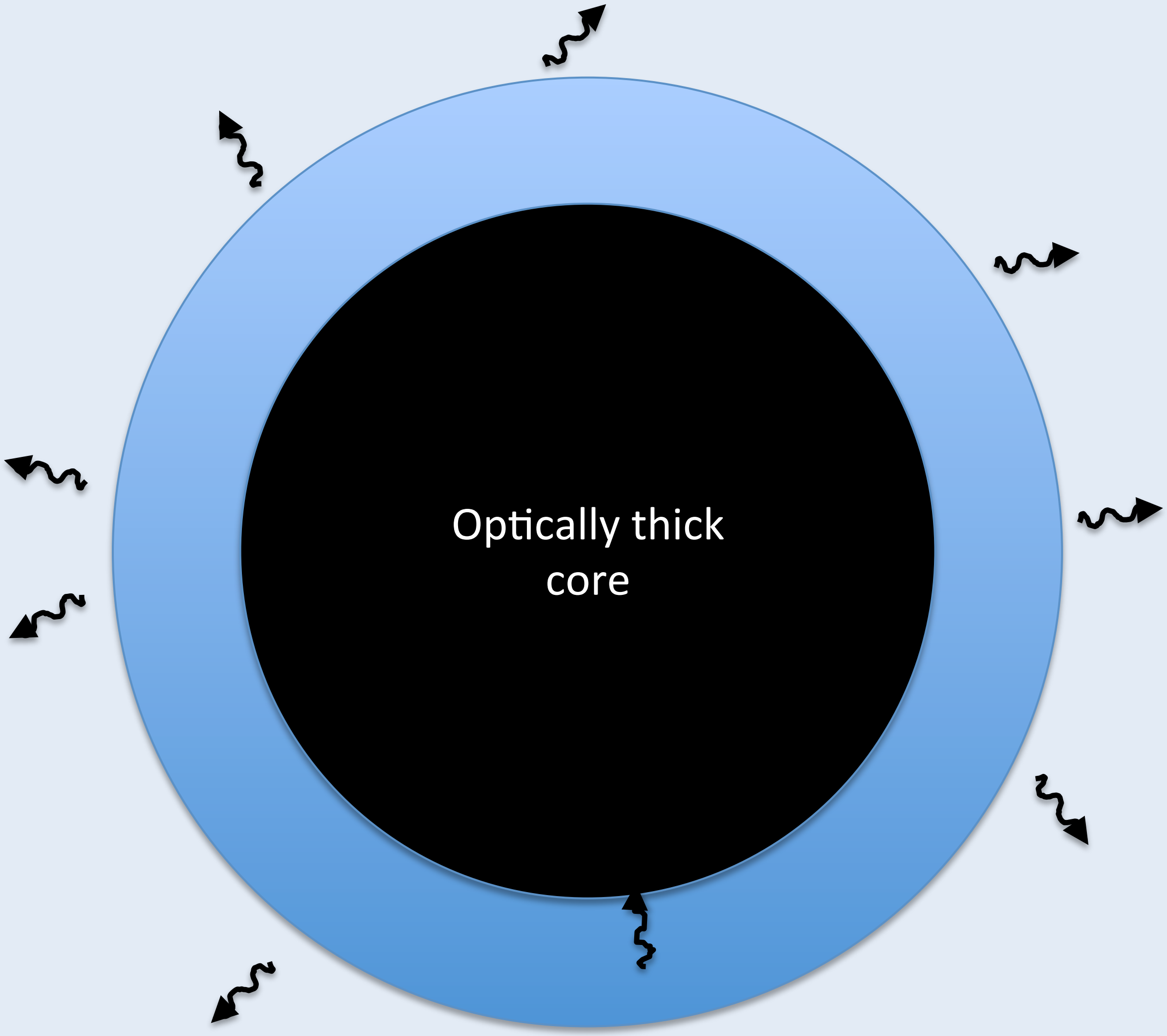

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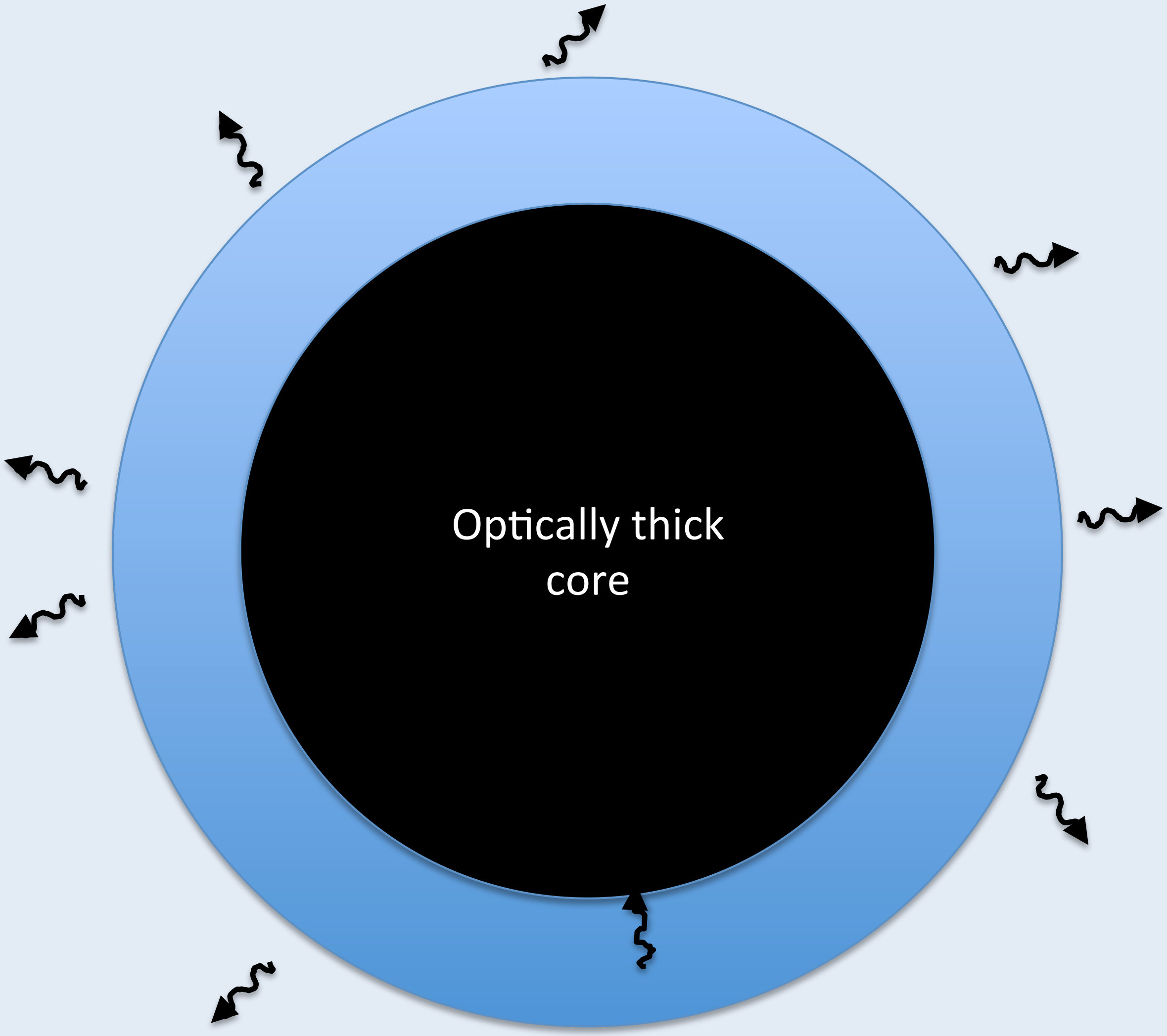


Need to iterate on atmosphere properties (ionization, temperatures)

Snap-shot spectral modelling

Mazzali & Lucy (1993), various later (e.g. TARDIS)

Propagation (as before, but no time steps):

1. Compute three time intervals:
   - To reach grid zone boundary
   - To reach (randomly selected) interaction point
2. Select shortest of these three distances and accordingly:
   - Move into next grid zone
   - Simulate the interaction
3. Rinse and repeat…until packet leaves simulation

Non-grey opacity

Non-grey opacity

Finding interaction points for MCRT method:

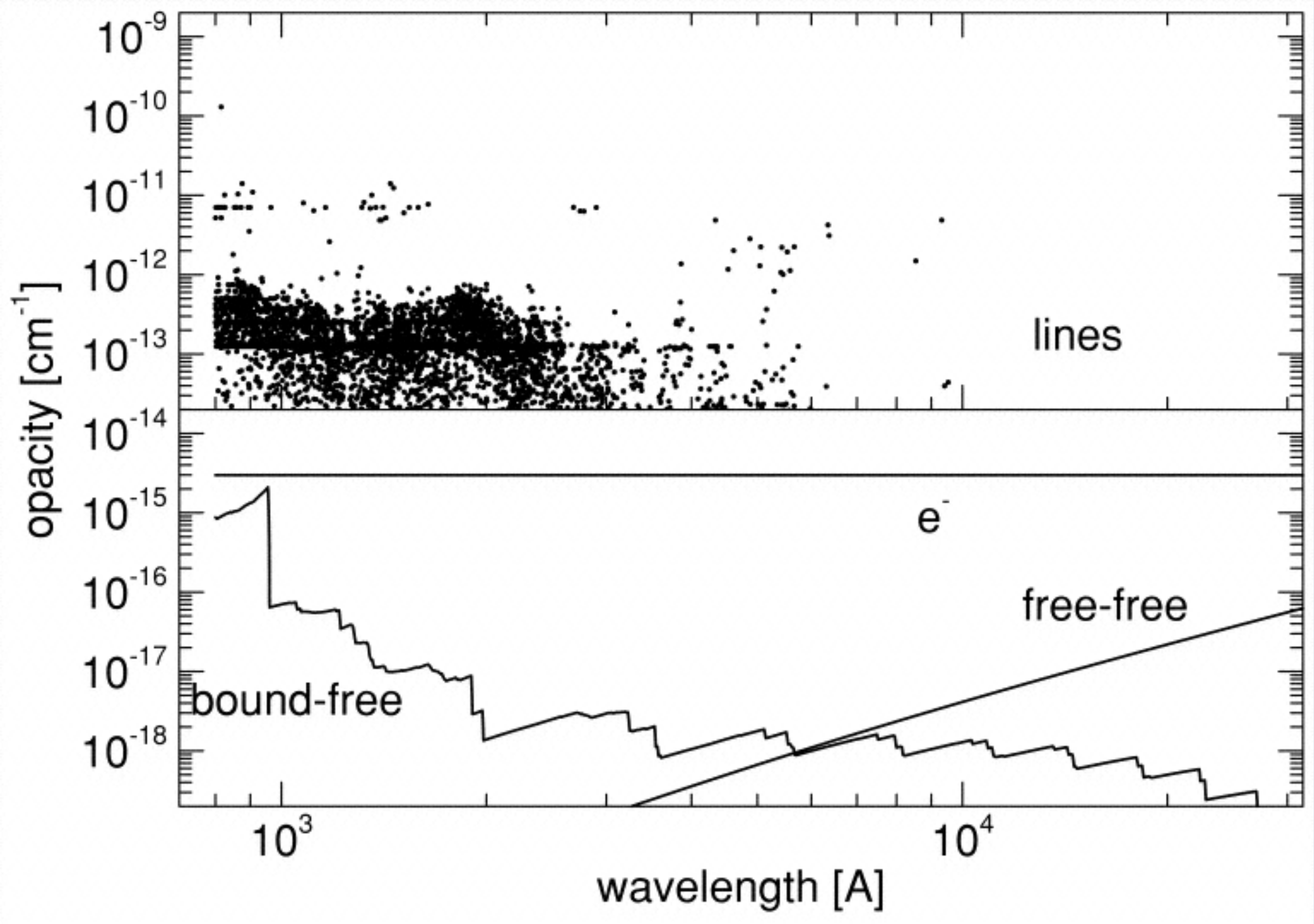
- Need to be able to compute rate at which optical depth is accumulated by propagating MC packet

$$\mathrm{d}\tau = \kappa\rho\mathrm{d}s$$

- Was trivial for grey opacity assumed in light curve above…

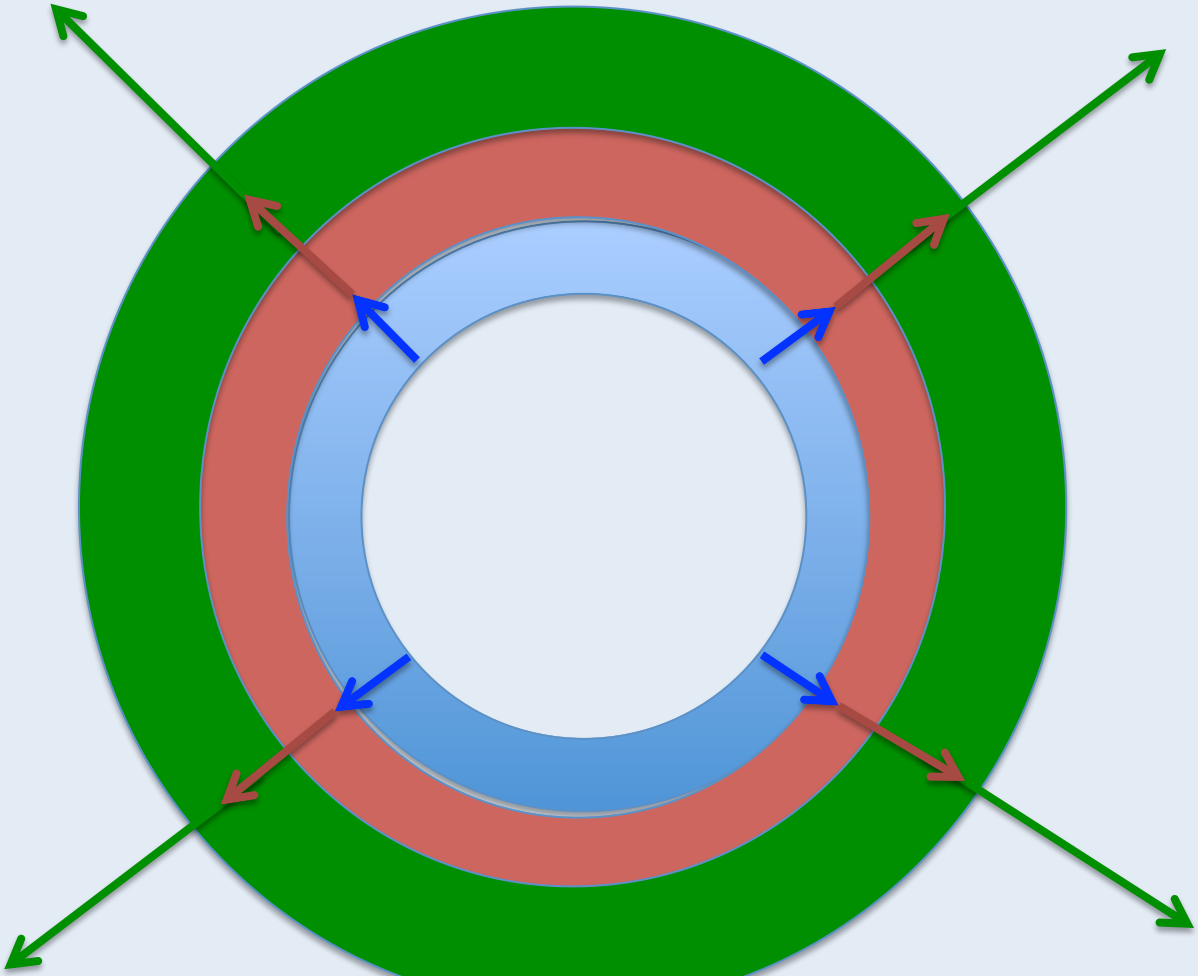
- …but still easily accomplished thanks to conditions / approximations appropriate to homologous supernovae ejecta

Non-grey opacity



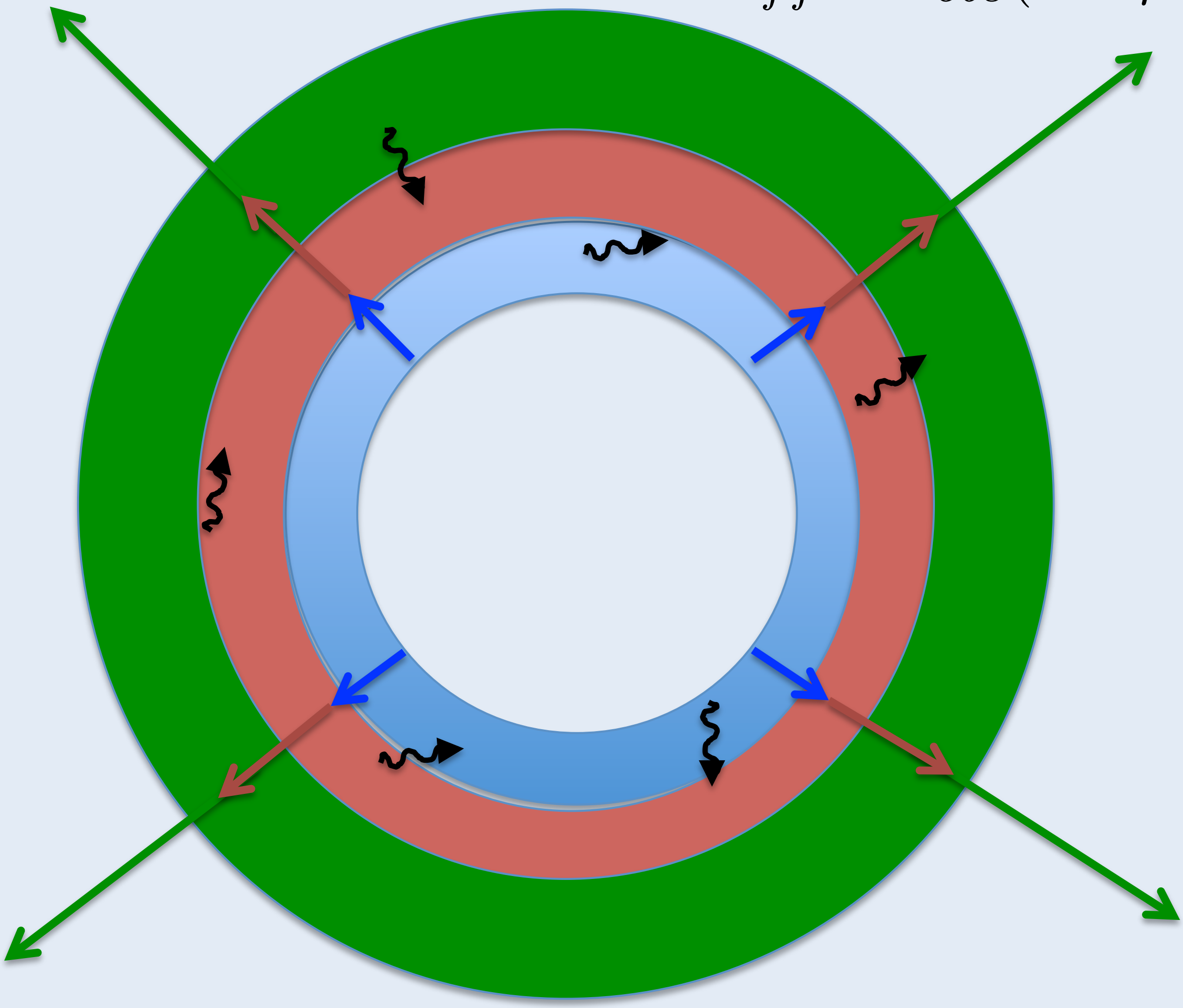
Pinto & Eastman 2000

Photons in expanding media



Photons in expanding media

$$\nu_{ff} = \nu_{obs}(1 - \mu_{obs}\beta)$$



Photons in expanding media

$$\nu_{ff} = \nu_{obs}(1 - \mu_{obs}\beta)$$

Fluid-frame frequency evolves in remarkably simple way for radial velocities:

$$\frac{\mathrm{d}\nu_{ff}}{\mathrm{d}s} = -\frac{\nu_{obs}}{c}\frac{\mathrm{d}}{\mathrm{d}s}\left(\mu_{obs}v(r)\right)$$

Use cosine rule to differentiate direction cosine along a path:

$$\frac{\mathrm{d}\nu_{ff}}{\mathrm{d}s} = -\frac{\nu_{obs}}{c}\left(\frac{v(r)}{r}(1 - \mu_{obs}^2) + \mu_{obs}^2\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)$$

Provided speed increases outward, always negative! For homologous flow, even simpler: independent of position and direction:

$$\frac{\mathrm{d}\nu_{ff}}{\mathrm{d}s} = -\frac{\nu_{obs}}{ct}$$

Photons in expanding media

Implication:

- Fluid-frame frequency of a propagating packet evolves at a constant rate to the red
- Will successively Doppler-shift in and out of resonance with line transitions in (inverse) frequency order

Sobolev approximation

Sobolev approximation:

- Simplification for dealing with line opacity in high velocity-gradient flows

Sobolev approximation

Sketch derivation (Sobolev 1957; see e.g. Lamers & Cassinelli 1999):
The absorption coefficient for a bound-bound line can be written:

$$\kappa\rho = \frac{B_{lu} h\nu_0}{4\pi} n_l \left(1 - \frac{n_u}{n_l}\frac{g_l}{g_u}\right) \phi(\Delta\nu_{ff})$$

The optical depth traversed by a photon along a short path is:

$$\mathrm{d}\tau = \kappa\rho \mathrm{d}s$$

So integrating along a path element

$$\tau_{ul} = \int_{s_0}^{s_1} \frac{B_{lu} h\nu_0}{4\pi} n_l \left(1 - \frac{n_u}{n_l}\frac{g_l}{g_u}\right) \phi(\Delta\nu_{ff})\, \mathrm{d}s$$

Use fact that frequency and path-length are related in flow

$$\tau_{ul} = \int_{\nu_{ff}(s=s_0)}^{\nu_{ff}(s=s_1)} \frac{B_{lu} h\nu_0}{4\pi} n_l \left(1 - \frac{n_u}{n_l}\frac{g_l}{g_u}\right) \frac{\mathrm{d}s}{\mathrm{d}\nu_{ff}} \phi(\Delta\nu_{ff})\, \mathrm{d}\nu_{ff}$$

If resonance region is small

$$\tau_{ul} = \frac{B_{lu} h\nu_0}{4\pi} n_l \left(1 - \frac{n_u}{n_l}\frac{g_l}{g_u}\right) \frac{\mathrm{d}s}{\mathrm{d}\nu_{ff}} \int_{\nu_{ff}(s=s_0)}^{\nu_{ff}(s=s_1)} \delta(\Delta\nu_{ff})\, \mathrm{d}\nu_{ff}$$

Sobolev approximation

Resulting optical depth for homologous flow:

$$\tau_{ul}^{S} = \frac{B_{lu} h c t \nu_0}{4\pi\nu_{obs}} n_l \left(1 - \frac{n_u}{n_l}\frac{g_l}{g_u}\right)$$

Leads to dramatic simplification:

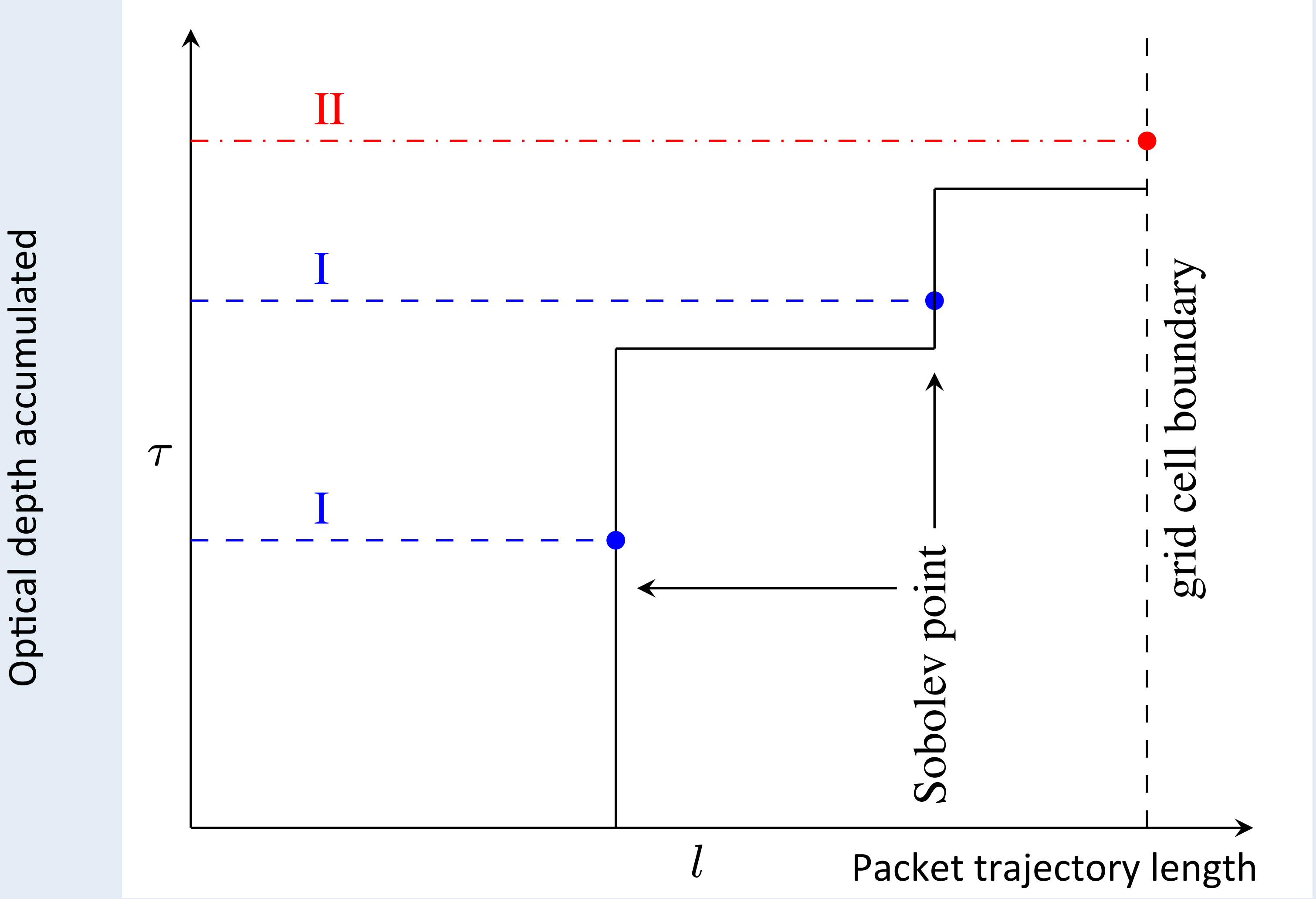
- Easy to compute total optical depth accumulated by packet that passes through resonance with a line
- In Sobolev limit, all this opacity encountered in spatially small region (approximated as Sobolev point in codes)
- Can be fairly-easily generalized e.g. to include continuum opacity
- …together with continuous red-shifting lends itself to simple algorithm with frequency-ordered line list

Issues:

- Overlapping lines
- Still need good level populations!

Sobolev approximation

Algorithm for finding interaction point (only lines):

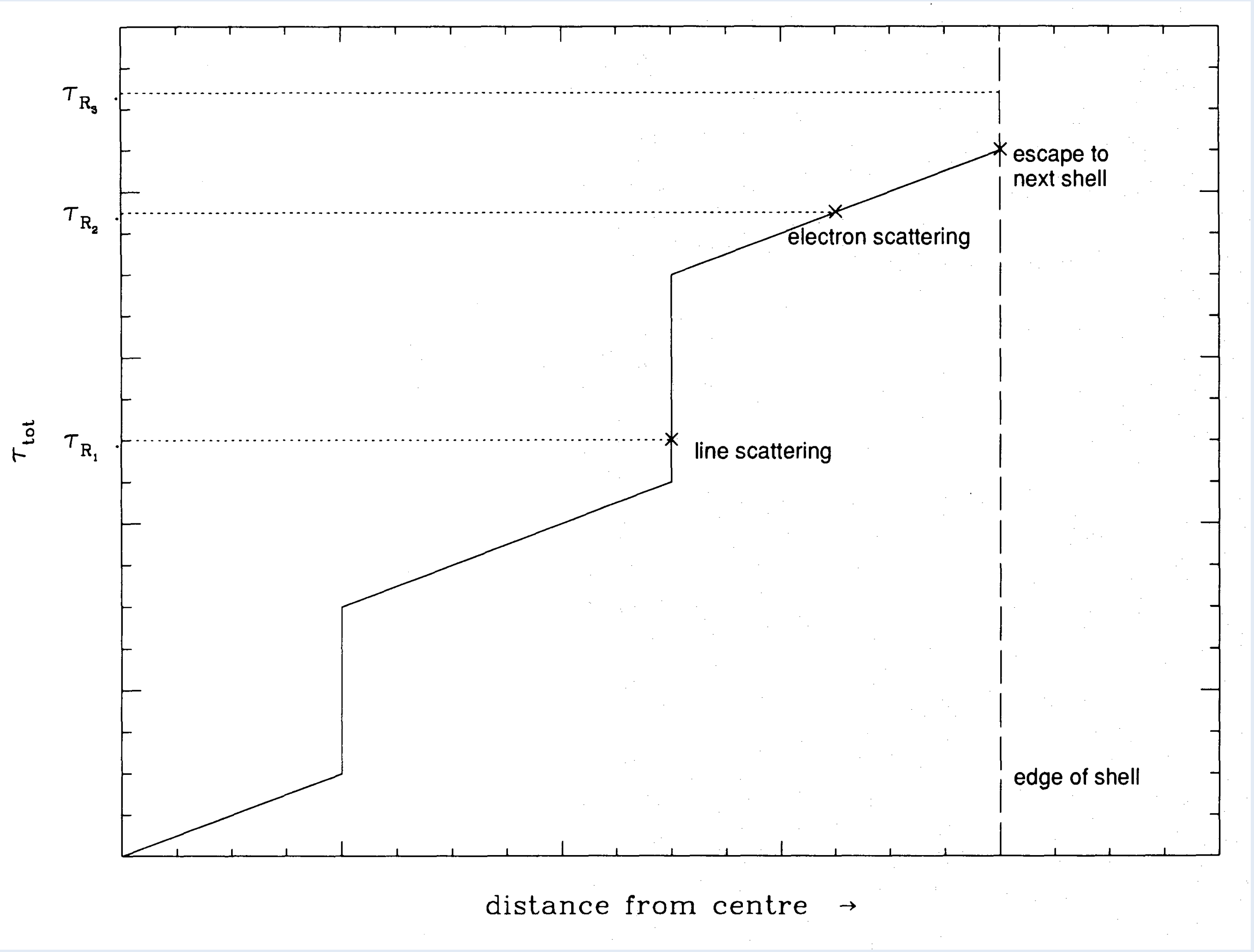


(figure from U. Noebauer)

Sobolev approximation

Process for finding interaction point (generalized to include continuum):

Optical depth accumulated



Packet trajectory length
(figure from Mazzali & Lucy 1993)

Line interaction events

Radiative equilibrium (indivisible packets) means any packet absorbed by a line transition must be re-emitted.

Extremely simple to use resonance scattering approximation:

- In homologous flow Sobolev escape probabilities are isotropic
- Empirically seems to do quite well for optical spectra of SNe Ia

Alternative schemes based on “down branching” (Mazzali & Lucy 1993) and Lucy’s (2002, 2003) “macro atom” / “k-packet” methods give more physical realism (second lecture).

Example implementation:

TARDIS code (Kerzendorf & Sim 14)

Example implementation:

TARDIS code (Kerzendorf & Sim 14)

Example implementation:

TARDIS code (Kerzendorf & Sim 14)

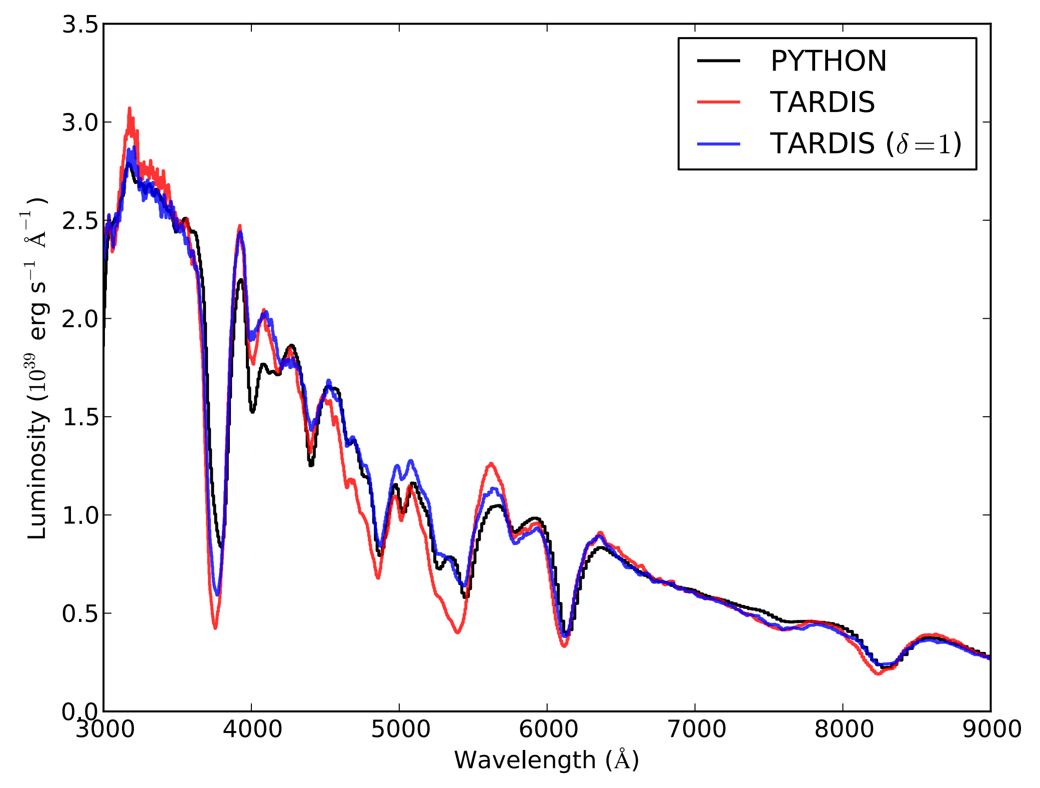
Motivation:

Many codes currently used, different approximations

- Goal: **facilitate comparison/testing of approaches**

With large datasets need for code that can make trade-offs between speed and accuracy

- Goal: **open source flexible tool using established methods**



What does TARDIS actually do?:

- Monte Carlo radiative transfer code
  - Uses well-established approach (Abbott & Lucy ‘85, Mazzali & Lucy 93, Lucy 99, Mazzali+ 00, Lucy 02, 03, Kasen+06, Kromer+09…)
  - Same basic algorithm as described above

Iterative calculation:

- Each zone has “plasma state” (envisage options from LTE to full nLTE for all ions)
  - From this, calculate opacities

- Radiation input at inner boundary (currently)

- Use radiation propagation to update plasma state

- Once solution converges, compute spectrum

TARDIS needs input:

- **Luminosity and epoch (based on observation?)**
- **“Model” for the ejecta layers being studied**
- **Atomic database (ships with Kurucz line list)**
- **Choices for how to treat ionization / excitation**
- **Numerical parameters (number of Monte Carlo quanta)**

TARDIS needs input:

- Luminosity and epoch (based on observation?)
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- Atomic database (ships with Kurucz line list)
- Choices for how to treat ionization / excitation
- Numerical parameters (number of Monte Carlo quanta)

“Model” for the ejecta:

Plausible goal of fitting spectra is to constrain the density/composition of the layers in which features form.

TARDIS needs density and composition information as a function of velocity in the homologous phase.

Currently 1D but otherwise fully flexible – anything from constant to full radial profiles of density and abundances.

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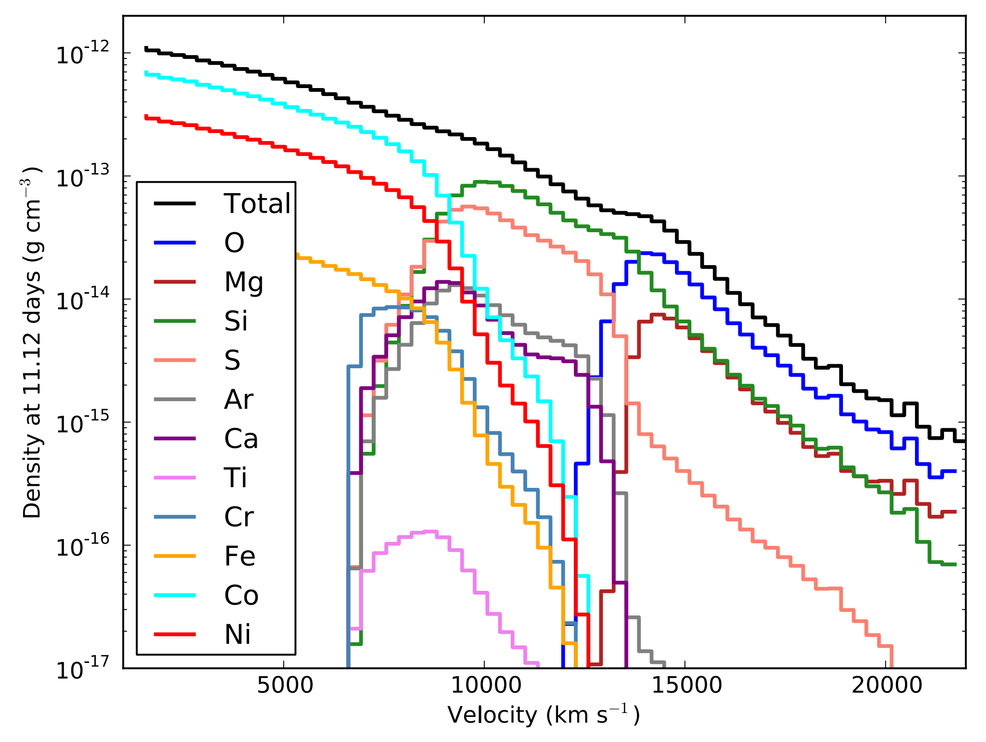

An example!

TARDIS status:

On public release (regular updates):

- Available on github,
  http://tardis.readthedocs.org/

Development:

- Initial goal was Ia focus: choices of opacity and ionization
- Working on bound-free / photoionization
- Planned extension to post-photospheric phases

Acknowledge support from GSoC in 2014 and 2015!

Questions/contact:

- User group (all interested can subscribe)
- s.sim@qub.ac.uk and wkerzendorf@gmail.com