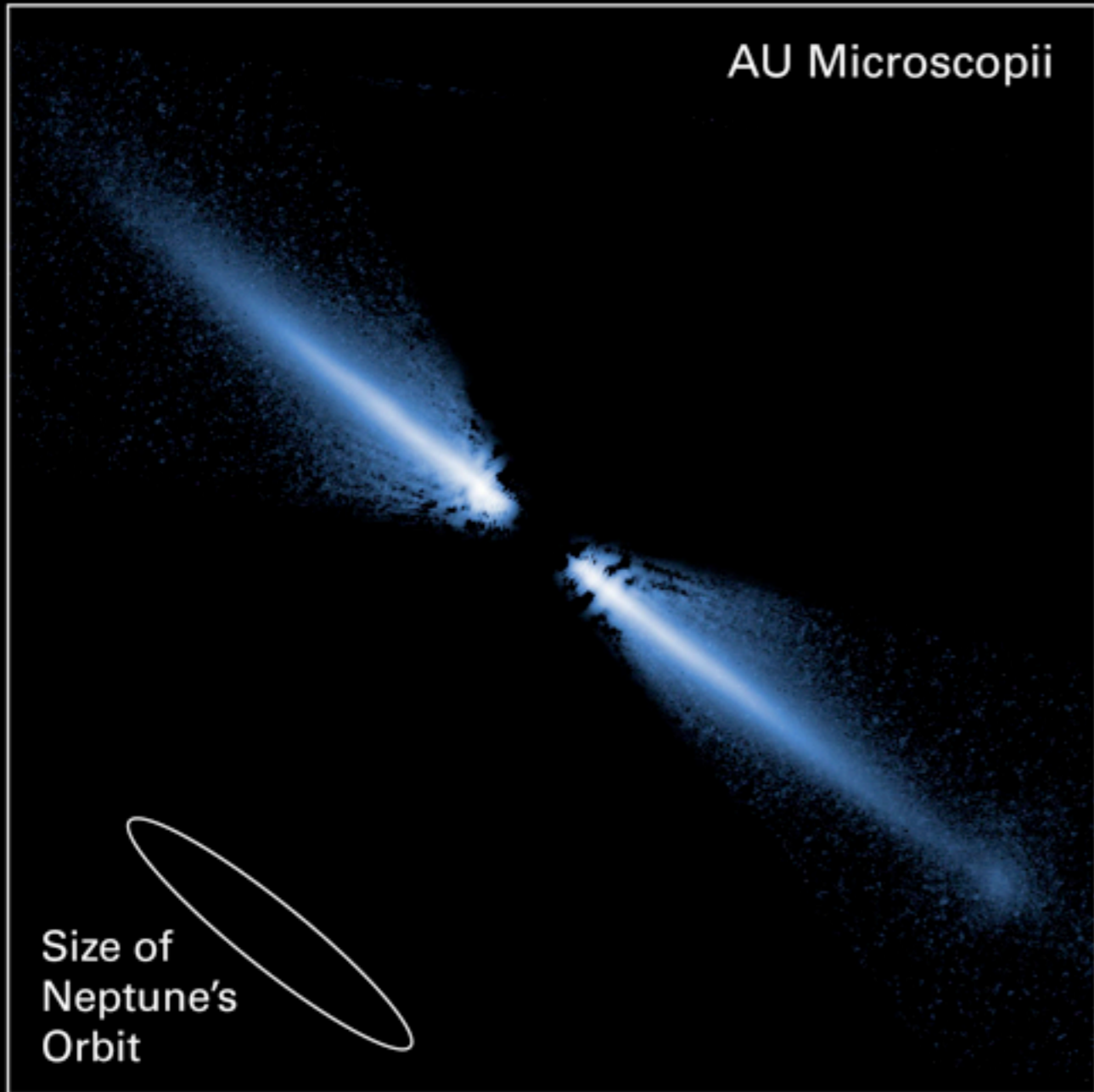


# Radiative Equilibrium

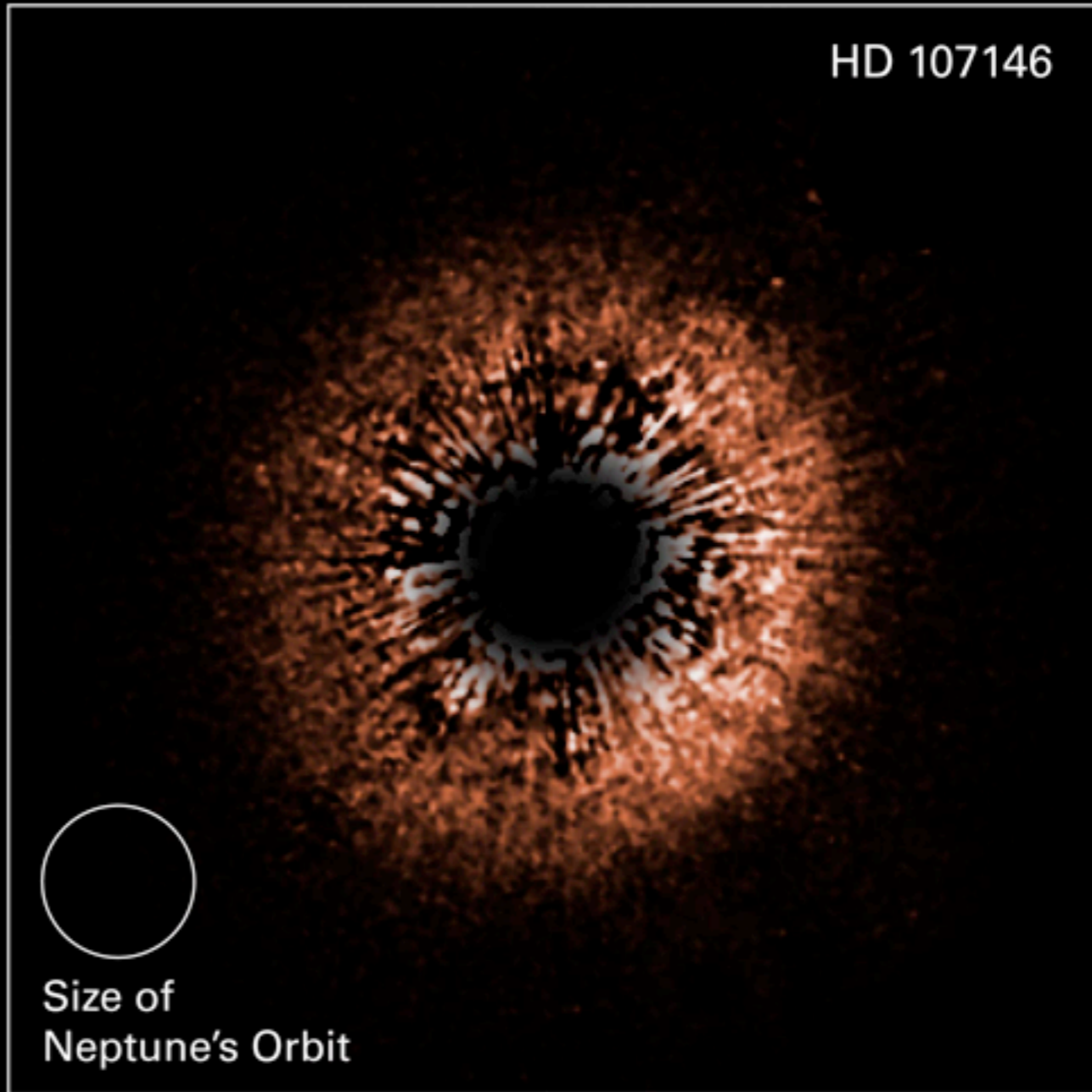


Thomas Robitaille (Max Planck Institute for Astronomy)

AU Microscopii



HD 107146

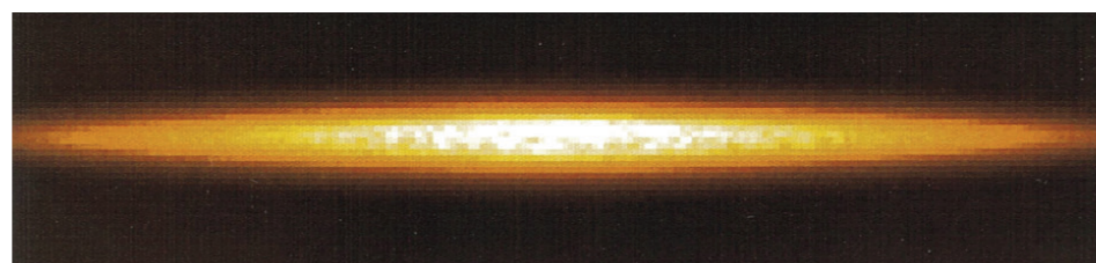
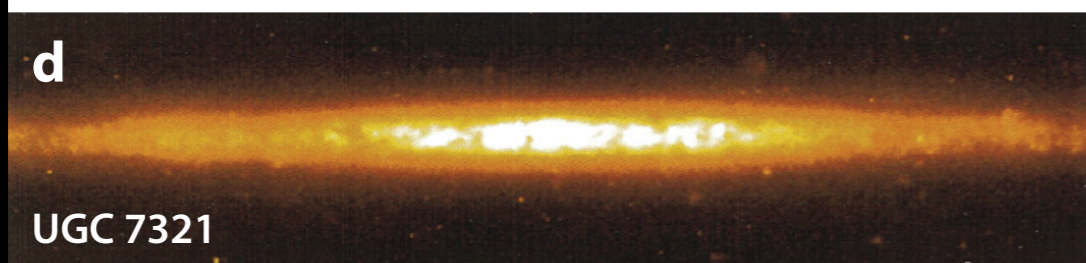
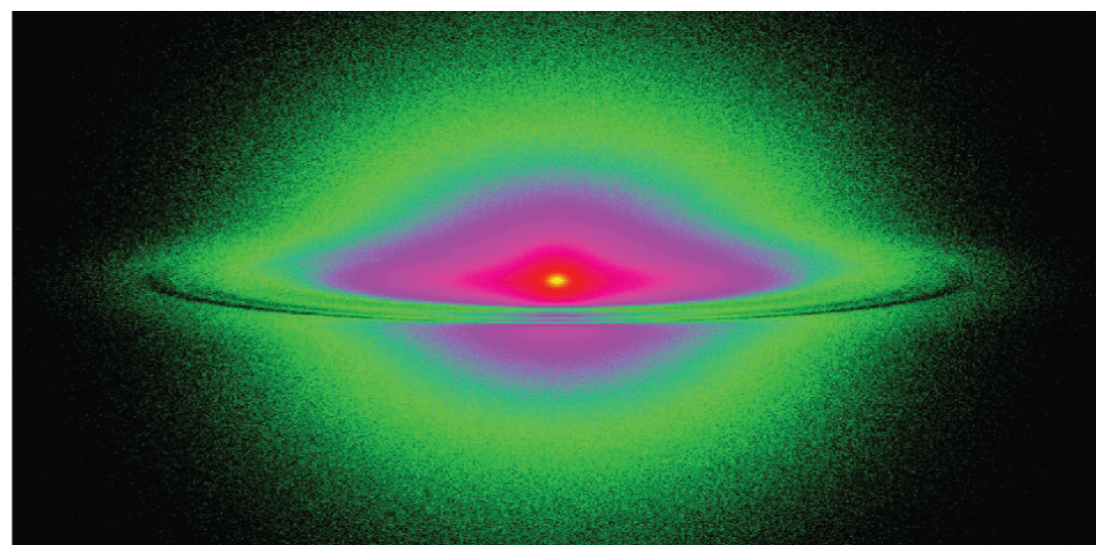
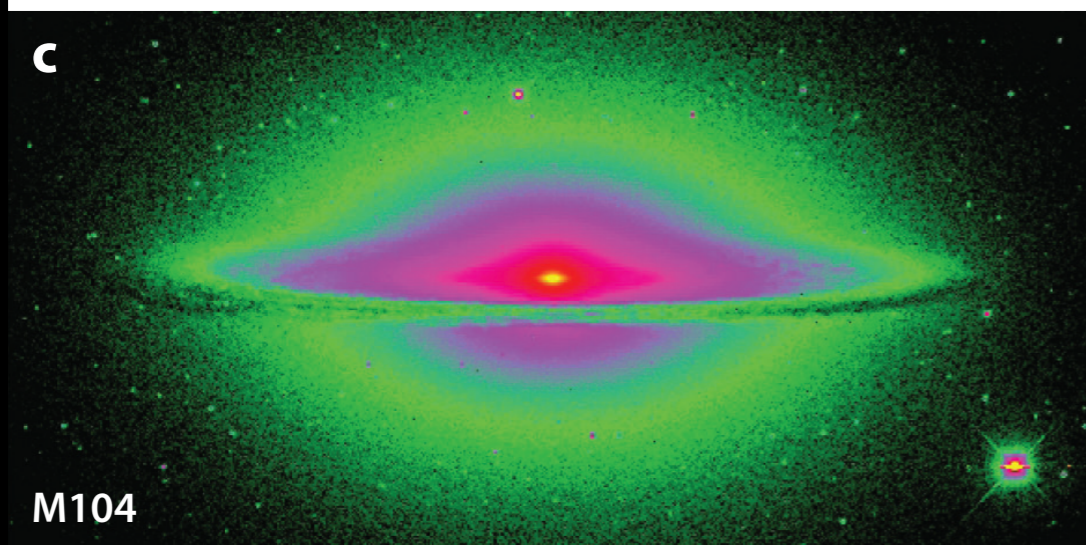
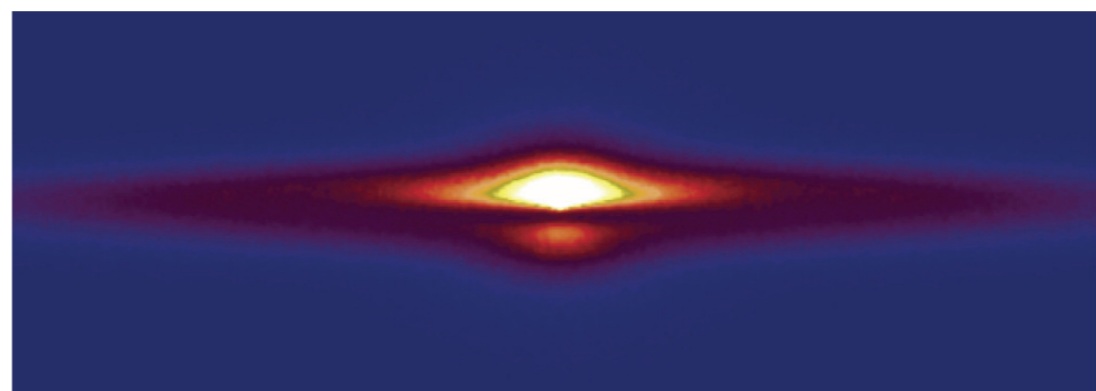
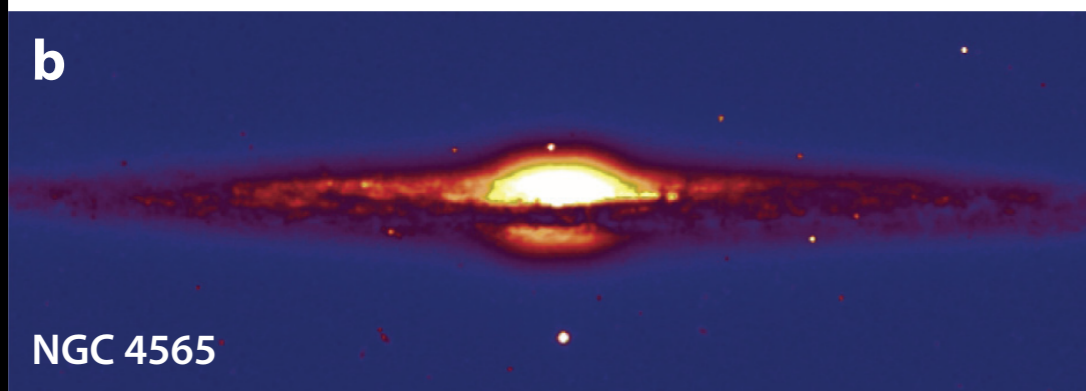
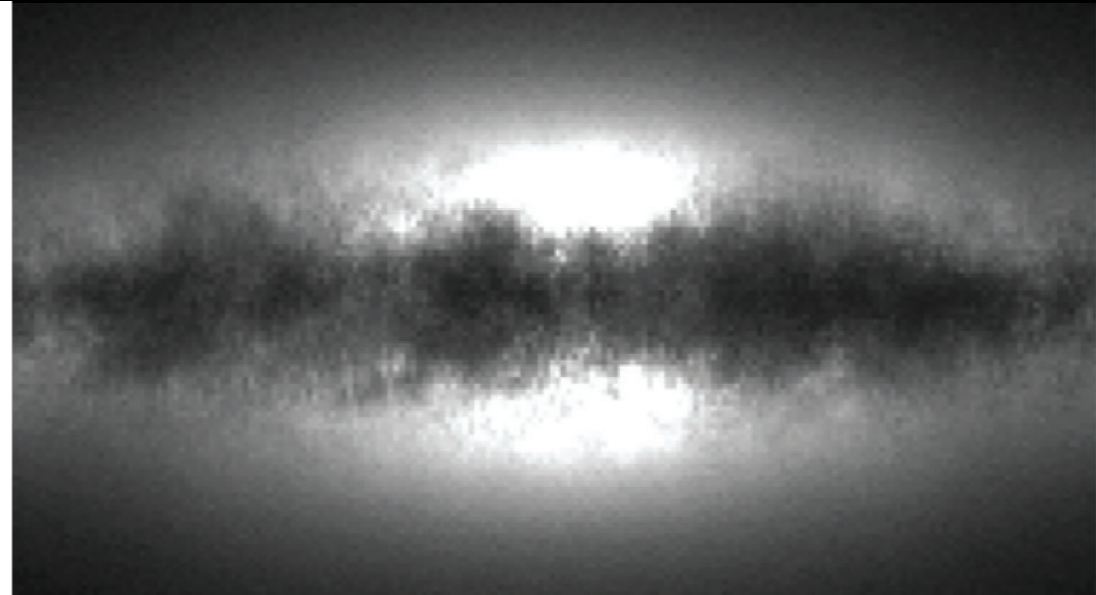
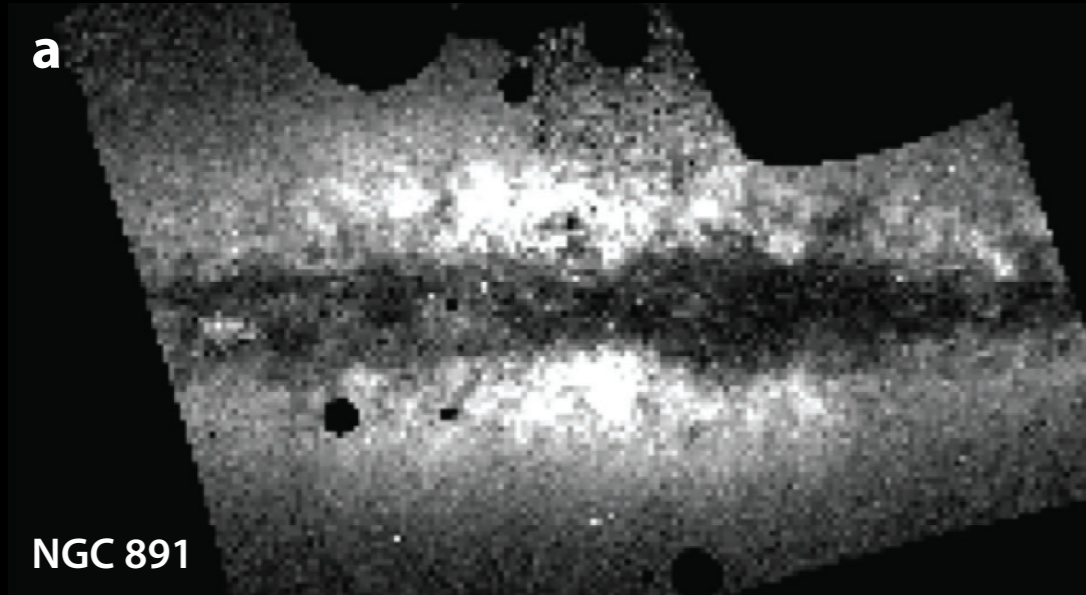


**Circumstellar Debris Disks**  
**Hubble Space Telescope • ACS HRC**

NASA, ESA, J. Krist (STScI/JPL), D.R. Ardila (JHU), D.A. Golimowski (JHU), M. Clampin (NASA/Goddard),  
 H. Ford (JHU), G. Hartig (STScI), G. Illingworth (UCO-Lick) and the ACS Science Team

STScI-PRC04-33a







1  $\mu\text{m}$

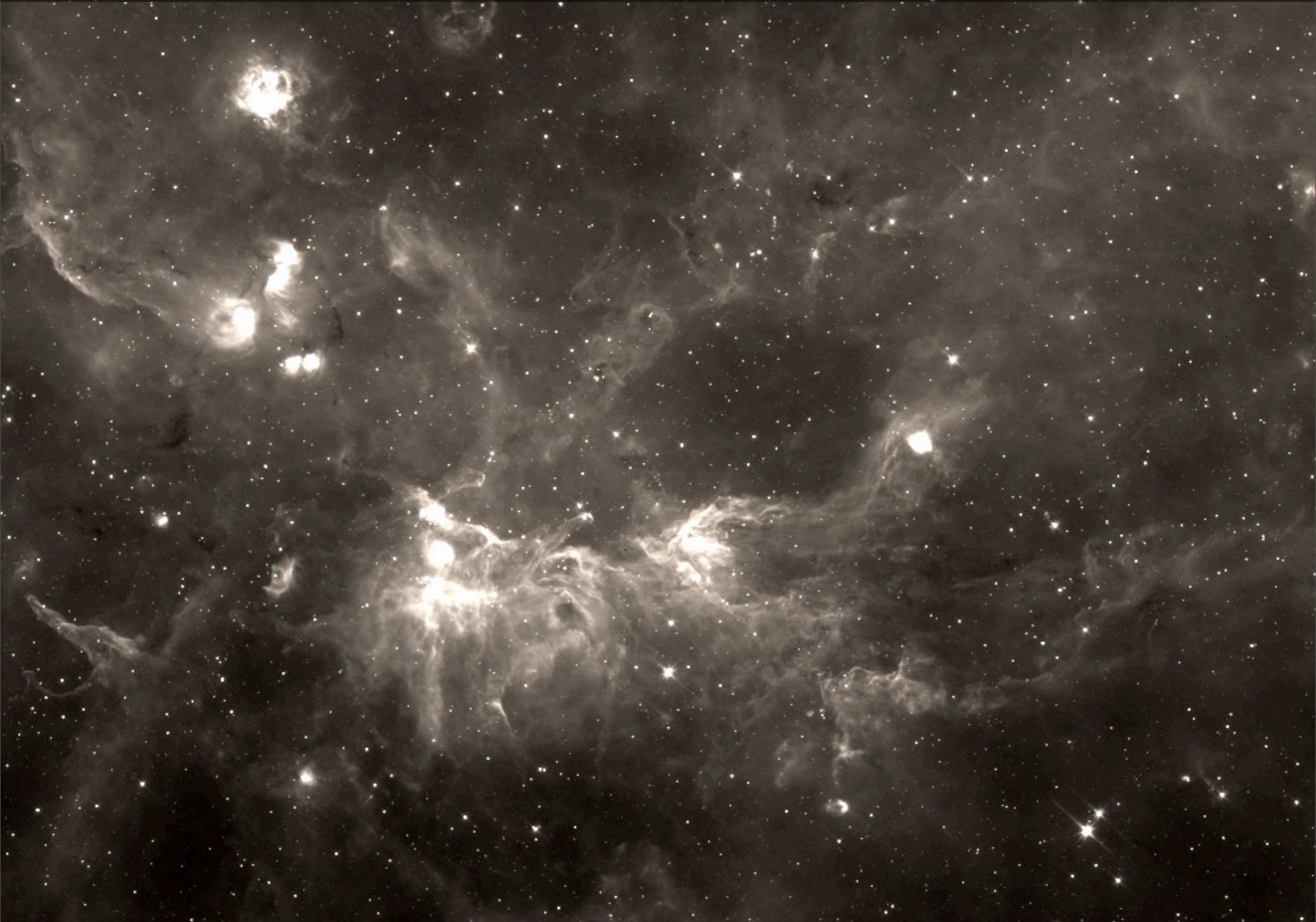
starlight, dust extinction





8  $\mu\text{m}$

direct and scattered starlight, hot dust ( $\sim 375\text{K}$ )





24  $\mu\text{m}$

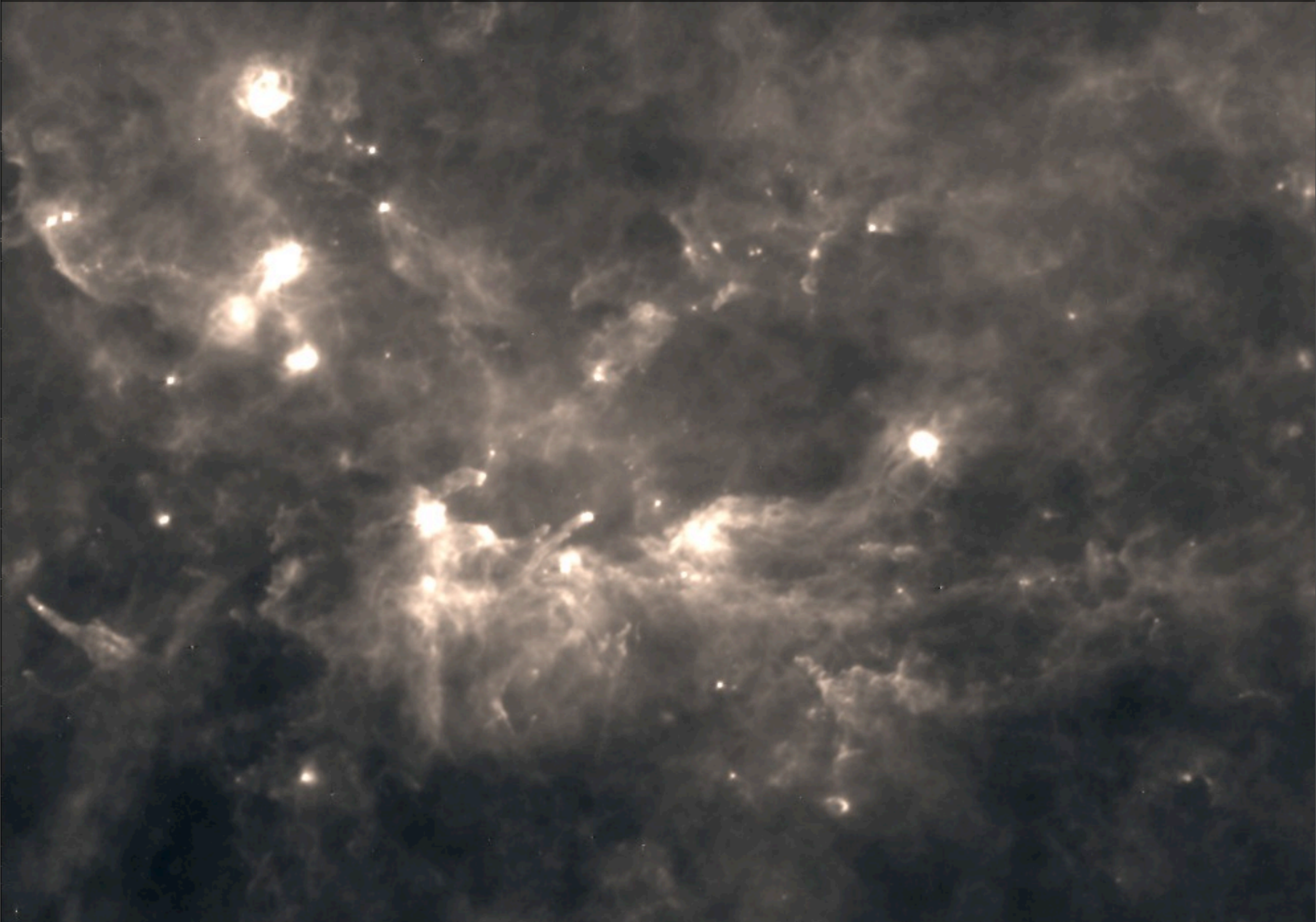
warm dust ( $\sim 125\text{K}$ )



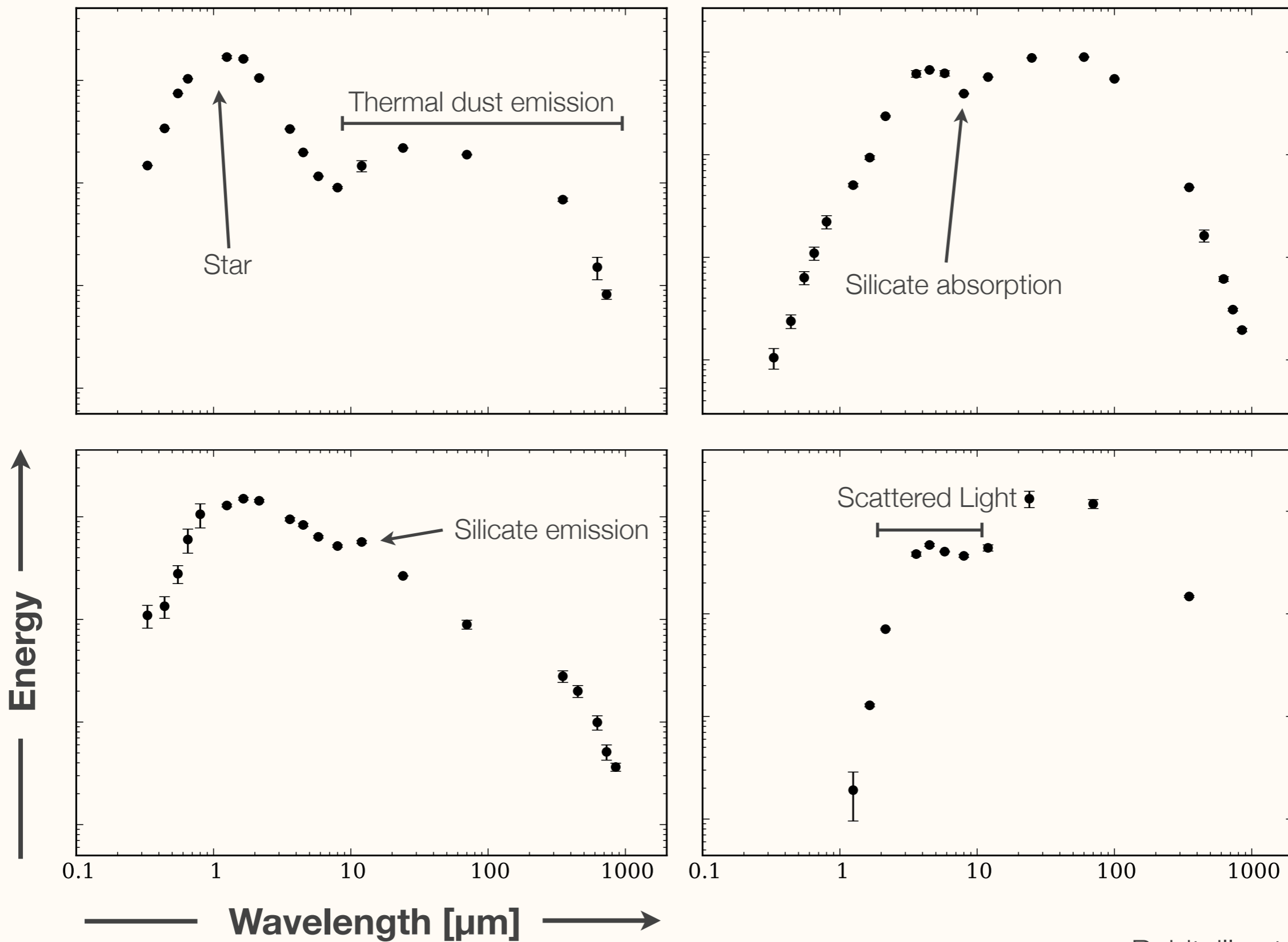


170  $\mu\text{m}$

cold dust (<20K)



# Spectral energy distributions of forming stars



Robitaille et al, (2007)  
Thomas Robitaille



$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$



$$\frac{dI_{x,y,z,\theta,\phi,\nu}}{ds} = -\alpha_{x,y,z,\nu} I_{x,y,z,\theta,\phi,\nu} + j_{x,y,z,\theta,\phi,\nu}(T)$$



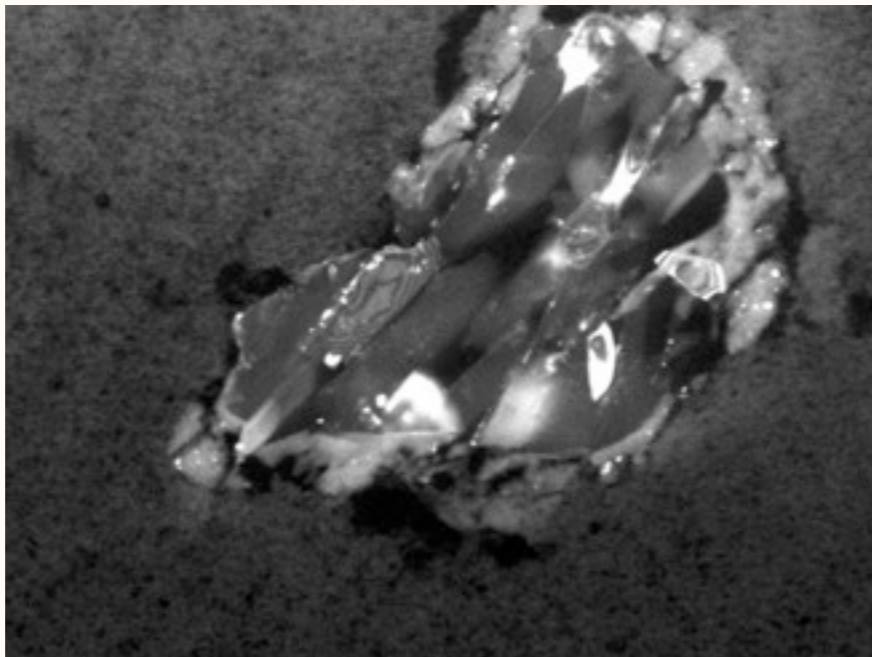
# Dust versus Gas

For dust in LTE, emissivities are given by  $\kappa_\nu B_\nu(T)$

For lines, need to take into account gas velocities, line profiles, level populations, etc.

In this lecture we focus on **dust** radiative transfer





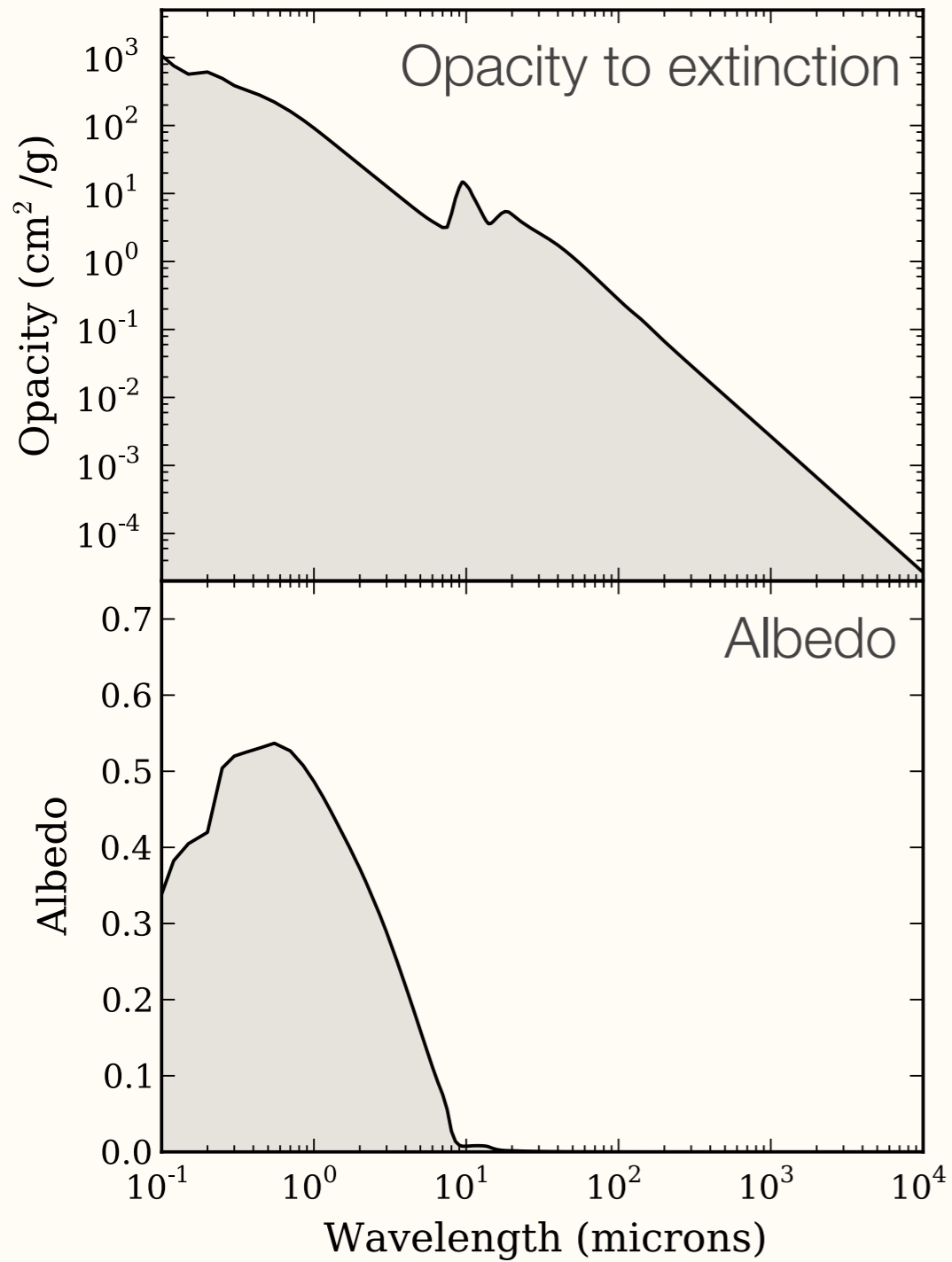
Forsterite crystal from Comet Wild 2 (NASA)

### Composition:

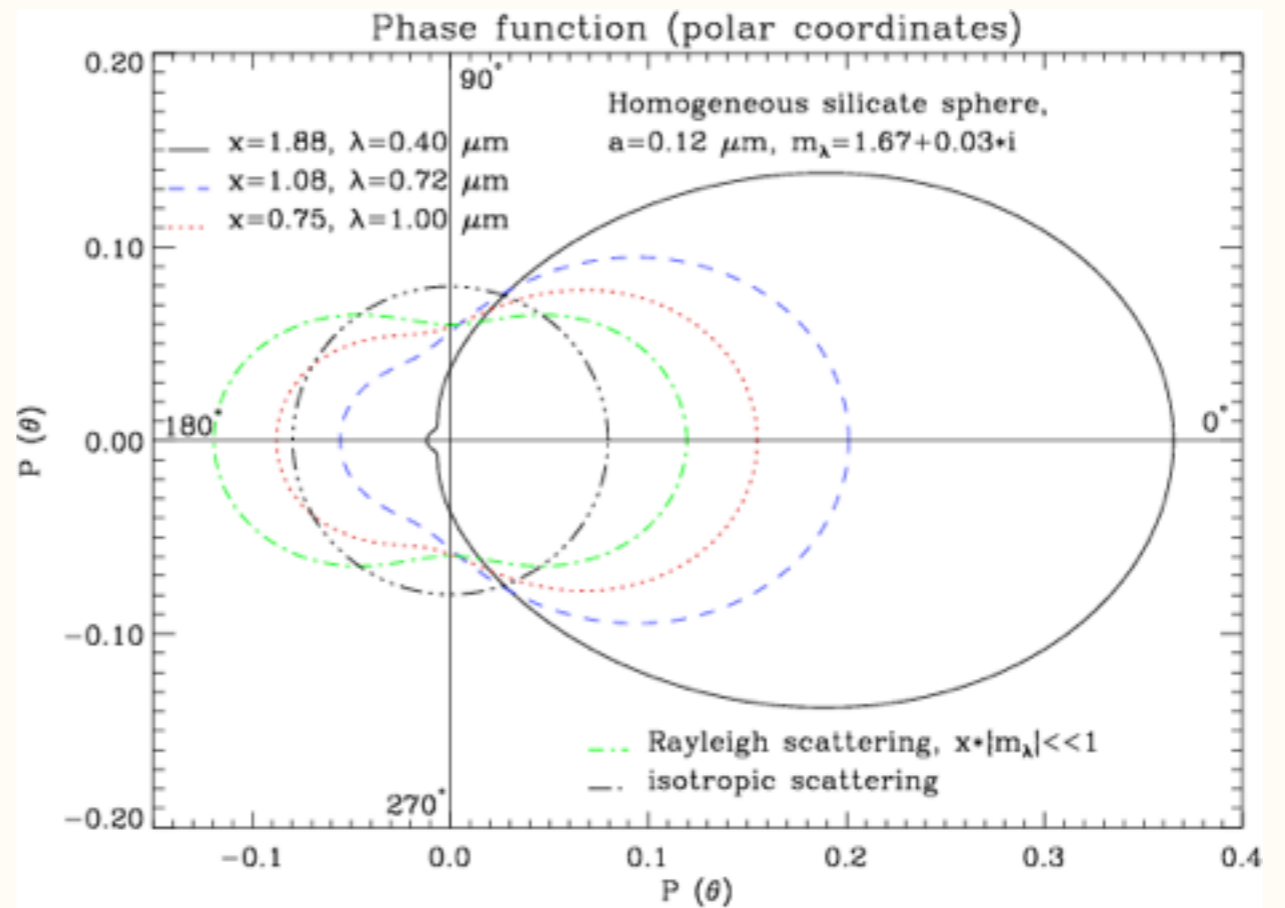
- Silicates
- Carbon
- Water
- Organic molecules

**Sizes** from nm to mm



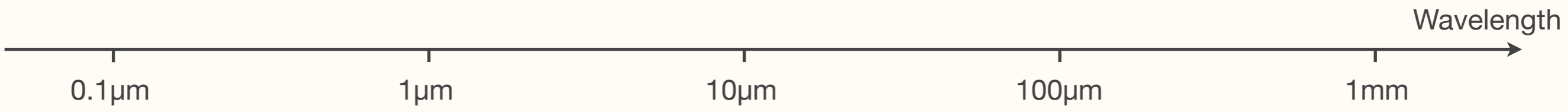


## Scattering Phase Function

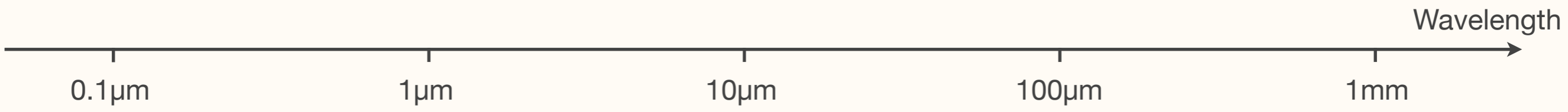


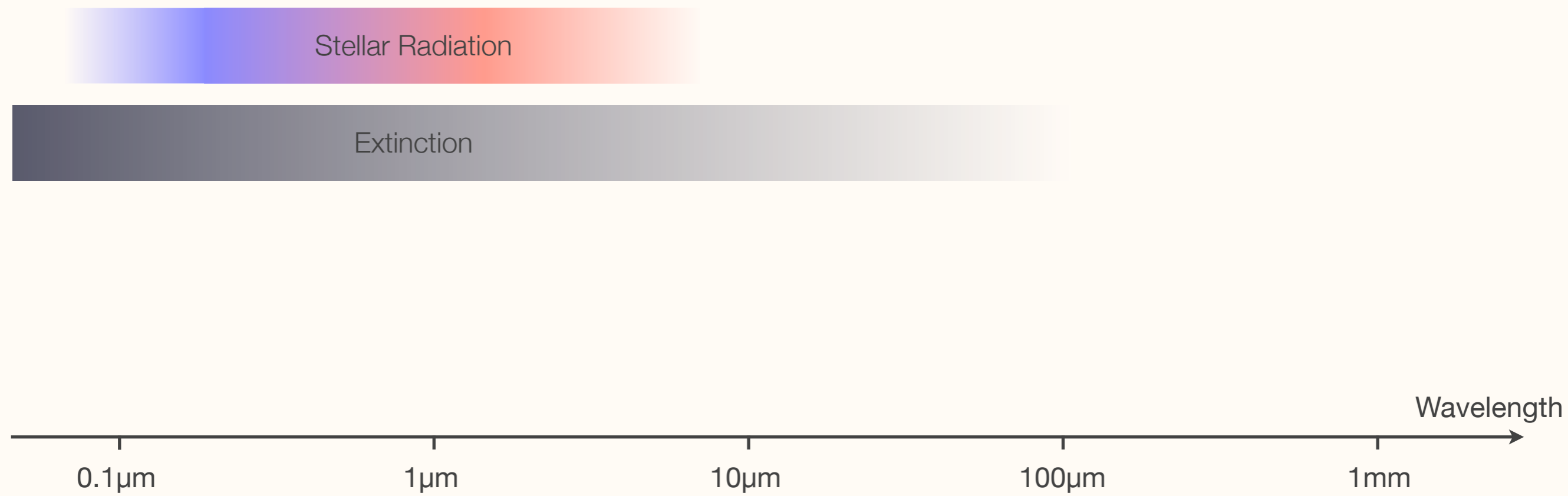
Steinacker et al. (2003)











Wavelength

0.1  $\mu\text{m}$

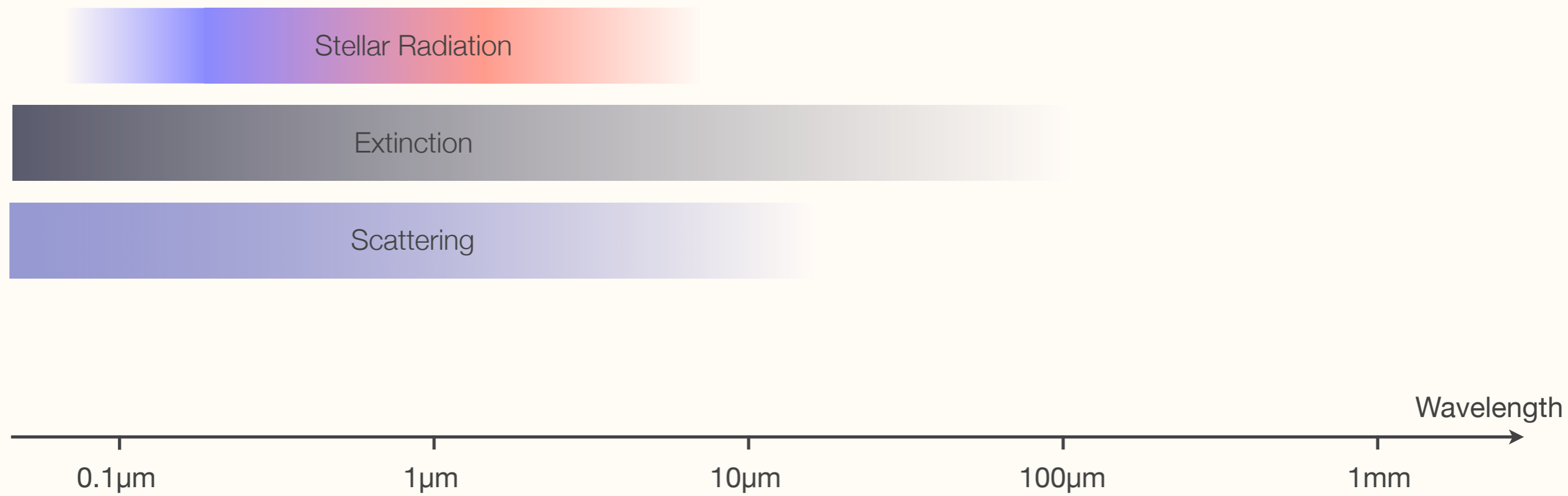
1  $\mu\text{m}$

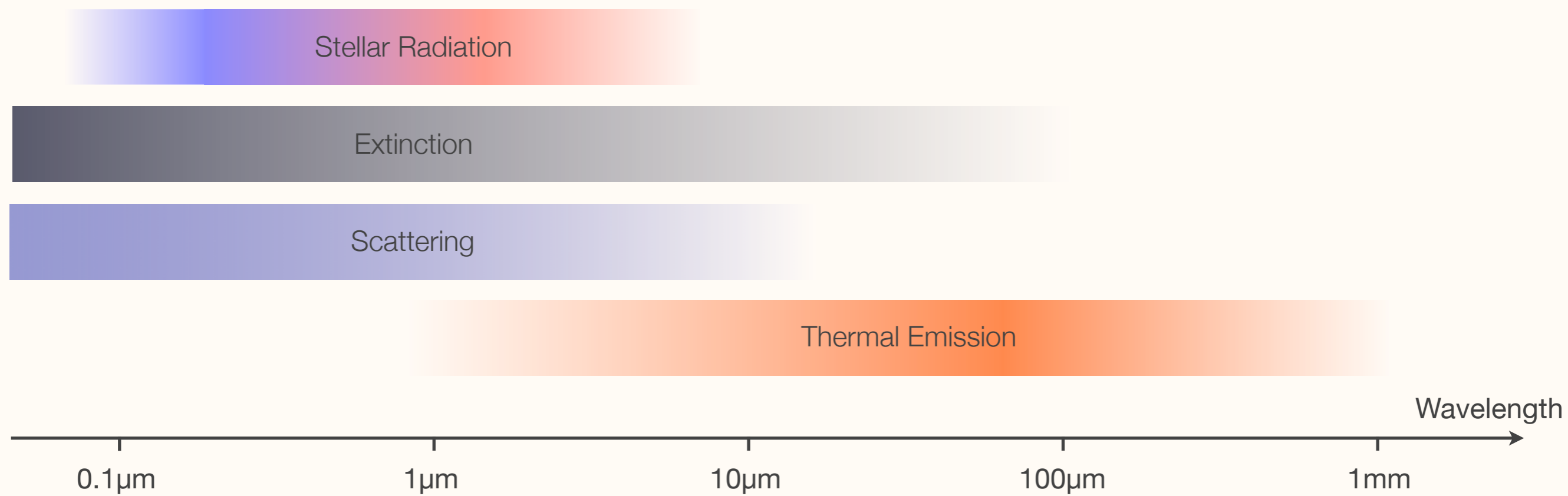
10  $\mu\text{m}$

100  $\mu\text{m}$

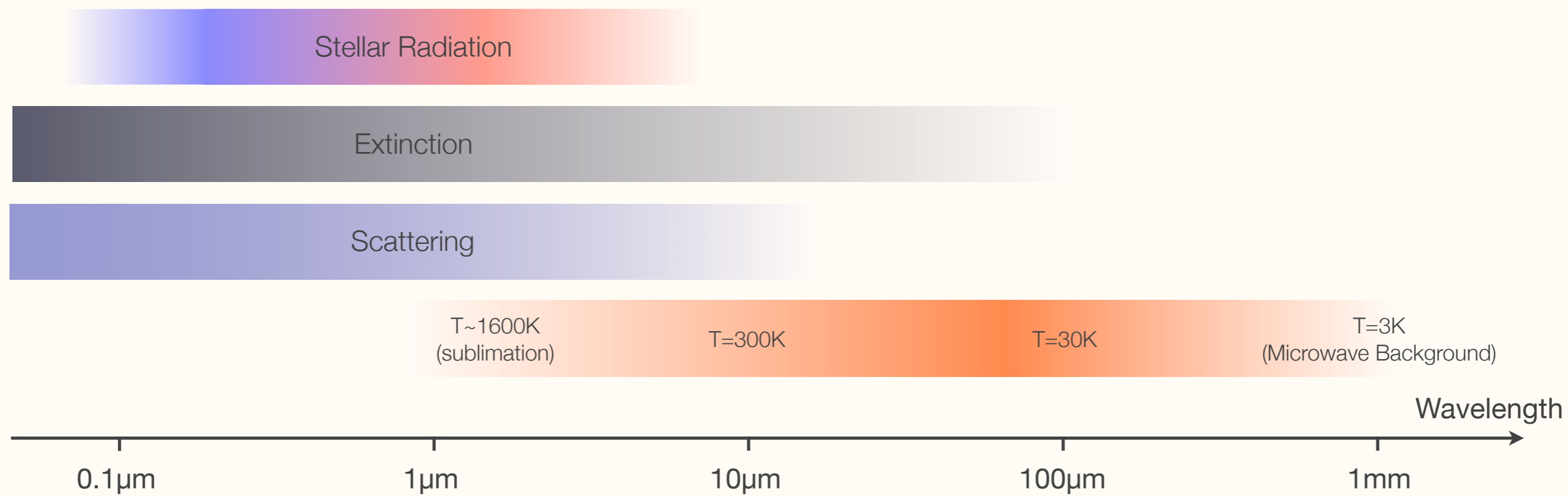
1 mm





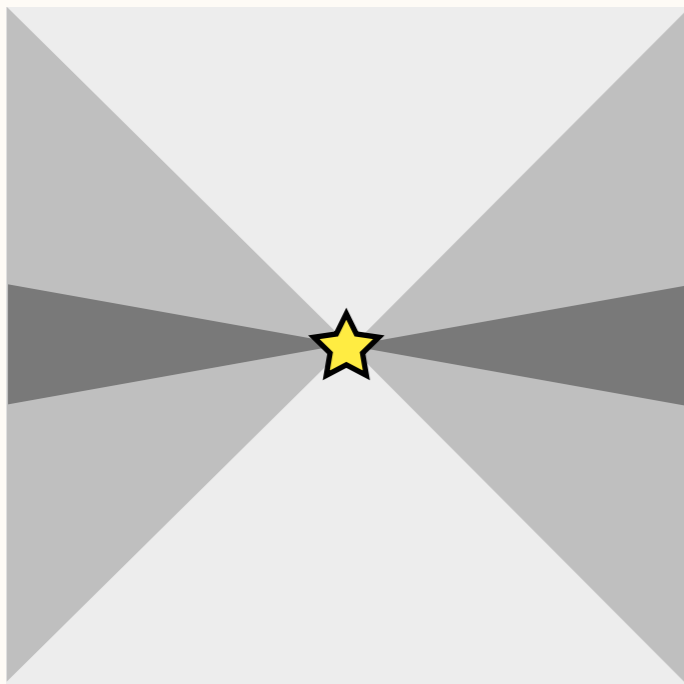






## Monte-Carlo Radiative Transfer:

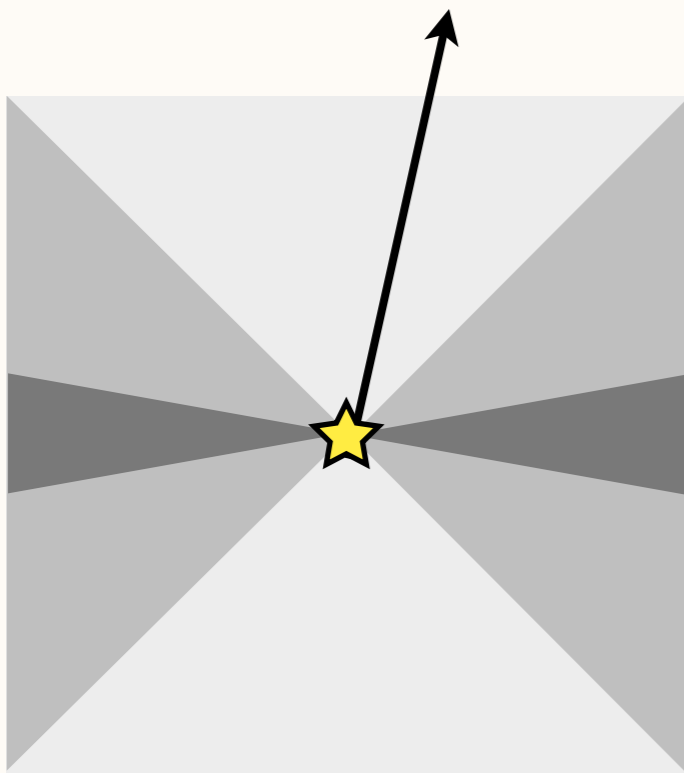
Propagate many photon packets by randomly sampling from probability density functions for directions, frequencies, path lengths, interactions.





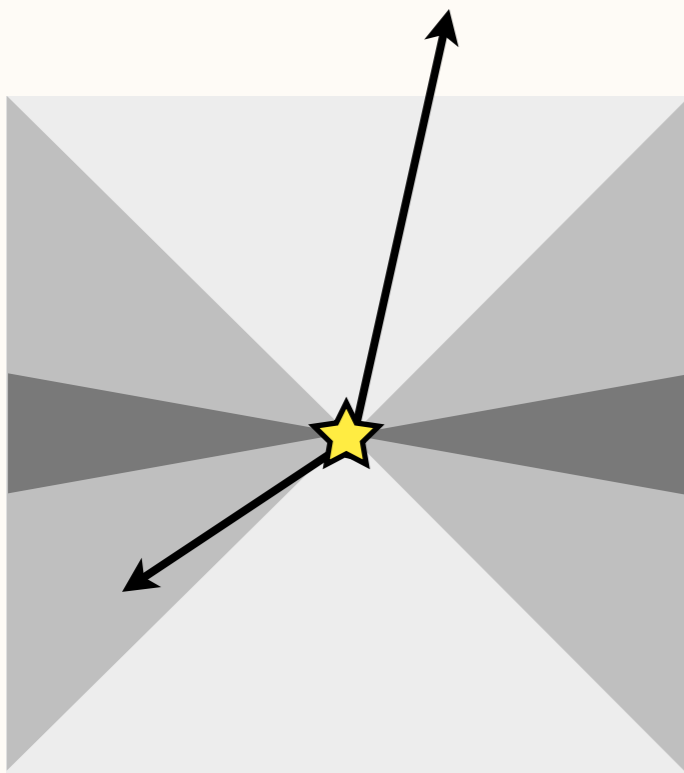
## Monte-Carlo Radiative Transfer:

Propagate many photon packets by randomly sampling from probability density functions for directions, frequencies, path lengths, interactions.



## Monte-Carlo Radiative Transfer:

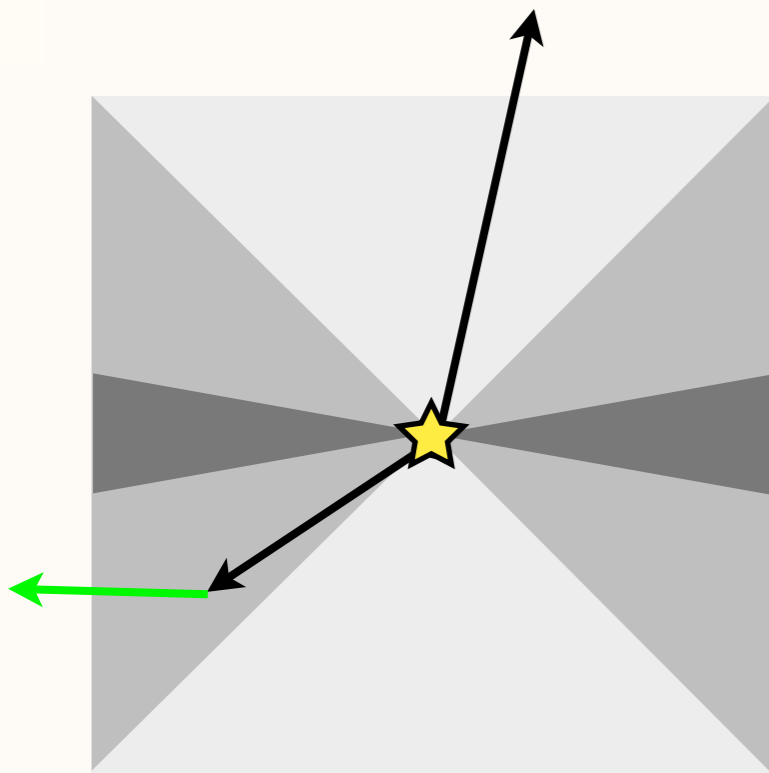
Propagate many photon packets by randomly sampling from probability density functions for directions, frequencies, path lengths, interactions.





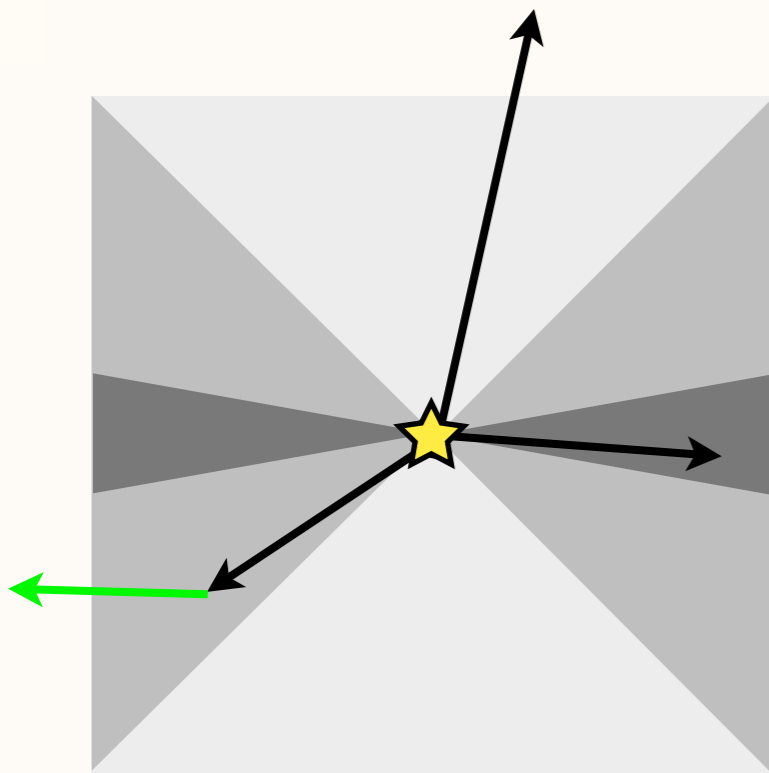
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## Monte-Carlo Radiative Transfer:

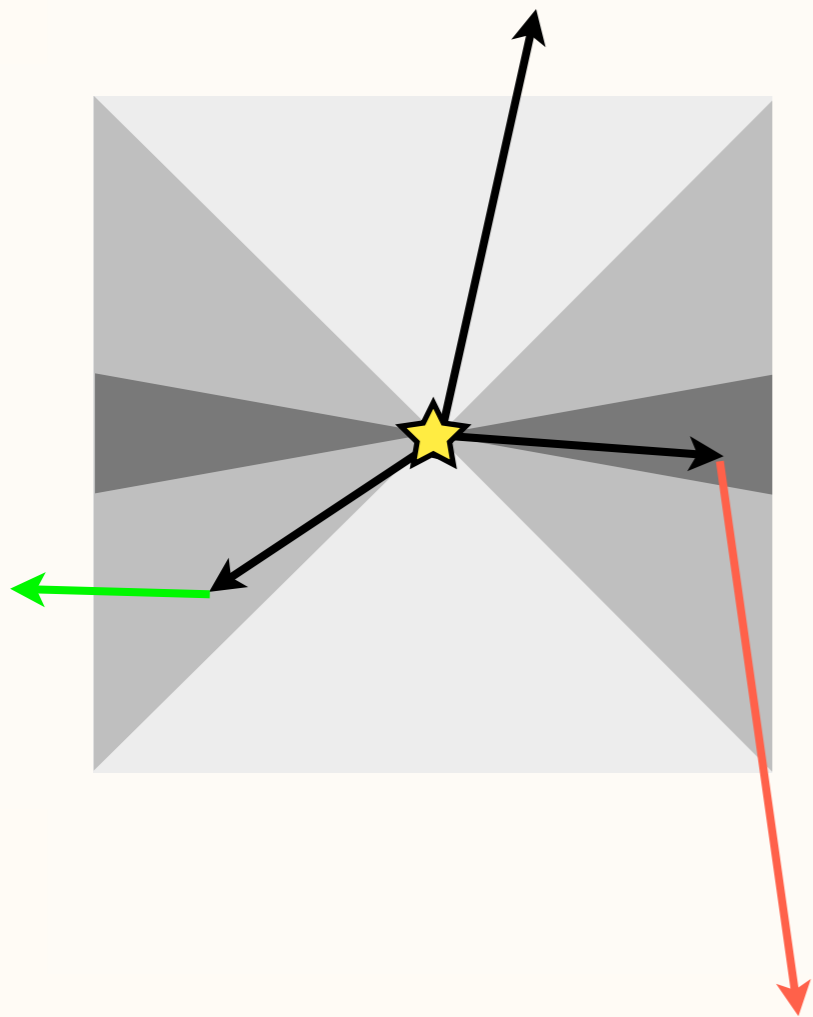
Propagate many photon packets by randomly sampling from probability density functions for directions, frequencies, path lengths, interactions.





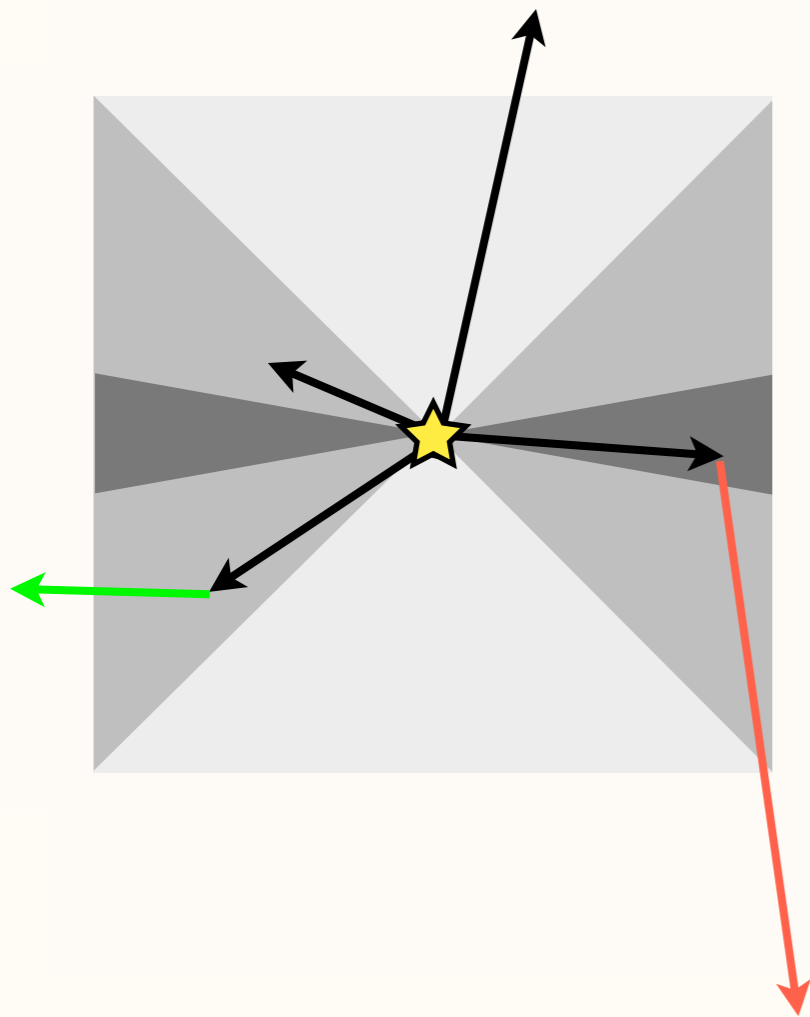
## Monte-Carlo Radiative Transfer:

Propagate many photon packets by randomly sampling from probability density functions for directions, frequencies, path lengths, interactions.



## Monte-Carlo Radiative Transfer:

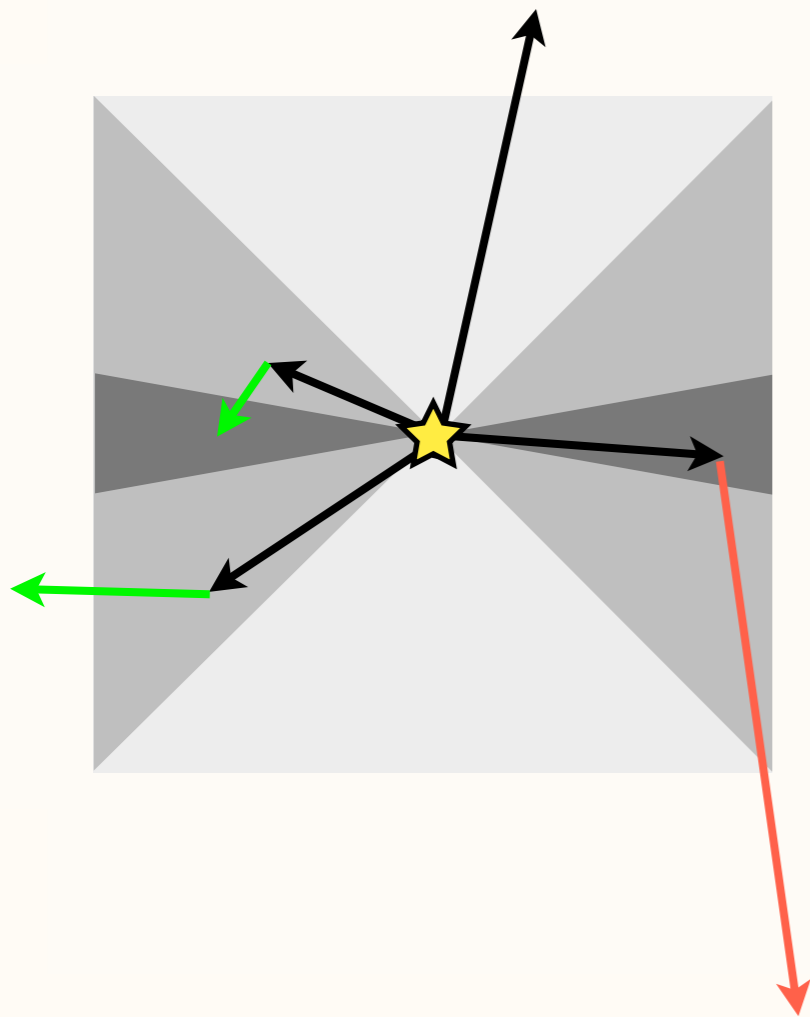
Propagate many photon packets by randomly sampling from probability density functions for directions, frequencies, path lengths, interactions.





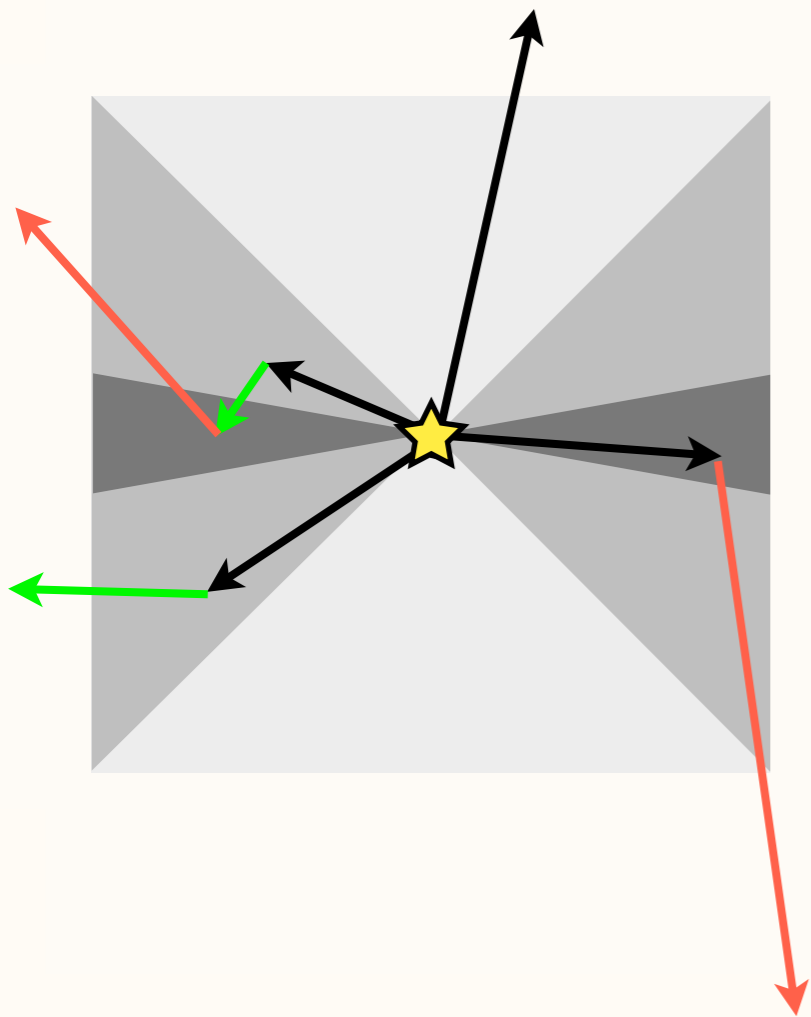
## Monte-Carlo Radiative Transfer:

Propagate many photon packets by randomly sampling from probability density functions for directions, frequencies, path lengths, interactions.



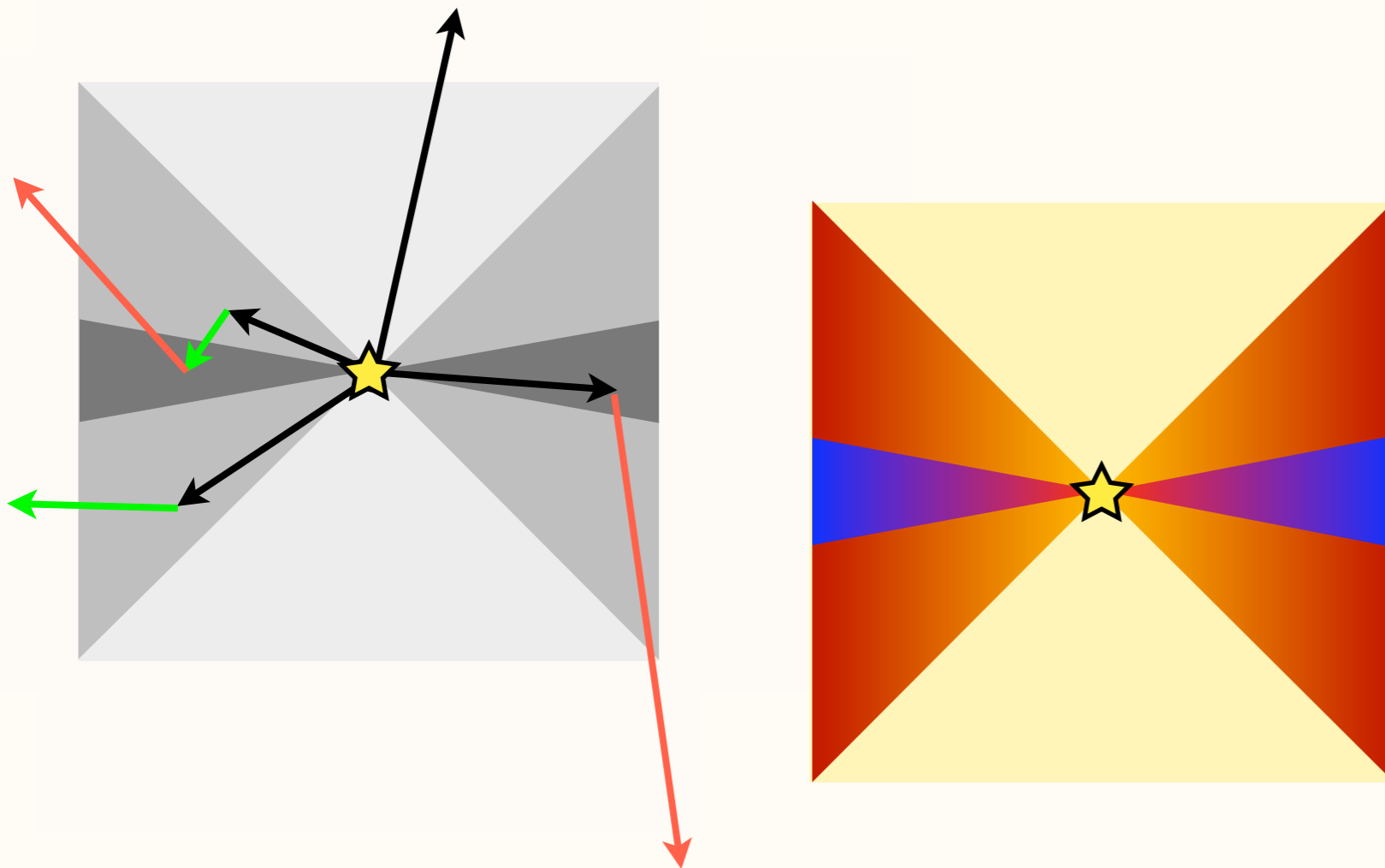
## Monte-Carlo Radiative Transfer:

Propagate many photon packets by randomly sampling from probability density functions for directions, frequencies, path lengths, interactions.



## Monte-Carlo Radiative Transfer:

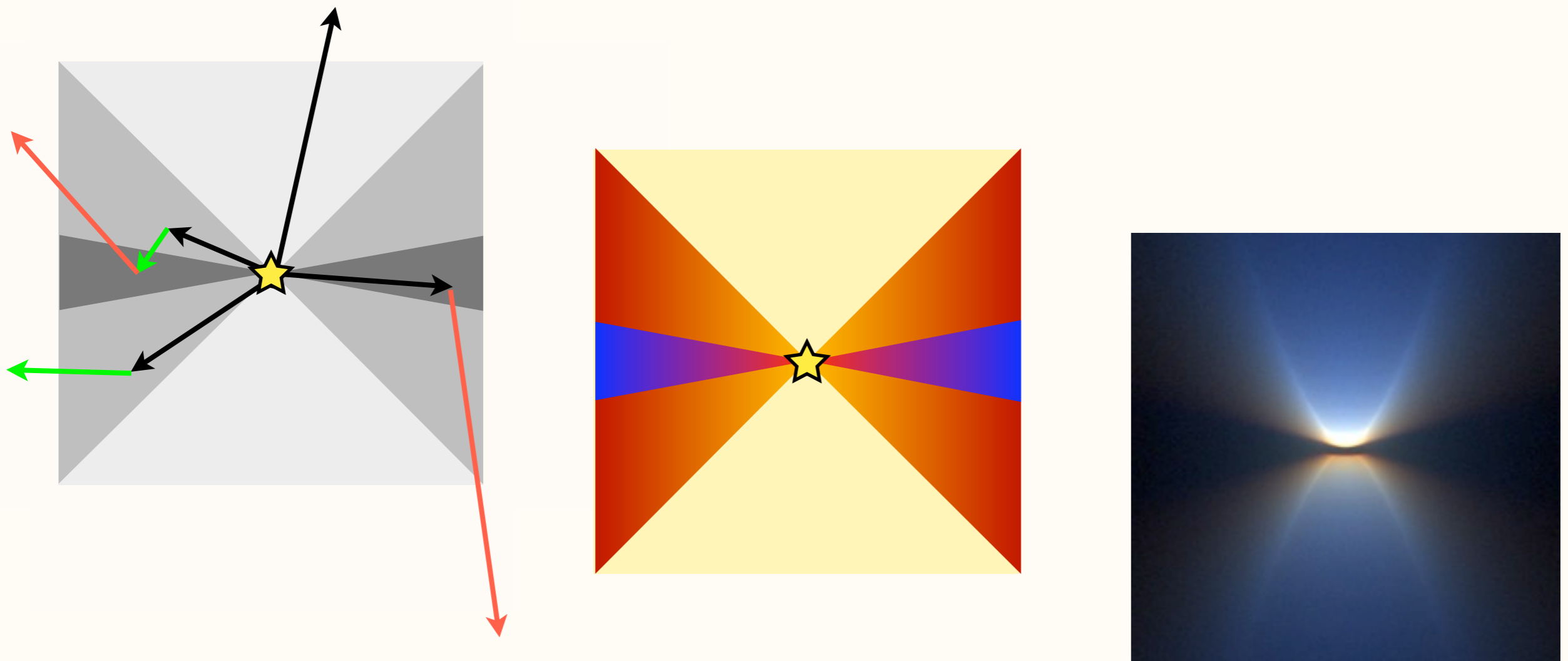
Propagate many photon packets by randomly sampling from probability density functions for directions, frequencies, path lengths, interactions.



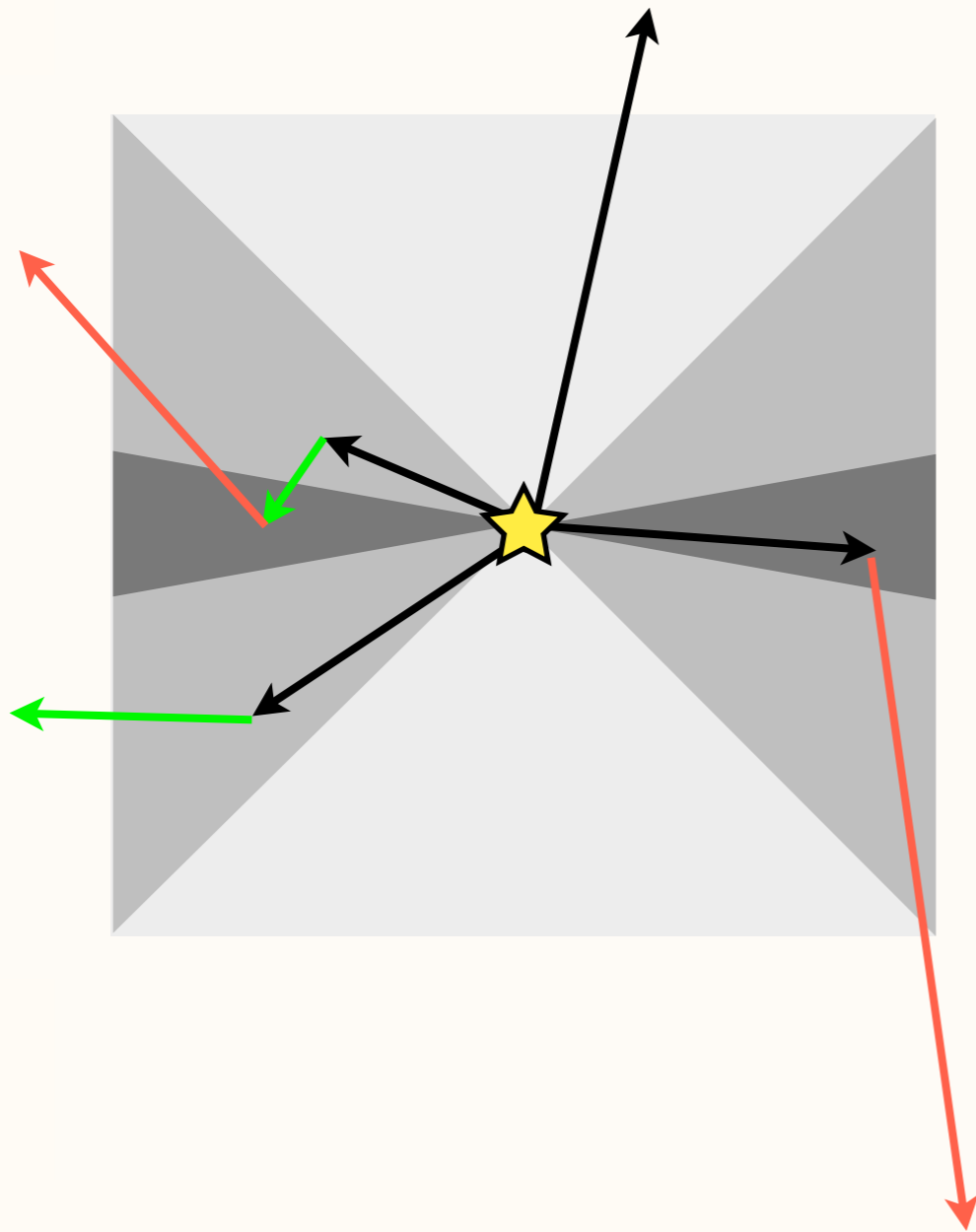


## Monte-Carlo Radiative Transfer:

Propagate many photon packets by randomly sampling from probability density functions for directions, frequencies, path lengths, interactions.



# Propagating photon packets on a grid

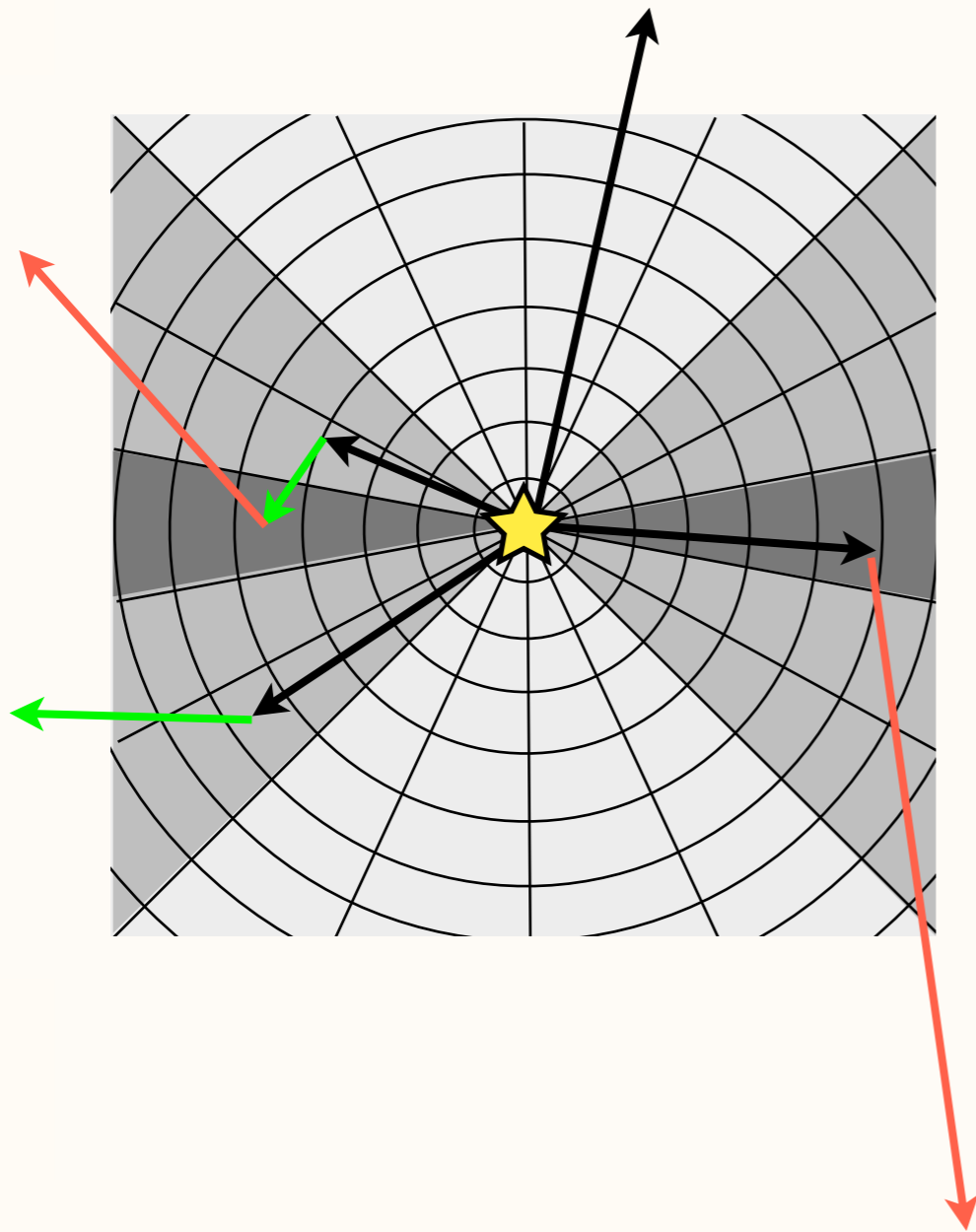


If density is constant, optical depth is proportional to distance.

For arbitrary 3-d model, not trivial!

We therefore split density structure into cells of **constant density**

# Propagating photon packets on a grid



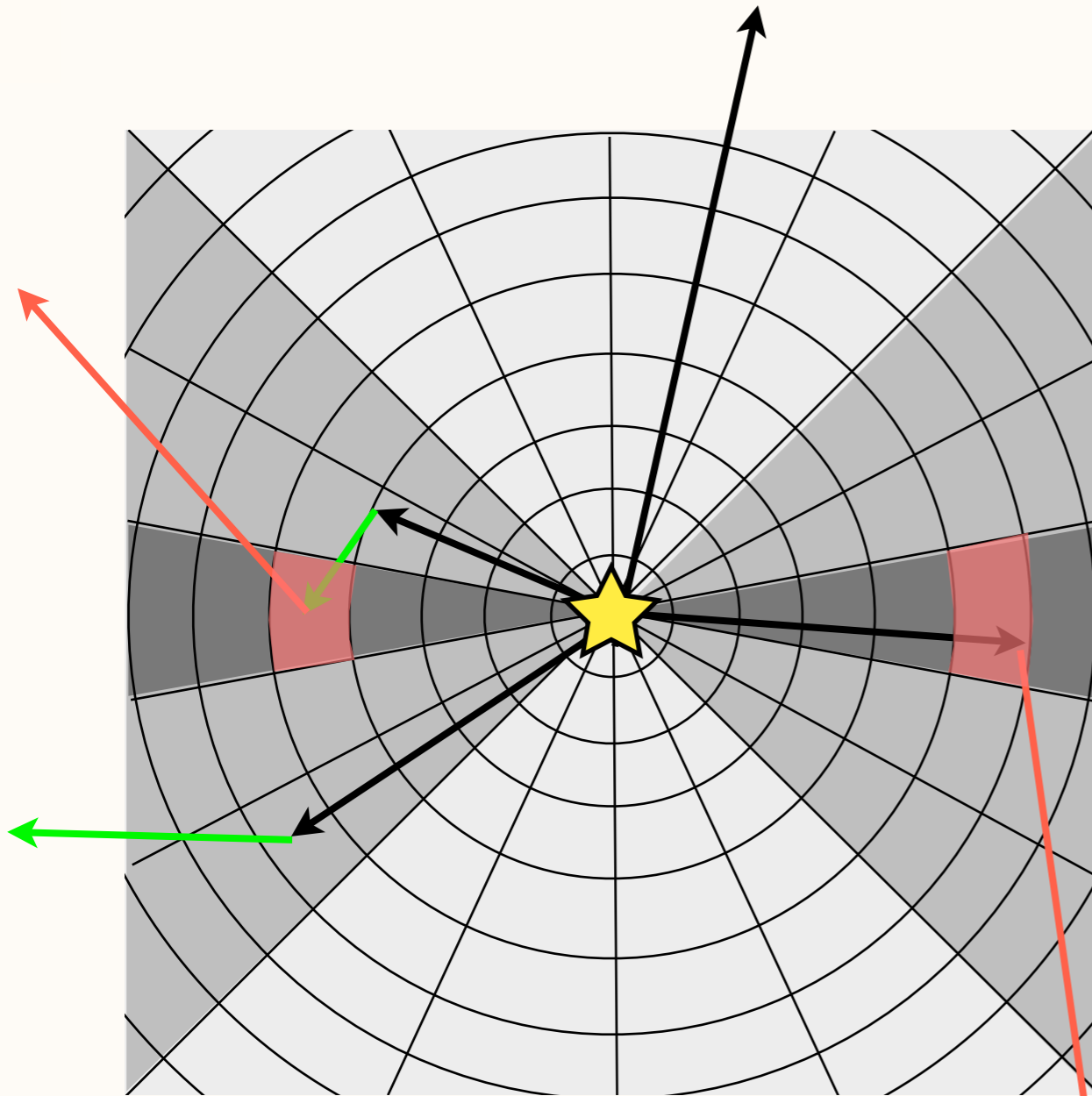
If density is constant, optical depth is proportional to distance.

For arbitrary 3-d model, not trivial!

We therefore split density structure into cells of **constant density**



# Radiative Equilibrium: Computing temperatures



**Naive algorithm:** count energy absorbed in each cell

In equilibrium, energy emitted should be equal to energy absorbed!

# Radiative Equilibrium: Computing temperatures

We can then convert specific absorbed energy rate to temperature:

$$4 \sigma \kappa_{\text{P}}(T) T^4 = \dot{A}$$

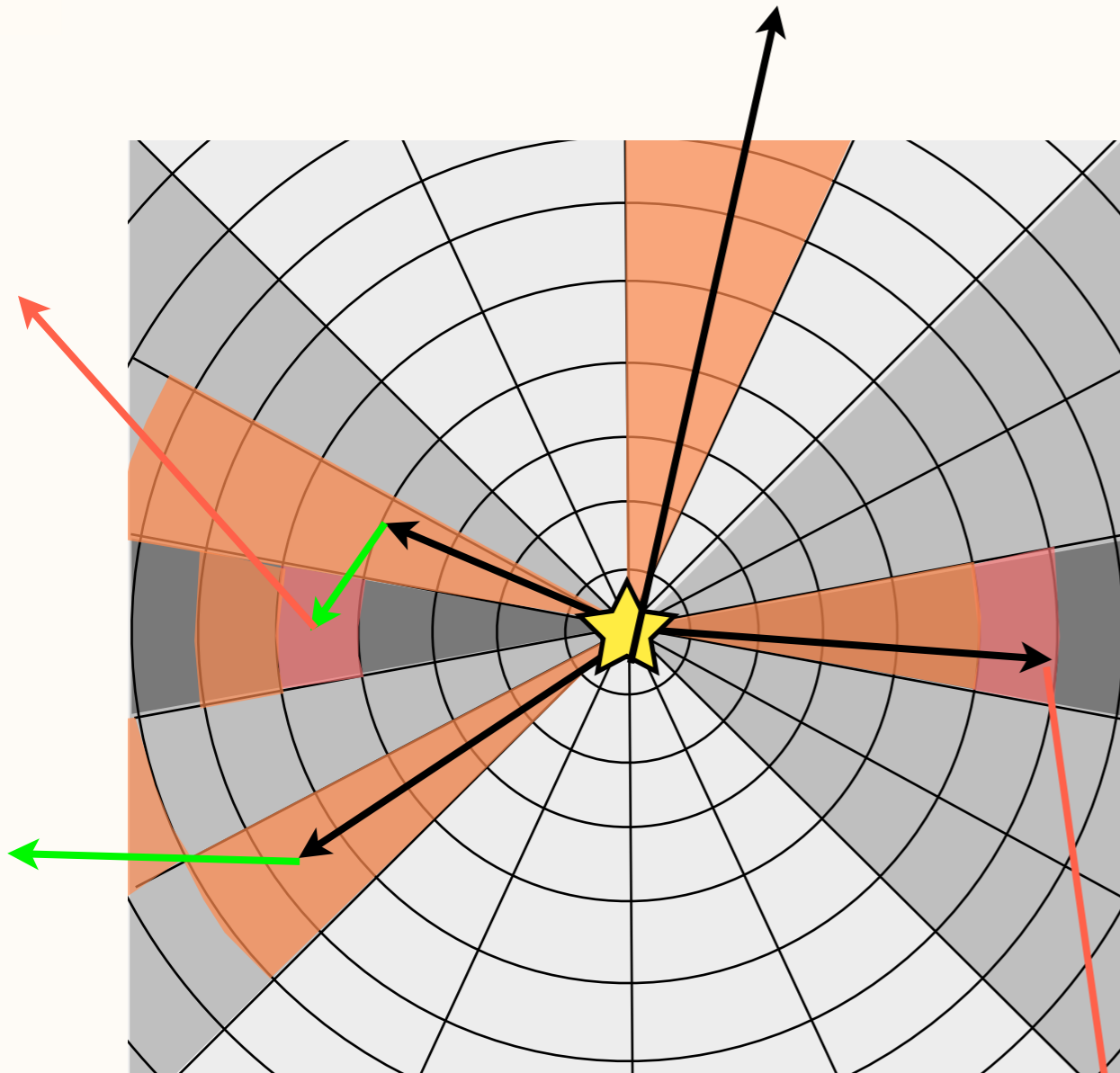
... but emissivity depends on temperature:

$$\frac{dI_{x,y,z,\theta,\phi,\nu}}{ds} = -\alpha_{x,y,z,\nu} I_{x,y,z,\theta,\phi,\nu} + j_{x,y,z,\theta,\phi,\nu}(T)$$

so once we have computed temperature, need to start over until temperature converges!

Alternatively, adjust emissivity on-the-fly (Bjorkman & Wood, 2001)

# Radiative Equilibrium: Computing temperatures



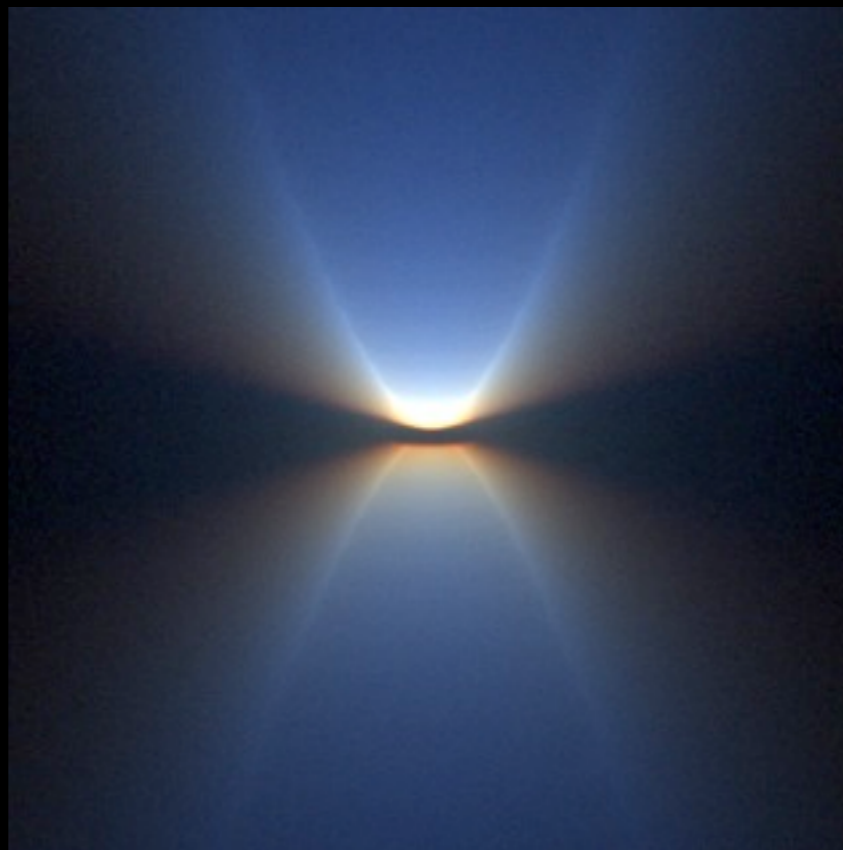
**Lucy (1999) algorithm:** count **possible** energy absorbed in each cell

$$= P(\text{absorption}) \times E_{\text{photon}}$$

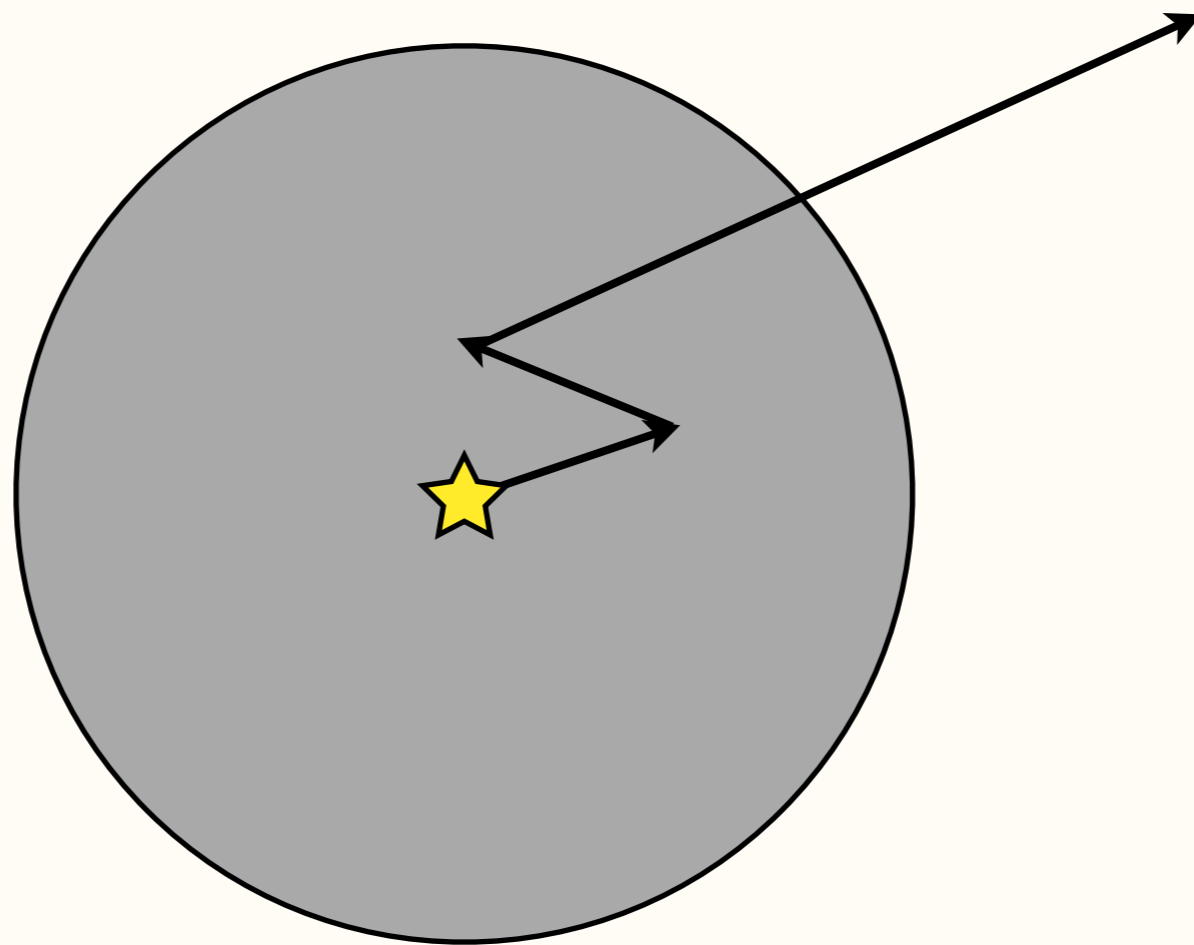
**Every** cell crossing counts



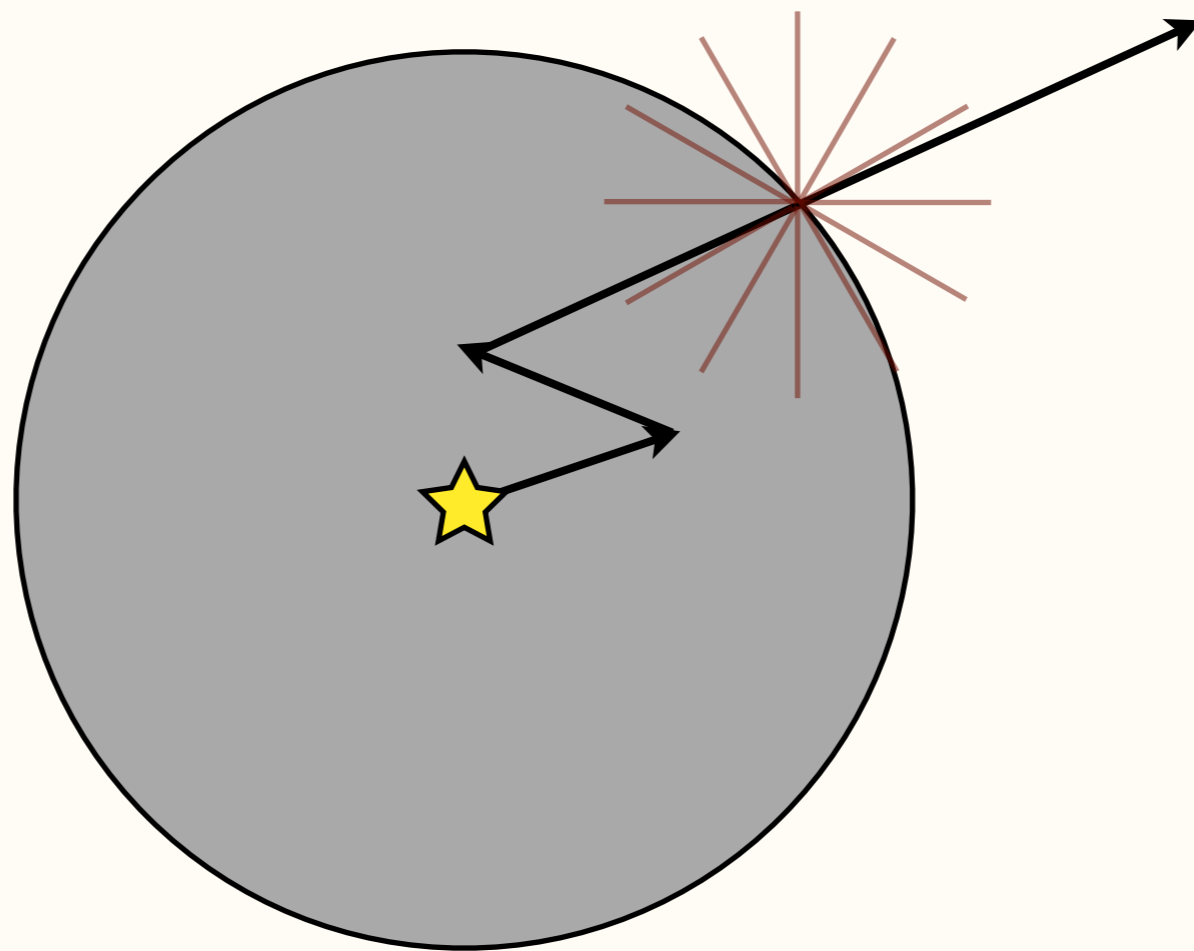
# Computing SEDs/Images



# Photon binning

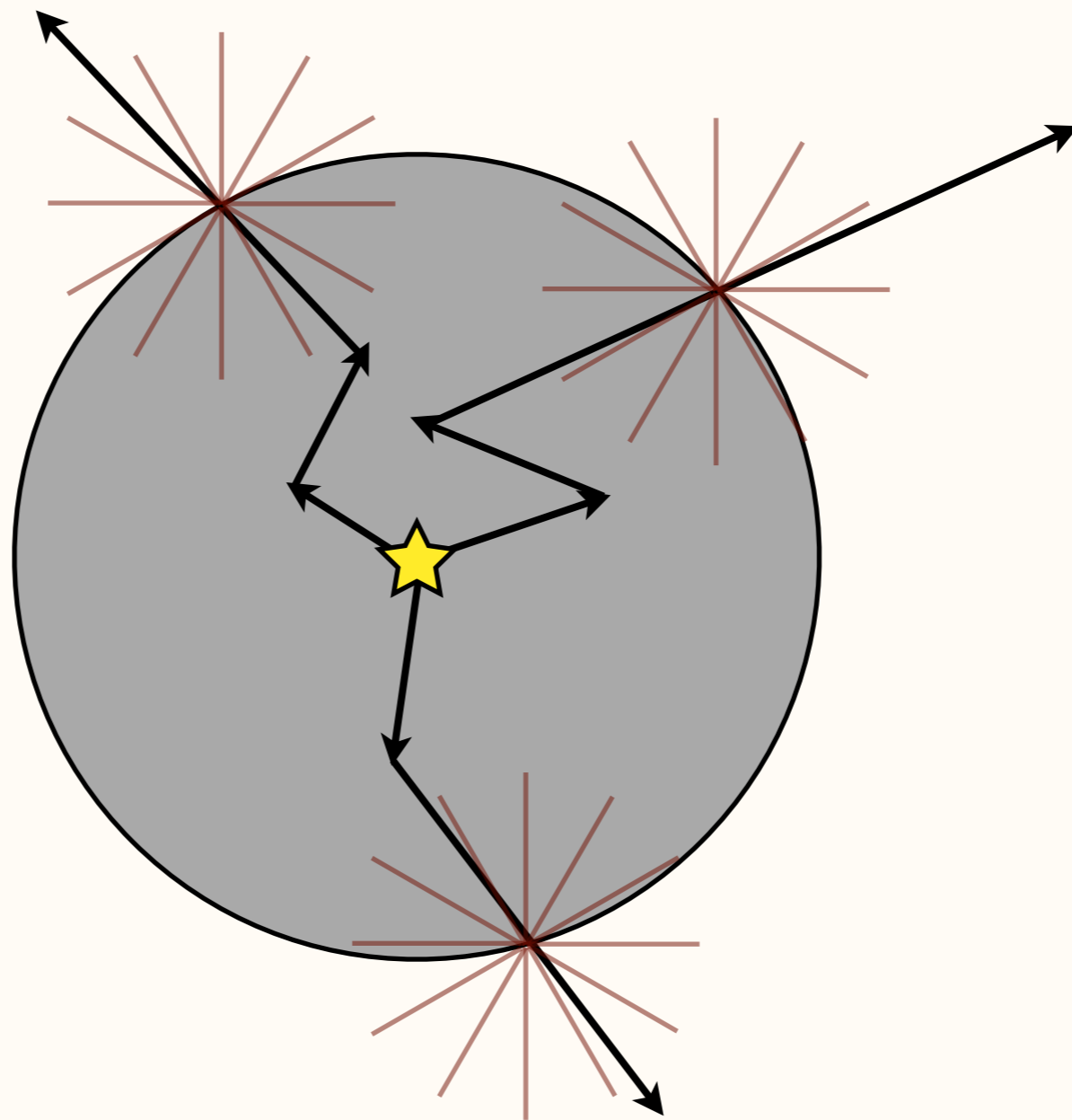


# Photon binning

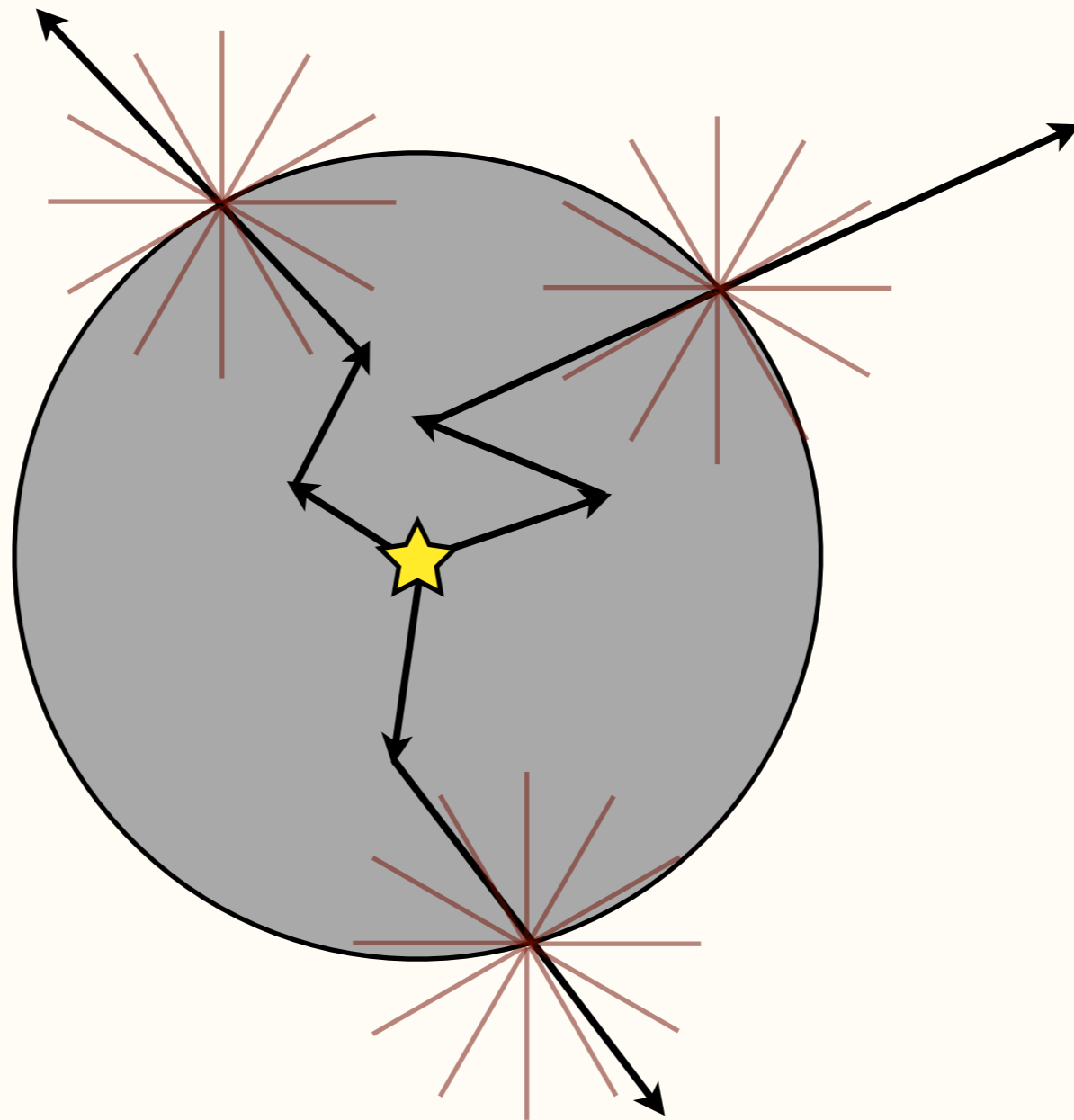




# Photon binning

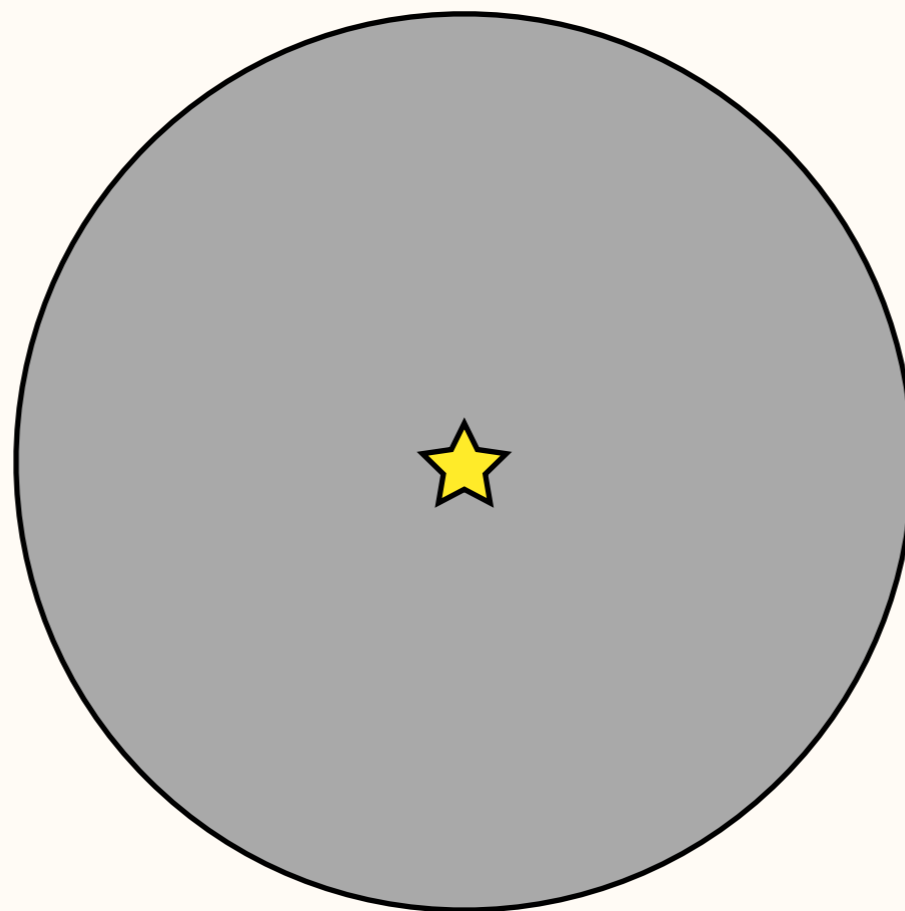


# Photon binning

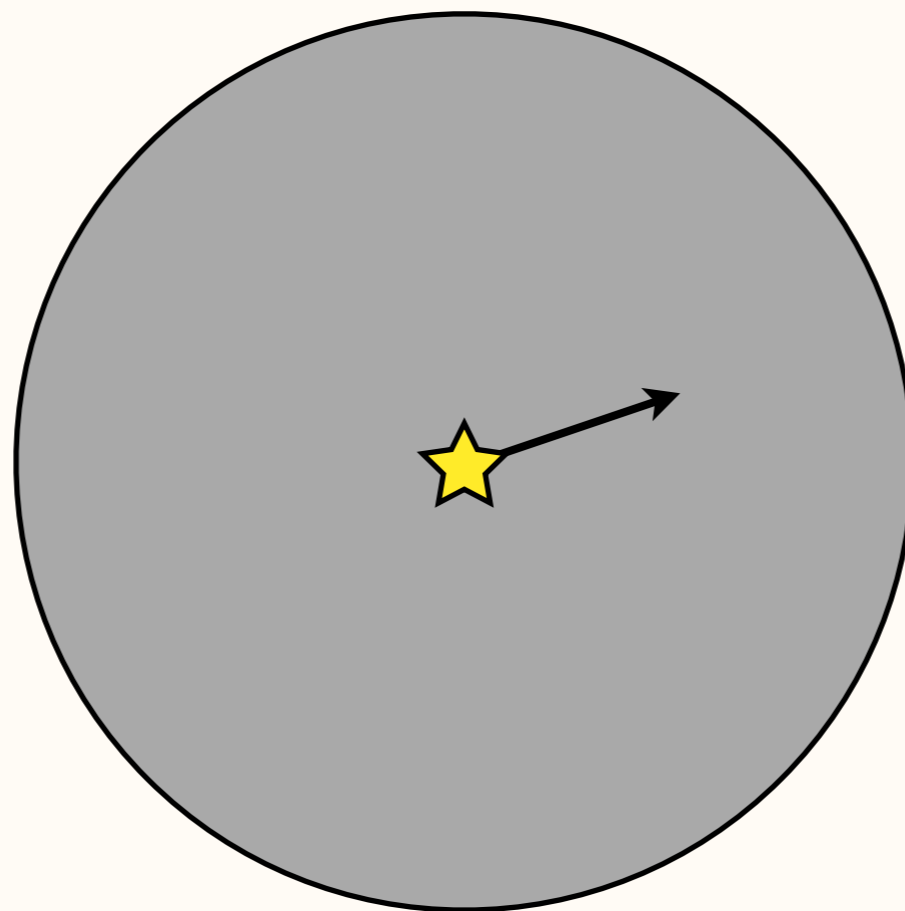


1 photon contributes to a single pixel, wavelength, and viewing angle

# “Peeling-off”

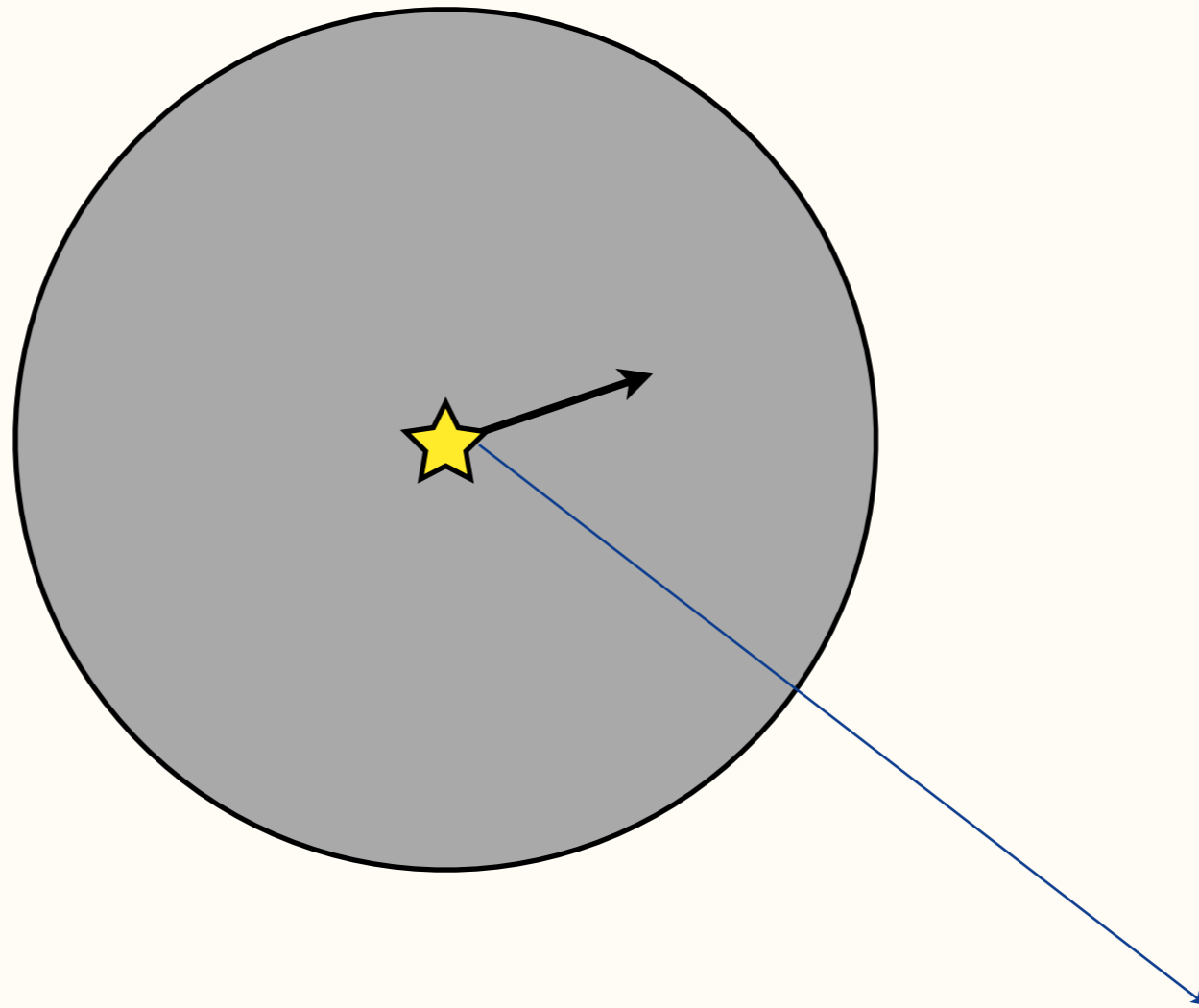


# “Peeling-off”



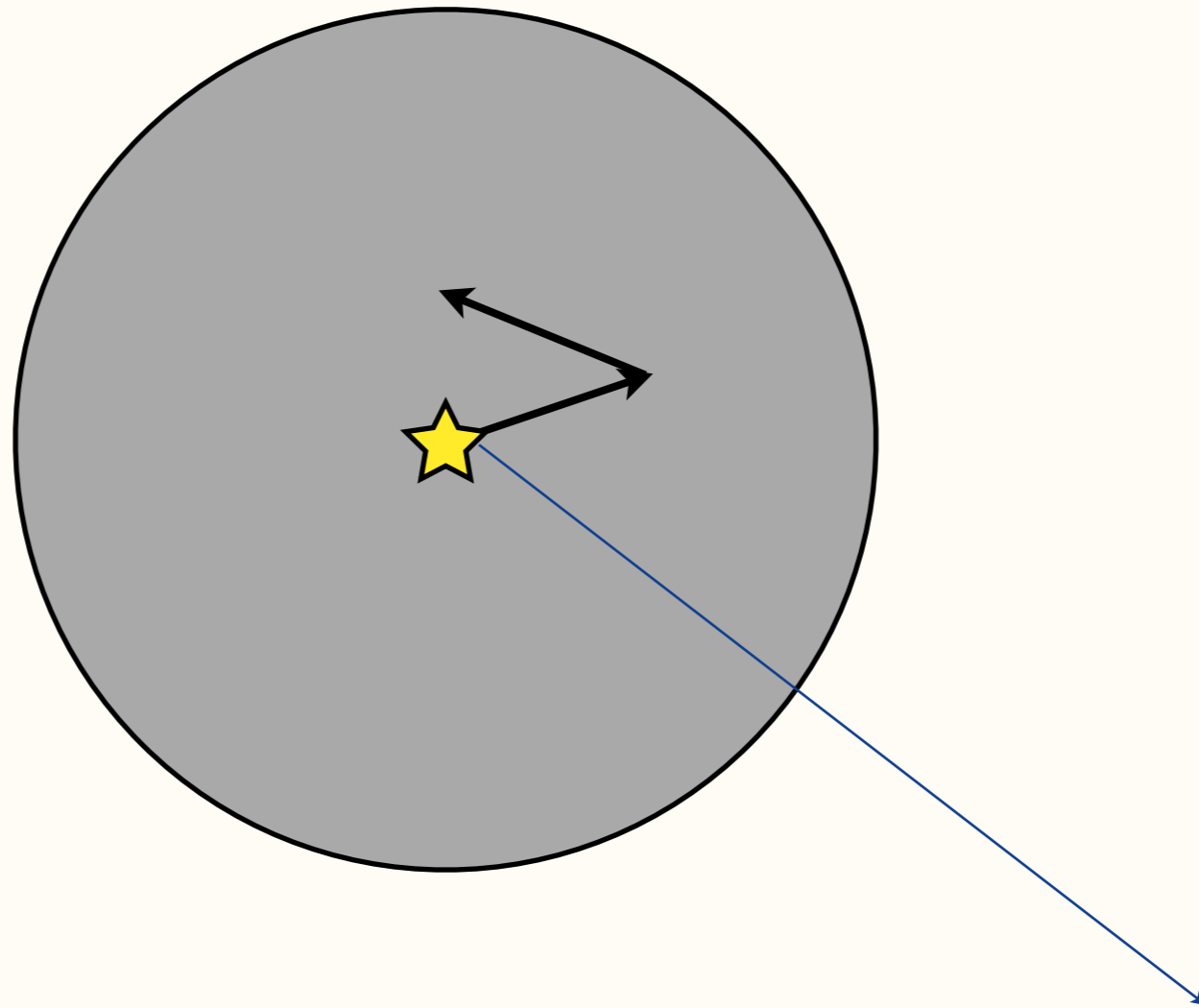


# “Peeling-off”



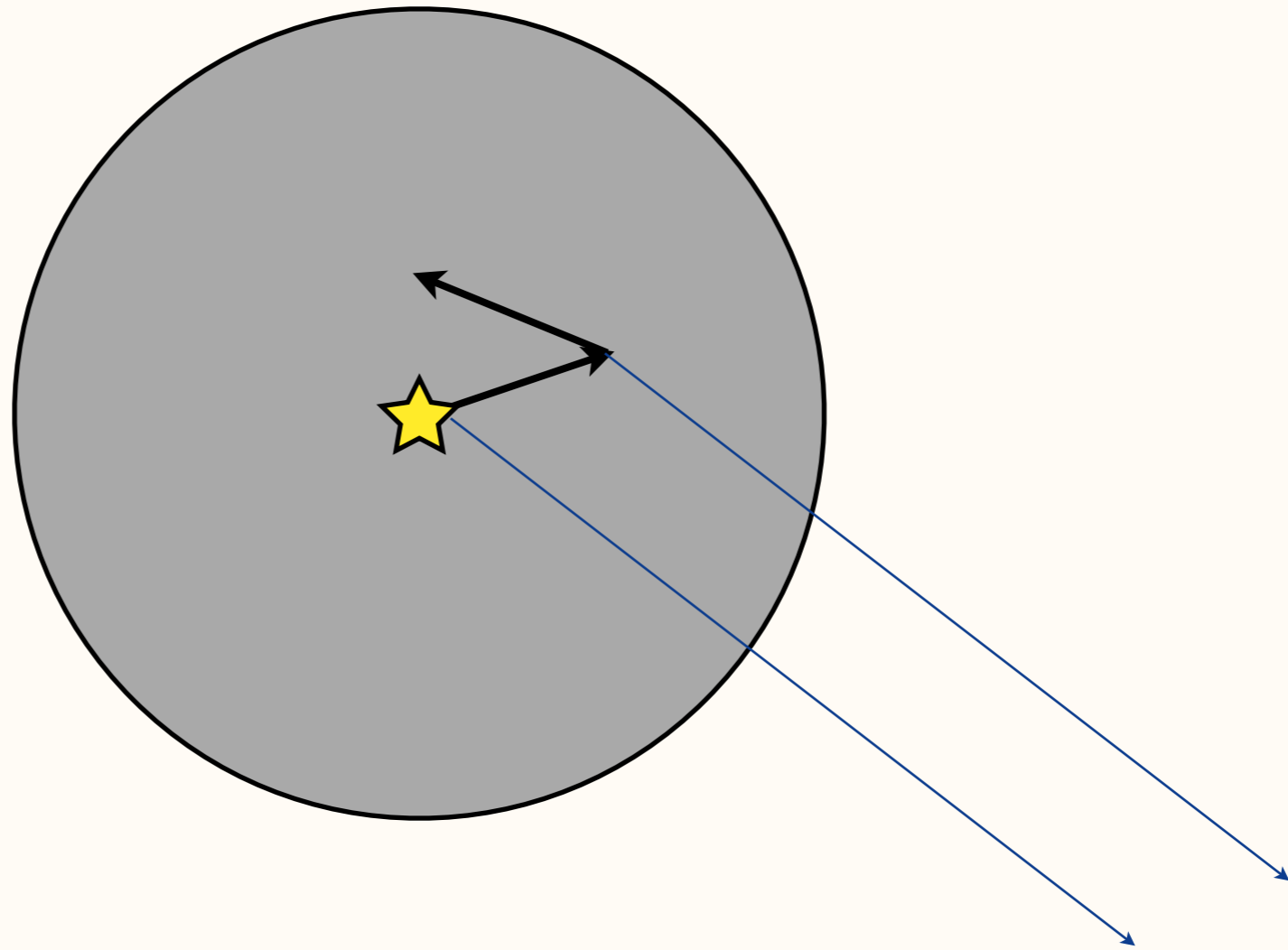
$$W = P(\theta, \phi) \cdot e^{-\tau_{\text{escape}}}$$

# “Peeling-off”



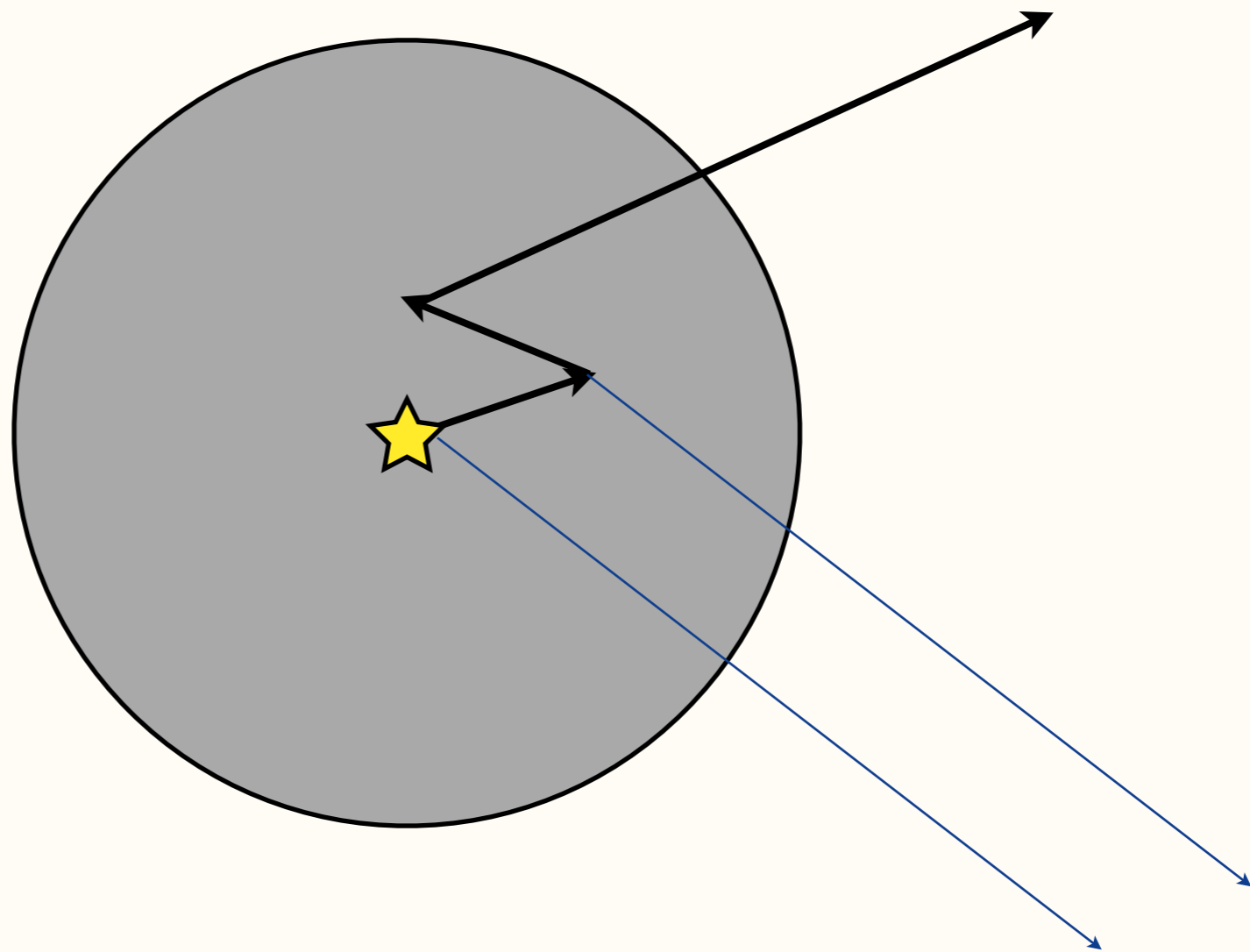
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# “Peeling-off”



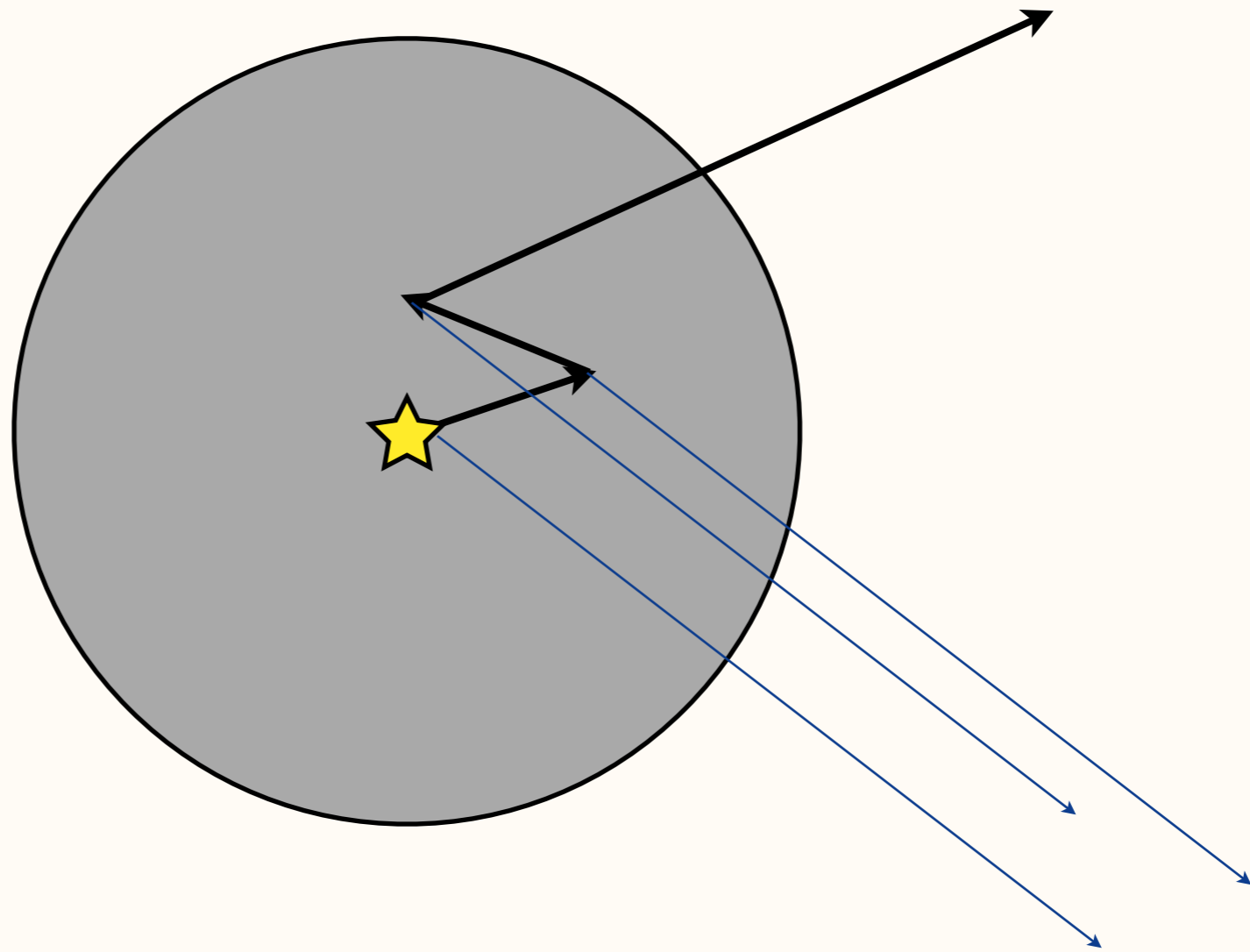
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# “Peeling-off”



$$W = P(\theta, \phi) \cdot e^{-\tau_{\text{escape}}}$$

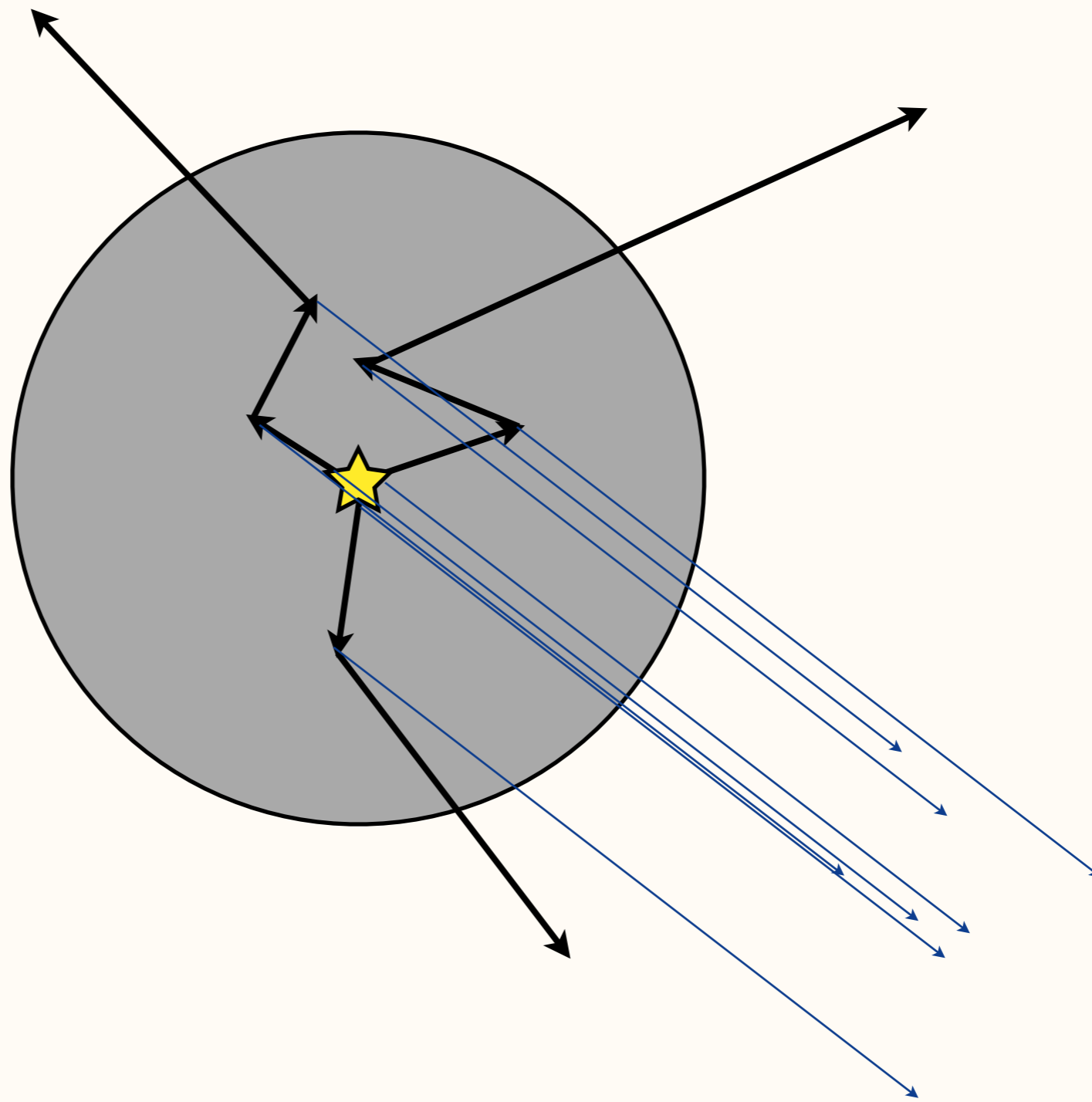
# “Peeling-off”



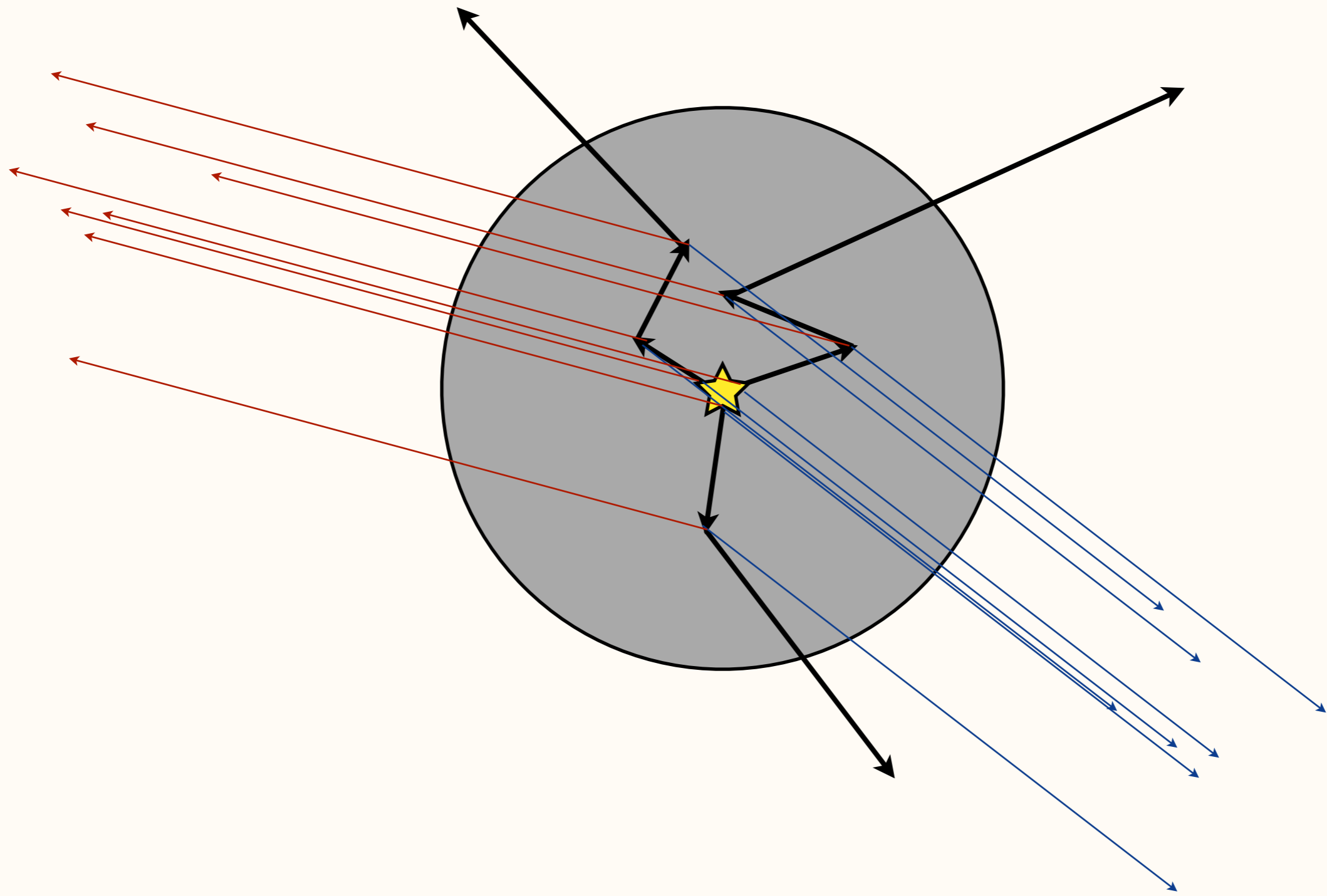
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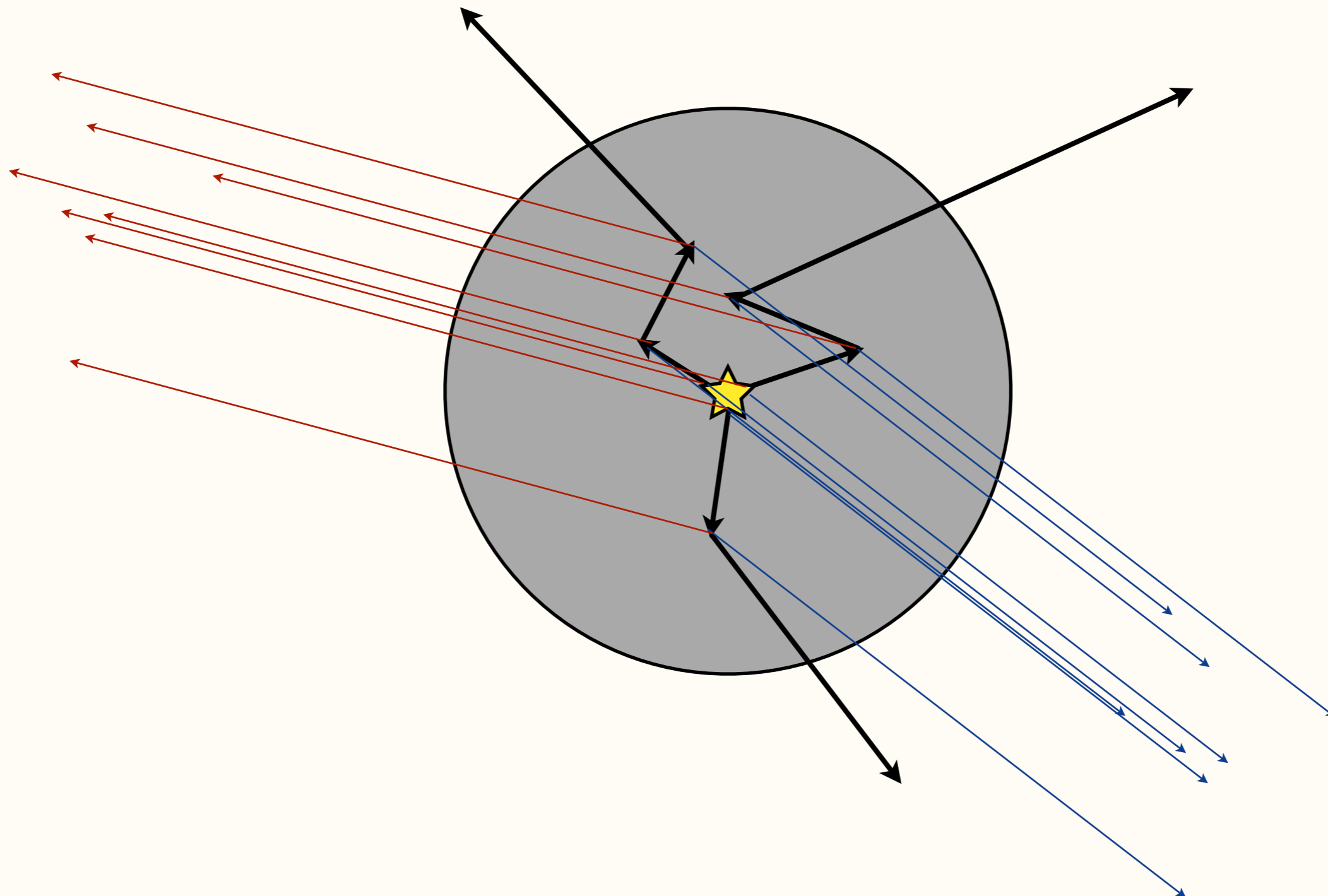
# “Peeling-off”



# “Peeling-off”

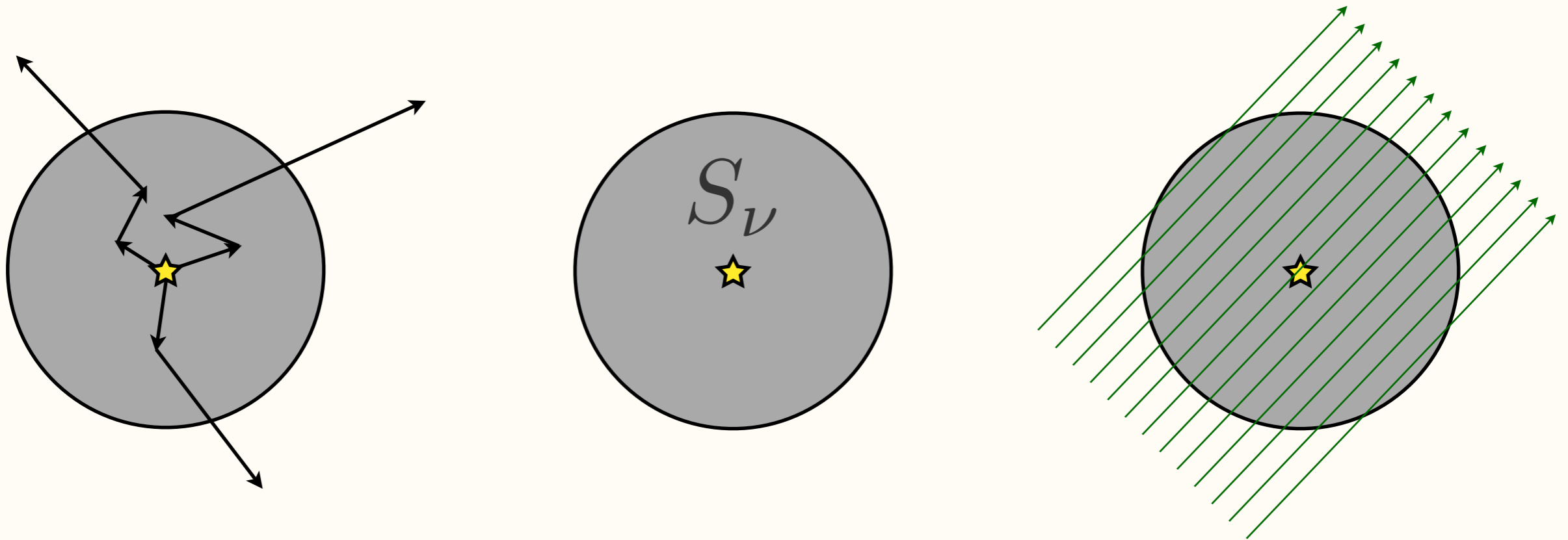


# “Peeling-off”



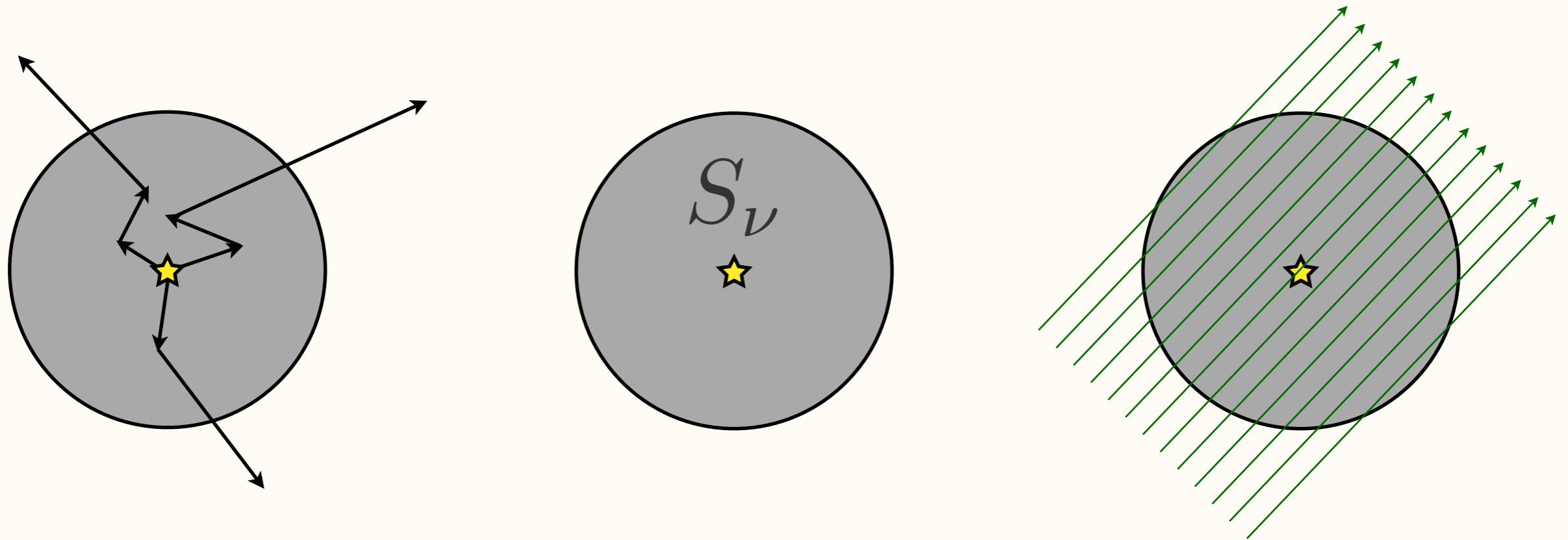
Every photon contributes multiple times to all viewing angles

# “Raytracing”



$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(t_\nu)e^{-(\tau_\nu - t_\nu)} dt_\nu$$

# “Raytracing”



$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(t_\nu)e^{-(\tau_\nu - t_\nu)} dt_\nu$$

Can solve all wavelengths at the same time, very efficient!

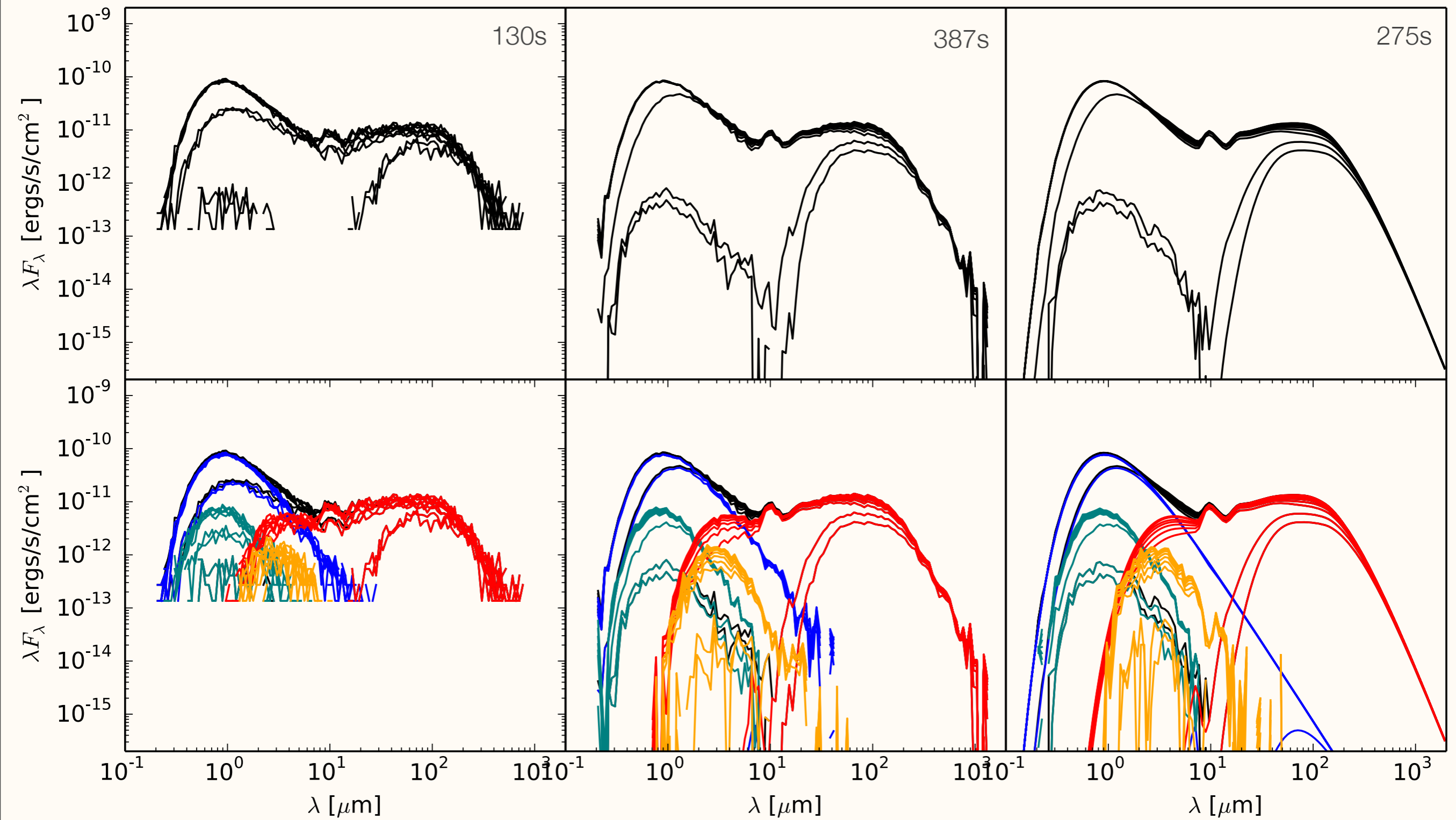


# “Raytracing”

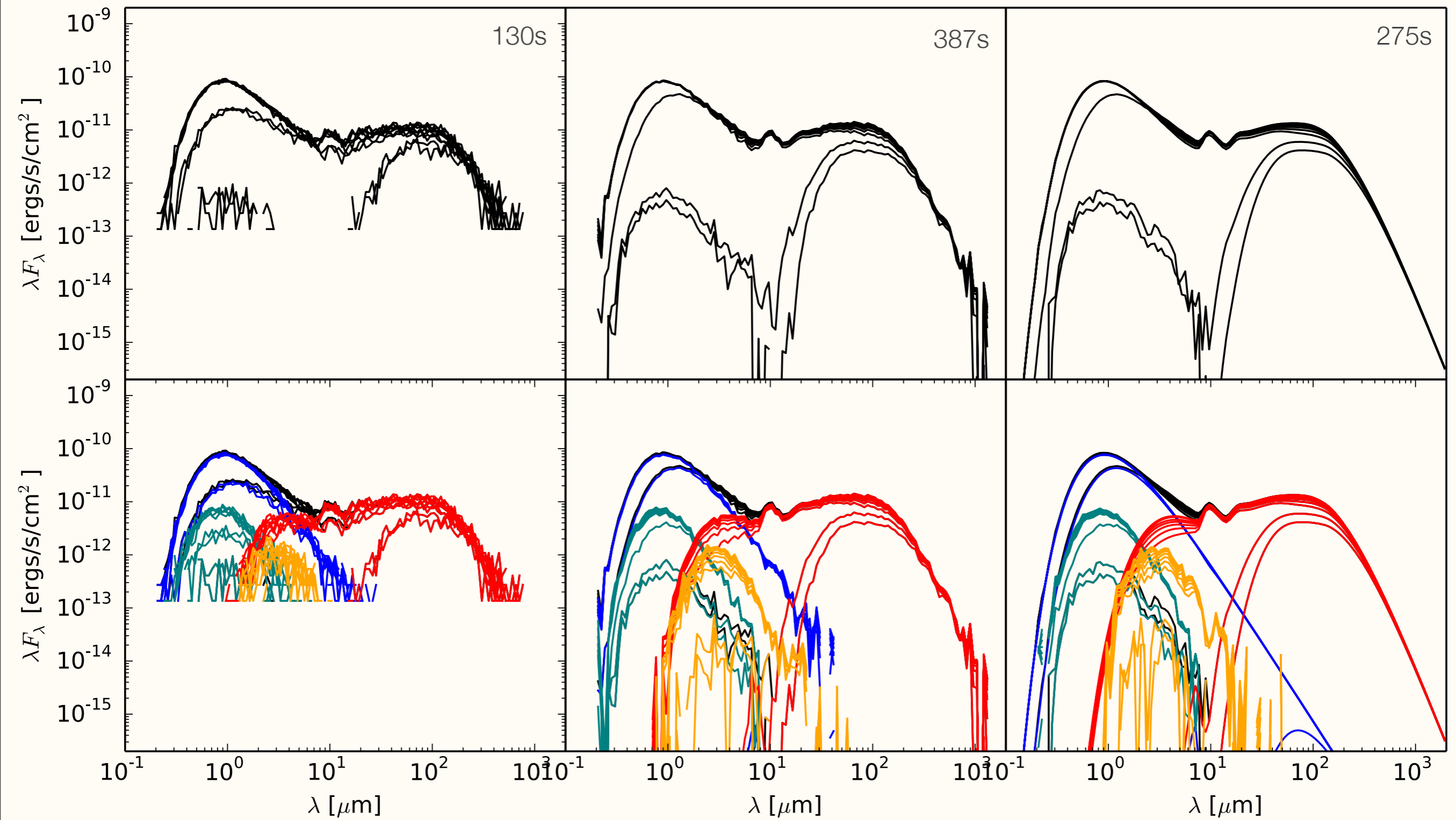
The downside of raytracing is that in the case of non-isotropic scattering, the source function can take up a LOT of memory:

$$S_\nu(\theta, \phi, x, y, z)$$

But for source and thermal emission, and for isotropic scattering, raytracing is **very** efficient.

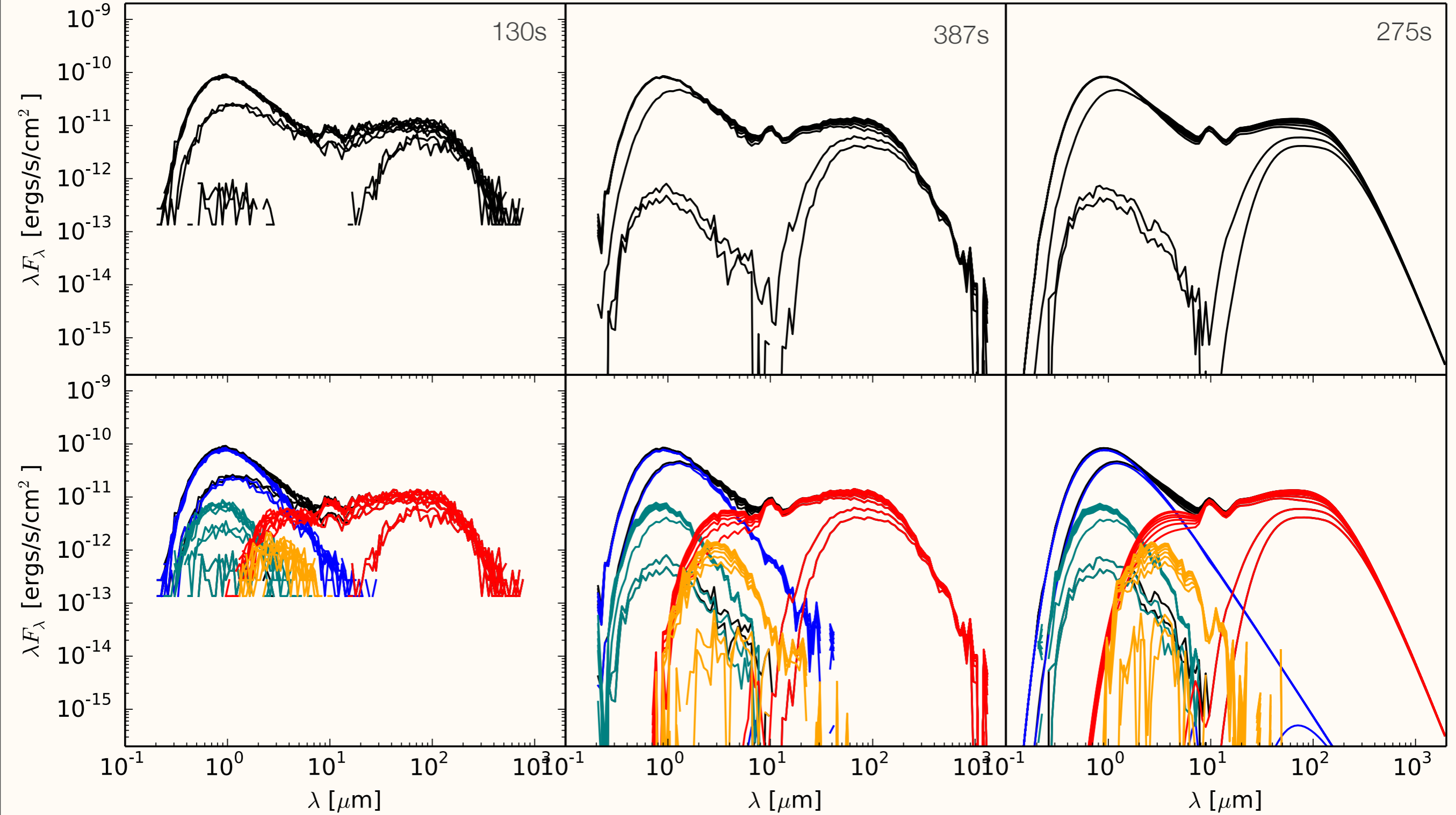


# Binned



Binned

“Peeling-off”



Binned

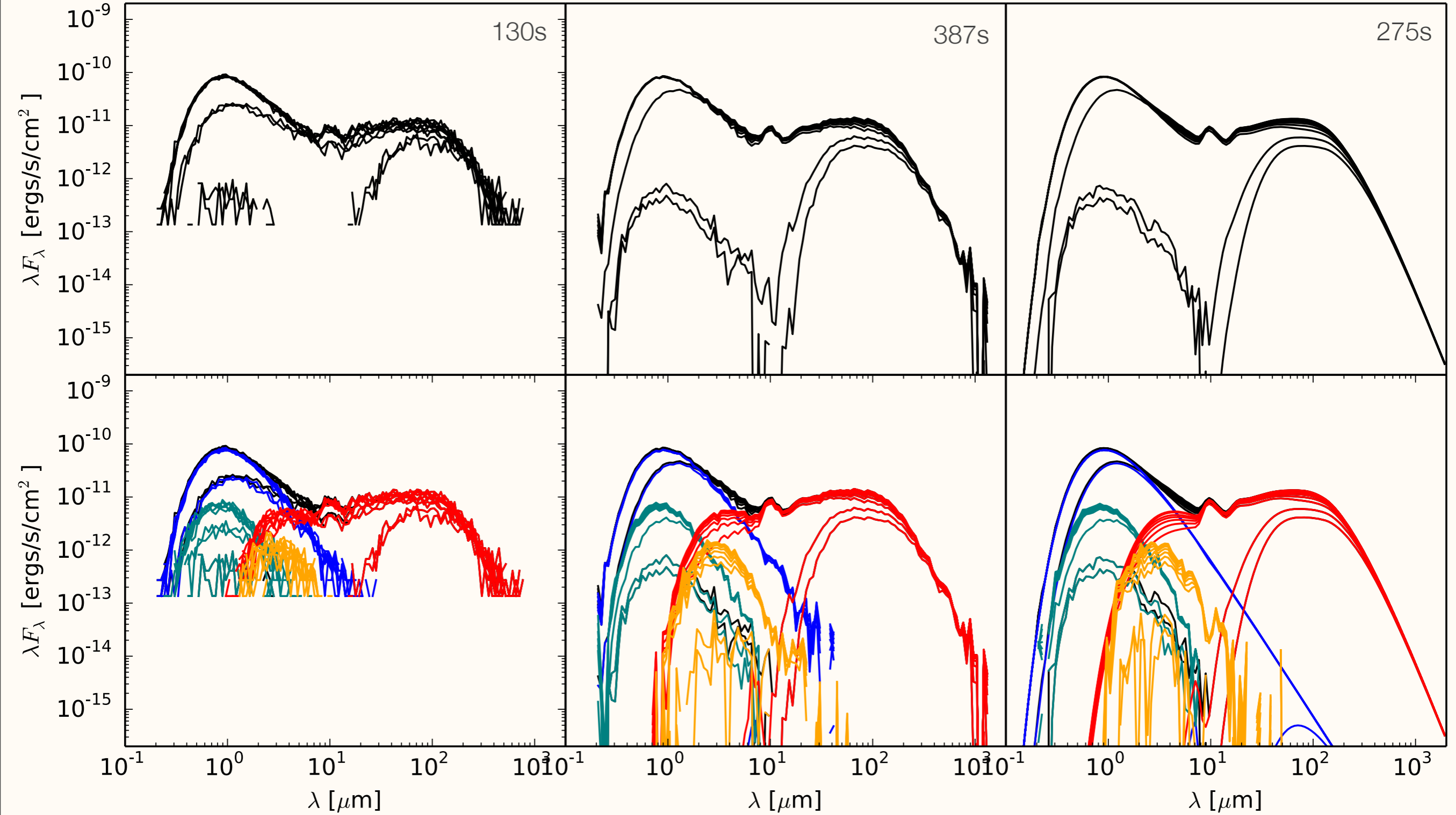
“Peeling-off”

“Raytracing”

130s

387s

275s

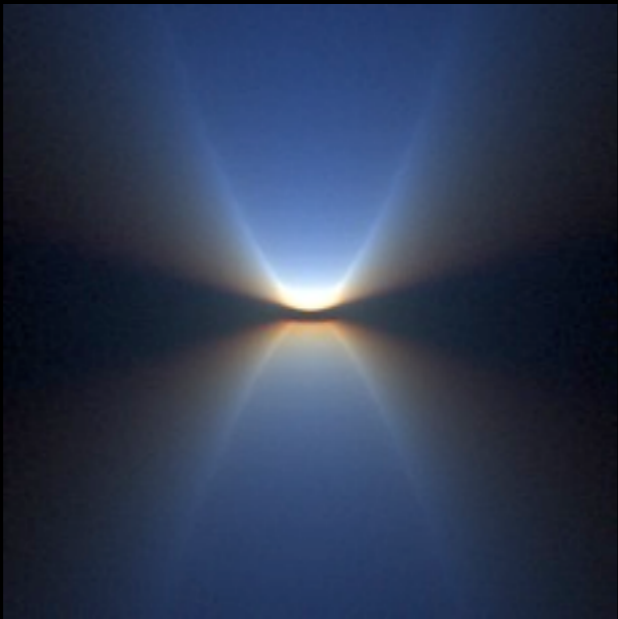




# Analytical model of a Young Stellar Object

1 - 3  $\mu\text{m}$

Scattered light + extinction



$6 \times 10^{14}$  m (4000 AU)

20 - 60  $\mu\text{m}$

Warm dust



400 - 800  $\mu\text{m}$

Cool dust

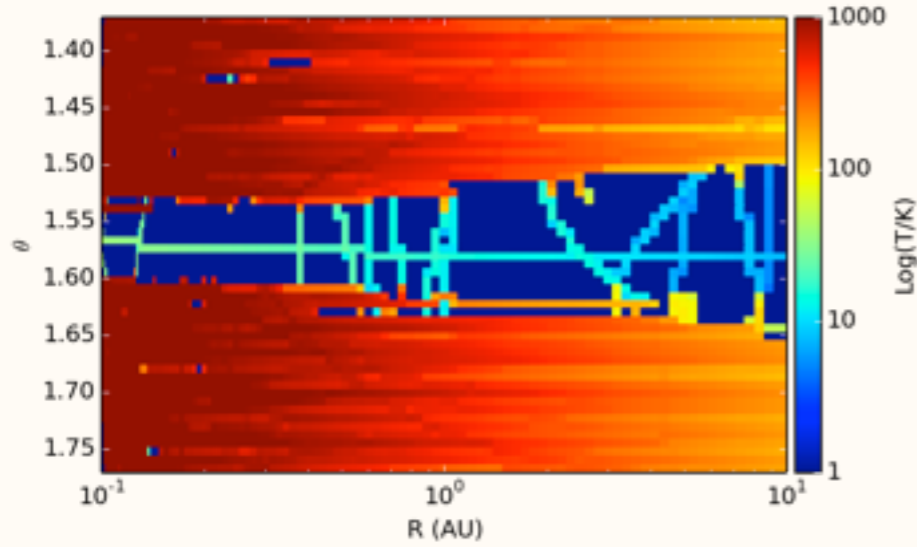
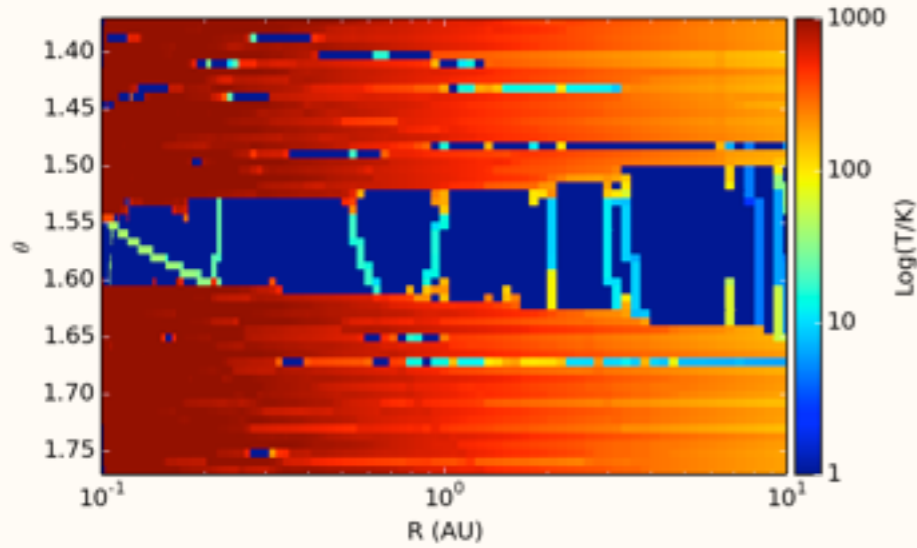
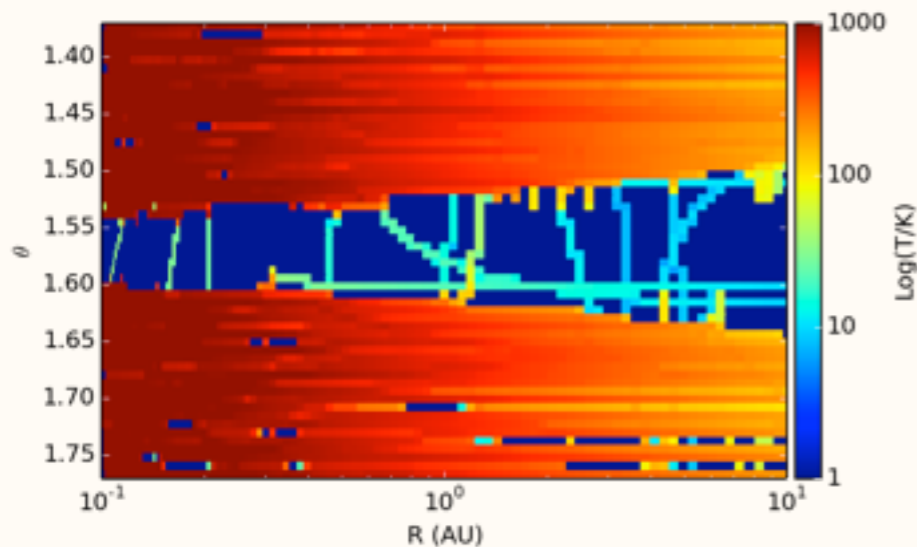


# Note on Parallelization

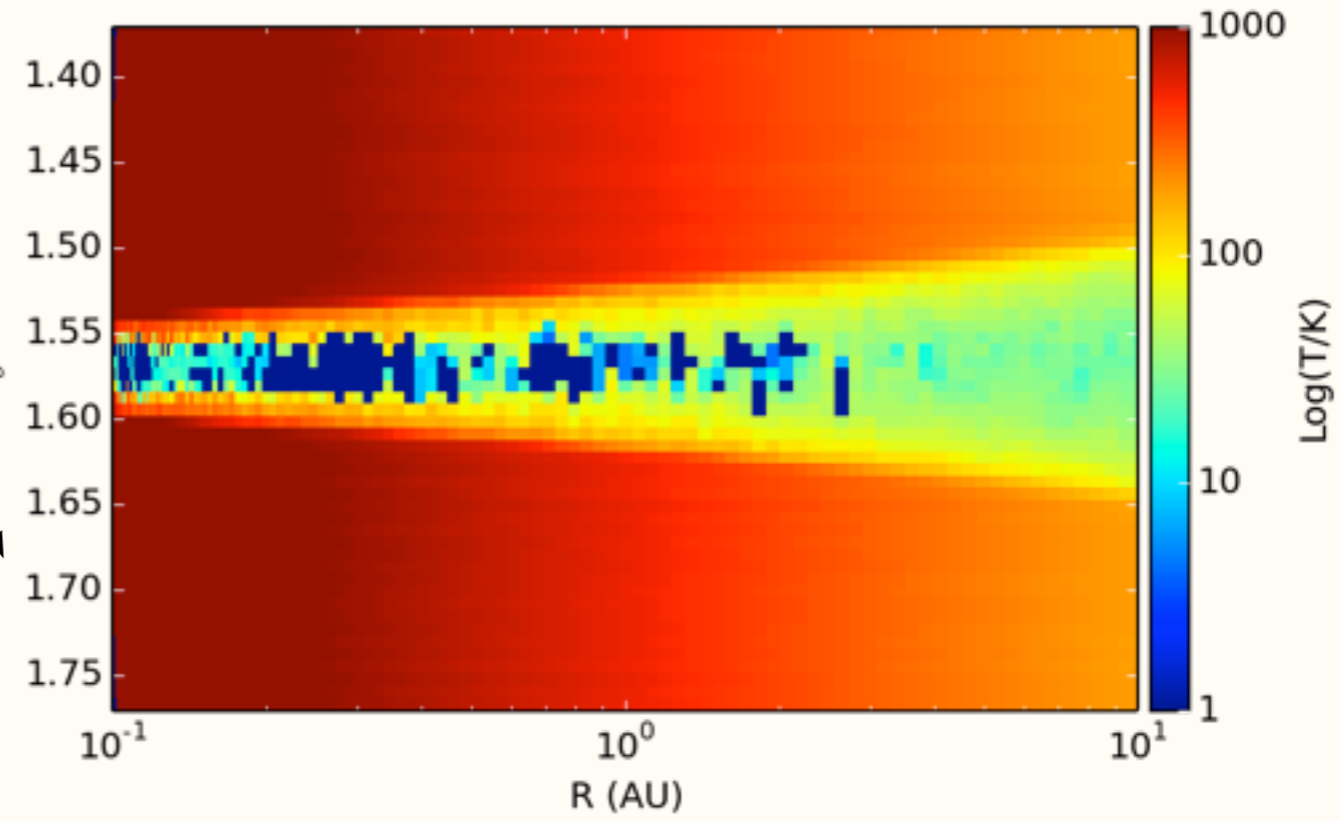
Monte-Carlo radiative transfer is extremely efficient in parallel (“embarrassingly parallel”)

Parallelization is **easiest and most efficient** with the **Lucy (1999, A&A 344, 282)** algorithm for temperature calculation, since the path lengths can be computed on separate cores and combined at the end of the iteration.

Simply split up photons by processes!



+97 more



and similarly for SEDs and images

Parallelization can provide **significant speedups!**