Gravity from Transactions: Fulfilling the Entropic Gravity Program

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We present new developments in Entropic Gravity in light of the Relativistic Transactional Interpretation (RTI). A transactional approach to spacetime events can give rise in a natural way to entropic gravity in the way originally proposed by Erik Verlinde, while also overcoming extant objections to that research program. The theory also naturally gives rise to a Cosmological Constant and to Modified Newtonian Dynamics (MOND) and thus provides a physical explanation for the phenomena historically attributed to "dark energy" and "dark matter".

Schlatter and Kastner (2023*), Journal of Physics Communications*,
https://iopscience.iop.org/article/10.1088/2399-6528/acd6d7

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- *“Space is in the first place a device introduced to describe the positions and movements of particles. Space is therefore literally just a storage space for information. This information is naturally associated with matter.”* --E. Verlinde
- At the quantum level, particles (really, quanta) are not localized (Heisenberg uncertainty principle; entanglement)
- Consider the quantum level as a substratum for the emergence of empirical spacetime—the *quantum substratum* (QS). This is a domain of physical possibilities (cf. Kastner, Kauffman, Epperson, 2018)
- Transactional formulation (RTI) describes how material quanta in the QS give rise to events that comprise empirical spacetime—the events are *actualities*.
- “Empirical spacetime” = an invariant set of actualized events capable of a covariant *description* in terms of a Lorentzian manifold. (The manifold is not to be identified with ontological spacetime.)
- **Quantifying the entropy cost of localization (relative to an inertial frame) quantifies the gravitational “force”**

Key terminology and concepts

1. quantum substratum: pre-spatiotemporal domain housing quantum systems with non-vanishing rest mass

2. spacetime: a storage device for measurement results and their structural connections (i.e., on-shell photons)

3. matter: quantum system with non-vanishing rest mass

4. holographic principle: description of a spacetime region can be thought of as encoded on a lower-dimensional boundary to the region

5. emitter: an elementary bound matter system in an excited internal state (e.g., atom or molecule)

6. absorber: an elementary bound matter system subject to excitation of its internal states (e.g., atom or molecule)

7. mass (m) M: a fermionic bound state in its local rest frame with rest mass, (m) M, possibly composed of N elementary systems

8. transaction: a non-unitary process ultimately leading to transfer of energy (and other conserved quantities) in the direct-action (absorber) theory of fields (not available in the standard QM formulation)

9. actualized transaction: the transfer of a real photon from one emitter to one absorber, generating an emission and absorption event and their invariant connection (null interval). The latter constitute elements of spacetime.

https://youtu.be/wVej0D0tPg8

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- In RTI, "spacetime" itself is just an invariant set of actualized events (consistent with Einstein's view) and their structural connections—*in effect, information "made available."* This set grows as new events are added with every transaction. A transaction effectively associates a material system with a localized "spacetime region", in terms of a Lorentzian manifold (which we do not reify but consider as a covariant *description* of invariant events).
- Holographic principle gains a new interpretation in this picture: the holographic screen can be thought of as the boundary between the quantum substratum and the newly emergent region of spacetime (where "region" just means the covariant description of the newly added event(s)).

In Schlatter & Kastner (2023), we show:

(1) that transactions are the physical basis of the entropic force, which is used in Verlinde (2011) to derive Newtonian gravity. (Our approach yields the effective Unruh acceleration.)

(2) that photons exchanged in transactions can serve as clocks, naturally inbuilt to the emergent sets of events, whose gauging leads most generally to Einstein's equations on a Lorentz-manifold.

(3) photon exchange leads to a cosmological term Λ corresponding to so-called “Dark Energy,” with a different physical origin than the “vacuum energy” presumed in standard QFT. Specifically, Λ arises from the effective radiation pressure of transferred photons, which drives the expansion.

(4) The synchronization of these light-clocks from the perspective of different observers further leads to the MOND correction of Newtonian gravity as calculated in Milgrom (1999, 2015).

Milgrom, M. (2015). MOND theory. *Can.J.Phys. 93:2*, 107.

Milgrom, M. (1999). The modified dynamics as a vacuum effect? *Phys.Lett.A 253*, 273.

Verlinde, E. (2011). On the origin of gravity and the laws of Newton. *J. High Energ. Phys.* 29. https://doi.org/10.1007/JHEP04(2011)029

(1) Transactional basis of entropic force

- Can assign a generic information-content to idealized points x_0 in a spatial hyperplane, $x_0 \in \mathbb{R}^3$, which can be associated with matter or not (as a result of a localizing transaction).
- The one-bit entropy associated with the information content of an idealized point x_0 is

$$S(x_0) = k_B I(x_0) = \frac{k_B}{2}$$

If we take into account that localization involves a finite Planck volume, the physical entropy for a single bit gains a factor of 4π,

$$S(\text{per bit}) = 2\pi k_B$$

Transactions, holographic principle, and entropy

Entropy S(R) of a sphere S_R of radius R enclosing a source mass M arises from a holographic principle, in terms of the number N of *possible absorptions on* S_R. Corresponds to the number of bits N(R), where each bit has information $I = \frac{1}{2}$ and Entropy $S_{bit} = 2\pi\ k_B$.

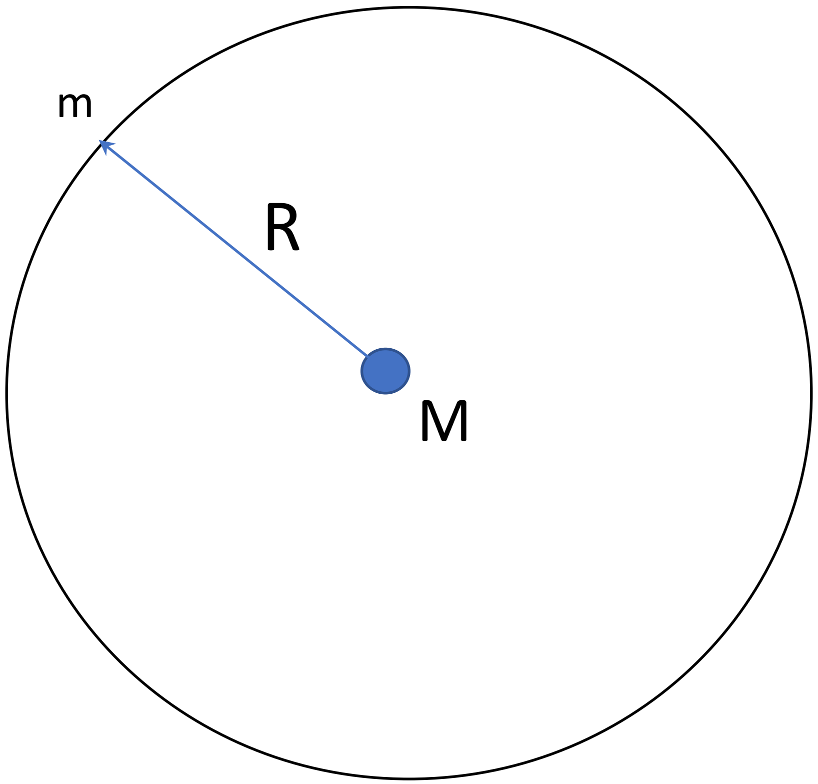
N(R) = A(R)/ $4\pi l^2_P$ = $\frac{R^2 c^3}{\hbar G}$

Thus, S(R) = $2\pi\ k_B N(R) = k_B \frac{2\pi R^2 c^3}{\hbar G}$

Can define a formal temperature T(M,R) in trms of the energy contained in M, and the entropy S(R) :

$$Mc^2 = S(R) \cdot T(M,R) = k_B \frac{A_R}{2 l_P^2} T(M,R).$$

$$\Rightarrow\ T(M,R) = \frac{\hbar MG}{2\pi c k_B R^2} = \frac{\hbar g_R}{2\pi c k_B}, \text{ where } g_R = \frac{GM}{R^2}$$

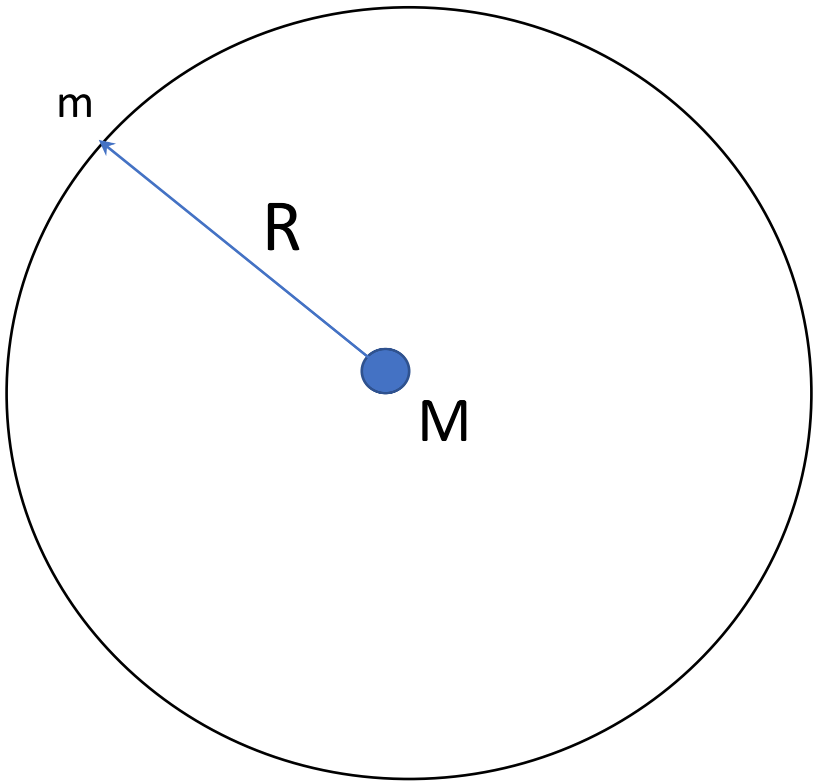


A transaction, resulting in exchange of a real photon from emitter to absorber, breaks unitary symmetry as well as time symmetry. Localizes emitter & absorber, generates a spacetime interval.

WRT a local inertial frame, a time interval Δt_R and distance R are generated by the transaction, where R= c $\Delta t_{R.}$.

Following Bekenstein's and Verlinde's argument, the information made available due to localization of the test mass *m* at some distance *r* within one Compton wavelength λ_C of S_R is proportional to its distance Δx from $S_{R:}$

$$\Delta S(r) = 2\pi k_B \frac{mc}{\hbar} \cdot \Delta x, 0 \leq \Delta x \leq \lambda_C$$



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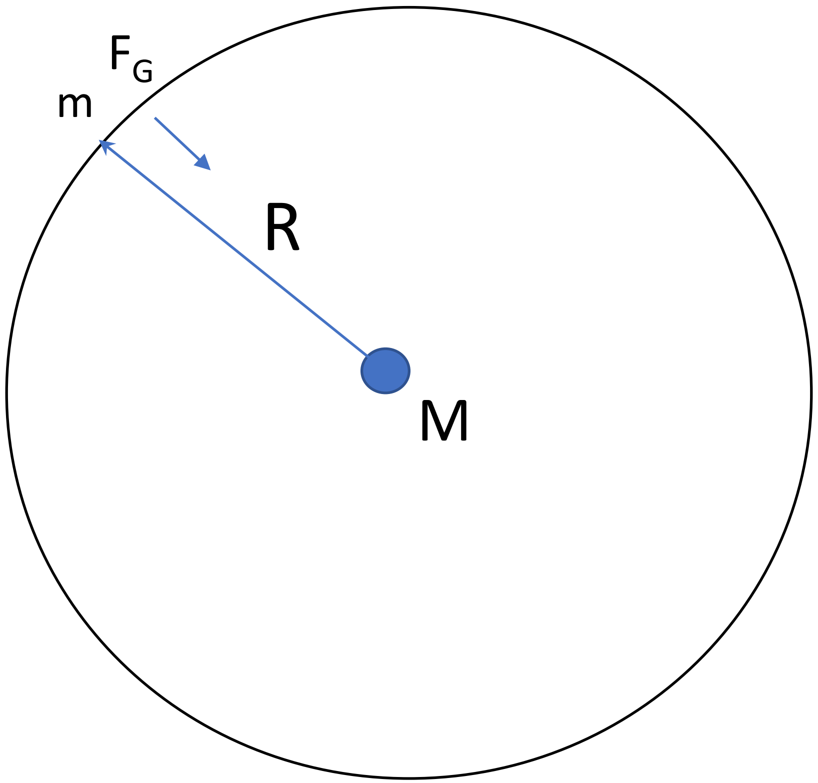
Localization of *m* can be thought of as the application of a constraint away from the equilibrium configuration of the combined system *M+m*, so that the entropic restoring force F_G points towards *M*.



Effective work due to entropic restoring force (F_G)

From E = ST, with T(M,R), we find the energy ΔE associated with ΔS:

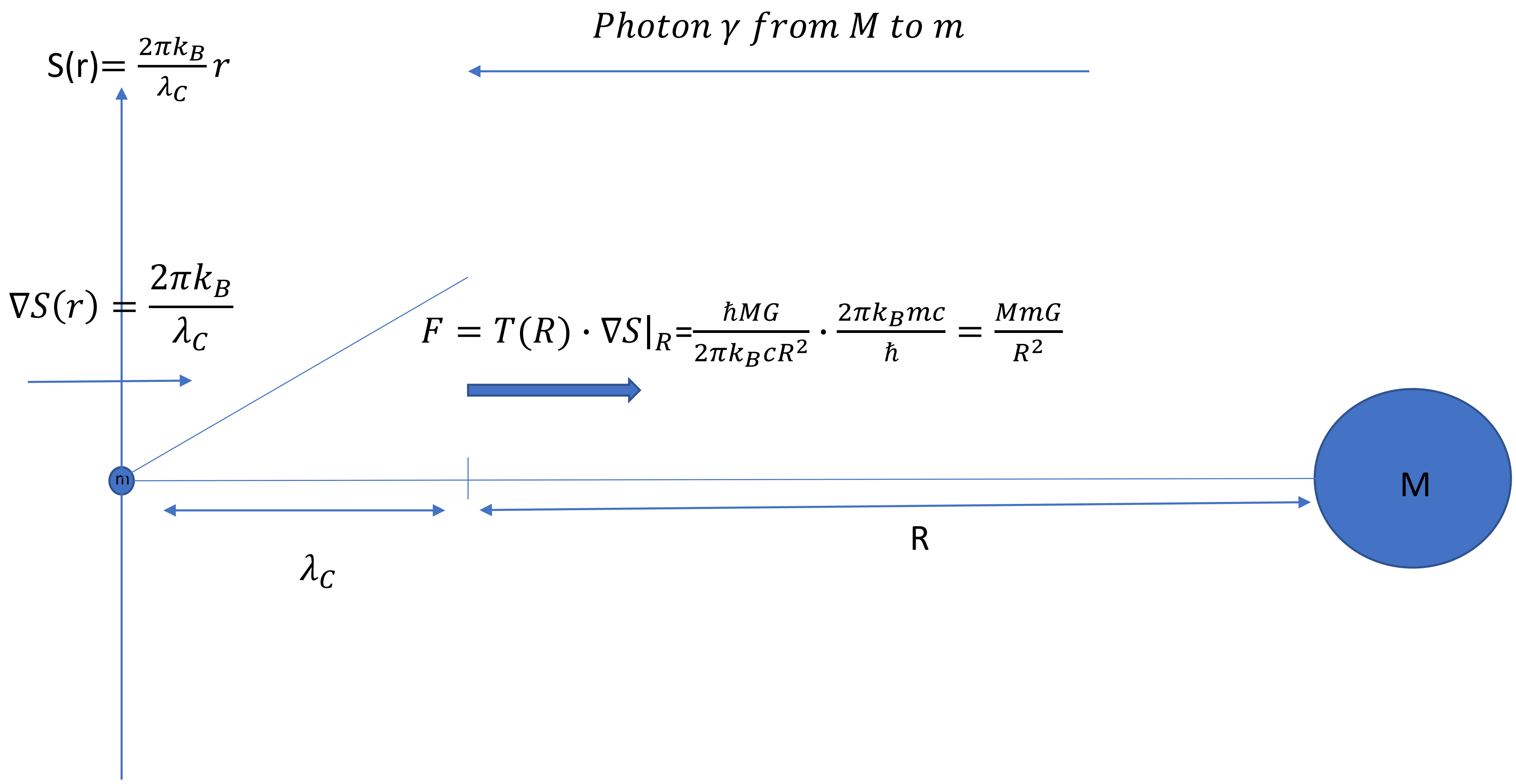
$$\Delta E = \Delta S \cdot T(M,R) = 2\pi k_B \frac{mc}{\hbar} \frac{\hbar M G}{2\pi c k_B R^2} \Delta x = F_G \Delta x,$$

where

$$F_G = m\, g(M,R) = \frac{mMG}{R^2}$$

Arriving at Verlinde's result by way of the information made available via the transaction, which acts as a constraint away from equilibrium.

Can also consider the restoring force as the entropy cost associated with the localization (2nd law; Landauer's conjecture)



$Photon\ \gamma\ from\ M\ to\ m$
S(r)= $\frac{2\pi k_B}{\lambda_C} r$
$\nabla S(r) = \frac{2\pi k_B}{\lambda_C}$
$F = T(R) \cdot \nabla S|_R = \frac{\hbar MG}{2\pi k_B c R^2} \cdot \frac{2\pi k_B mc}{\hbar} = \frac{MmG}{R^2}$
m
M
λ_C
R

(2) Thermal clock + transactions ⇒Einstein's equations

- Transactions, by the exchanged photons, embody naturally inbuilt clocks—*thermal clocks*. These measure the rhythm of becoming by an invariant gauge c, the speed of light. **This provides a way to derive a metric structure of spacetime, locally governed by the Einstein equations.**
- (Also provides a reason for the clock-hypothesis: that duration, measured by clocks, is proportionate to the length of their respective world lines)
- Thermal environment of temperature T_0 (supported by ongoing transactions) defines a uniform time step Δt for a system to change state:* $\Delta t = \frac{h}{4k_B T_0}$
- Can define a "Thermal time" τ that measures proper time ds in units of Δt :

$$d\tau = \frac{ds}{\Delta t} = \frac{4}{h} k_B T_0 ds$$

*E.g.: Margolus & Levitin , 1998

Consider a test system with mass m at a sufficiently close distance R from a mass $M \gg m$.

Assume that both systems are instantaneously at relative rest. The test system will; feel an acceleration g_R in a thermal environment $T_{g_R} = T(M, R)$.

In Rindler coordinates: (θ, ϱ, y, z); $\varrho = \frac{c^2}{g_R}$

Thermal time becomes: $d\tau = \frac{ds}{\Delta t} = \frac{4}{h} k_B T_0 c \, d\vartheta$

Multiply $\frac{d\tau}{d\vartheta}$ by $\frac{c}{g_R}$, we get a velocity:

$$v_{g_R} = \frac{4}{h} k_B T_{g_R} \frac{c^2}{g_R}$$

Taking into account that the acceleration g_R arises from the exchange of photons, this velocity must be identified with that of the light-clock*:

$$\frac{c}{\pi^2} = \frac{4}{h} k_B T_{g_R} \frac{c^2}{g_R}$$

*See Schlatter (2022) for factor of π^2

Solving for T gives the Unruh temperature: $T_{g_R} = \frac{\hbar g_R}{2\pi k_B c}$

Thus, the "gravitational field" as expressed by g_R can be understood as arising from the accelerating effect of transactions in a thermal environment defined by the gravitating mass M.

Starting point for deriving Einstein's equations

Recalling $E = Mc^2 = S(S_R) \cdot T(M,R) = k_B \frac{A_R}{2l_P^2} T(M,R)$

Putting in the expressions for T(M,R) due to Unruh acceleration g_R (and l_P) gives us:

$$g_R A_R = \frac{4\pi G}{c^2} E \quad (*)$$

Time development of (*) and increasing generalization yields Einstein's equations:

If $V_R(t)$ denotes the volume of a small ball of test systems at radius $R(t)$ around M, with $R(0) = R, \dot{R}(0) = 0$ and $\ddot{R}(0) = g_R$, (*) becomes:

$$\frac{d^2}{dt^2}|_{t=0} V_R(t) = \frac{4\pi G}{c^2} E.$$

Introduce zero-component of the energy- momentum tensor via $T_{00} = \lim_{R\to 0} \frac{E_R}{V_R}$ for energy density at origin (RHS); introduce zero-component of the Ricci tensor (LHS) : $\frac{\ddot{V}_R}{V_R}|_{t=0} \xrightarrow{R\to 0} c^2 R_{00}$.

(*) becomes:

$$R_{00} = \frac{4\pi G}{c^4} T_{00}$$

(3) Cosmological constant Λ arises due to the effective energy density of ongoing transactions

$$\Lambda = -\frac{4\pi G}{c^4}\overline{P}_\gamma = \frac{4\pi G h}{c^3}\varrho_\gamma = 4\pi^2 l_P^2 \varrho_\gamma,$$

"the thermal excitations responsible for the de Sitter entropy constitute the positive dark energy"-Verlinde (2017)

where $\overline{P}_\gamma$= pressure (energy density) due to photon momentum,

$$\overline{P}_\gamma = -c \cdot h \cdot \varrho_\gamma.$$

ϱ_γ = avg. transaction rate

This generalizes the Einstein equations to: $R_{00} + \Lambda\delta_{00} = \frac{4\pi G}{c^4} T_{00}$

Including nonvanishing matter flow, standard transformations wrt all local inertial frames allows full generalization:

$$R_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}\right),$$

(4) MOND correction

Consider deSitter metric with horizon R_0, Hubble parameter H_0, horizon acceleration a_∞:

$$a_\infty = cH_0 = \frac{c^2}{R_0} = c^2\sqrt{\frac{\Lambda}{3}}$$

Can define a general de Sitter acceleration: $a(r) = \frac{r}{R_0}a_0$, where $a_0 = \frac{a_\infty}{2}$

and related de Sitter entropy from $S(r) = \frac{k_B A_r}{4l_P^2}$: $S_{dS}(r) = \frac{r}{R_0}S(r)$

(Verlinde 2017 arrives at same expression from different reasoning)

As shown in Verlinde (2017), addition of a mass M (yielding Schwarzschild-de Sitter metric) reduces the horizon and the entropy:

$$\overline{S}_{dS}(r) = S_{dS}(r) + \Delta S_{dS}(r) = \frac{k_B r}{R_0}\left(\frac{A_r}{4l_P^2} - \frac{2\pi c M R_0}{\hbar}\right)$$

The effective acceleration in Schwarzschild-de Sitter spacetime,

$$\bar{a}(r) = \frac{r}{R_0} a_0$$

describes the point of view of a non-inertial observer for whom the flow of matter is subject to a combined vacuum-gravity acceleration.

But we could also take the viewpoint of an observer co-moving with the vacuum expansion and only experiencing gravitative pull $g(r)$. Both accounts must be consistent. The consequences can be developed via the concept of a thermal clock whose temperature is dictated by the source mass M. The consistency condition is then:

$$T_{\overline{a}(r)} \overline{S}_{dS}(r) = T_{g(r)} S(r).$$

Evaluating this leads finally to the expression

$$\overline{a}(r) = a_0 |\Phi(r)| \left(1 + \sqrt{1 + \frac{c^4}{M G a_0}} \right).$$

Original MOND correction found:

Recall the result for the Schwarzchild-DeSitter acceleration: $\overline{a}(r) = a_0|\Phi(r)|\left(1+\sqrt{1+\frac{c^4}{MGa_0}}\right)$.

This is valid for the limit $r \geq r_0$, where r_0 is defined by the requirement that the consistency condition $T_{\overline{a}(r)}\overline{S}_{dS}(r) = T_{g(r)}S(r)$ makes sense; i.e., the left hand side must not vanish.

This means that $\overline{S}_{dS}(r) \geq 0$, where the equality defines r_0. Thus, by the expression for $\overline{S}_{dS}(r)$,

$$\frac{4\pi r_0^2}{4l_P^2} = \frac{2\pi cMR_0}{\hbar}, \qquad \text{and we find } r_0 = \sqrt{\frac{MG}{a_0}}. \qquad \text{(recalling that } R_0 = \frac{c^2}{2a_0}\text{)}$$

For $r \geq r_0$, vacuum expansion dominates; for $r < r_0$ Newtonian gravity dominates.

Consistency checks for $r = r_0$ are provided in Schlatter and Kastner (2023).

Specifically. for $r > r_0$, $|\Phi(r)|a_0 \ll 1$ and so $\overline{a}(r) \approx |\Phi(r)|a_0\sqrt{\frac{c^4}{MGa_0}} = \frac{\sqrt{MGa_0}}{r}$;

the original MOND correction offered by Milgrom.