Three ways of looking at the radial acceleration relation

Harry Desmond

Portsmouth

w/ Richard Stiskalek, Deaglan Bartlett & Pedro Ferreira

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Overview

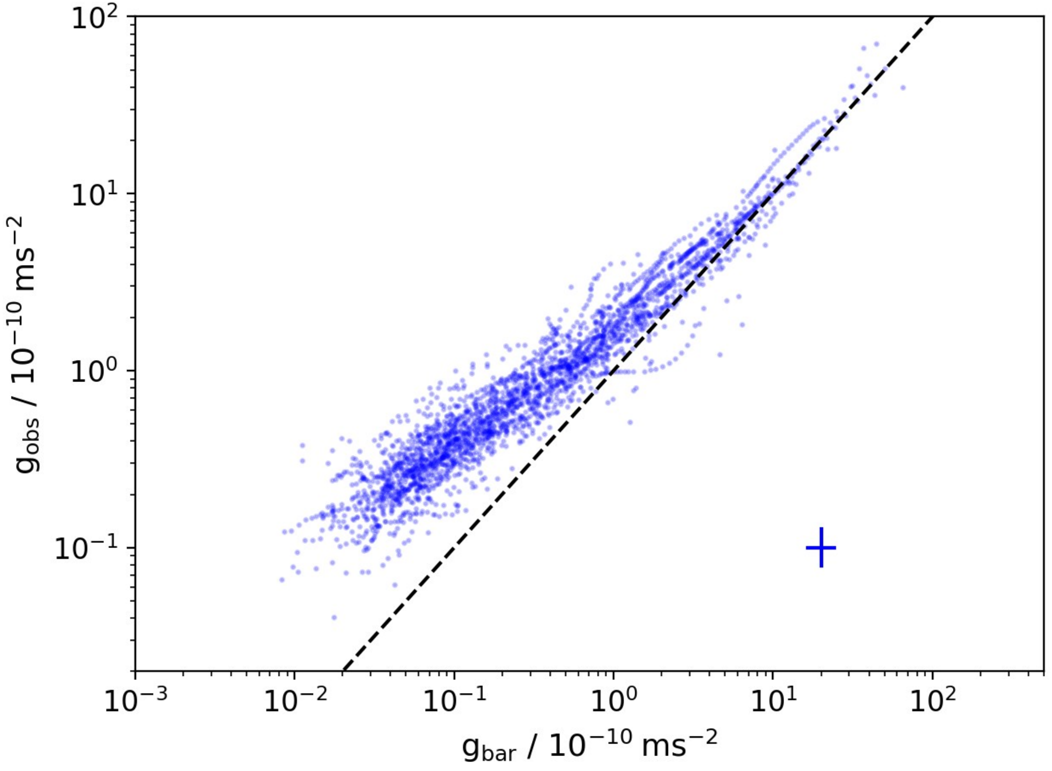
- The RAR and its significance
- The underlying RAR: optimising galaxy nuisance parameters
- The functional form of the RAR assessed by symbolic regression
- The RAR as the fundamental correlation of late-type galaxy dynamics
- Conclusions

Introduction. The radial acceleration relation

- Relates acceleration sourced by baryons (g_{bar}) to total acceleration as measured by rotation velocity (g_{obs})

- 2,696 points from 147 late-type galaxies (SPARC sample)

- Regularity and low scatter hard to understand in ΛCDM



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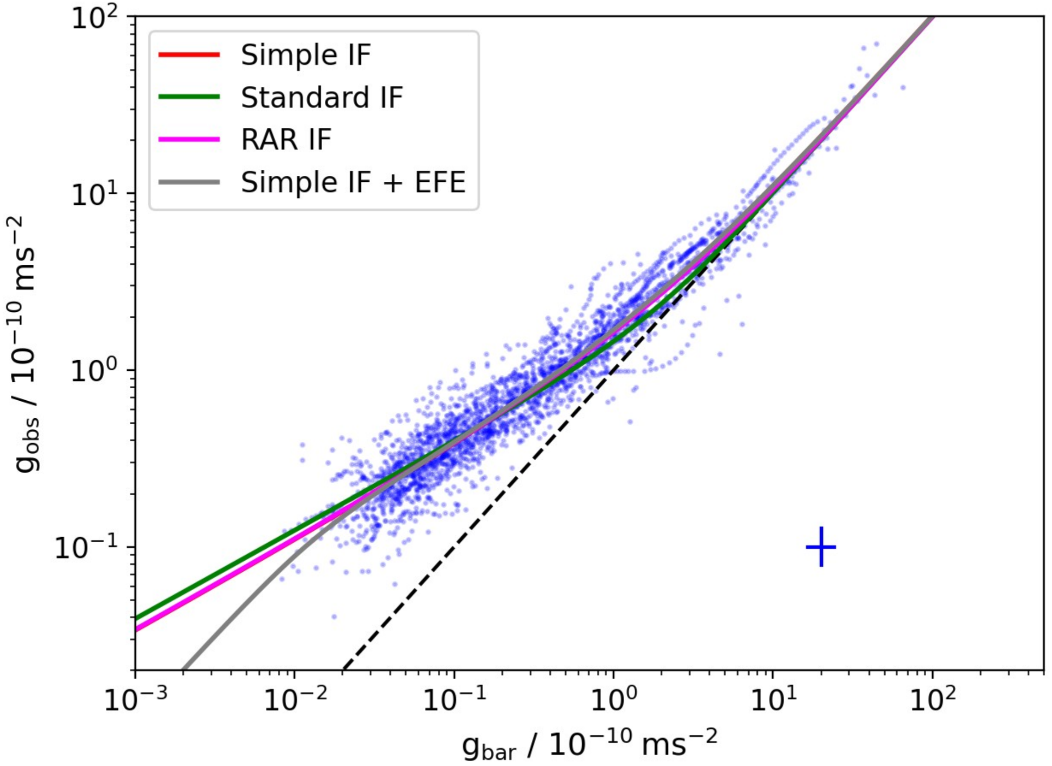

MOND Interpolating Functions (IFs)

Simple — $g_{obs} = g_{bar}/2 + \sqrt{g_{bar}^2/4 + g_{bar}g_0}$

Standard — $g_{obs} = \frac{1}{\sqrt{2}}\sqrt{g_{bar}^2 + \sqrt{g_{bar}^2\left(g_{bar}^2 + 4g_0^2\right)}}$

RAR — $g_{obs} = g_{bar}/(1 - \exp(-\sqrt{g_{bar}/g_0}))$

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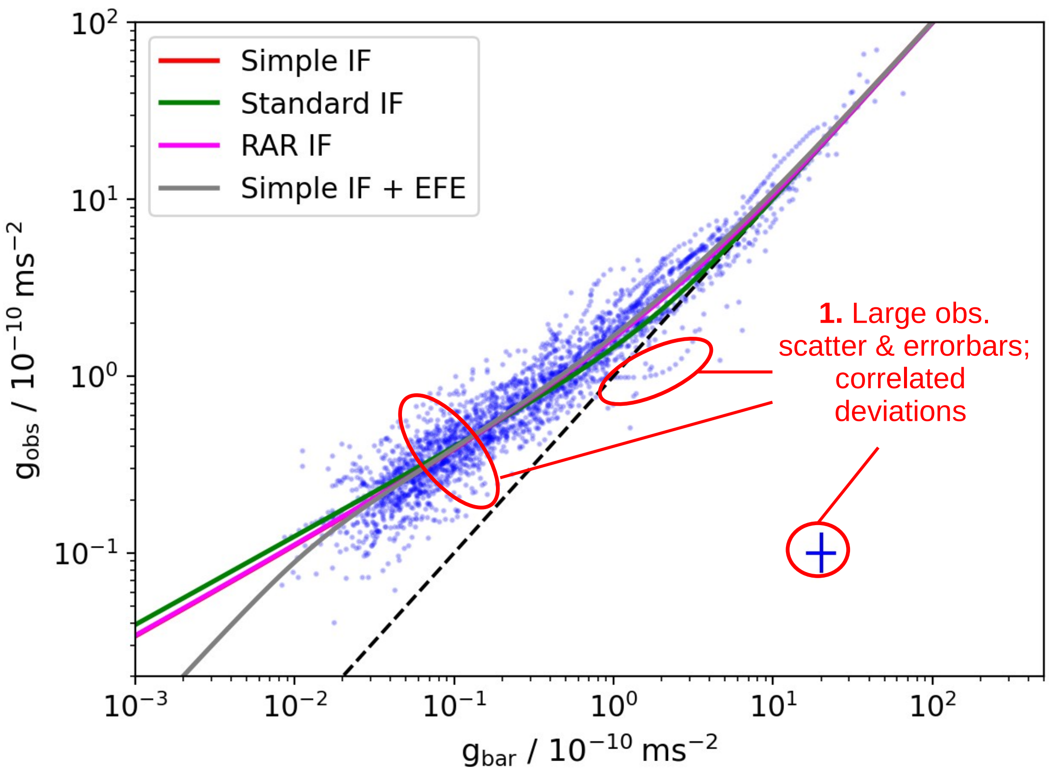

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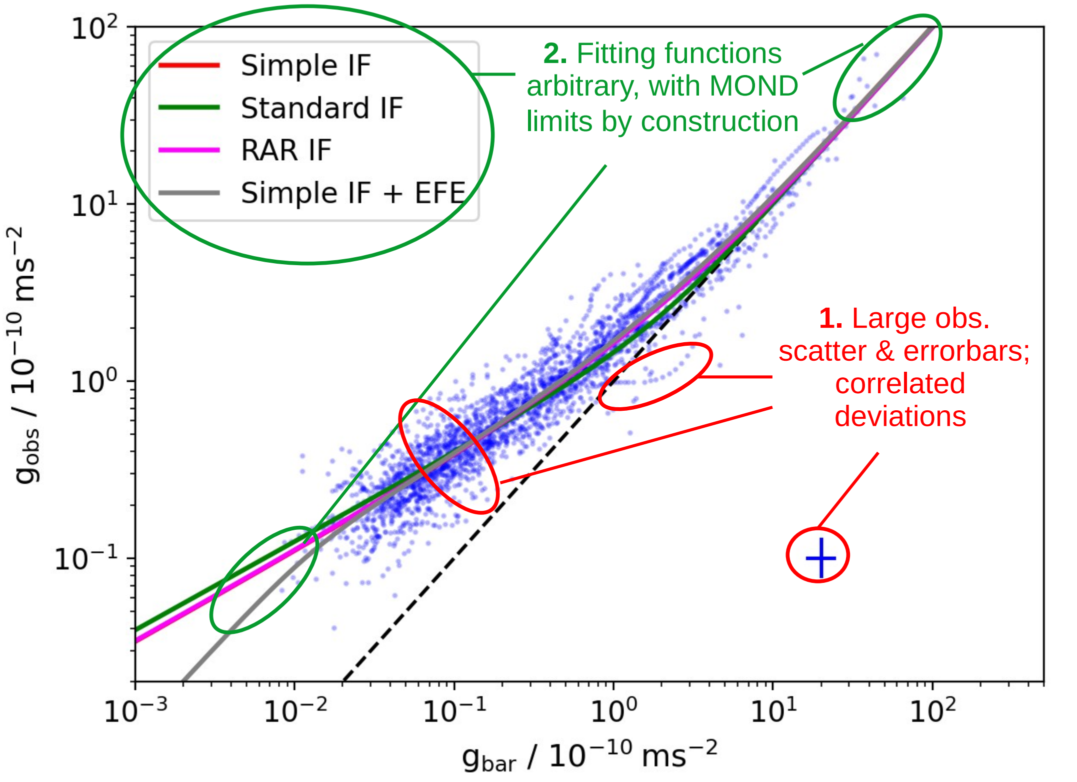

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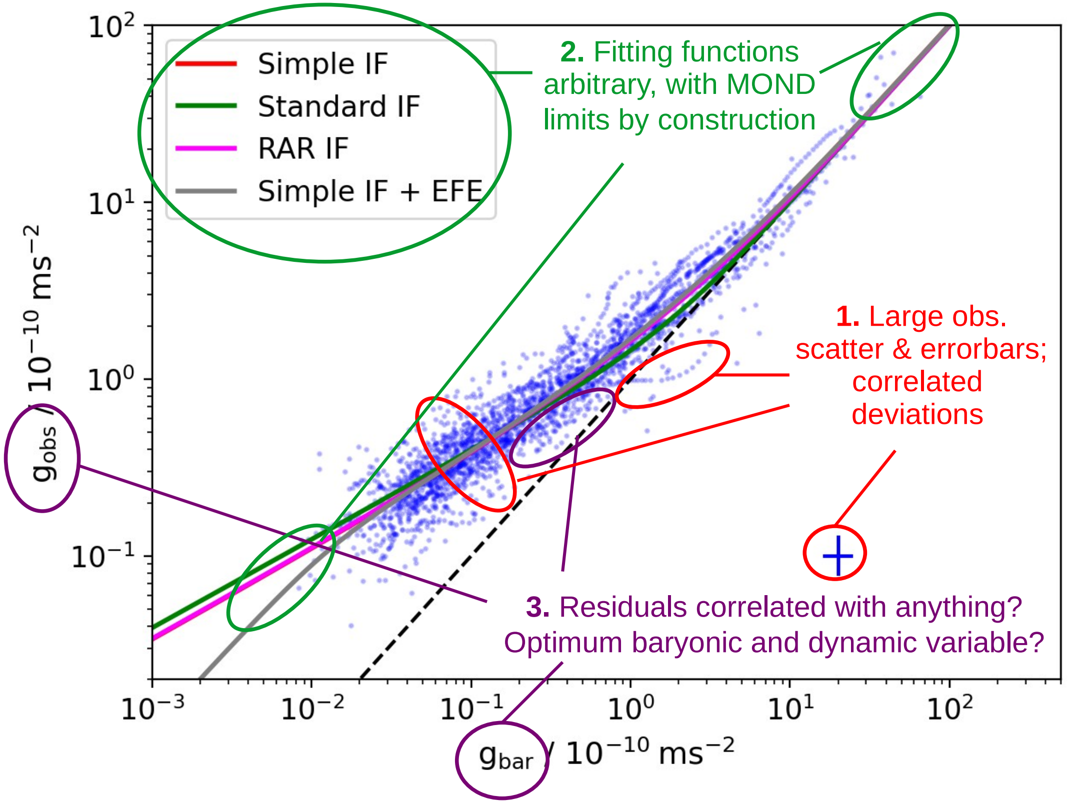

1. The underlying RAR

$$g_{\rm bar} = \frac{\left(\Upsilon_{\rm gas} V_{\rm gas}^2 + L_{36}/\bar{L}_{36}\left(\Upsilon_{\rm disk} V_{\rm disk}^2 + \Upsilon_{\rm bulge} V_{\rm bul}^2\right)\right)}{r}$$

$$g_{\rm obs} = \frac{V_{\rm obs}^2}{r} \, \frac{\sin(\bar{i})^2}{\sin(i)^2} \, \frac{\bar{D}}{D}$$

- Previous RAR results use maximum a priori galaxy parameters and propagate their uncertainties as statistical
- Incorrectly assumes uncorrelated deviations in gbar and gobs, leading to “bands” on RAR plane
- Proper analysis jointly infers ~768 parameters → need HMC

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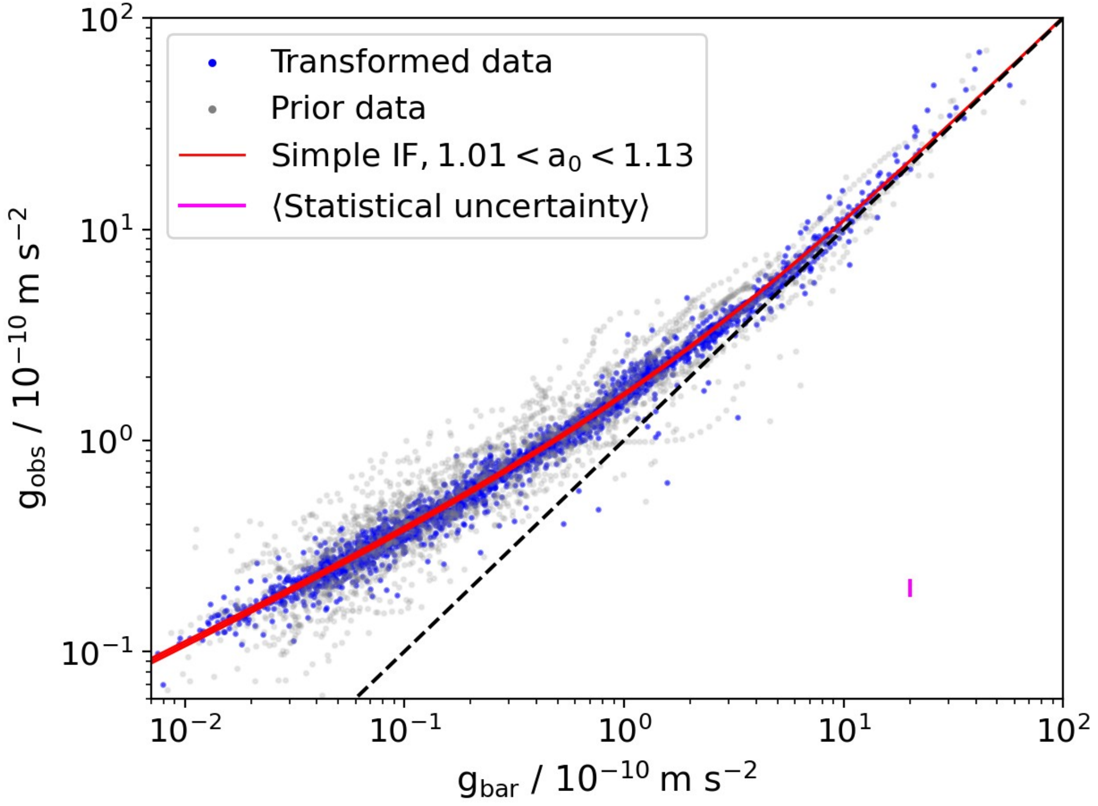

$$a_0 = (1.19 \pm 0.04\,(\mathrm{stat}) \pm 0.09\,(\mathrm{sys}))\ \times\ 10^{-10}\ \mathrm{m\ s^{-2}}$$

$$\sigma_{\mathrm{int}} = (0.034 \pm 0.01\,(\mathrm{stat}) \pm 0.01\,(\mathrm{sys}))$$ dex

- 8% uncertainty reducible to 0 with additional plausible extra data uncertainties

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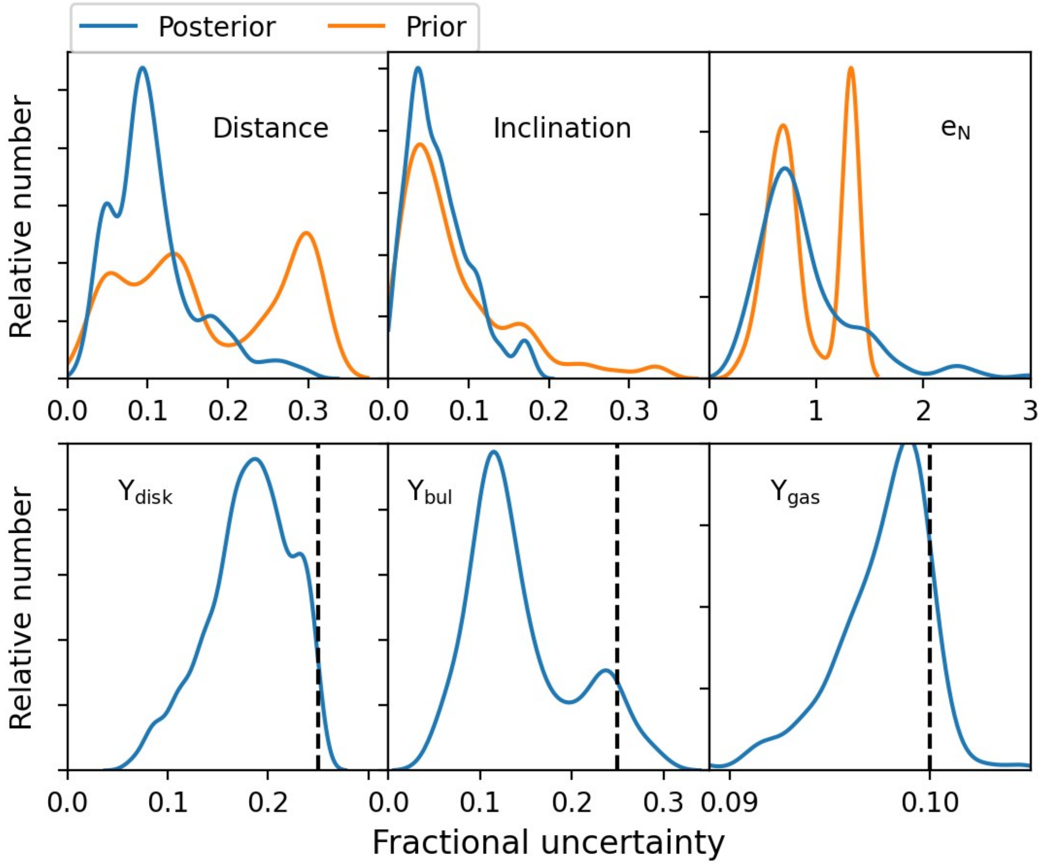
- *Byproduct:* galaxy-by-galaxy measurement of distance / inclination / mass-to-light. E.g. uRAR gives 10% distance measurement!



2. The functional form of the RAR

1) Are the MOND IFs optimal descriptions of the RAR?

2) Do optimal functional fits obey the MOND limits (and hence may be considered new IFs)?

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Symbolic Regression

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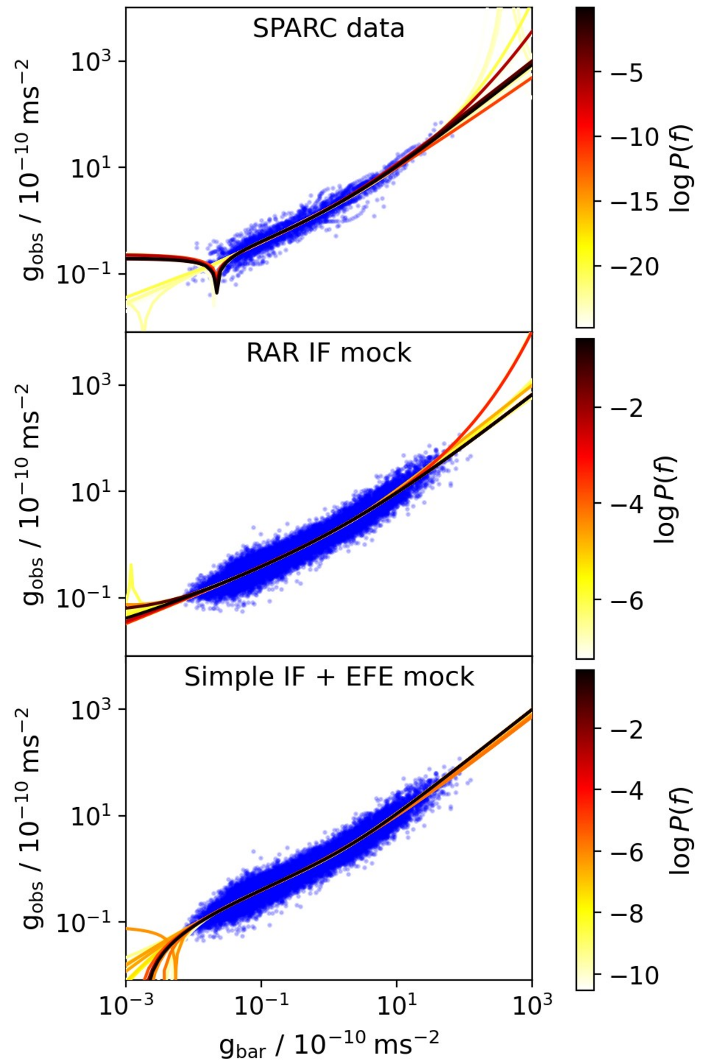
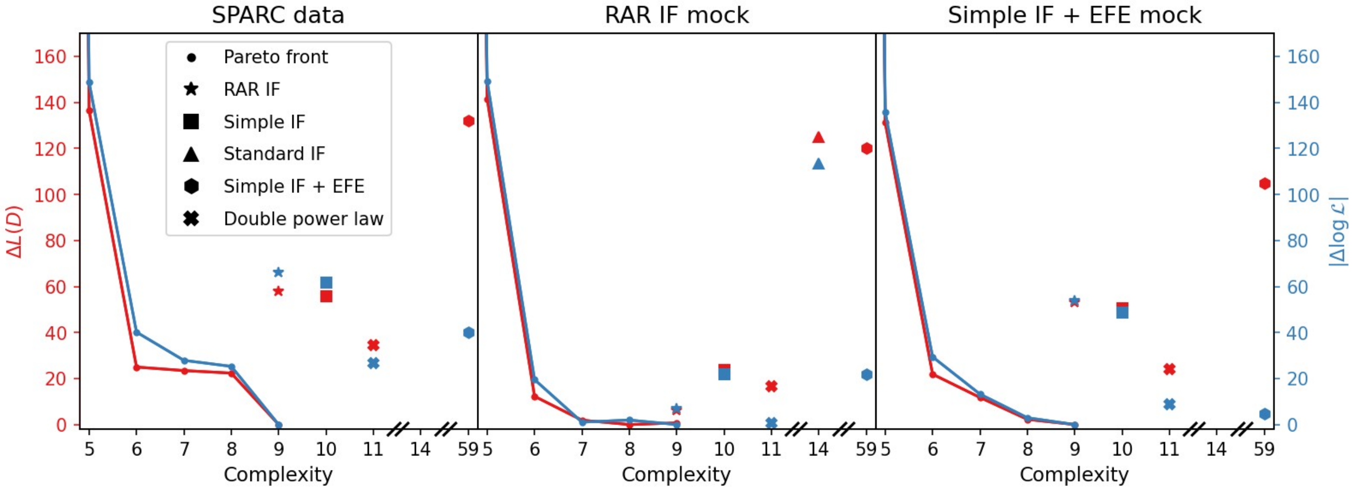
2) Do optimal functional fits obey the MOND limits (and hence may be considered new IFs)?

→ *Exhaustive Symbolic Regression (ESR)*

- *All* functions up to complexity 9 from operator set {+, -, *, /, pow, exp, sqrt, sq, inv, gbar, θ}
- Judged according to information efficiency in compressing the data (*Minimum Description Length principle*)
- Papers: arxiv:2211.11461 - arxiv:2301.04368 - arxiv:2304.06333
- Code & data: https://github.com/DeaglanBartlett/ESR - https://zenodo.org/record/7339113

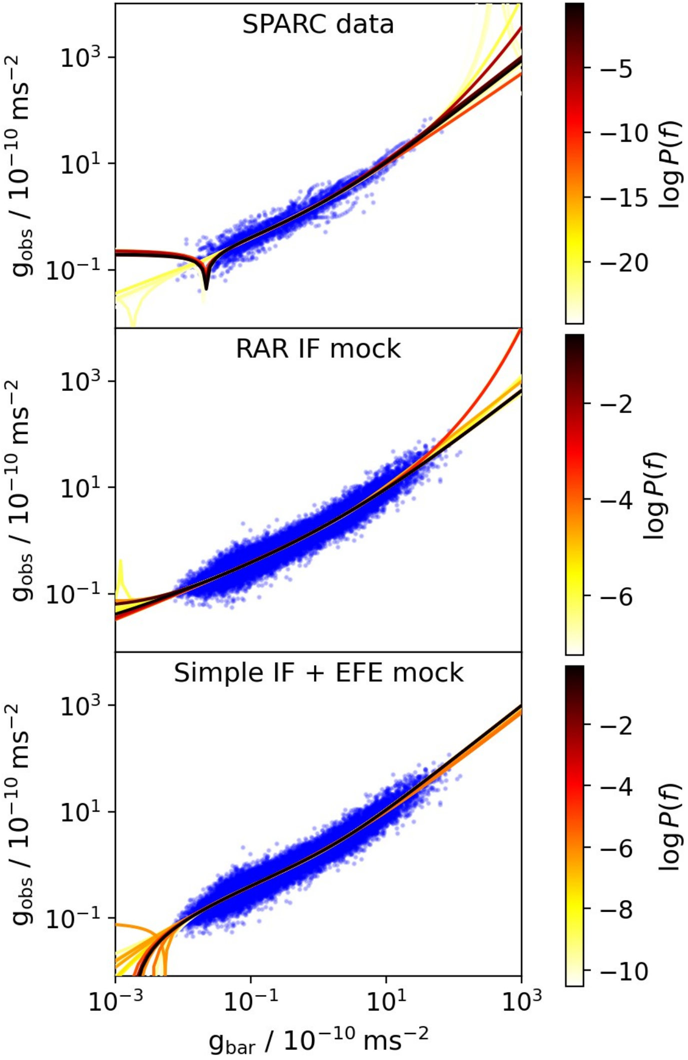


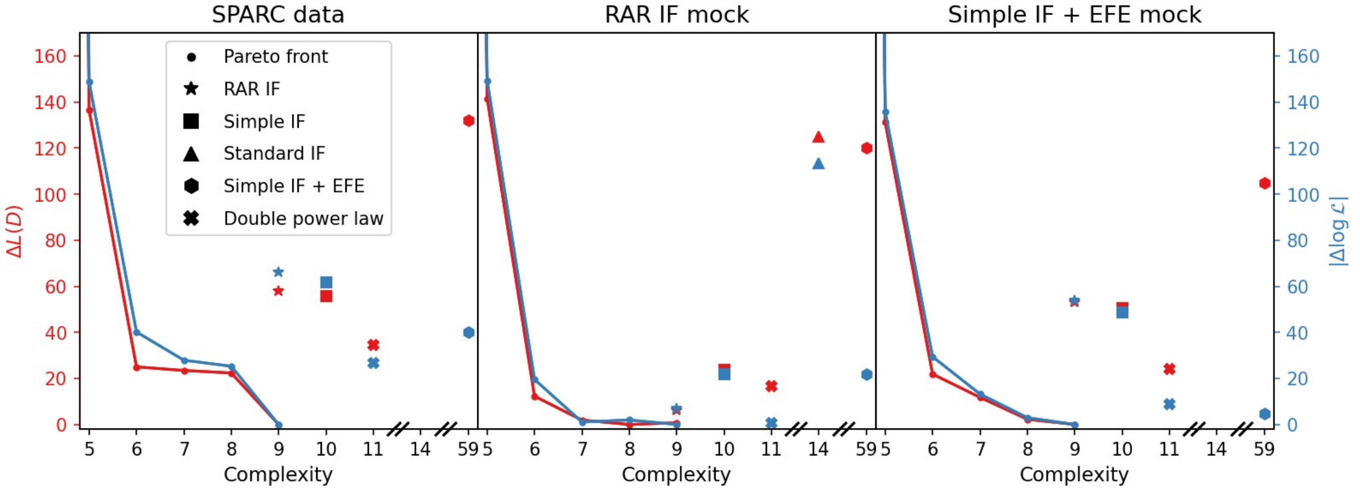


$$\theta_0\left(|\theta_1+x|^{\theta_2}+x\right)$$

$$\theta_0+\theta_1 x+\sqrt{x}$$

$$\theta_0+\sqrt{x^2+2x}$$



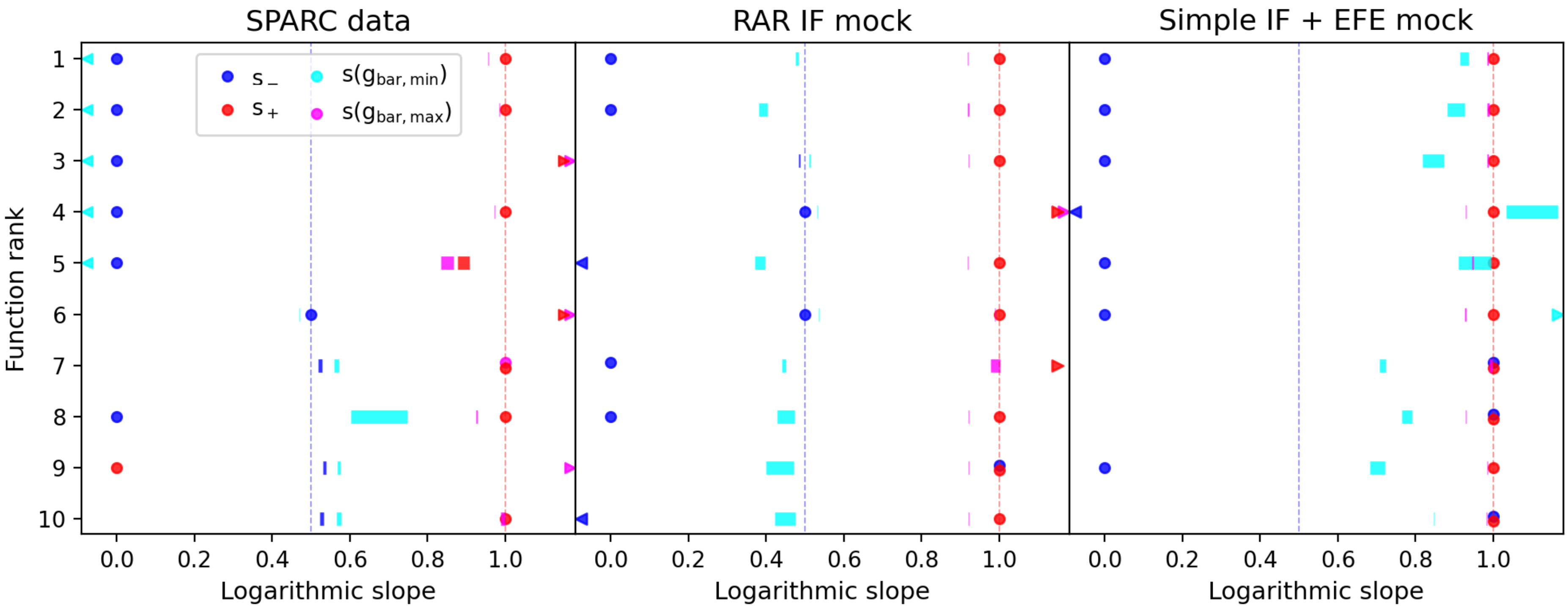


$$\theta_0\left(|\theta_1 + x|^{\theta_2} + x\right)$$

$$\theta_0 + \theta_1 x + \sqrt{x}$$

$$\theta_0 + \sqrt{x^2 + 2x}$$

- ESR functions superior to IFs and double power law
- ESR does not recover the generating function on mocks
- Best functions on real data do not have deep-MOND limit



- Newtonian limit often found; deep-MOND limit rarely
- Can’t recover MOND behaviour even from MOND mocks!
  → Uncertainties and dynamic range of data insufficient

3. Fundamentality of the RAR

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1. No residual correlations with any other variable
2. Tightest projection of galaxies' dynamical parameter space
3. Can account for all other correlations
4. No other correlation possesses these properties

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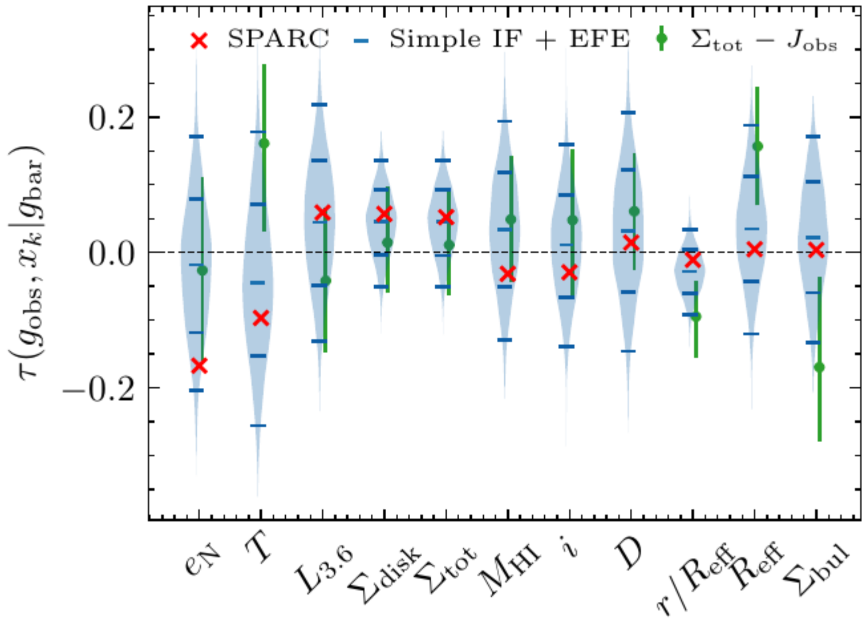
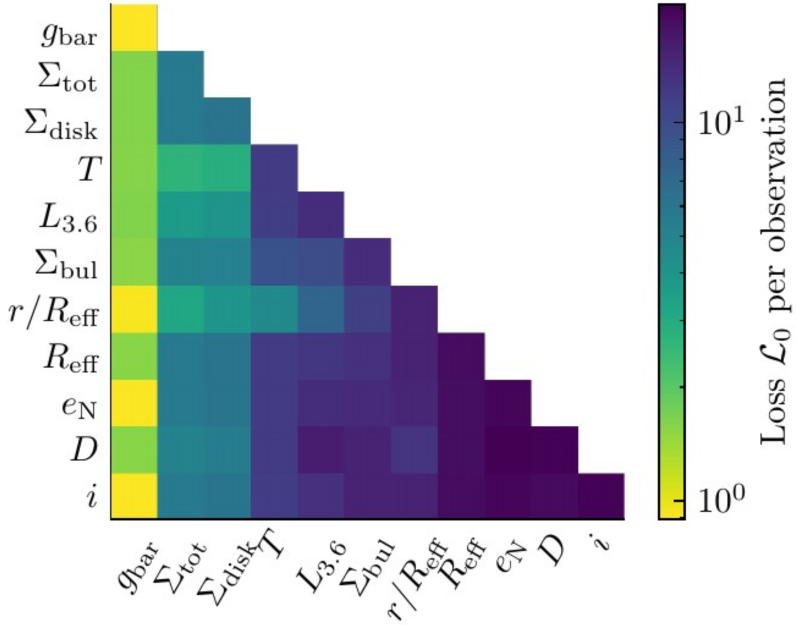
Residual correlations?

Partial Kendall coefficient



ExtraTrees regressor



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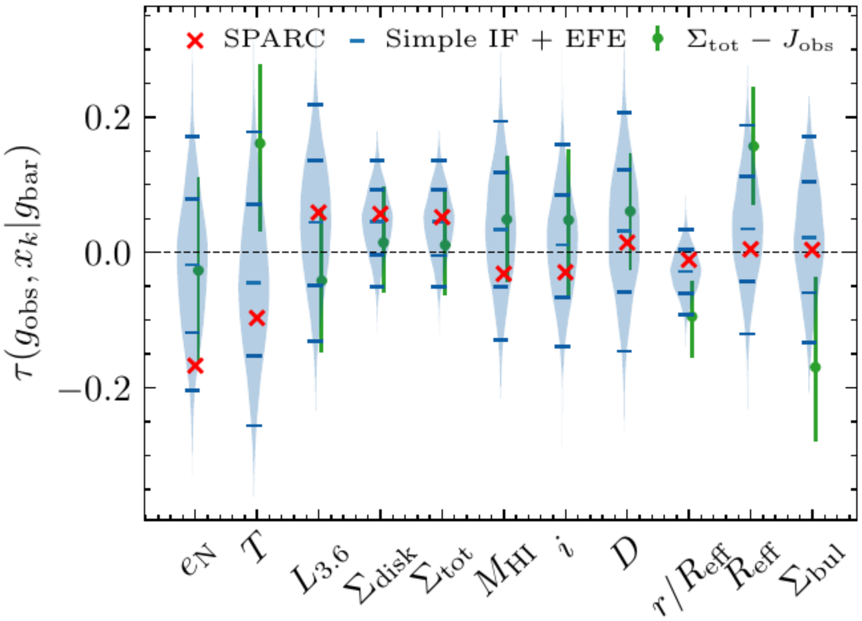
Residual correlations? **No.**

Partial Kendall coefficient



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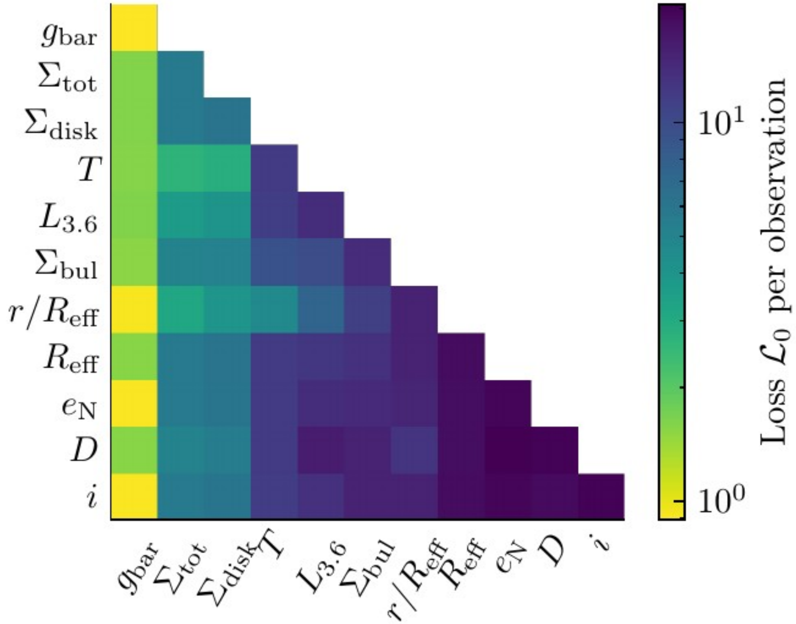

Tightest projection accounting for all other correlations?

- Form general dynamical variable

$$\mathcal{D}(\alpha, \beta) \equiv \frac{V_{\rm obs}^{\alpha}}{r^{\beta}} \qquad \tan\theta \equiv \frac{\beta}{\alpha}$$

- How predictable is *D* from baryonic variables at any θ?

Can any other correlation do this?

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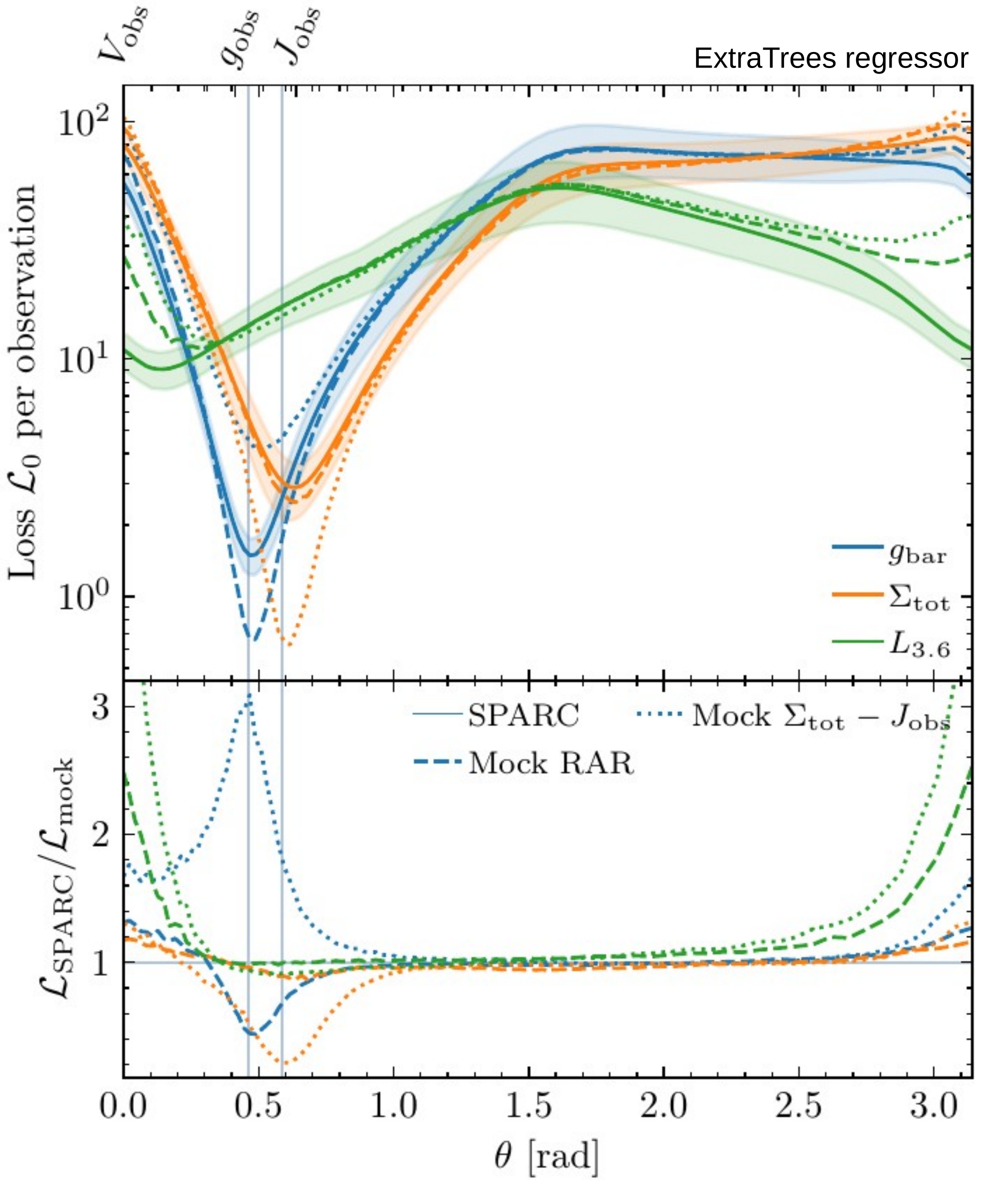
- θ=arctan(1/2) (gobs) is most predictable, from gbar alone



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Tightest projection accounting for all other correlations? **Yes.**

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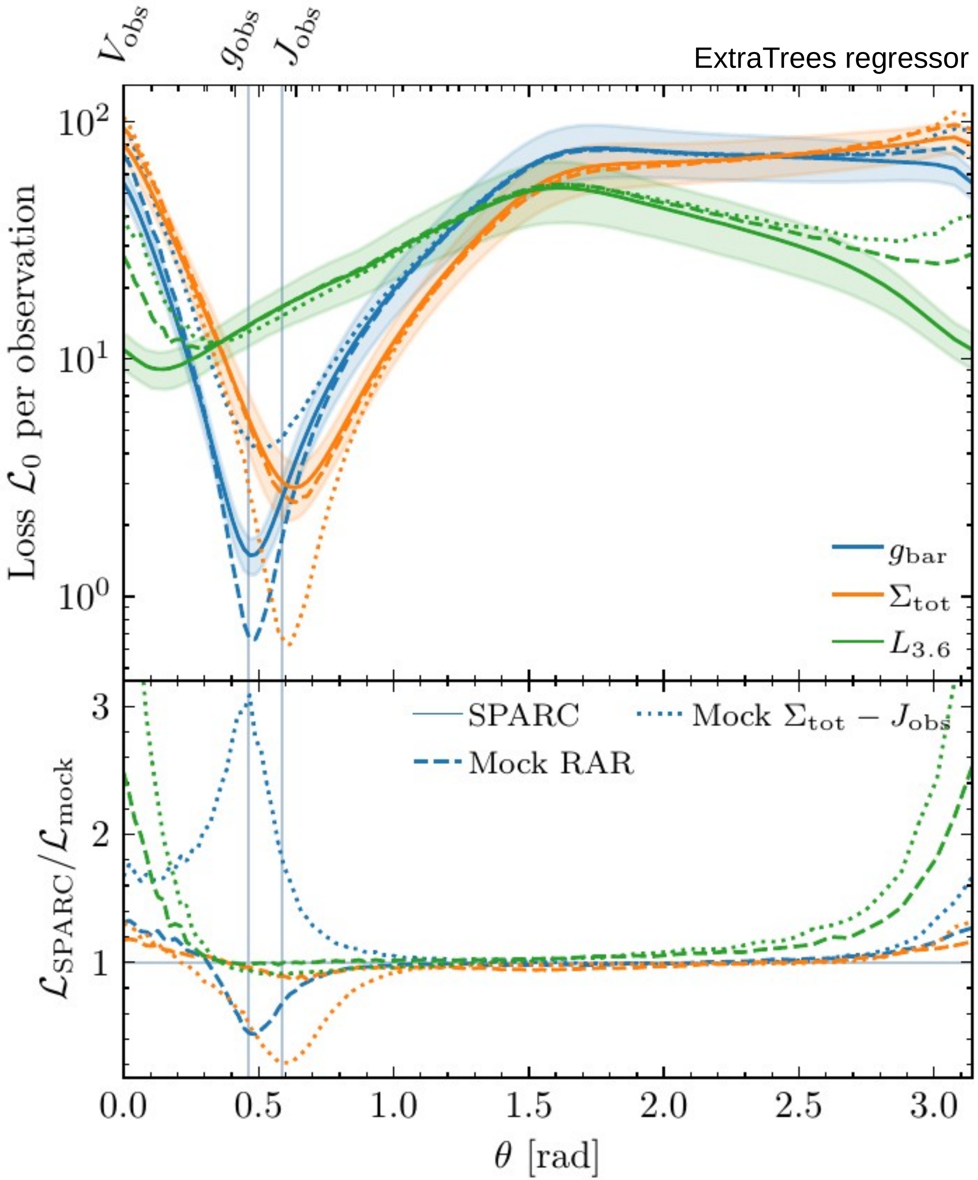
- How predictable is *D* from baryonic variables at any θ?
- θ=arctan(1/2) (gobs) is most predictable, from gbar alone
- RAR mock data (dashed) can account for all correlations of *D* with any baryonic quantity at any θ
- Mock data from the 2nd strongest correlation, Sigma–Jobs, cannot

Can any other correlation do this? **No.**



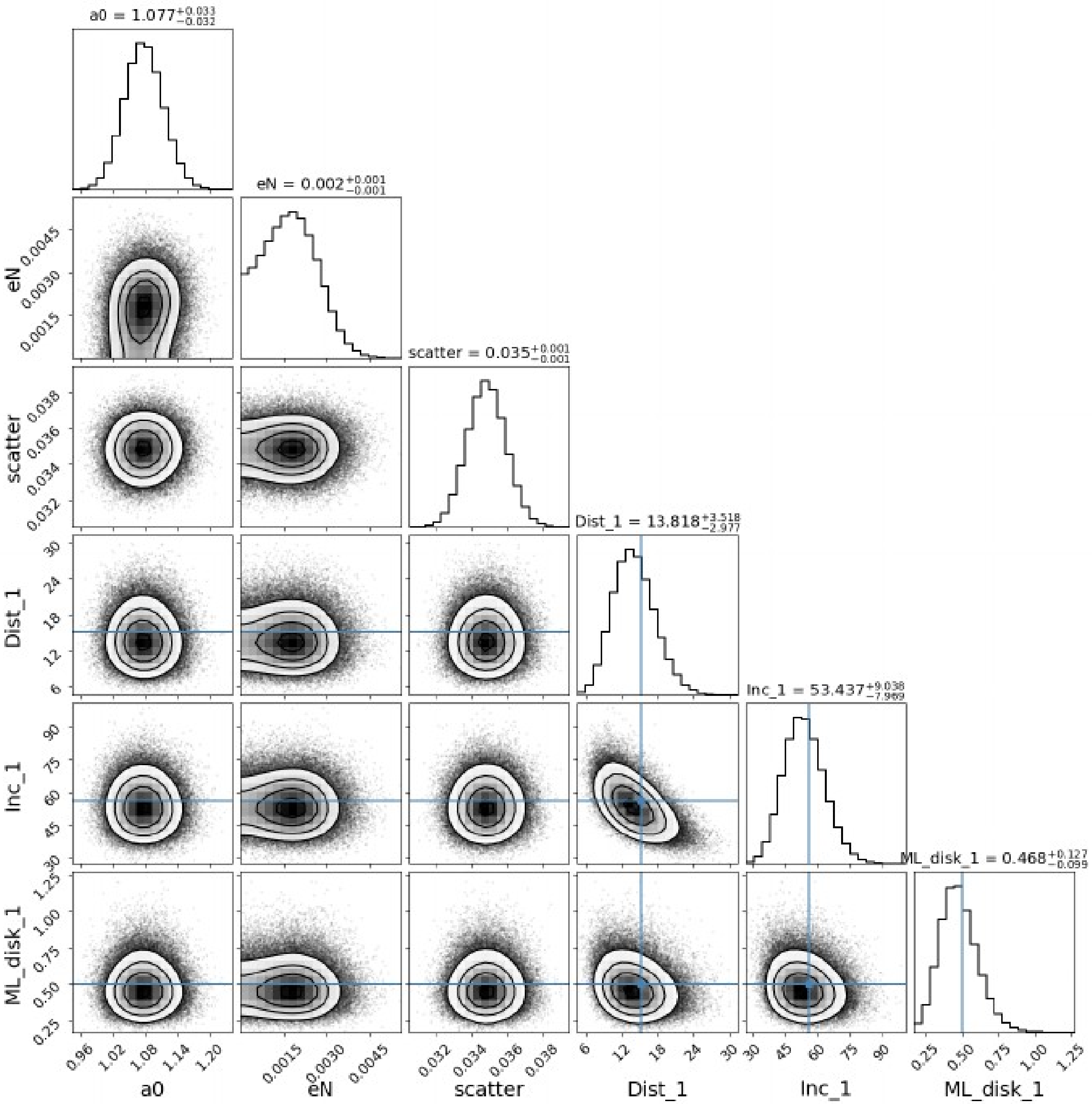
Conclusions

- The *underlying RAR* − derived by optimising galaxy nuisance parameters to propagate their uncertainties correctly − has intrinsic scatter 0.034 ± 0.002 dex and shows no evidence of deviation from the Simple or RAR IF
- The data is not currently powerful enough for *symbolic regression* based on the minimum description length principle to establish the optimal function form of the RAR or whether it has MONDian limits
- The RAR accounts for all correlations of radial dynamics with baryonic properties, making it *the fundamental* relation of late-type galaxy dynamics

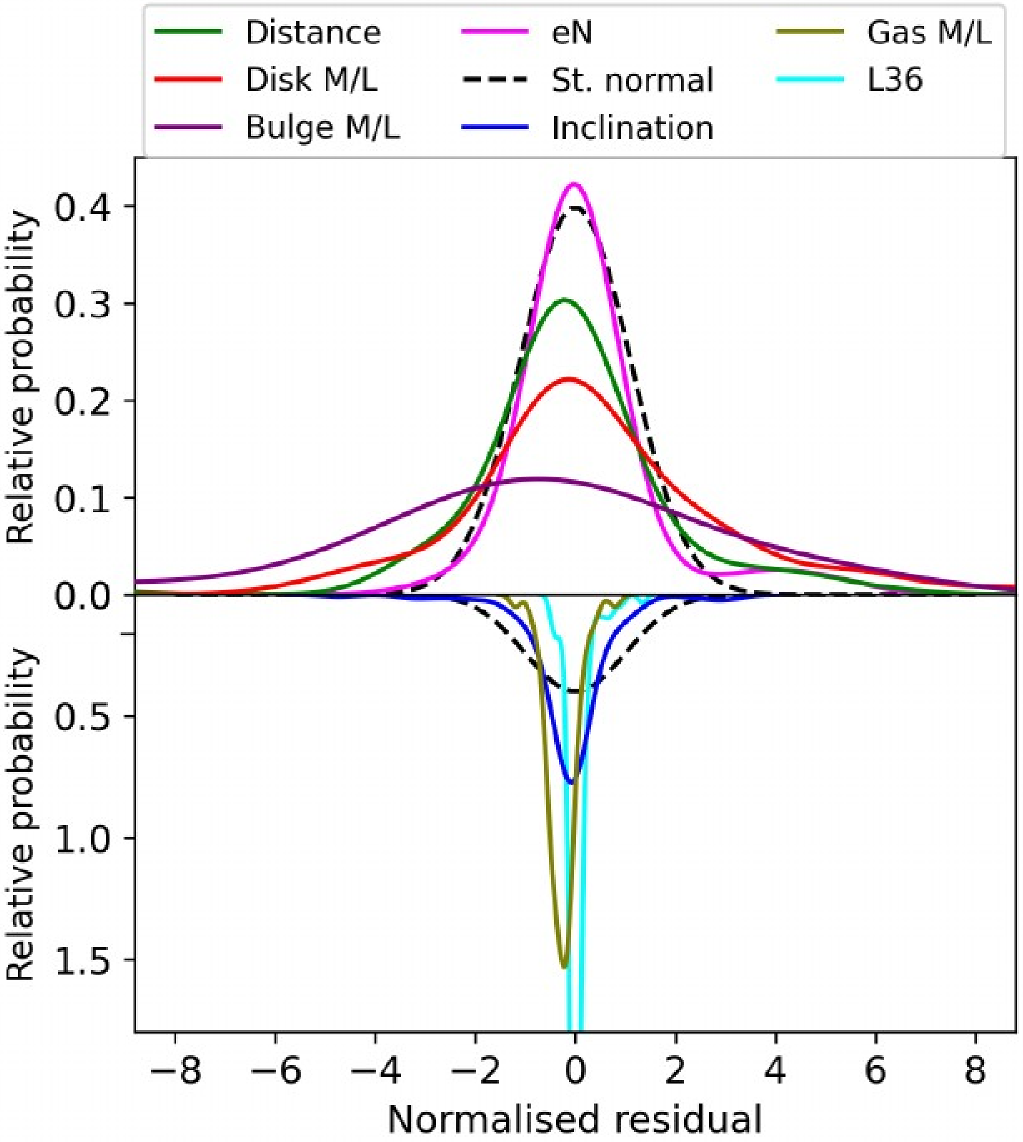
Extra Slides

Model	$\mathbf{a_0}$	$\mathbf{e_\mathrm{N}}$	σ_int	$\Delta \ln(\hat{L})$	$\Delta \ln(\hat{P})$	ΔBIC
No scatter, no EFE	$1.134^{+0.028}_{-0.027}$	—	—	0	0	0
No scatter, global EFE	$1.138^{+0.028}_{-0.027}$	$0.0016^{+0.0005}_{-0.0005}$	—	-3.44	3.24	14.8
No scatter, max-clustering EFE	$1.309^{+0.039}_{-0.037}$	$0.0050^{+0.0203}_{-0.0033}$	—	292	55.8	577
No scatter, avg-clustering EFE	$1.307^{+0.038}_{-0.036}$	$0.0022^{+0.0231}_{-0.0017}$	—	275	-19.2	611
Scatter, no EFE	$1.070^{+0.032}_{-0.031}$	—	$0.035^{+0.001}_{-0.001}$	2020	2381	-4032
As above, boosted uncertainties	$1.121^{+0.035}_{-0.034}$	—	< 0.0020	1936	2354	-3864
Scatter, global EFE	$1.077^{+0.033}_{-0.032}$	$0.0017^{+0.0009}_{-0.001}$	$0.035^{+0.001}_{-0.001}$	2019	2375	-4023
Scatter, max-clustering EFE	$1.236^{+0.043}_{-0.041}$	$0.0049^{+0.0125}_{-0.003}$	$0.033^{+0.001}_{-0.001}$	2052	2220	-2935
As above, boosted uncertainties	$1.275^{+0.047}_{-0.044}$	$0.0047^{+0.0103}_{-0.0029}$	< 0.0019	1948	2182	-2727
Scatter, avg-clustering EFE	$1.272^{+0.047}_{-0.045}$	$0.0020^{+0.0134}_{-0.0016}$	$0.033^{+0.001}_{-0.001}$	2074	2161	-2978

Table 3. Constraints on RAR parameters and goodness-of-fit statistics for the models considered. For the maximum-clustering and average-clustering EFE models the quoted e_N constraints describe the stacked posteriors over all galaxies. The final three columns are the maximum log-likelihood, maximum log-posterior and Bayesian information criterion relative to the first model.



a0 = 1.077+0.033 −0.032
eN = 0.002+0.001 −0.001
scatter = 0.035+0.001 −0.001
Dist_1 = 13.818+3.518 −2.977
Inc_1 = 53.437+9.038 −7.969
ML_disk_1 = 0.468+0.127 −0.099
eN
scatter
Dist_1
Inc_1
ML_disk_1
a0
eN
scatter
Dist_1
Inc_1
ML_disk_1



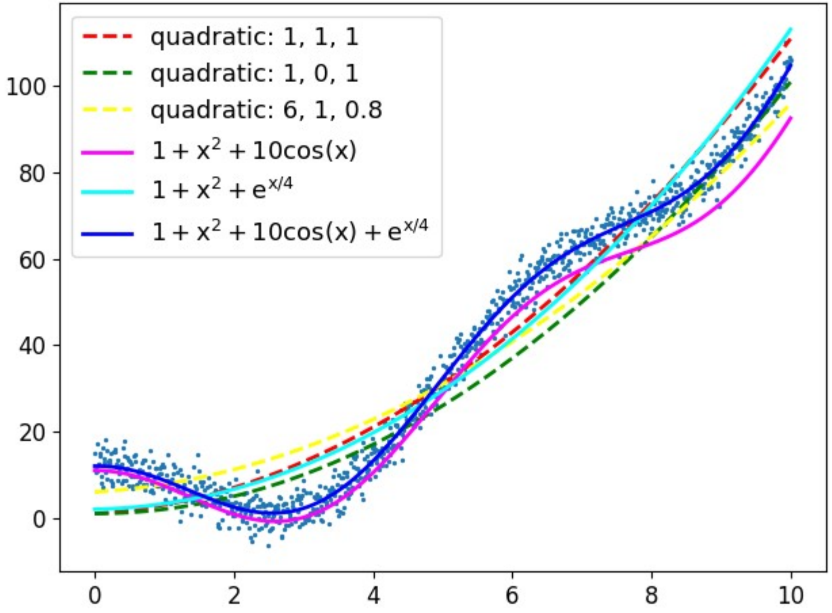
Distance
Disk M/L
Bulge M/L
eN
St. normal
Inclination
Gas M/L
L36
Relative probability
Relative probability
0.4
0.3
0.2
0.1
0.0
0.5
1.0
1.5
−8
−6
−4
−2
0
2
4
6
8
Normalised residual

Symbolic Regression overview

- Discover *functions* describing a dataset rather than parameters of predefined function
- Difficulties:
  - Larger search space makes convergence harder
  - Optimisation methods of numerical regression not applicable
- Advantages:
  - Much more general (reduces confirmation bias)
  - Easy to prevent overfitting
  - Highly interpretable

Numerical regression: $y = 6 + 1x + 0.8x^2$

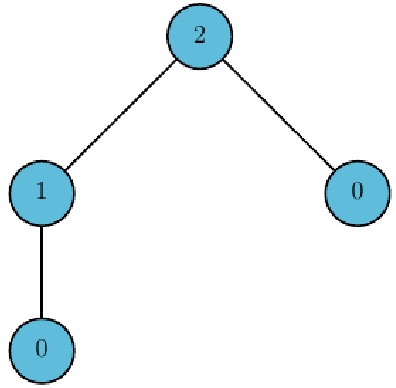
Symbolic regression: $y = 1 + x^2 + 10\cos(x)$



Exhaustive Symbolic Regression

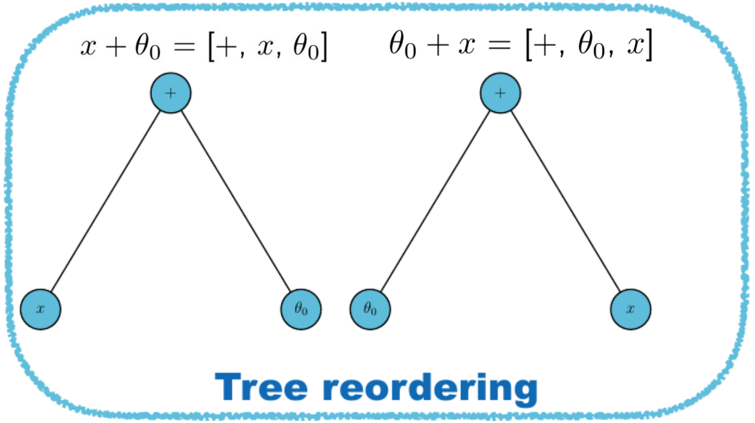
I. Function generation & optimisation

1) Generate all possible trees with given complexity = #nodes, with placeholder operators labelled by arity (number of arguments to operator)



2) Decorate with all operator permutations

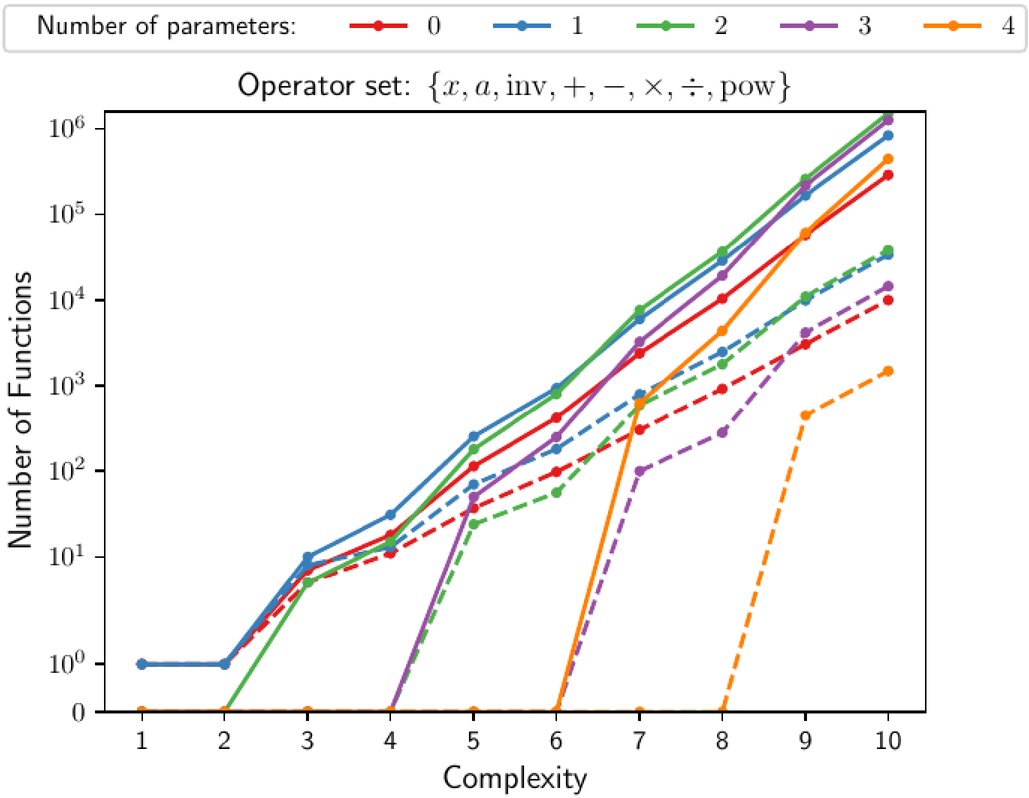
3) Simplify and remove duplicates (*tree reordering*, *parameter permutations*, *simplifications*, *reparametrisation invariance*, *parameter combinations*)



4) Calculate maximum-likelihood parameter values

5) Repeat for other desired complexities

Simplifications make an exhaustive search feasible



168 million

Naïve estimate: $\sum_{j=1}^{n} j^k = H_n^{(-k)}$

5.2 million

Total number of decorated trees

134,234

Number of unique equations

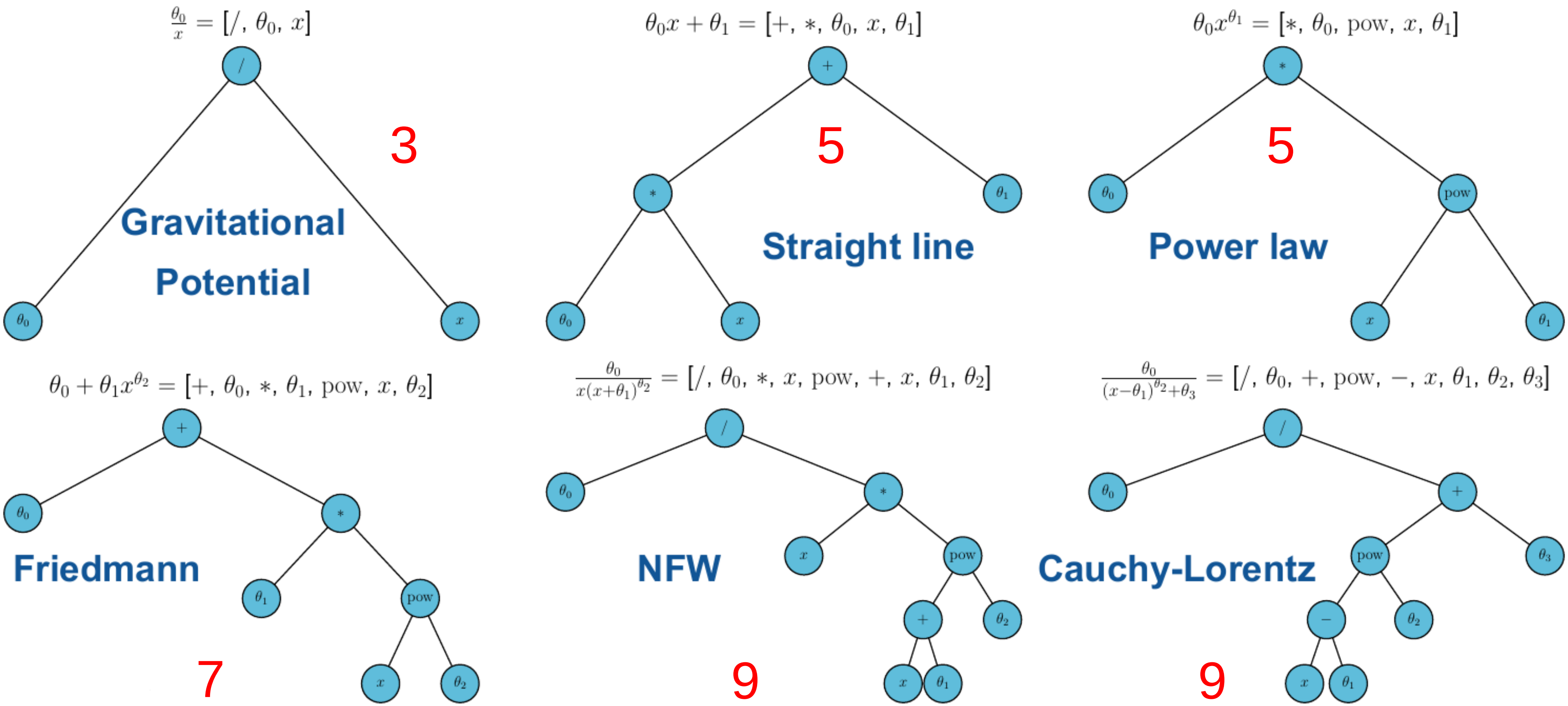
119,861

Number of equations containing at least one parameter

1400

Times fewer equations to consider than our naïve guess

Many physics functions have complexity < 10



Exhaustive Symbolic Regression

II. Model selection principle: *minimum description length*

$$L(D) = L(H) + L(D \mid H)$$

Description length — Hypothesis — Residuals

$$L(D) = -\log(\mathscr{L}(\hat{\theta})) + k\log(n) + \sum_i^p \log(|\hat{\theta}_i|/\Delta_i) + p\log(2) + \sum_j \log(c_j)$$

Shannon-Fano coding

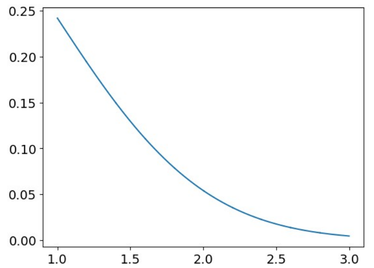
k nodes with n basis functions needs $\log\left(n^k\right)$ nats

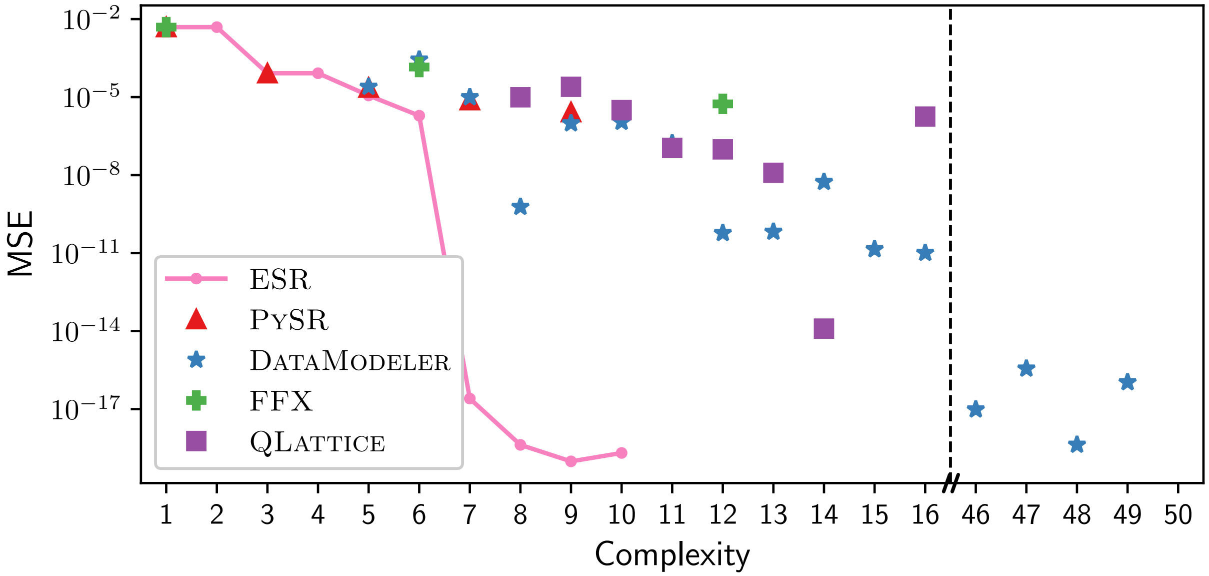
Send p parameters, each with precision Δ_i

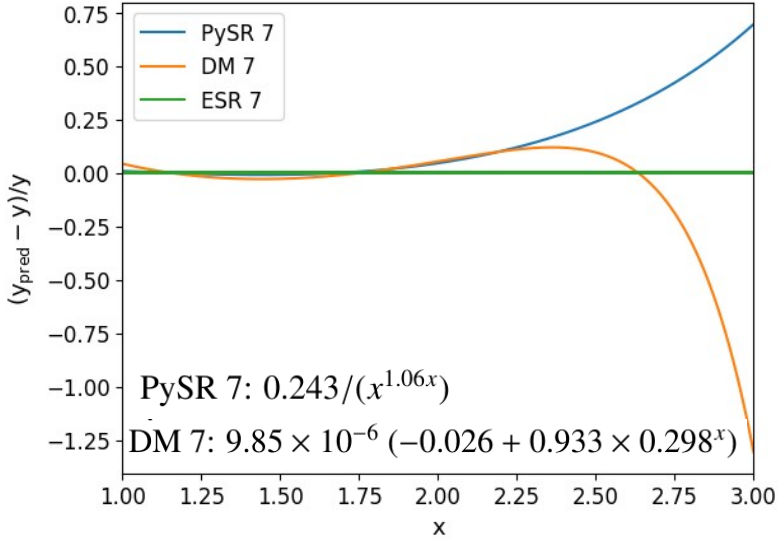
Integer constants

- Purpose of functional fit is *data compression*
- Most information-efficient function has minimum *L(D)*
- Both accuracy and complexity expressed in nats $\longrightarrow$ can be combined
- Accounts for both functional and parametric complexity. Accuracy is likelihood.

Test case 0: Benchmarking







- *feynman_I_6_2a* dataset from the *SRBench 2022 Competition*
- Not only does ESR get by far the lowest error... it discovers the standard normal!

$$y = \theta_1 \theta_0^{x^2}$$
$$\theta_0 = 1/\sqrt{e}$$
$$\theta_1 = 1/\sqrt{2\pi}$$

SPARC data

Rank	Function	Comp.	$P(f)$	Parameters				Description length			
				θ_0	θ_1	θ_2	θ_3	Resid.[1]	Func.[2]	Param.[3]	Total
1	$\theta_0\left(\|\theta_1+x\|^{\theta_2}+x\right)$	9	9.3×10^{-1}	0.84	-0.02	0.38	—	-1279.1	14.5	14.0	-1250.6
2	$\|\|\theta_1\|^x+\theta_0\|^{\theta_2}+x$	9	6.4×10^{-2}	-0.99	0.64	0.36	—	-1279.9	12.5	19.6	-1247.9
3	$\|\theta_0\|^{\|\theta_1-x\|^{\theta_2}-\theta_3}$	9	2.0×10^{-3}	-1.4×10^2	0.02	0.14	0.89	-1276.4	12.5	19.5	-1244.4
4	$\|\theta_0(\theta_1+x)\|^{\theta_2}+x$	9	1.4×10^{-4}	0.35	-0.02	0.34	—	-1268.9	14.5	12.7	-1241.7
5	$\left\|\theta_0-\|\theta_1-x\|^{\theta_2}\right\|^{\theta_3}$	9	1.0×10^{-5}	-0.30	0.02	0.42	2.14	-1271.1	12.5	19.5	-1239.1
6	$\sqrt{x}\exp\left(\frac{\|\theta_0+x\|^{\theta_1}}{2}\right)$	9	1.5×10^{-9}	-0.02	0.36	—	—	-1257.9	17.5	10.0	-1230.3
7	$\left(\frac{\|\theta_0\|^x}{x}\right)^{\theta_1}+x$	9	2.4×10^{-10}	1.87	-0.52	—	—	-1250.6	14.5	7.6	-1228.5
8	$\sqrt{\|\theta_0+x\|}+\theta_1 x$	8	1.8×10^{-10}	-1.8×10^{-3}	0.72	—	—	-1245.6	12.9	4.5	-1228.2
9	$\left\|\theta_0+\frac{1}{\sqrt[4]{x}}\right\|^{\theta_1}$	8	9.6×10^{-11}	-0.22	-2.14	—	—	-1251.1	14.3	9.2	-1227.6
10	$\left(\sqrt{x}+\frac{1}{x}\right)^{\theta_0}+x$	9	8.2×10^{-11}	-0.53	—	—	—	-1248.3	16.1	4.8	-1227.4
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
17	$x/(\exp(\theta_0)-\|\theta_1\|^{\sqrt{x}})$	9	2.2×10^{-11}	0.03	0.44	—	—	-1250.9	17.5	7.3	-1226.1
—	Double power law	11	9.7×10^{-16}	4.65	3.96	1.03	0.60	-1252.3	17.7	18.5	-1216.1
—	Simple IF	10	5.5×10^{-25}	1.11	—	—	—	-1217.3	18.6	3.9	-1194.8
—	RAR IF	9	6.7×10^{-26}	1.13	—	—	—	-1212.8	16.1	3.9	-1192.7
—	Simple IF + EFE	59	5.0×10^{-69}	1.16	6.8×10^{-3}	—	—	-1238.9	139.9	5.6	-1093.4
—	Standard IF	14	9×10^{-150}	1.54	—	—	—	-939.5	27.9	4.1	-907.5

[1] $-\log\mathcal{L}(\hat{\boldsymbol{\theta}})$ [2] $k\log(n)+\sum_j\log(c_j)$ [3] $-\frac{p}{2}\log(3)+\sum_i^p(\log(I_{ii})^{1/2}+\log(\|\hat{\theta}_i\|))$

RAR IF mock

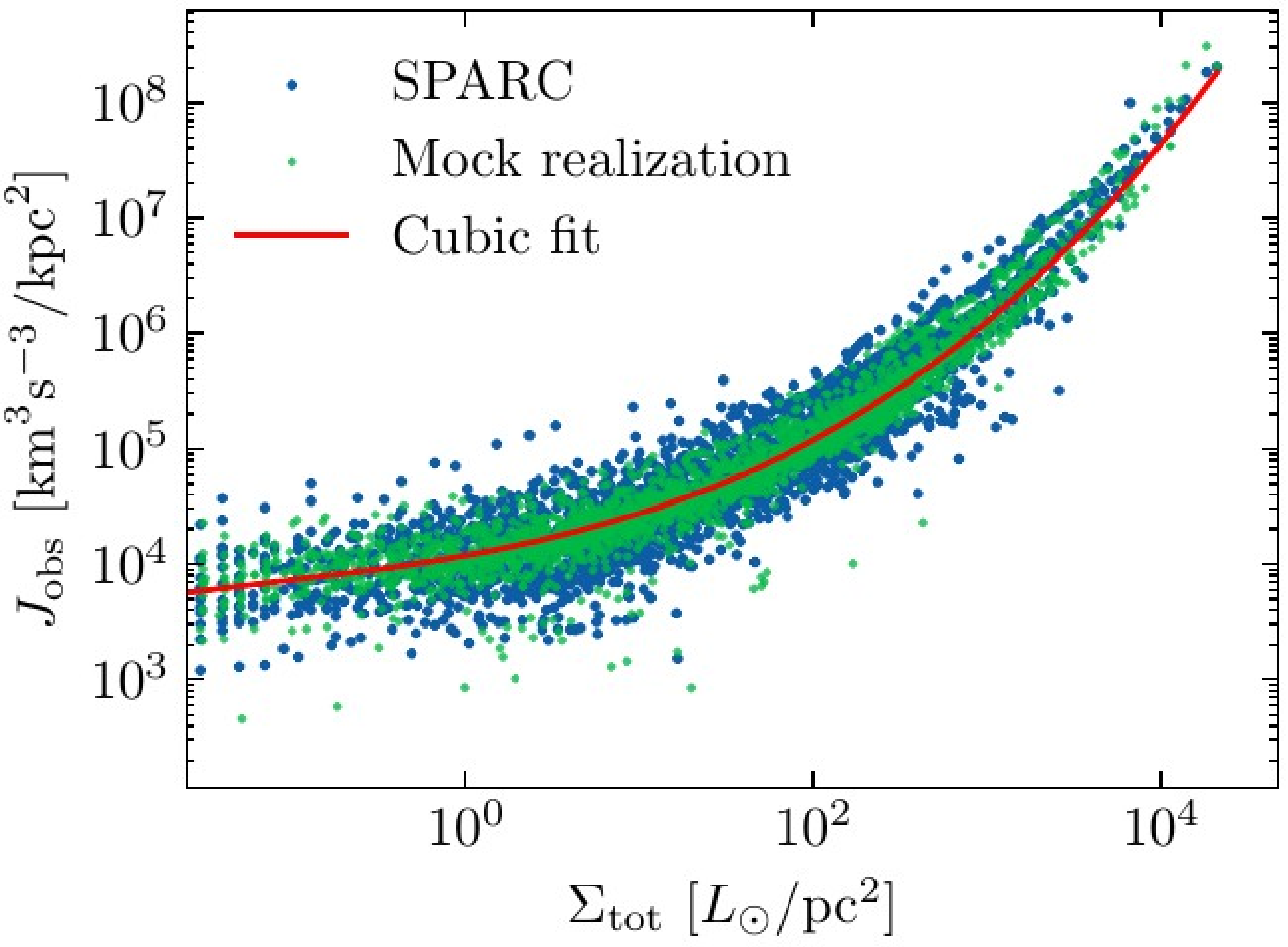
Rank	Function	Comp.	$P(f)$	Parameters				Description length			
				θ_0	θ_1	θ_2	θ_3	Resid.[1]	Func.[2]	Param.[3]	Total
1	$\theta_0 + \theta_1 x + \sqrt{x}$	8	5.6×10^{-1}	9.1×10^{-3}	0.63	—	—	-2045.2	12.9	4.9	-2027.4
2	$\sqrt{\lvert\theta_0 + x\rvert} + \theta_1 x$	8	2.8×10^{-1}	3.0×10^{-3}	0.64	—	—	-2044.4	12.9	4.8	-2026.7
3	$\theta_0 x + x^{\theta_1}$	7	8.2×10^{-2}	0.64	0.49	—	—	-2045.2	11.3	8.5	-2025.5
4	$\sqrt{x}\exp\left(\frac{x^{\theta_0}}{2}\right)$	7	3.5×10^{-2}	0.36	—	—	—	-2040.7	12.5	3.5	-2024.7
5	$(\theta_0 + x)\left(\theta_1 + \frac{1}{\sqrt{x}}\right)$	9	1.1×10^{-2}	1.3×10^{-3}	0.64	—	—	-2044.5	16.1	4.8	-2023.5
6	$\frac{1}{\sqrt{\lvert\theta_0 + \frac{1}{x}\rvert}} + x$	8	8.8×10^{-3}	1.74	—	—	—	-2038.5	12.9	2.3	-2023.3
7	$(x\lvert\theta_0\rvert)^{(x\lvert\theta_1\rvert)^{\theta_2}}$	9	3.1×10^{-3}	-2.09	-1.4×10^{-4}	0.04	—	-2045.3	12.5	10.6	-2022.2
8	$\theta_0 x + \lvert\theta_1 + x\rvert^{\theta_2}$	9	2.4×10^{-3}	0.64	1.4×10^{-3}	0.49	—	-2045.4	14.5	8.9	-2022.0
9	$x\left(\lvert\theta_0 - x\rvert^{\theta_1} - \theta_2\right)$	9	2.3×10^{-3}	1.2×10^{-3}	-0.51	-0.64	—	-2045.3	14.5	8.9	-2021.9
10	$(\theta_0 - x)\left(\theta_1 - x^{\theta_2}\right)$	9	2.2×10^{-3}	-6.5×10^{-4}	-0.64	-0.51	—	-2045.4	14.5	9.0	-2021.9
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
27	$x/(\exp(\theta_0) - \exp(-\sqrt{x}))$	9	3.2×10^{-4}	-0.01	—	—	—	-2039.3	17.5	1.9	-2020.0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
41	$x/(\exp(\theta_0) - \lvert\theta_1\rvert^{\sqrt{x}})$	9	1.1×10^{-4}	-5.0×10^{-3}	0.38	—	—	-2042.1	17.5	5.7	-2018.9
—	RAR IF	9	1.0×10^{-3}	1.14	—	—	—	-2041.1	16.1	3.9	-2021.1
—	Double power law	11	3.4×10^{-8}	1.25	1.47	0.90	0.54	-2047.2	17.7	18.7	-2010.8
—	Simple IF	10	2.8×10^{-11}	1.12	—	—	—	-2026.2	18.6	3.9	-2003.7
—	Standard IF	14	2.9×10^{-55}	1.54	—	—	—	-1934.4	27.9	4.1	-1902.4
—	Simple IF + EFE	59	5.9×10^{-64}	1.12	0	—	—	-2026.2	139.9	3.9	-1882.4

[1] $-\log \mathcal{L}(\hat{\boldsymbol{\theta}})$ [2] $k\log(n) + \sum_j \log(c_j)$ [3] $-\frac{p}{2}\log(3) + \sum_i^p (\log(I_{ii})^{1/2} + \log(\lvert\hat{\theta}_i\rvert))$

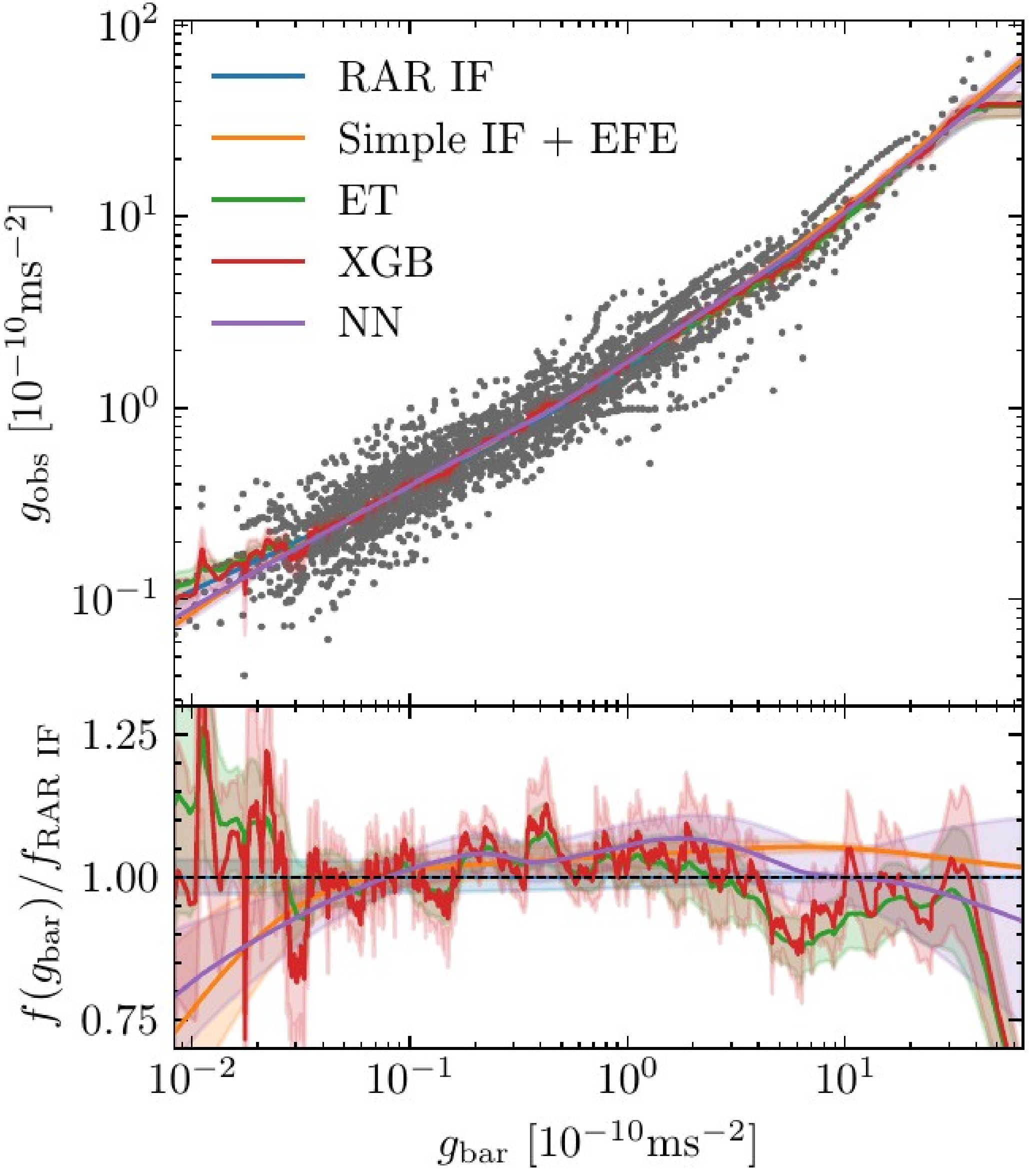
Simple IF + EFE mock

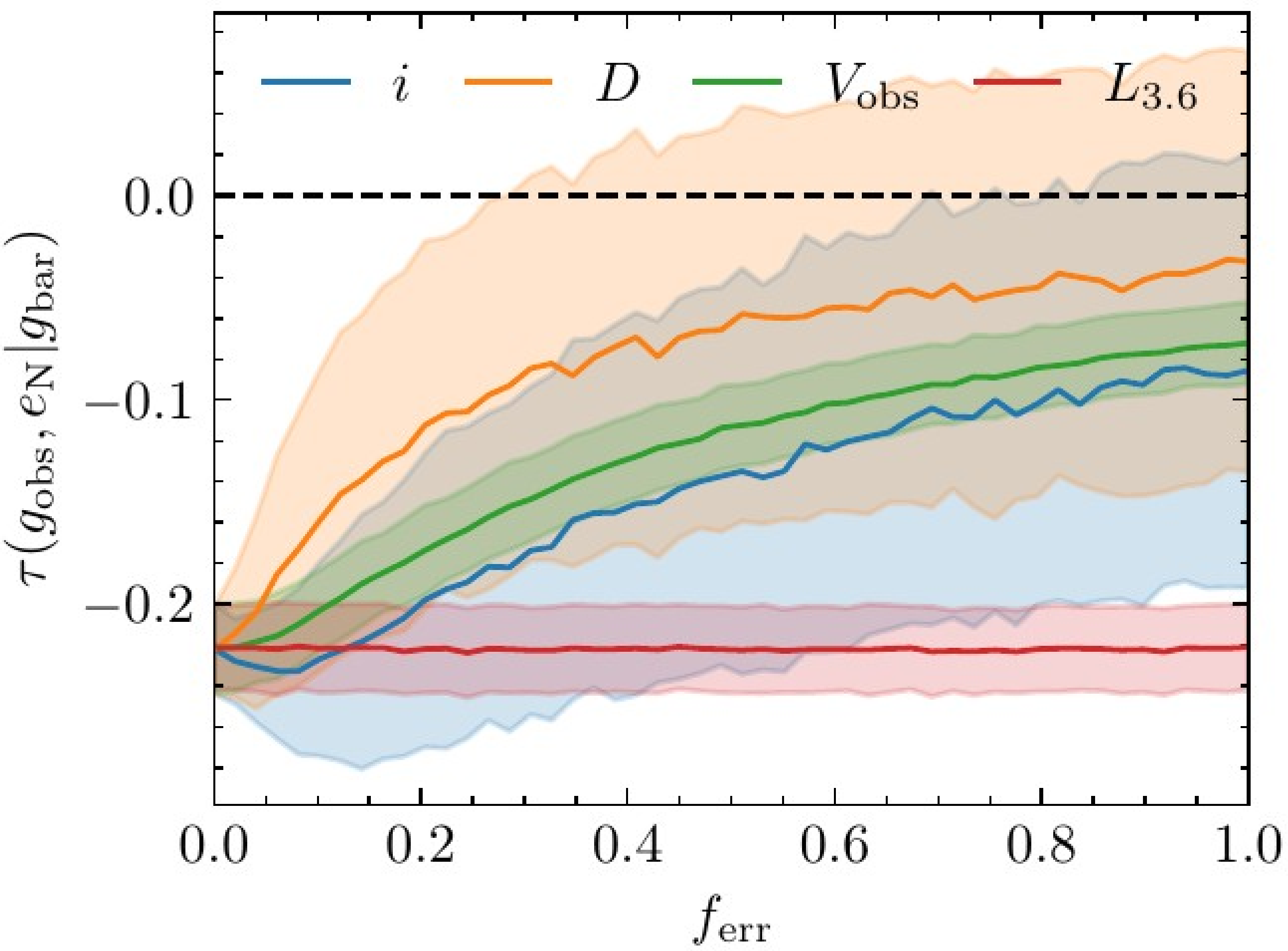
Rank	Function	Comp.	$P(f)$	Parameters				Description length			
				θ_0	θ_1	θ_2	θ_3	Resid.[1]	Func.[2]	Param.[3]	Total
1	$\theta_0 + \sqrt{x^2 + 2x}$	9	8.9×10^{-1}	-0.06	—	—	—	-2017.7	14.5	3.1	-2000.0
2	$\theta_0 + \sqrt{x\lvert\theta_1 + x\rvert}$	8	9.3×10^{-2}	-0.06	1.97	—	—	-2017.9	12.9	7.3	-1997.8
3	$-\lvert\theta_0\rvert^{\sqrt{x}} + \theta_1 + x$	8	5.6×10^{-3}	0.26	0.95	—	—	-2017.9	12.9	10.1	-1995.0
4	$(\theta_0 - x)\left(\theta_1 - x^{\theta_2}\right)$	9	3.3×10^{-3}	3.1×10^{-3}	-0.71	-0.53	—	-2019.7	14.5	10.7	-1994.4
5	$x^{\theta_0} - \theta_1(\theta_2 - x)$	9	2.4×10^{-3}	0.39	0.79	0.12	—	-2020.9	14.5	12.3	-1994.1
6	$\lvert\theta_0 - x\rvert^{\theta_1} - \theta_2 x$	9	2.0×10^{-3}	5.5×10^{-3}	0.48	-0.71	—	-2019.1	14.5	10.6	-1994.0
7	$x\lvert\theta_0\rvert^{-\lvert\theta_1\rvert^{x^{\theta_2}}}$	9	1.7×10^{-3}	0.04	-0.16	0.33	—	-2018.1	12.5	11.9	-1993.8
8	$x\left(\theta_0 + \lvert\theta_1 + x\rvert^{\theta_2}\right)$	9	1.5×10^{-3}	0.71	0.01	-0.53	—	-2018.7	14.5	10.6	-1993.7
9	$\lvert\theta_0\rvert^{\lvert\theta_1\rvert^{x^{\theta_2}}} + x$	9	6.5×10^{-4}	7.0×10^{-6}	0.03	0.17	—	-2016.7	12.5	11.4	-1992.8
10	$\exp\left(\theta_0 - \frac{1}{\sqrt[4]{x}}\right) + x$	9	5.5×10^{-4}	0.57	—	—	—	-2014.0	17.5	3.9	-1992.6
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
21	$x/(\exp(\theta_0) - \lvert\theta_1\rvert^{\sqrt{x}})$	9	1.8×10^{-5}	0.03	0.44	—	—	-2014.2	17.5	7.4	-1989.3
—	Double power law	11	3.4×10^{-11}	3.53	3.31	0.98	0.60	-2012.3	17.7	18.6	-1976.0
—	Simple IF	10	1.2×10^{-22}	1.11	—	—	—	-1972.1	18.6	3.9	-1949.6
—	RAR IF	9	7.0×10^{-24}	1.13	—	—	—	-1966.9	16.1	3.9	-1946.8
—	Simple IF + EFE	59	3.8×10^{-57}	1.19	8.6×10^{-3}	—	—	-2016.0	139.9	5.9	-1870.2
—	Standard IF	14	2×10^{-141}	1.54	—	—	—	-1708.3	27.9	4.1	-1676.3

[1] $-\log\mathcal{L}(\hat{\boldsymbol{\theta}})$ [2] $k\log(n) + \sum_j \log(c_j)$ [3] $-\frac{p}{2}\log(3) + \sum_i^p (\log(I_{ii})^{1/2} + \log(\lvert\hat{\theta}_i\rvert))$

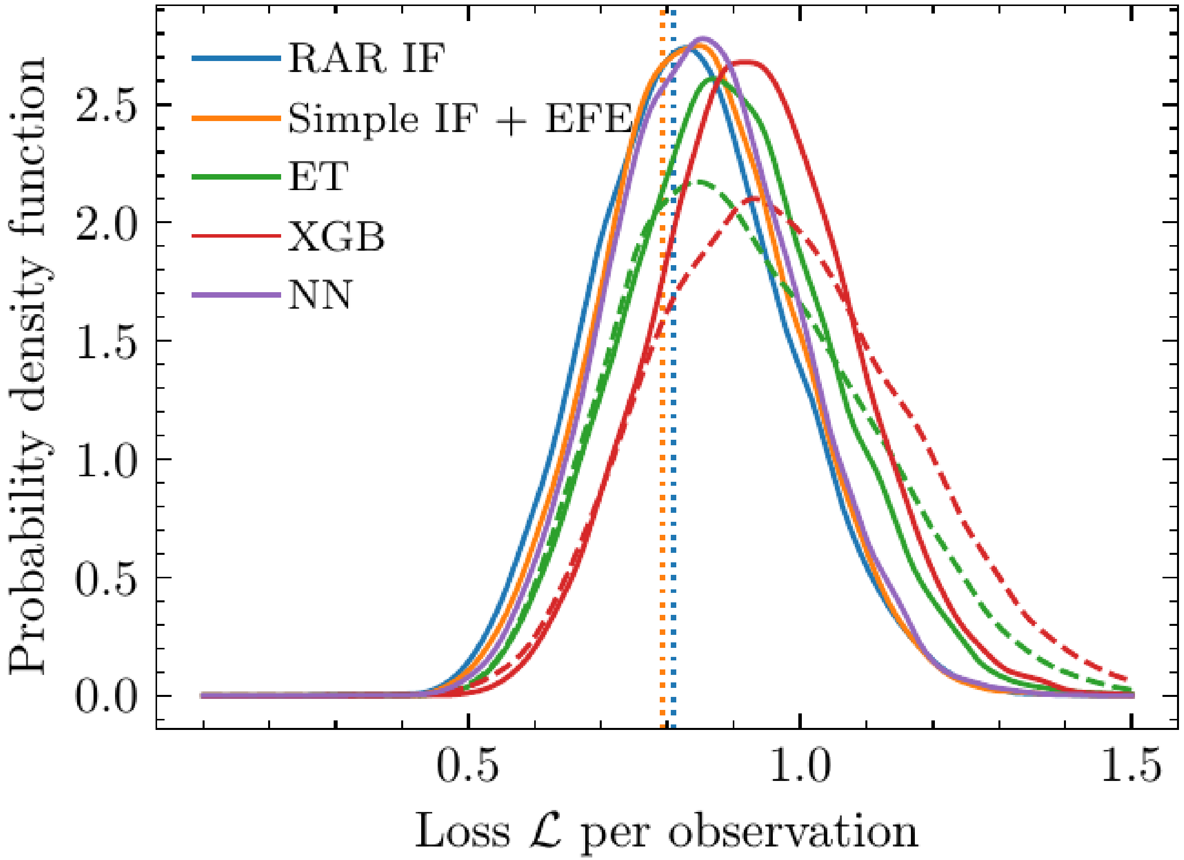


SPARC
Mock realization
Cubic fit
10^3
10^4
10^5
10^6
10^7
10^8
$J_{\rm obs}$ [km^{3}s^{-3}/kpc^2]
10^0
10^2
10^4
$\Sigma_{\rm tot}$ [$L_\odot$/pc^2]

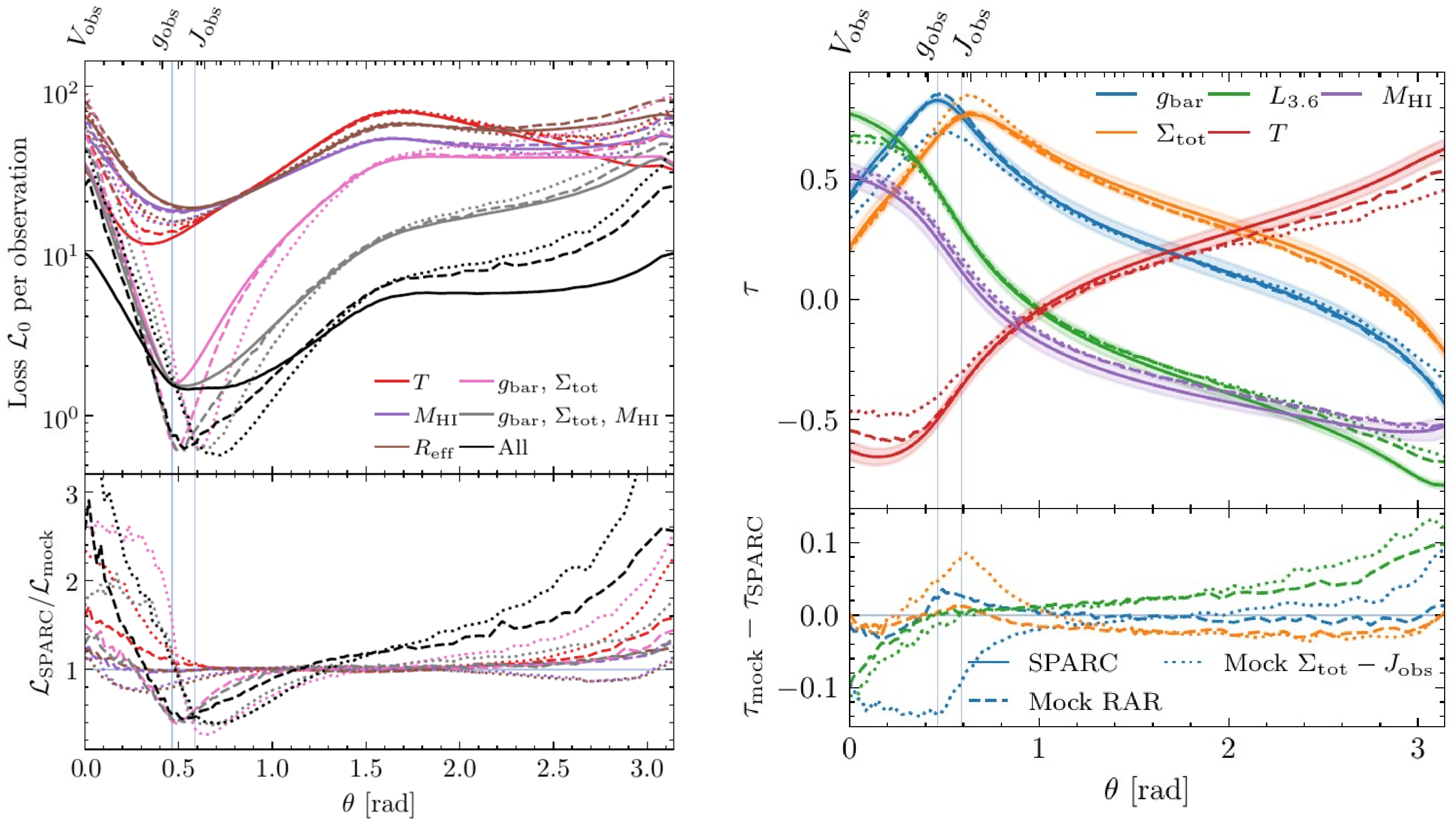

RAR IF
Simple IF + EFE
ET
XGB
NN
$g_{\rm obs}$ $[10^{-10}{\rm ms}^{-2}]$
$f(g_{\rm bar})/f_{\rm RAR\ IF}$
$g_{\rm bar}$ $[10^{-10}{\rm ms}^{-2}]$



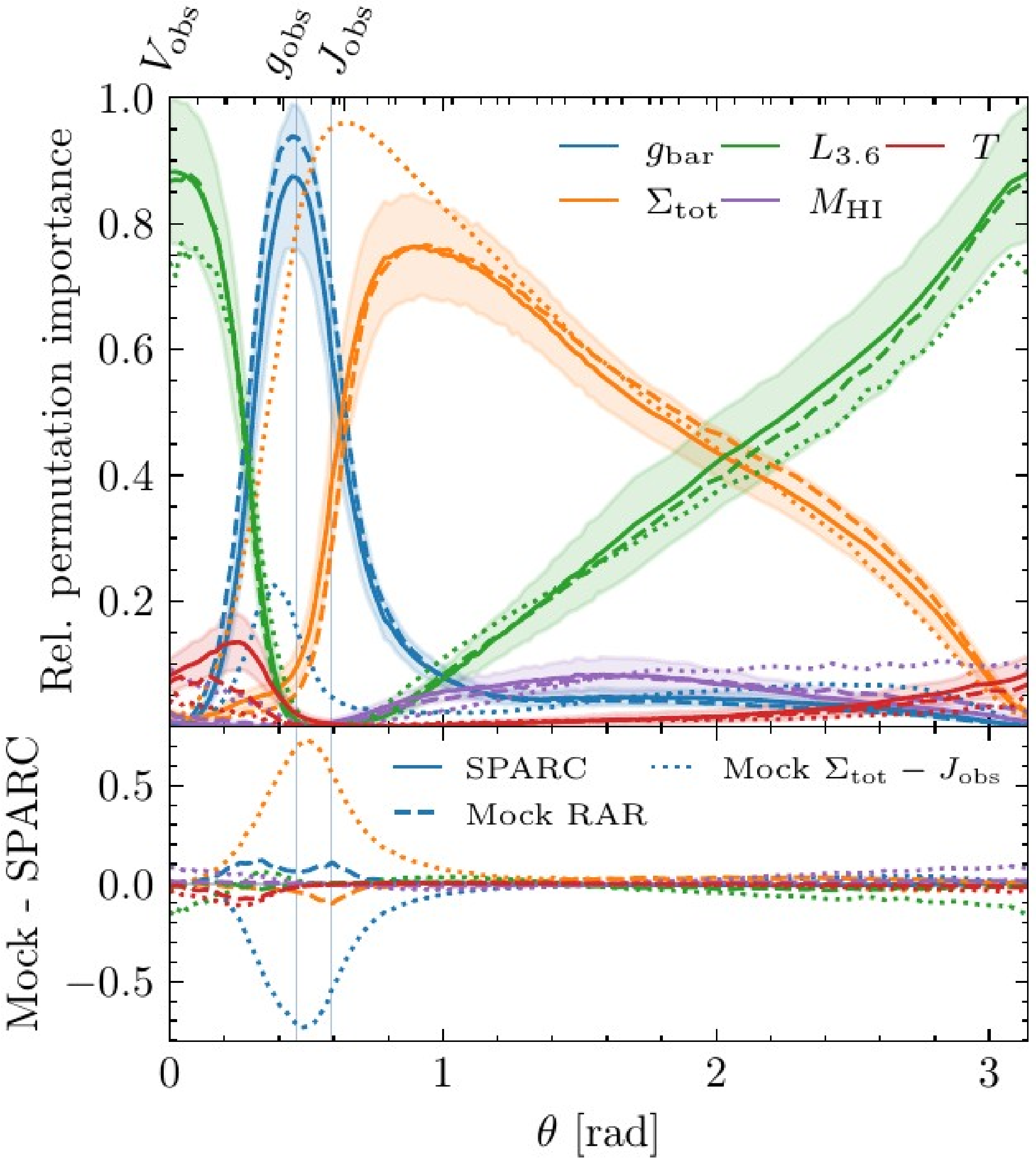
i
D
$V_{\rm obs}$
$L_{3.6}$
0.0
−0.1
−0.2
$\tau(g_{\rm obs}, e_{\rm N}|g_{\rm bar})$
0.0
0.2
0.4
0.6
0.8
1.0
$f_{\rm err}$



RAR IF
Simple IF + EFE
ET
XGB
NN
Probability density function
0.0
0.5
1.0
1.5
2.0
2.5
0.5
1.0
1.5
Loss $\mathcal{L}$ per observation



V_{obs}
g_{obs}
J_{obs}
Loss $\mathcal{L}_0$ per observation
T
g_{bar}, Σ_{tot}
M_{HI}
g_{bar}, Σ_{tot}, M_{HI}
R_{eff}
All
$\mathcal{L}_{SPARC}/\mathcal{L}_{mock}$
θ [rad]
τ
g_{bar}
$L_{3.6}$
M_{HI}
Σ_{tot}
T
$\tau_{mock} - \tau_{SPARC}$
SPARC
Mock $\Sigma_{tot} - J_{obs}$
Mock RAR
θ [rad]



V_{obs}
g_{obs}
J_{obs}
g_{bar}
$L_{3.6}$
T
Σ_{tot}
M_{HI}
Rel. permutation importance
1.0
0.8
0.6
0.4
0.2
SPARC
Mock RAR
Mock $\Sigma_{tot} - J_{obs}$
Mock - SPARC
0.5
0.0
−0.5
0
1
2
3
θ [rad]