What is MOND?

(a minimalist definition)

MOND is an algorithm that permits calculation of the radial distribution of force in an object from the observable distribution of baryonic matter with only one additional fixed parameter having units of acceleration.

It works! (at least for galaxies)

And this is problematic for dark matter.

Moreover, explains systematic aspects of galaxy photometry and kinematics, and…..
makes predictions! (cdm gets it wrong)

The Algorithm: (acceleration based)

(Milgrom 1983) $\mathbf{g}\ \mu(g/a_0) = \mathbf{g_N}$

g -- true gravitational acceleration

g_N -- Newtonian acceleration

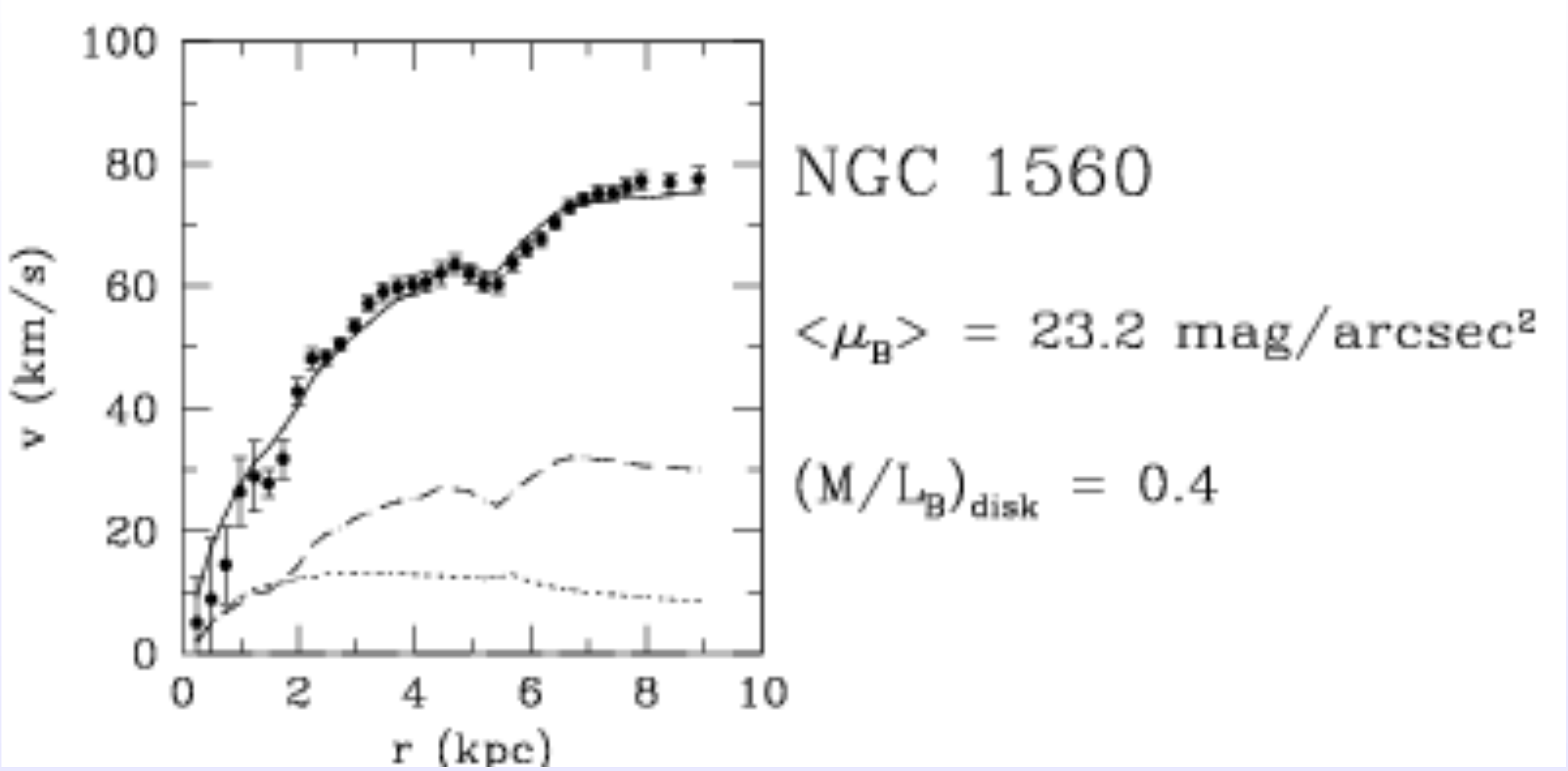
a_0 -- fixed acceleration parameter

$\mu(x) = 1 \quad x \gg 1$

$\mu(x) = x \quad x \ll 1$

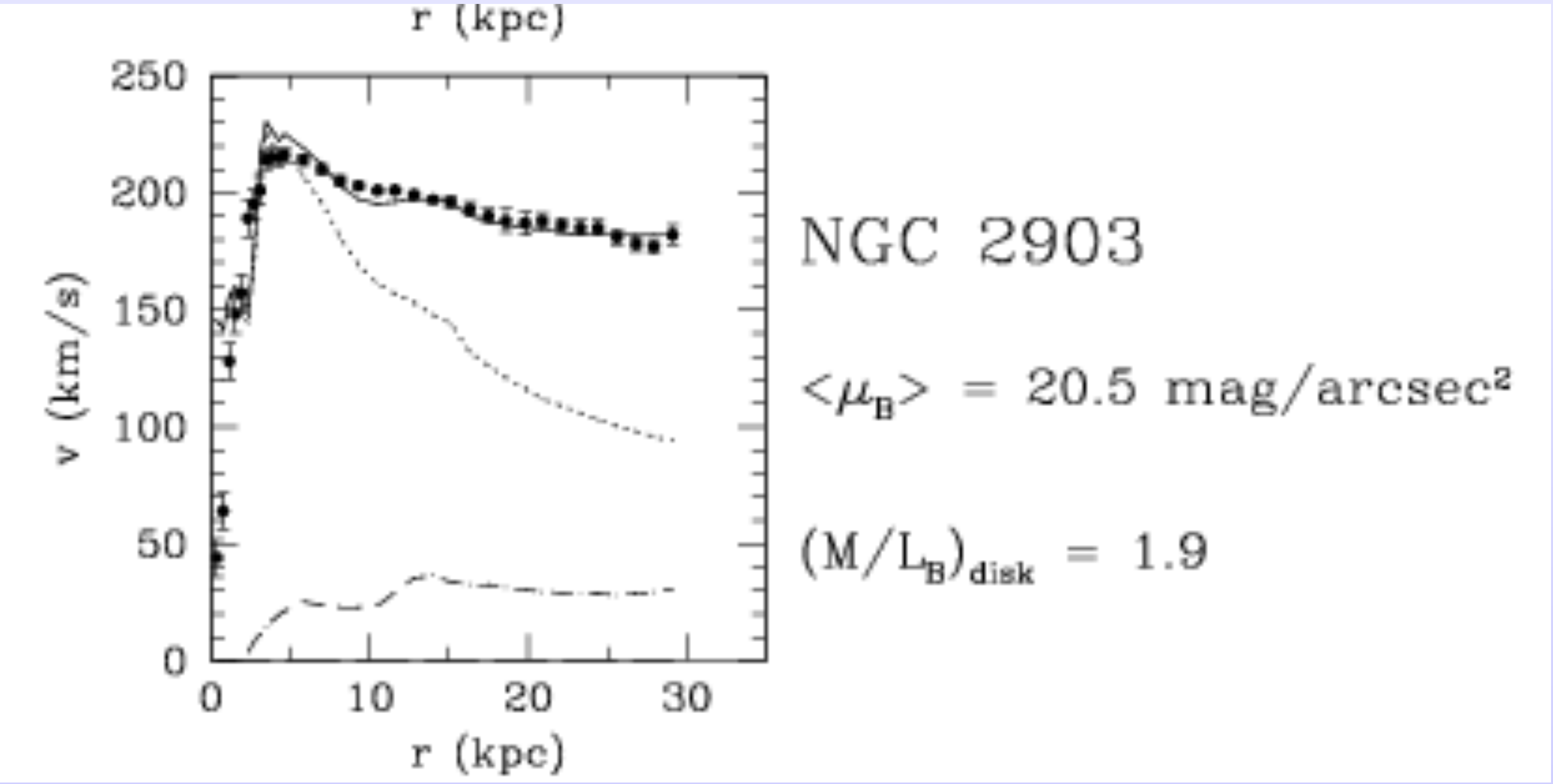
LSB: $\Sigma < \Sigma_c$

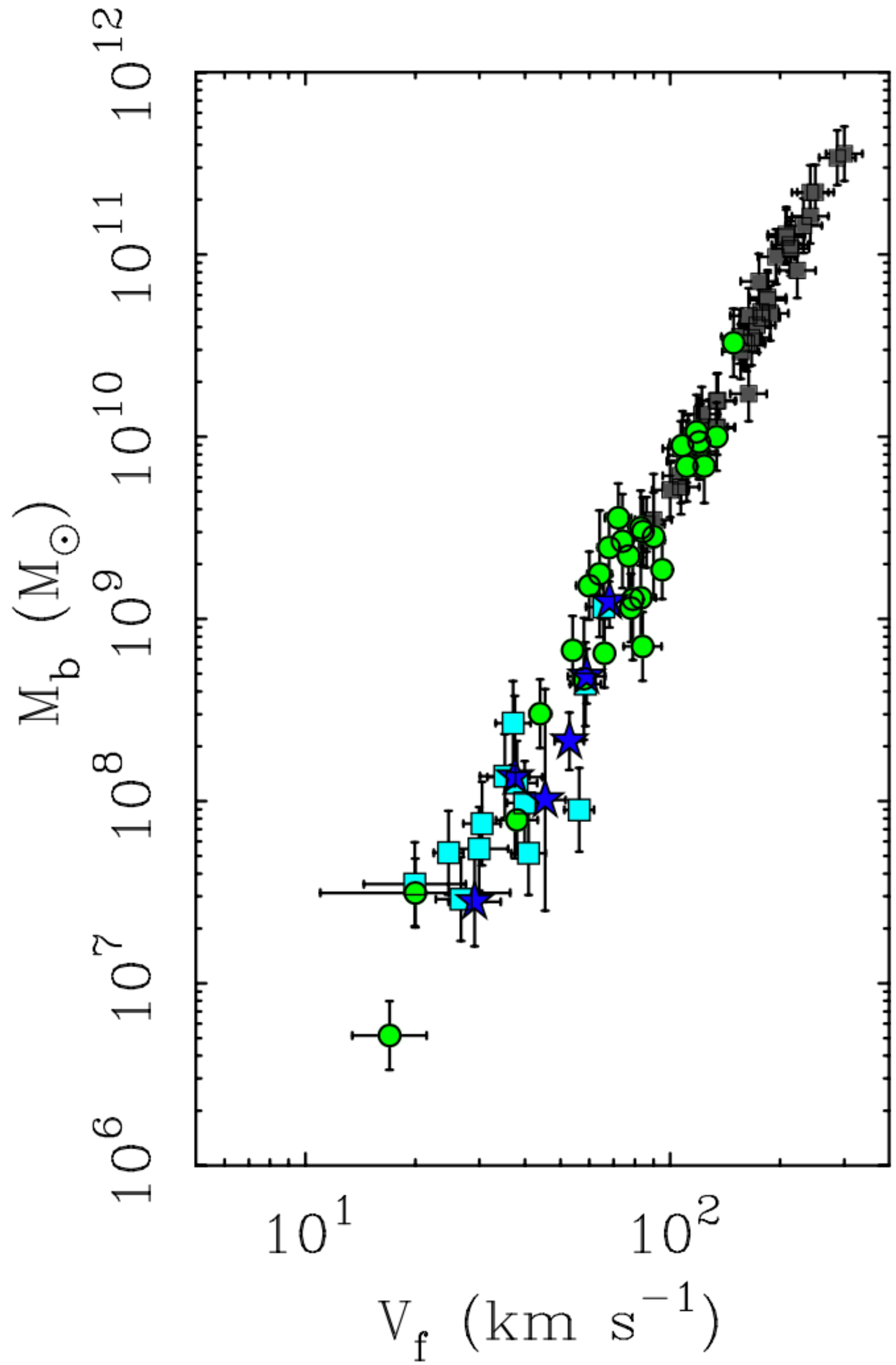
(Broeils)



HSB: $\Sigma > \Sigma_c$

(Begeman)





$$V^2/r = (GM_b a_0)^{1/2}/r$$

$$V^4 = GM_b a_0$$

Baryonic TF relation. — McGaugh 2000

Successes of MOND

- Predicts observed form of galaxy rotation curves from observable mass distribution. Near IR photometry — no free parameters.

- Baryonic TR relation for spirals, FJ relation for ellipticals.

- Presence of preferred surface density in spirals (Freeman law). LSB– large discrepancy,

  HSB– small discrepancy
- Subsumes the "radial acceleration relation" (McGaugh, Lelli et al.)

- All with $a_o \approx cH_o$

- But underpinned by new physics?

MOND workers circa 1997

Jacob Bekenstein (1947-2015)

PhD. with John Wheeler at Princeton

Fundamental work on black holes: entropy $\propto$ surface area

Information stored on surfaces not volume   Hawking rad.

N-1   N   Holographic principle.

Contributions to theoretical basis of Milgromian dynamics.

Steps to Tensor-Vector-Scalar theory

TeVeS – Bekenstein astro-ph/0412652

Relativistic MOND $\Longrightarrow \left\{ \begin{array}{l} \text{Gravitational lensing} \\ \text{Cosmology} \\ \text{Structure formation} \\ \text{CMB} \\ \text{Gravitational waves} \end{array} \right.$

Phenomenologically driven theory– 3 fields, 3 free parameters, 1 free function.

Bottom-up– attributes are added in response to particular pathologies or phenomenological requirements.

MOND as a modification of gravity:

Bekenstein & Milgrom (1984) aquadratic Lagrangian

$$S_f = -\int d^3r \left[\rho\phi + (8\pi G)^{-1} {a_o}^2 F\left(\frac{\nabla\phi^2}{{a_o}^2}\right)\right]$$

$$\Rightarrow \qquad \nabla \cdot \left[\mu\left(\frac{|\nabla\phi|}{a_o}\right)\nabla\phi\right] = 4\pi G\rho$$

Where $\mu = \frac{dF(x)}{dx} = \begin{cases} 1 & x > 1 \\ x^{1/2} & x < 1 \end{cases}$

Modified Poisson equation-- Conservative!

Lagrangian is invariant to space-time translations and rotations.

$\Longrightarrow$ Theory conserves linear momentum, energy, angular momentum.

Theory in MOND limit is conformally invariant.

Conformal transformation: angle preserving, space-time dependent scale transformations:

Covariant Extension of AQUAL:

Scalar field Lagrangian:

$$L_s = \frac{1}{16\pi G} F\left[\frac{\phi_{,\alpha}\phi^{,\alpha} c^4}{{a_o}^2}\right]$$

Interaction Lagrangian:

$$L_I = L_I[\xi(\phi^2) g_{\mu\nu} \ldots]$$

$$\Longrightarrow \quad \nabla \cdot \left[\mu\left(\frac{|\nabla\phi|}{a_o}\right)\nabla\phi\right] = 4\pi G\rho$$

But now ϕ is a scalar field. Complete theory includes $g_{\mu\nu}$ and Hilbert-Einstein action of GR:

$$S_g = \int R\sqrt{-g}\, d^4x$$

As before: $\mu = \frac{dF(x)}{dx}$ $x = \frac{\phi_{,\alpha}\phi^{,\alpha}c^4}{a_o^{\;2}}$

so-- $F(x) = \frac{2}{3}x^{3/2}$ $x < 1$ $(|\nabla\phi| < a_o)$

and.. $F(x) = \omega x$ $x > 1$ $(|\nabla\phi| > a_o)$

This is a non-standard scalar-tensor theory; in the limit of large scalar field gradients $\Longrightarrow$ Brans-Dicke theory.

$\omega > 10^4$ to be consistent with solar system experiments

$x < 0$ problem for cosmology– more later.

AQUAL not unfamiliar: K-inflation, K-essence (kinetically driven)

But a problem with form required for MOND-- for scalar waves

$v = \sqrt{2}c$ parallel to direction of $\nabla\phi$

Acausal propagation of scalar waves!

For JB this was a no-go.

How does one construct a scalar tensor theory that is causal?

Immediate problem: Lensing.

Constraints on gravity theory:

Strong limits on violation of WEP implies metric theory. i.e., particles (relativistic and non-relativistic) follow geodesics of a physical metric (i.e., motion is independent of composition of internal structure.)

So --

$$d\tilde{\tau} = -\tilde{g}_{\mu\nu} dx^{\mu} dx^{\nu}$$

describes incremental path of a particle in 4-d space-time.

But there is also a field given by solution to Einstein field equations:

$g_{\mu\nu}$ the gravitational (or Einstein) metric.

In GR $g_{\mu\nu} = \tilde{g}_{\mu\nu}$

Gravitational waves follow null geodesics of the gravitational metric, em waves follow null geodesics of physical metric. In GR these are identical so

$$C_{gw} = C_{em}$$

But what about modified theories of gravity?

Simplest modification is to add a new field -- a scalar field -- ϕ, but WEP requires that we maintain a metric theory ϕ cannot couple directly to particles but jointly through the Einstein metric: i.e., physical metric becomes

$$\tilde{g}_{\mu\nu} = f^2(\phi) g_{\mu\nu}$$

(conformal transformation)

Null geodesics coincide:

$$d\tilde{\tau} = -\tilde{g}_{\mu\nu} dx^\mu dx^\nu = 0 \equiv -g_{\mu\nu} dx^\mu dx^\nu$$

So again $$C_{gw} = C_{em}$$

This means that em waves and gravitational waves do not "feel" the scalar field (not effected).

But non-relativistic particles are affected $\mathrm{d}\tilde{\tau} \neq 0 \neq d\tau$

The point is: if you want to use a scalar field as an additional component of the gravitational field, then with the usual conformal coupling it affects the motion of slow moving particles but not relativistic particles. Gravitational lensing only "sees" the baryonic matter and not the dark matter,

Absolute contradiction with observations of lensing by clusters of galaxies (and individual galaxies).

A solution: two metrics are related by a "disformal" transformation).

Pick out a particular direction in space-time for additional stretching. Since space should be isotropic, that additional direction is ‘time’ in some preferred frame (cosmic frame).

This can be done by adding a unit vector (normalized to unity:

$$\mathrm{A}^{\nu} = \delta_t^{\nu}$$

Breaks Lorentz invariance by picking out pf!

$$\tilde{g}_{\mu\nu} = e^{-2\eta\phi} g_{\mu\nu} - 2 sinh(2\eta\phi) A_{\mu} A_{\nu}$$

Correct defection of light by gravitating masses.

Disformal transformation -- “stratified theory” (Ni 1972)

With aquadratic Lagrangian $\Longrightarrow$ MOND & enhanced lensing.

$$\Delta\theta = \frac{2}{c^2}\int f_\perp dl$$

as in GR (Sanders 1997)

But, non dynamic vector field violates General Covariance.

(a toy theory)

No conserved $T_{\mu\nu}$

The cure-- dynamical vector field

$\Rightarrow TeVeS$ (Bekenstein 2004)

Tensor-Vector-Scalar theory (TeVeS)

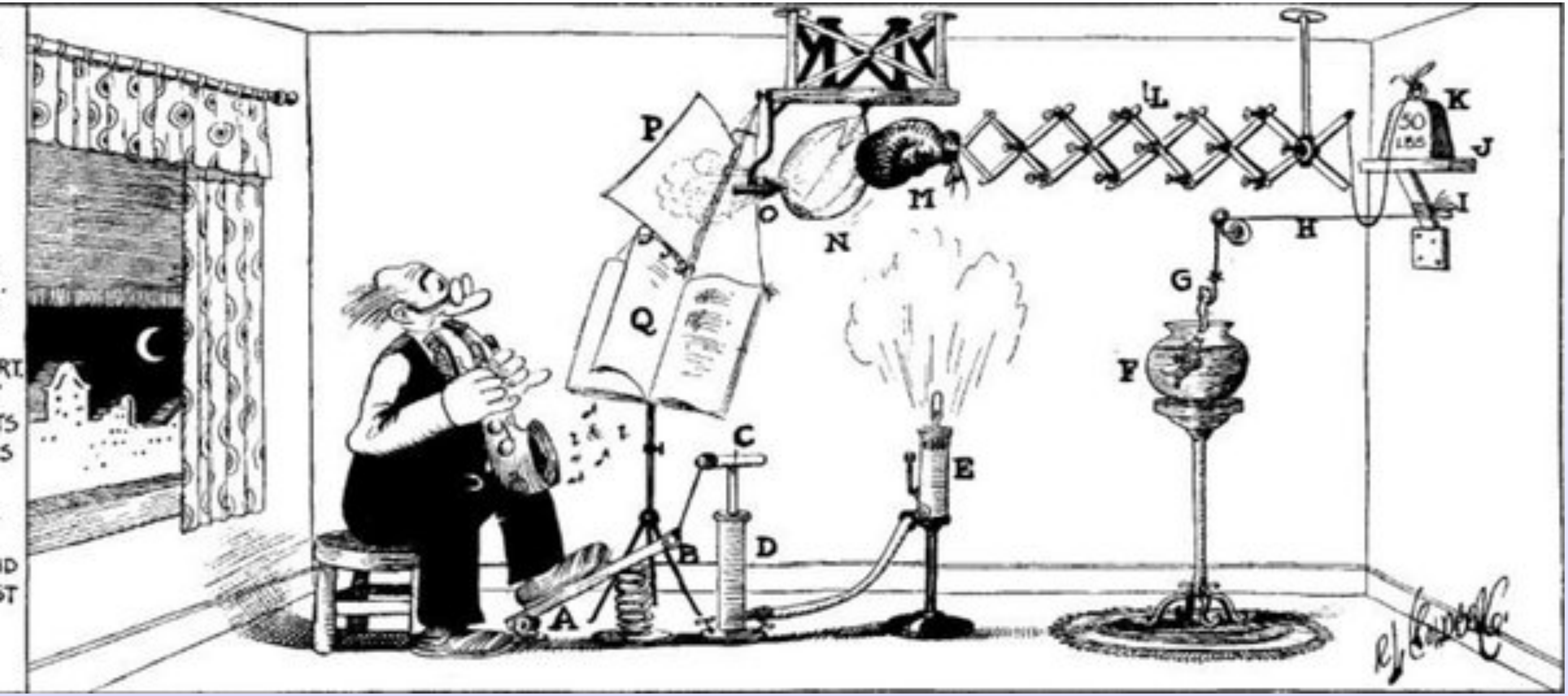
Bekenstein 2004

Tensor:	$g_{\mu\nu}$	Einstein tensor-- must reduce to GR
Vector:	A_α	Dynamical, necessary for lensing
Scalar:	ϕ	Non-standard Lagrangian. Provides MOND force

Messy-- 3 new parameters and one free function.

Does not approach GR as $a_0 \longrightarrow 0$

Phenomenological problems: Vector establishes a preferred frame. PF effects cannot be naturally suppressed.



Automatic music page turner

Avoid Rube Goldberg effect if possible.

(complicated machine to perform a simple task)

Successes of TeVeS

1. MOND phenomenology with enhanced gravitational lensing
   (by construction– disformally related $\tilde{g}_{\mu\nu}$ and $g_{\mu\nu}$

2. Static Post-Newtonian parameters identical to GR.

   $$\beta = \gamma = 1$$

   (consistency with deflection, precession, radar echo delay)
   But ether-drift parameters require fine tuning (Sagi 2012)

3. Scalar waves are causal ($v_s = c$) because $h^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}$ replaces $g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}$ as invariant object.

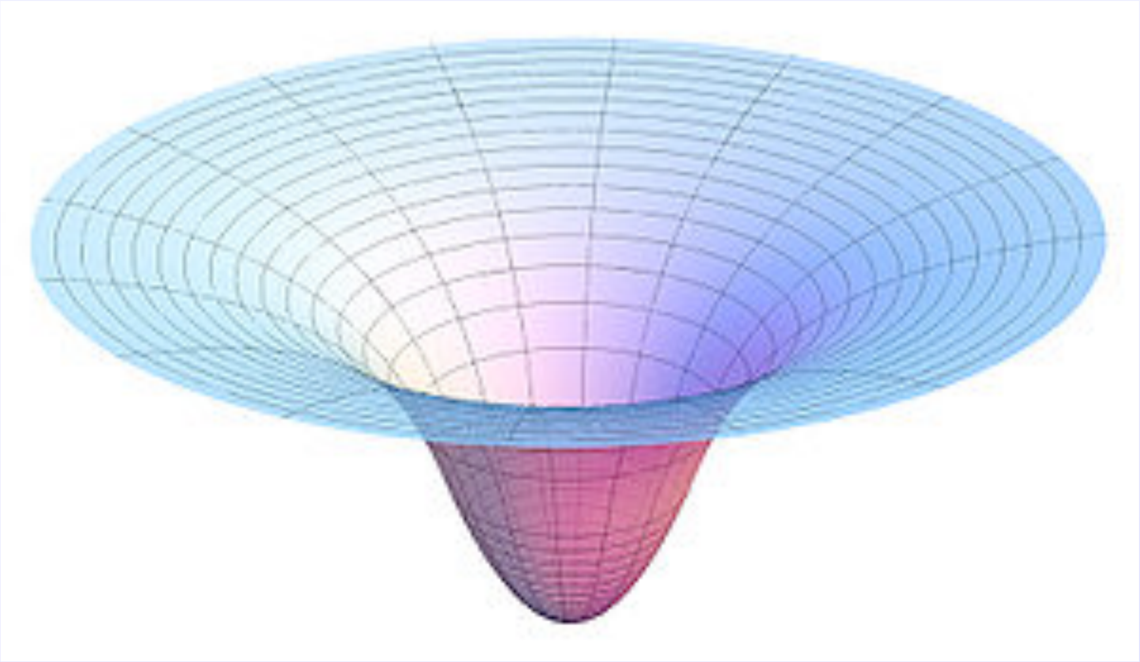
4. Gravitational waves are also causal if $\phi > 0$.
   In fact $v_g = ce^{-\eta\phi} < c$. A test?

5. Cosmology is standard FLRW– with variable and extra source terms.

AND -- null geodesics of Einstein metric no longer correspond to null geodesics of physical metric
EM waves feel effective dark matter, gravitational waves do not.

$$d\tau = 0 \qquad\qquad d\tilde{\tau} \neq 0$$

One-way Shapiro delay



Due to ST curvature, effective distancs traversed by photon deep in potential well is greater than in flat space.

GW170817 LIGO-Virgo

A Spectacular Event -- Multi-messenger astronomy

$$\nu \approx \frac{c^3}{6\pi GM} \approx 500\ \mathrm{Hz}$$

$$\rightarrow M \approx 1M_\odot$$ binary pulsar.

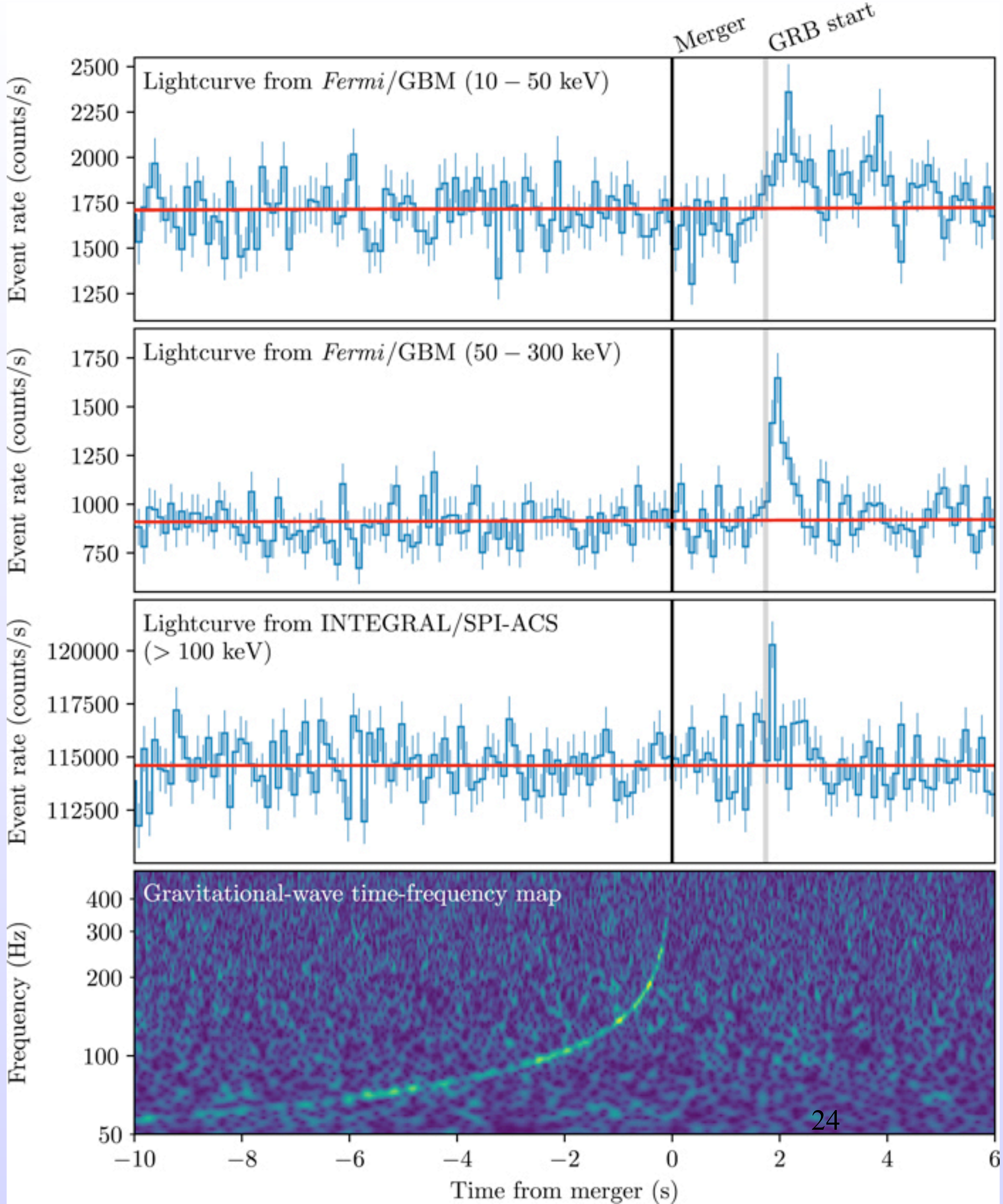
$$\mathrm{L}_{GW} \approx fc^5/G$$ with measured flux ($f \approx 10^{-4} - 10^{-3}$)

$$\rightarrow D \approx 40\mathrm{Mpc}$$

Followed by Fermi gamma ray short duration gamma ray burst identified with early-type galaxy (NGC 4993) at a distance of 40 Mpc

Enormous implications!

Fermi
GB 1708017a



Ligo-Virgo
GW 170817

One-way Shapiro delay (previously academic)

$$\Delta t = (1+\gamma)\frac{GM}{c^3}\ln r_2/r_1$$

$$\mathrm{M}{=}10^{12} M_\odot \quad r_2, r_1 = 400\,\mathrm{kpc} \quad 8\,\mathrm{kpc}$$

$$\Delta t \approx 445 \ \ \mathrm{days}$$

Total including host galaxy ~ 1000 days

$$\Delta t_{obs} = 1.7\,\mathrm{s}$$

Consistent with $\tilde{g}_{\mu\nu} = g_{\mu\nu}$

Falsifies theories with disformally related metrics.

(Desai et al. 2008)

Alternatives

Einstein-Aether Theory

(vector-tensor)

(Zlosnik et al. 2007, Skordis & Zlosnik 2021

With V-T theory four invariants of the Vector field can be added—
4 new parameters. TeVeS can be written as a EA theory.

Specific relation between parameters imply correct lensing.
Energy density in vector field can provide effective dm clumps
That can agree with CMB anisotropies,

The Scenario:

Cosmology is described by preferred frame theory in which force is mediated by a long range scalar field.

In regions of high field gradients ($|\nabla\phi| > a_o$) dynamics is that of GR (preferred frame effects are suppressed)

Outskirts of galaxies are transition regions between preferred frame cosmology and GR –dominated local dynamics.

Transition observable as MOND!

Jacob Bekenstein in retrospect

Classical relativist — would like to emulate Einstein.
Enumerate grand principles — deduce MOND.

But actual approach was bottom-up: elements added in response to demands of phenomenology.

Hear more about both approaches.

Personal qualities: Self-confident but modest. Impatient with colleagues who were out of depth. Cosmologists who knew little about GR. Large-scale structure industry — engineering.