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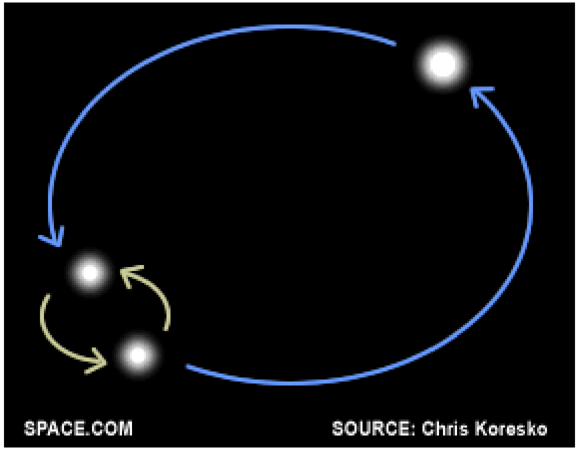
Strong constraints on weak gravity from Gaia DR3 wide binaries (Banik+, submitted π day 2023)

ABSTRACT

We test Milgromian dynamics (MOND) using wide binary stars (WBs) with separations of $2-30$ kAU. Locally, the WB orbital velocity in MOND should exceed the Newtonian prediction by $\approx 20\%$ at asymptotically large separations given the Galactic external field effect (EFE). We investigate this with a detailed statistical analysis of *Gaia* DR3 data on 10290 WBs within 300 pc of the Sun. Orbits are integrated in a rigorously calculated gravitational field that directly includes the EFE. We also allow line of sight contamination and undetected close binary companions to the stars in each WB. We interpolate between the Newtonian and Milgromian predictions using the parameter α_{grav}, with 0 indicating Newtonian gravity and 1 indicating MOND. Directly comparing the best Newtonian and Milgromian models reveals that Newtonian dynamics is preferred at 18σ confidence. Using a complementary Markov Chain Monte Carlo analysis, we find that $\alpha_{\text{grav}} = -0.036^{+0.064}_{-0.055}$, which is fully consistent with Newtonian gravity but excludes MOND at 16σ confidence. This is in line with the similar result of Pittordis and Sutherland using a somewhat different sample selection and less thoroughly explored population model. We show that although our best-fitting model does not fully reproduce the observations, an overwhelmingly strong preference for Newtonian gravity remains in a considerable range of variations to our analysis. Adapting the MOND interpolating function to explain this result would cause tension with rotation curve constraints. We discuss the broader implications of our results in light of other works, concluding that MOND must be substantially modified on small scales to account for local WBs.



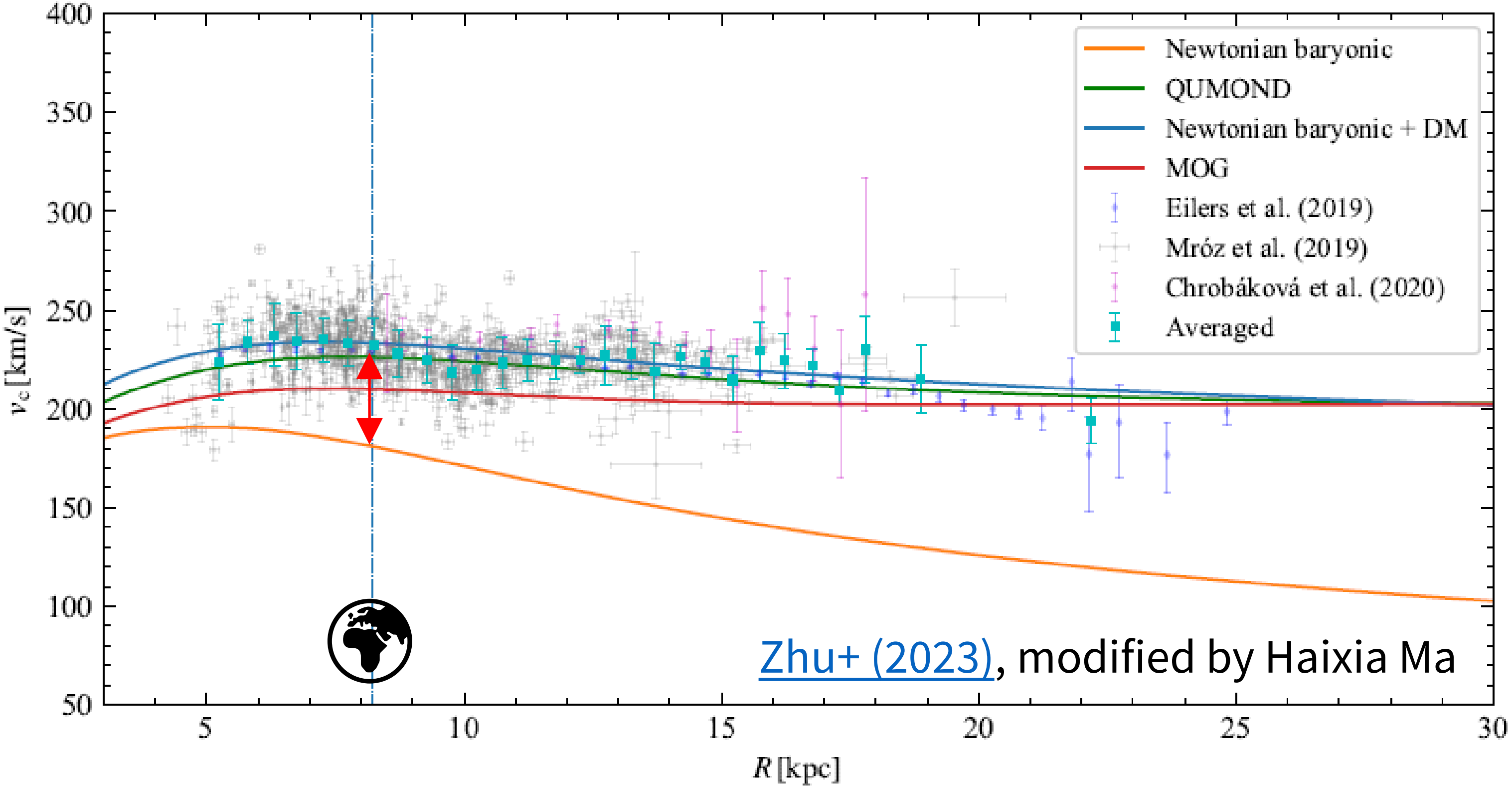
Testing gravity independently of dark matter

$$g_N \ll a_0 \text{ implies that } r \gg r_M \equiv \sqrt{GM/a_0}$$

- The MOND radius of the Sun is r_M = 7 kAU = 0.034 pc, much smaller than the Galaxy
  - ➢ Dark matter would not affect a local wide binary (WB), even if massive CDM halos exist
- But low accelerations imply that in MOND, anomalous effects are expected in the Proxima Centauri WB (Beech 2009, 2011) and other WBs (Hernandez+ 2012)
- Projection and orbital phase effects important in individual systems, so need statistical test: WBT
- Need to consider distribution of parameter $\tilde{v} \equiv v_{rel} \div \sqrt{\frac{GM}{r}}$ with only sky-projected quantities
- In Solar neighbourhood, **MOND enhances WB orbital velocity by 20%** in the regime of large separations due to Galactic external field effect (EFE; Banik & Zhao 2018, MNRAS, 480, 2660)
- Factor almost the same as that by which Newtonian baryonic rotation curve must be enhanced.

MOND at the Solar circle: a non-negligible effect

- Newtonian baryonic rotation curve falls short of the observed curve and declines much more steeply, so changing stellar M/L not sufficient
- Enhancement is 25%, but complications of MOND mean it enhances local WB velocities by only 20%.

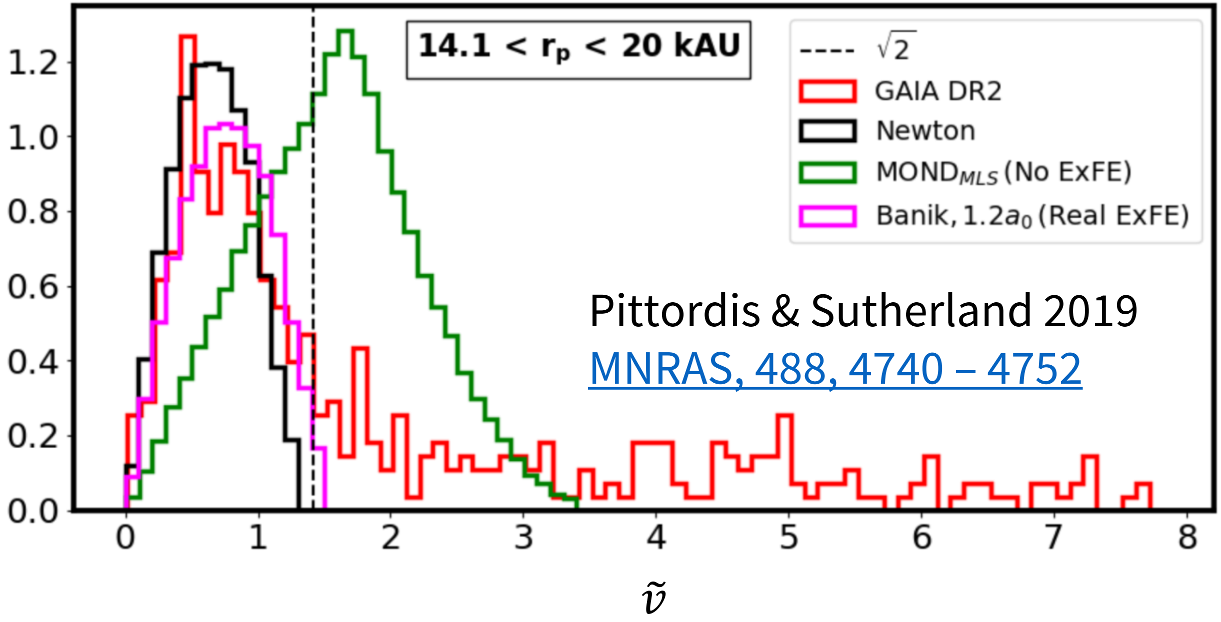


Zhu+ (2023), modified by Haixia Ma

Hierarchical systems

- Neglecting the Galactic EFE leads to predicted distribution shown in green, in drastic disagreement with data (red)
- Notice the extended high-velocity tail
- Close binaries (CBs) expected to be dominant systematic concern with the WBT *a priori* (Banik & Zhao 2018)
- Declining tail implies it is not due to line of sight (LOS) contamination
- *Gaia* measurement errors too small

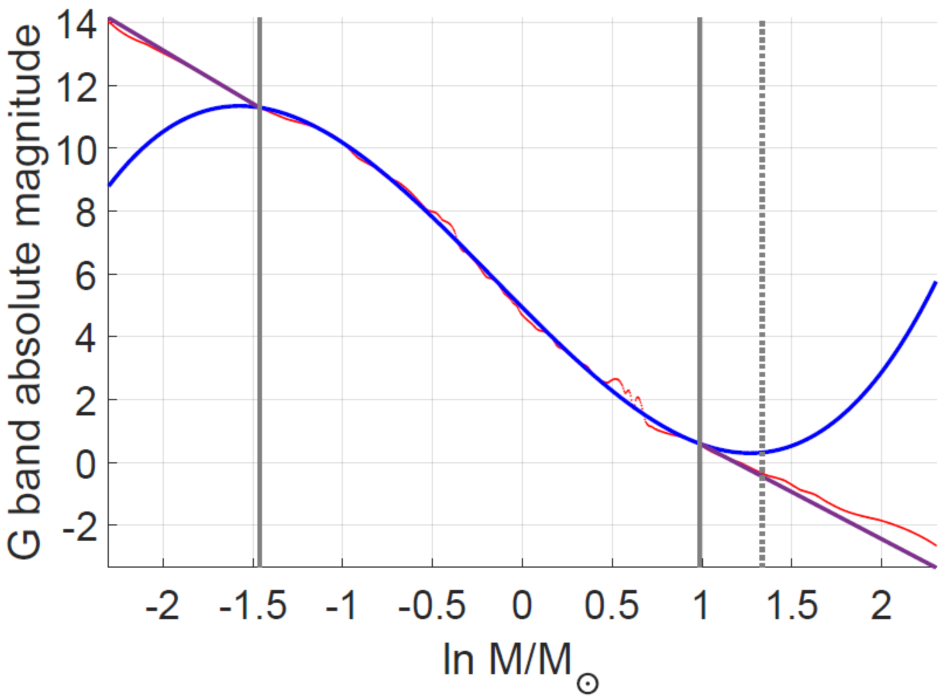
➢ Most likely cause is CBs, as suggested by Belokurov+ 2020 and Clarke 2020.



Pittordis & Sutherland 2019
MNRAS, 488, 4740 – 4752

Obtaining $\tilde{v}$ and its uncertainty

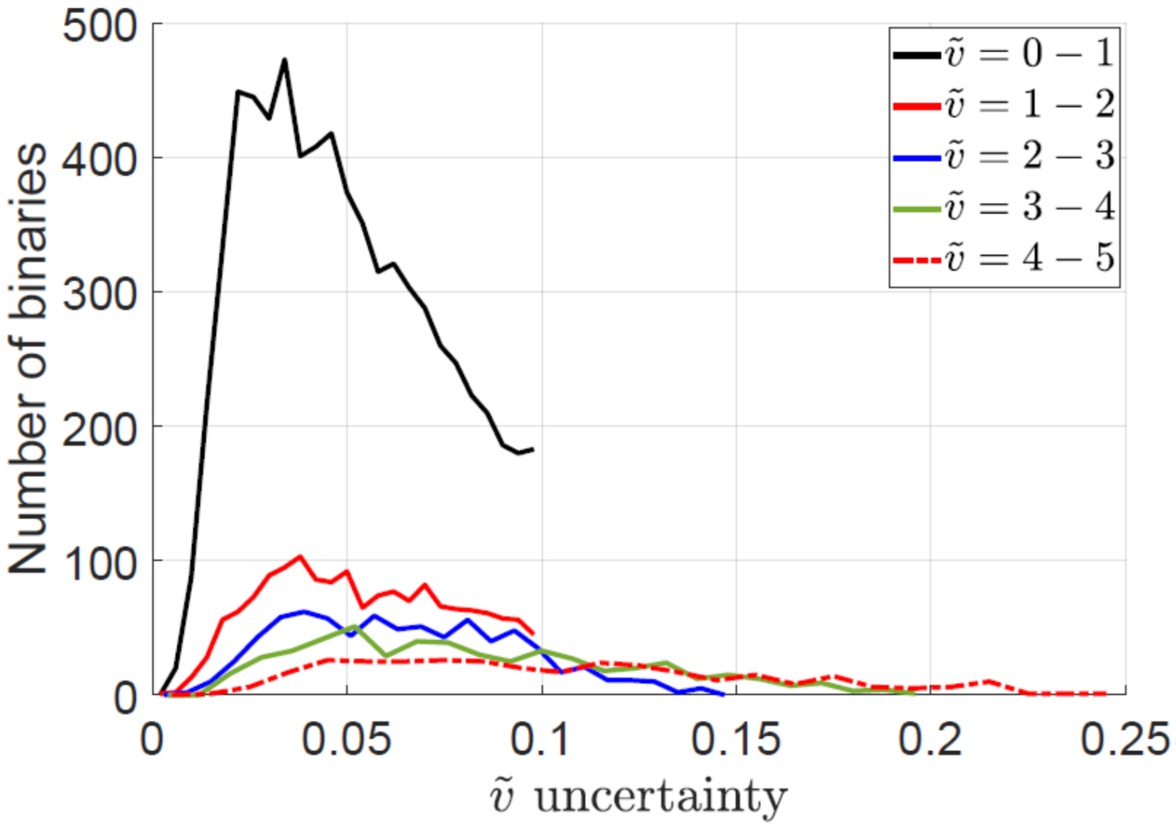
- Mass estimated from absolute Gaia-band magnitude using mass-luminosity relation from Pecaut & Mamajek (2013), data table updated in March 2021
- MOND boost equivalent to ≈1.8 magnitudes
- Systemic radial velocity (RV) needed as small part of it within sky plane: recession reduces separation
- Parallax uncertainties mean 3D structure of WB unknown, but 3D separation inferred statistically from r_{sky}, the 2D projected separation (Banik 2019)
- Full 5×5 *Gaia* covariance matrix used to propagate uncertainties in parallax and proper motions
- Detailed plan posted in advanced to mitigate moral hazards associated with WBT (Arxiv:2109.03827).



Uncertainties in $\tilde{v}$ are small in Gaia DR3

- Initial sample based on Pittordis & Sutherland 2023 (Open Journal of Astrophysics), low Galactic latitudes & regions of known star clusters rejected, other standard quality cuts applied
- Max. allowed error: 0.1 max(1, $\tilde{v}$/2)
- Main peak is near 0.5, so broadening by maximum uncertainty of 0.1 would raise width to $\sqrt{0.5^2 + 0.1^2} = 0.52$
- This is much smaller than the 20% broadening predicted by MOND

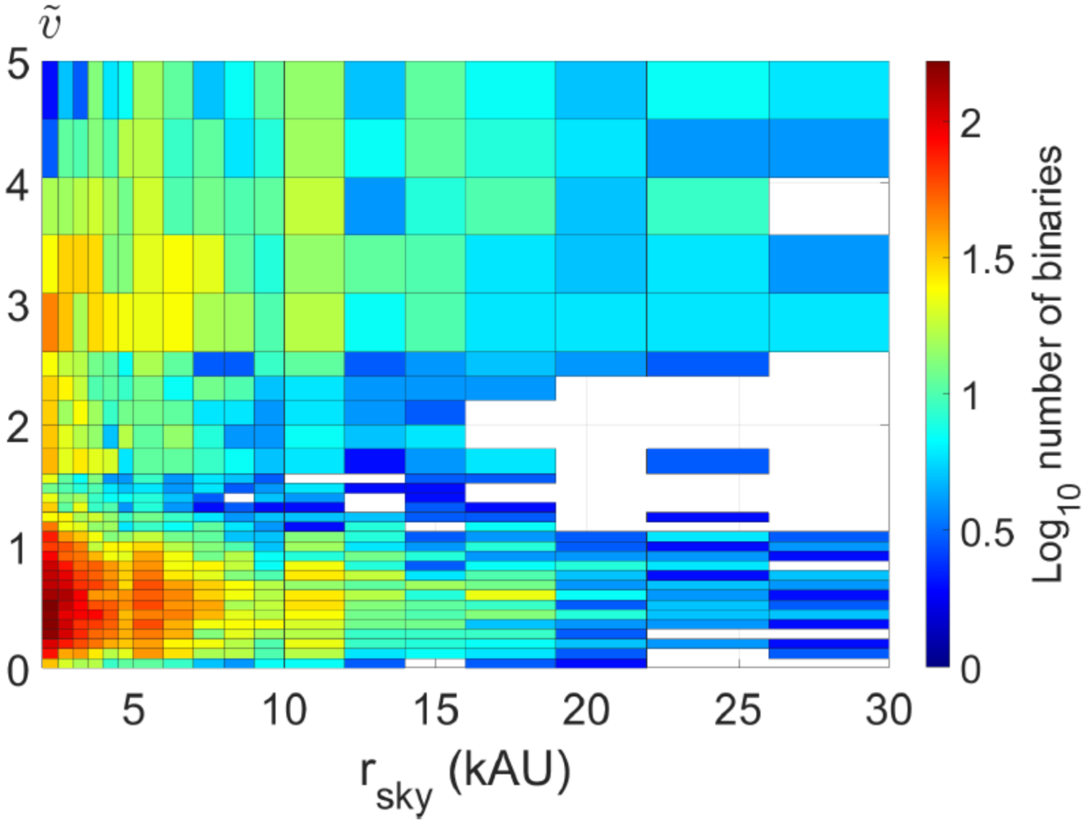
➢ **Gaia errors far smaller than the predicted MOND signal.**



The observed (r_{sky}, $\tilde{v}$) distribution

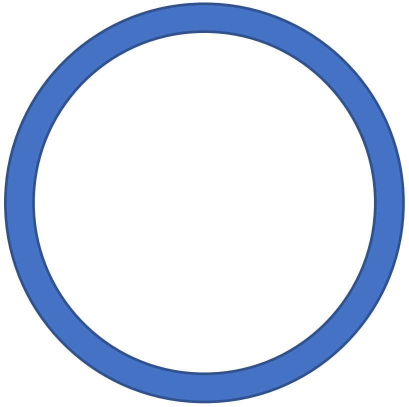
- 10290 systems pass further quality cuts, e.g., RV of at least one star known
- Extended CB tail diluted over larger $\tilde{v}$ range at large r_{sky} because Newtonian v_c of WB lower, but CB motion independent of WB separation
- LOS contamination only becomes important towards high $\tilde{v}$ as this is really a 2D quantity: think concentric circles

➢ Gap in $\tilde{v}$ distribution at high r_{sky} beyond main peak of $\tilde{v}$ distribution, before LOS contamination at high $\tilde{v}$ kicks in.



Line of sight (LOS) contamination

- Chance alignments of field stars too rare to much affect WBT (Pittordis & Sutherland 2019)
- But co-eval birth of stars in star cluster could lead to enhanced likelihood that unbound stars pass near each other and masquerade as a WB
- Assume relative velocity can greatly exceed WB orbital velocity
- Since both r_{sky} and v_{sky} are 2D quantities, get linear distribution in each. But when using $\widetilde{v}$, dependence on r_{sky} cancels out
- Fit this distribution to data to infer the LOS contamination fraction f_{LOS}, expect a few percent.



$$\frac{dN_{\mathrm{LOS}}}{dr_{\mathrm{sky}}\,dv_{\mathrm{sky}}} \propto r_{\mathrm{sky}} v_{\mathrm{sky}} \, . \tag{24}$$

Bearing in mind that $v_{\mathrm{sky}} \propto 1/\sqrt{r_{\mathrm{sky}}}$ at fixed $\widetilde{v}$ (Equation 3), the population distribution of LOS contamination is

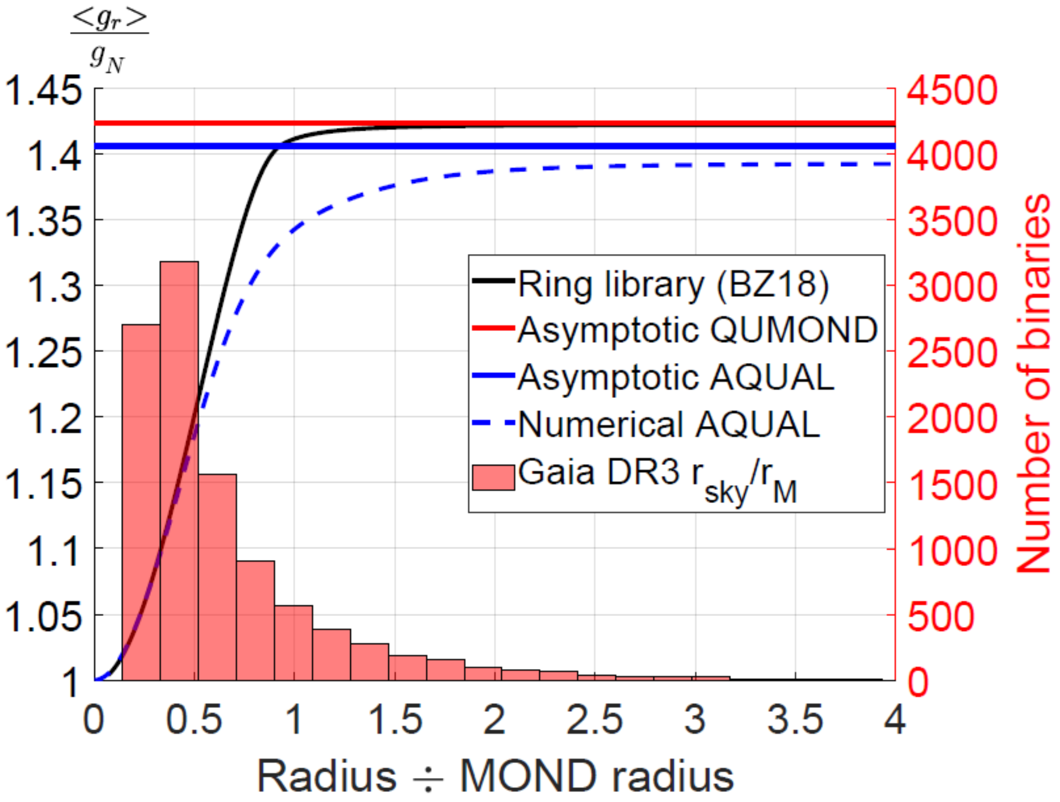
$$\frac{dN_{\mathrm{LOS}}}{dr_{\mathrm{sky}}\,d\widetilde{v}} \propto \widetilde{v} \, . \tag{25}$$

The attraction of local WBs in MOND

- Radial gravity depends on angle relative to external field; there is also some tangential gravity (unlike Newton; Banik & Zhao 2018)
- Numerical QUMOND result (black curve) agrees excellently with asymptotic limit for large separations (horizontal red line)
- Numerical AQUAL results from Chae & Milgrom 2022 are valid towards left (weak EFE), but their assumed asymptotic formula is not applicable to WBs

➢ AQUAL and QUMOND give similar results

➢ **Gaia WBs extremely sensitive to MOND.**

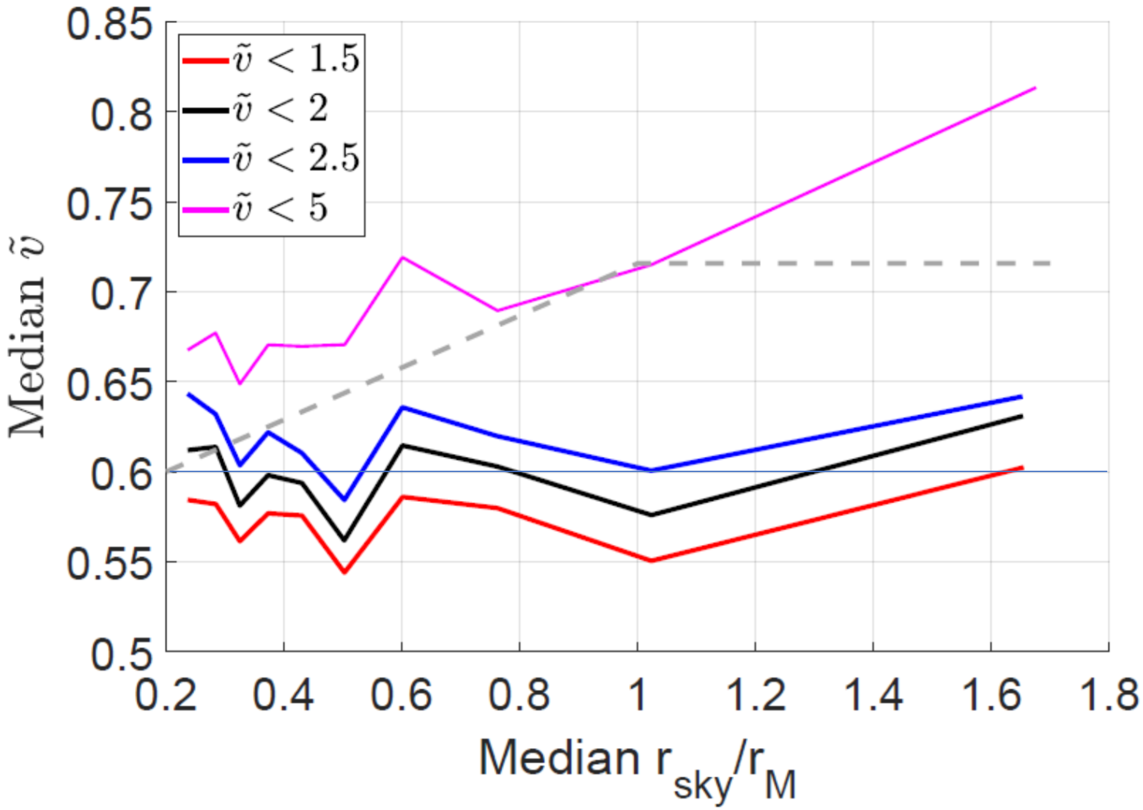


Simple median analysis

- r_{sky}/r_M is proxy for internal WB acceleration: make 10 bins in this with equal sample size
- Any genuine MOND signal would be in the main peak region at $\tilde{v} < 2$ or so (limit of $\sqrt{2}$ in Newtonian gravity rises to 1.7 in MOND, so genuine WBs should not be much beyond this: consider limits of 1.5, 2, 2.5)
- Including higher $\tilde{v}$ only allows in more contamination, which becomes more important at high r_{sky} and thus lower acceleration → fake MOND signal

➢ Expected Milgromian trend not apparent

➢ Flat Newtonian expectation is confirmed.

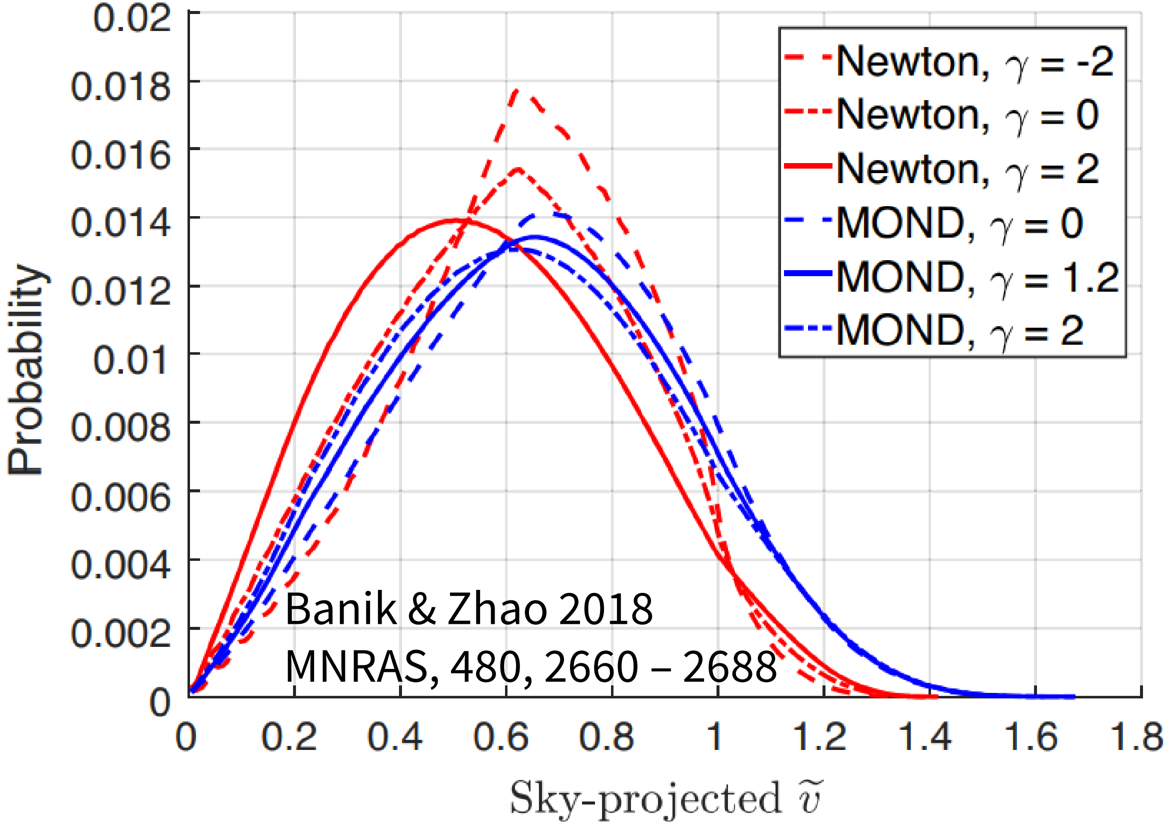


Modelling WB orbital motion & projection effects

- Compute 2D gravitational field for point mass with fixed EFE, treat WB mass as entirely within one of the stars
- Consider 20 revolutions, dense 2D grid of viewing angles at each timestep
- Higher γ means orbits typically more eccentric, which affects the tail slightly
- But gravity law is much more important

➢ Clear water between Newtonian and Milgromian predictions for the WBT

➢ Interpolate between them with gravity law parameter α_{grav}, which is 0 for Newtonian gravity and 1 for MOND.



Banik & Zhao 2018

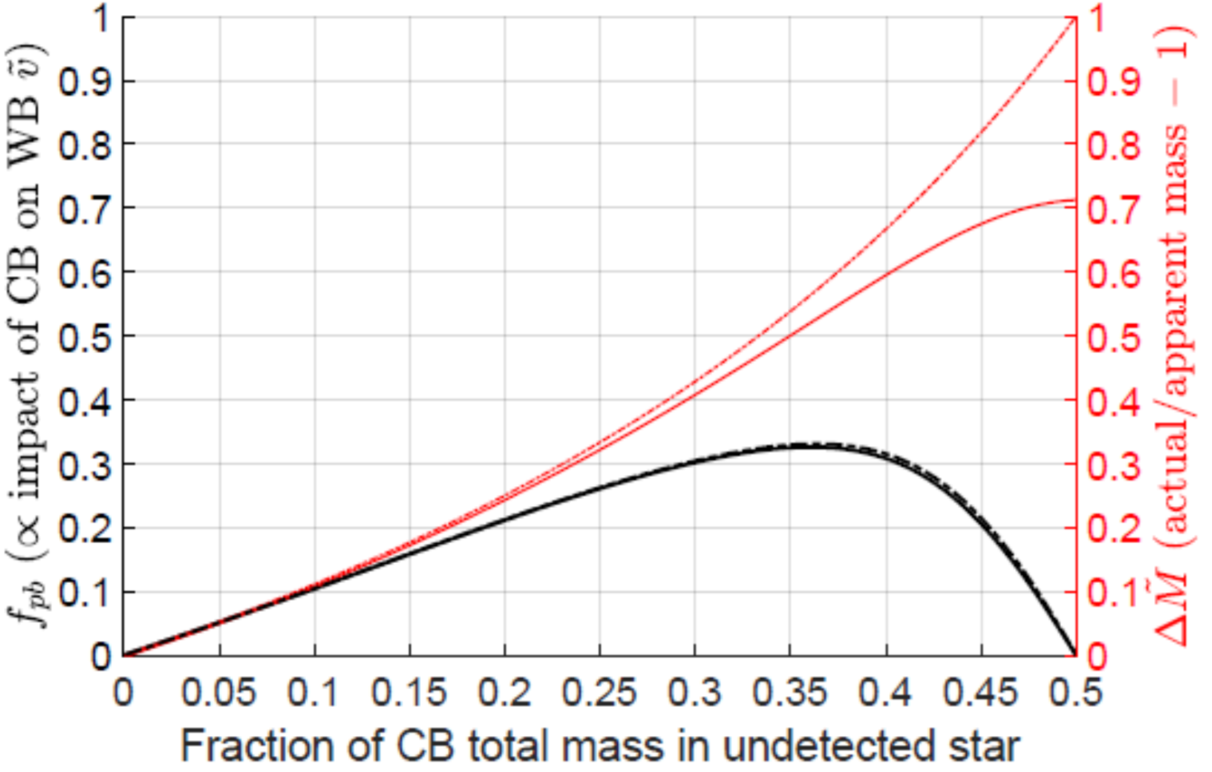
MNRAS, 480, 2660 – 2688

The importance of close binaries (CBs)

- Banik & Zhao 2018 argued *a priori* that main systematic with WBT likely to be CB companions
- Amplitude of extended high-$\tilde{v}$ tail drops with r_{sky} and also with $\tilde{v}$ (Pittordis & Sutherland 2019), strongly rejecting idea that the tail is LOS contamination. Pattern similar to WB population
- Clarke 2020 suggested undetected close binaries with a high occurrence rate
- Belokurov+ 2020 noticed that WBs with high $\tilde{v}$ have poorer *Gaia* astrometric fits, which only consider parallax and proper motion. Long-period CBs would also induce astrometric acceleration, not included in model (astrometric time series in *Gaia* DR4)
- Manchanda+ 2023 showed that most CB companions can be identified with follow-up
- To model CBs, each star is assumed to have some likelihood f_{CB} of having an undetected CB companion with semi-major axis a_{CB} > 0.1% of the maximum allowed: fixed fraction of a_{WB}
- Most expensive steps arise from possibility of CB companions to both stars in a WB.

The effect of close binaries

- Light from the 'undetected' star is assumed to be blended with light from the contaminated star
- Due to steep mass-luminosity relation, extra light inflates the inferred mass, but by less than the companion mass
- CBs thus introduce **hidden mass** (red)
- They also cause **recoil velocity** (black)
- We see the photocentre of the CB but want its barycentre: key quantity is the photocentre-barycentre offset. This vanishes for an exactly equal mass CB.



Model parameters and the likelihood function

- Wide binaries: α_{grav}, Slope P_a, a_{break} (Öpik 1924 law 1/a assumed for distribution of semi-major axis a, but steeper decline with log-log slope of Slope P_a beyond a_{break})
- Wide and close binaries: γ (common eccentricity distribution), defined so $P(e) = (\gamma + 1)e^{\gamma}$
- Close binaries: f_{CB}, k_{CB} (maximum allowed a_{CB}/a_{WB})
- LOS contamination fraction across full sample: f_{LOS}

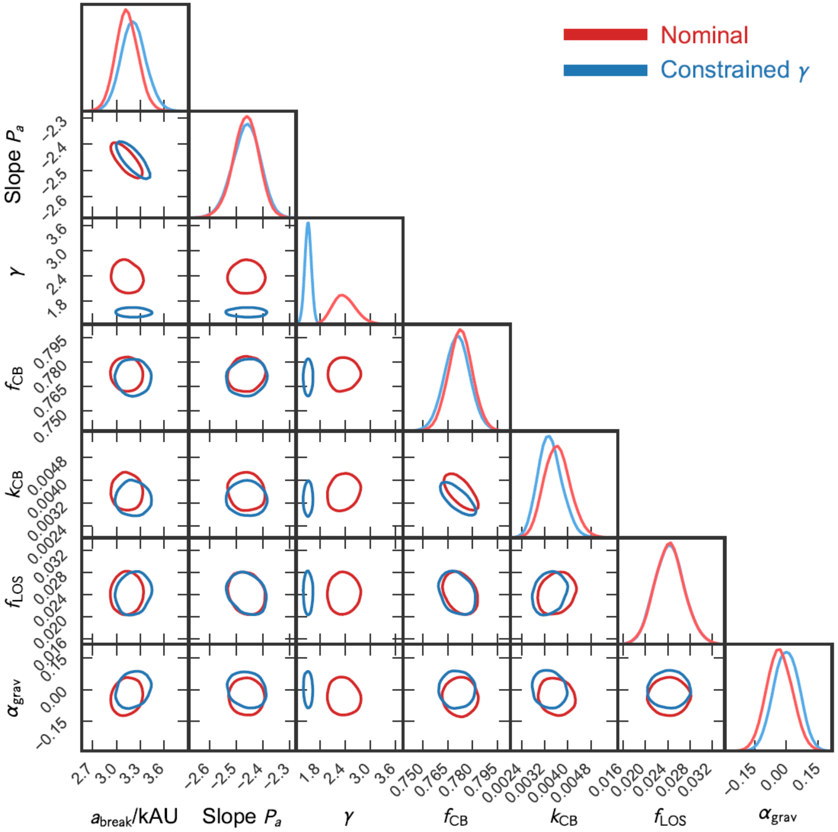
- Gradient ascent used first, followed by MCMC (similar to Asencio+ 2022)
- Binomial statistics used at pixel level, probabilities multiplied together
- Calculations done in log-space due to large number of pixels and WBs
- Many code optimisations involved, planning and construction took >1yr.

Nominal results: triangle plot

- Wide binaries: α_{grav} (Newton: 0, MOND: 1), Slope P_a, a_{break} (Öpik 1924 law 1/a assumed for distribution of semi-major axis a, but steeper decline beyond a_{break})
- Wide and close binaries: γ (eccentricity distribution)

$$P(e) = (\gamma + 1)e^{\gamma}$$

- Close binaries: f_{CB}, k_{CB} (maximum allowed a_{CB}/a_{WB})
- LOS contamination fraction across full sample: f_{LOS}
- Main result is α_{grav} = –0.04±0.06, which is consistent with Newtonian gravity but rules out MOND at 16σ.
- γ is free (0–4), but [Hwang+ 2022] argue that distribution of angles between projected separation and relative velocity constrains γ to 1.32±0.09



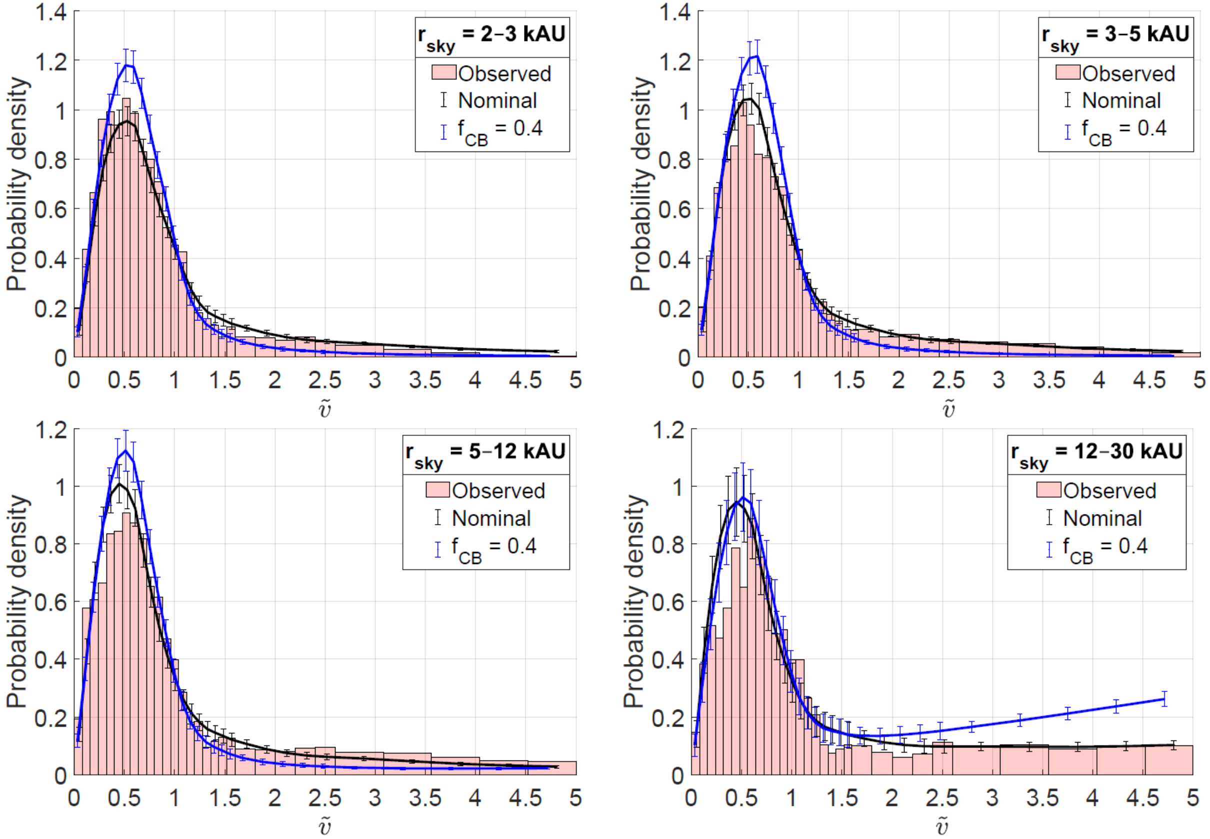
Revised model assumptions

Altered assumption	$a_{\rm break}$ (kAU)	β	γ	$f_{\rm CB}$ (%)	$k_{\rm CB}$ (‰)	$f_{\rm LOS}$ (%)	$\alpha_{\rm grav}$	Best $\ln P$
Definition	Eq. 14	Eq. 14	Eq. 15	Sec. 3.2.3	Sec. 3.2.2	Sec. 3.3	Sec. 3.1.3	Sec. 3.4
Prior	$(1, 15)$	$(-15, -1)$	$(0, 4)$	$(0, 99)$	$(0, 750)$	$(0, 60)$	$(-2, 3.6)$	–
Nominal	$3.12^{+0.15}_{-0.13}$	$-2.47^{+0.05}_{-0.04}$	$2.27^{+0.34}_{-0.19}$	$77.12^{+0.84}_{-0.58}$	$3.62^{+0.41}_{-0.47}$	$2.45^{+0.23}_{-0.29}$	$-0.036^{+0.064}_{-0.055}$	-1576.91
Flat q for CBs	$3.12^{+0.15}_{-0.13}$	$-2.47^{+0.04}_{-0.05}$	$2.48^{+0.28}_{-0.27}$	$77.69^{+0.60}_{-0.70}$	$3.37^{+0.31}_{-0.31}$	$2.45^{+0.30}_{-0.25}$	$-0.064^{+0.073}_{-0.048}$	-1578.78
Linear q for CBs	$3.14^{+0.13}_{-0.15}$	$-2.47^{+0.05}_{-0.05}$	$2.36^{+0.27}_{-0.25}$	$77.32^{+0.78}_{-0.66}$	$3.33^{+0.54}_{-0.33}$	$2.39^{+0.31}_{-0.20}$	$-0.035^{+0.062}_{-0.061}$	-1579.74
$\gamma = 1.32 \pm 0.09$	$3.22^{+0.13}_{-0.18}$	$-2.46^{+0.06}_{-0.05}$	$1.51^{+0.08}_{-0.06}$	$76.99^{+0.79}_{-0.67}$	$3.32^{+0.40}_{-0.37}$	$2.43^{+0.27}_{-0.29}$	$0.012^{+0.039}_{-0.067}$	-1587.77
$M = (1-2)\,M_\odot$	$3.32^{+0.14}_{-0.18}$	$-2.57^{+0.05}_{-0.08}$	$2.21^{+0.33}_{-0.23}$	$77.97^{+0.67}_{-0.98}$	$3.21^{+0.51}_{-0.39}$	$2.34^{+0.26}_{-0.34}$	$-0.094^{+0.051}_{-0.073}$	-1406.24
No WDs, $A_V < 0.1$	$2.77^{+0.24}_{-0.12}$	$-2.44^{+0.07}_{-0.07}$	$2.59^{+0.66}_{-0.33}$	$77.14^{+0.88}_{-1.16}$	$3.62^{+0.93}_{-0.65}$	$2.47^{+0.43}_{-0.46}$	$-0.126^{+0.079}_{-0.118}$	-1156.22
$k_{\rm CB} = 0.2$	$2.89^{+0.12}_{-0.12}$	$-2.57^{+0.04}_{-0.06}$	$4.00^{+0.00}_{-0.14}$	$74.63^{+0.97}_{-0.94}$	200	$4.89^{+0.29}_{-0.31}$	$-0.254^{+0.083}_{-0.064}$	-1916.24
$f_{\rm CB} = 0.4$	$3.42^{+0.17}_{-0.15}$	$-2.62^{+0.06}_{-0.07}$	$0.77^{+0.10}_{-0.08}$	40	$59.5^{+15.8}_{-4.3}$	$7.47^{+0.40}_{-0.31}$	$0.192^{+0.037}_{-0.057}$	-2363.71

- Alterations to CB mass ratio distribution or eccentricity distribution have little effect on gravity law
- Biases reduced with narrower range of WB total mass and thus MOND radius, but little effect
- Most promising scenario is to substantially reduce the CB contamination fraction.

Halving the likelihood of CB companions

- Nominal analysis prefers that likelihood f_{CB} of a star having an undetected companion is 77±1%
- Reduce this to 40%
- Reduced role of CBs means analysis may try to broaden the $\tilde{v}$ distribution by changing the gravity law
  - ➢Inferred gravity law becomes $0.19^{+0.04}_{-0.06}$, which is still far closer to Newtonian gravity than to MOND
  - ➢Overall fit much poorer than nominal, estimated at 40σ significance.

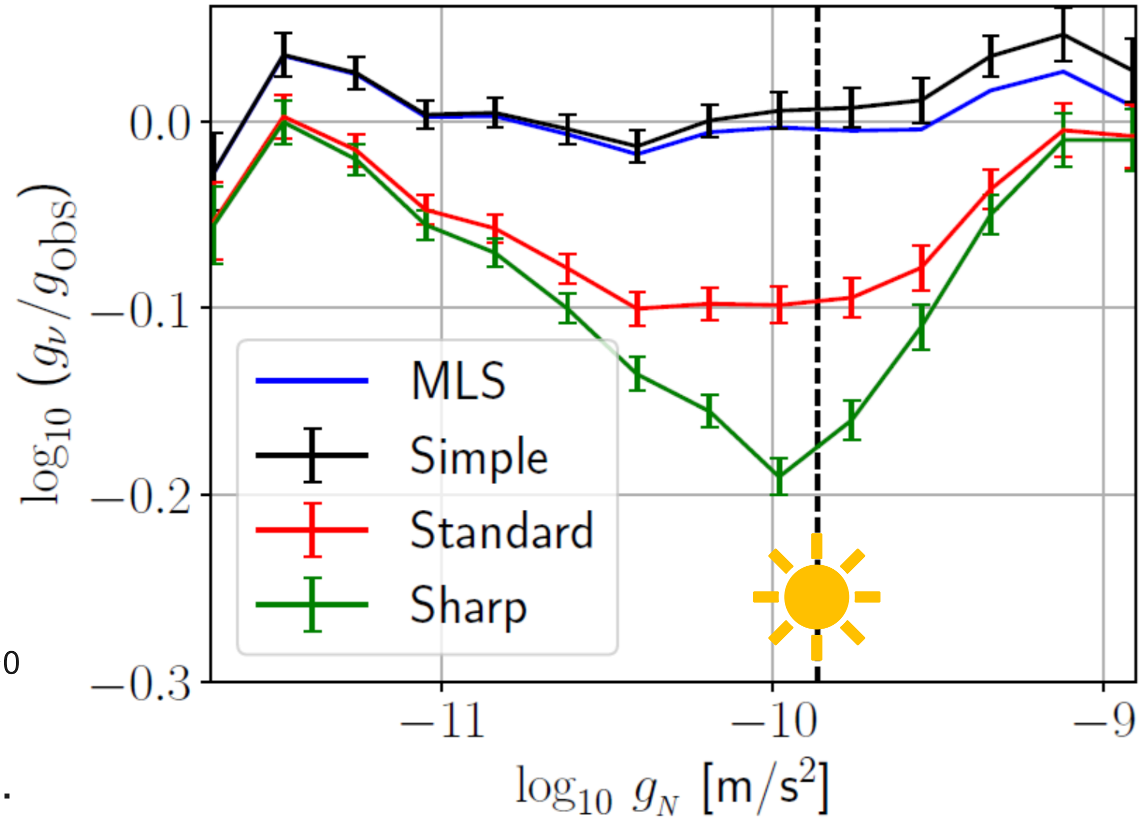


Wide binaries in light of galaxy rotation curves

Interpolating function	AQUAL η	AQUAL α_{grav}	QUMOND η	QUMOND α_{grav}
Simple	1.4056	0.96	1.4228	1
MLS	1.3508	0.84	1.3692	0.88
Standard	1.0661	0.17	1.0726	0.18
Sharp	1	0	1	0

- Standard interpolating function marginally OK with WBT, but completely fails to match the observed RAR (red curve)
- WBT really prefers an infinitely sharp transition (green), which works well with local WBs as Galactic EFE slightly above a_0

➢ No interpolating function simultaneously consistent with WBT and disc galaxy RAR.



Broader implications of the WBT

- Consider MOND gravity as Newtonian gravity of baryons plus 'phantom dark matter' (PDM)
- Model of Babichev+ 2011 similar to a limit on the PDM density
- More generally, Vainshtein-like screening mechanism not so unusual in modified gravity theories
- The WBT implies PDM density $<20\ M_\odot/\mathrm{pc}^3$, about 0.1% of Galactic halo density in CDM
- Since MOND radius $r_M \propto M^{1/2}$, phantom density at this location $\propto M/r_M{}^3 \propto M^{-1/2}$
- This means MOND effects suppressed down to 10^{-6} of MW mass, so about $10^5\ M_\odot$
- Galactic scale tests only cover down to $10^6\ M_\odot$ using tidal stability of Fornax Cluster dwarfs (Asencio+ 2022) or velocity dispersions of isolated Local Group dwarfs (McGaugh+ 2021)
- Mild tension with NGC 2419 globular cluster as it is consistent with Newtonian velocity dispersion despite outer halo location and weak EFE from MW (Ibata+ 2011a,b; but see Sanders 2012a,b)
- NGC 2419 mass is $9 \times 10^5\ M_\odot$, hinting that MOND effects are suppressed below this mass.

Conclusions

- In MOND, local WBs at kAU separations should orbit 20% faster than Newtonian expectations
- This strong prediction is falsified at 16σ confidence; result similar to Pittordis & Sutherland 2023
- No way to reconcile MOND as modified gravity with disc galaxy rotation curves & RAR
- But MOND as modified inertia also ruled out at 6.9σ confidence (Chae 2022, ApJ, 941, 55)
- New fundamental constant required beyond a_0, perhaps new maximum phantom density scale?
- No obvious tension with MOND successes in galaxy dynamics
- MOND designed for disc galaxy RCs, so difficulties extending it to smaller and larger scales cast doubt on its overall validity.

Outlook for MOND after forty years

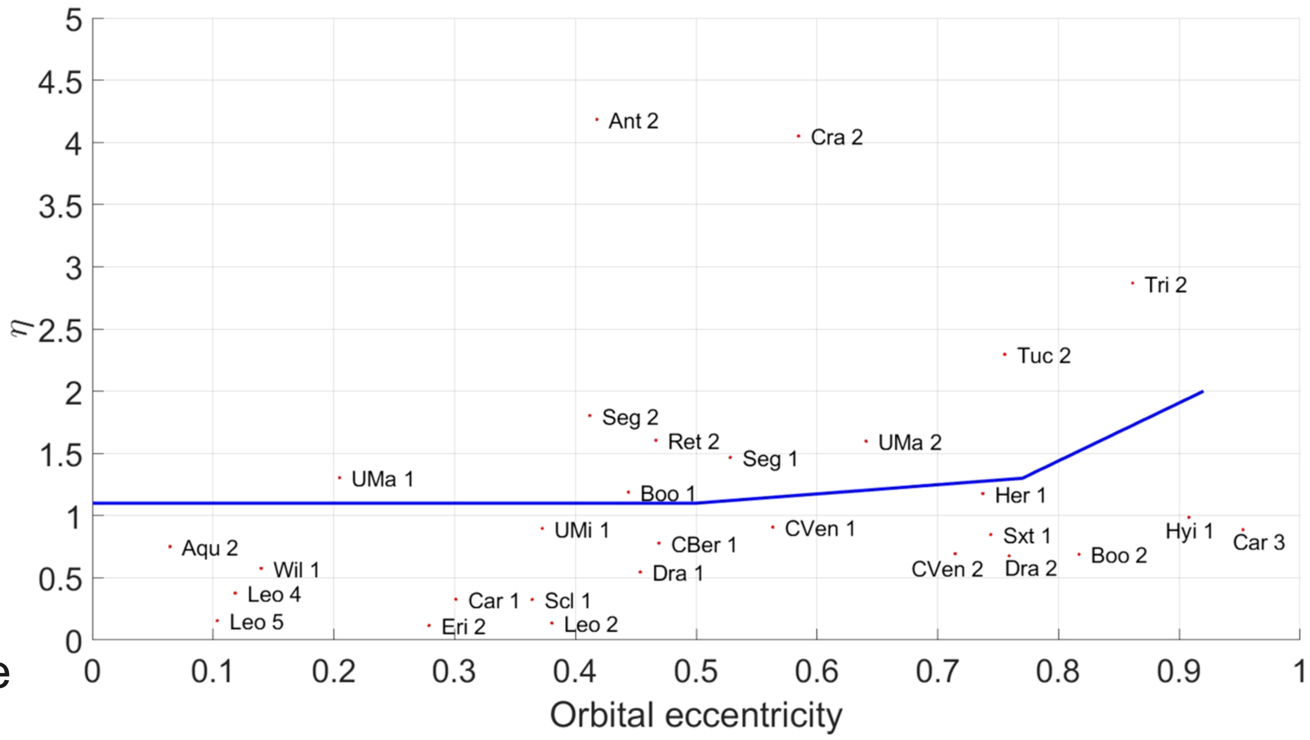
- Imagine MOND as a clock initially pointing 1 o'clock: it was designed to fit galaxy rotation curves
- The clock then stopped working
- Using Sun to estimate time to within a few hours shows the clock to be accurate at times of day around 1 o'clock, but not at other times
- Analogous to failures of MOND when tested on scales very different to equilibrium galaxy dynamics

➢ It may be impossible to extend MOND beyond the scales it was originally designed for, casting strong doubt on its validity.

Tidal stability of ultrafaint MW satellites

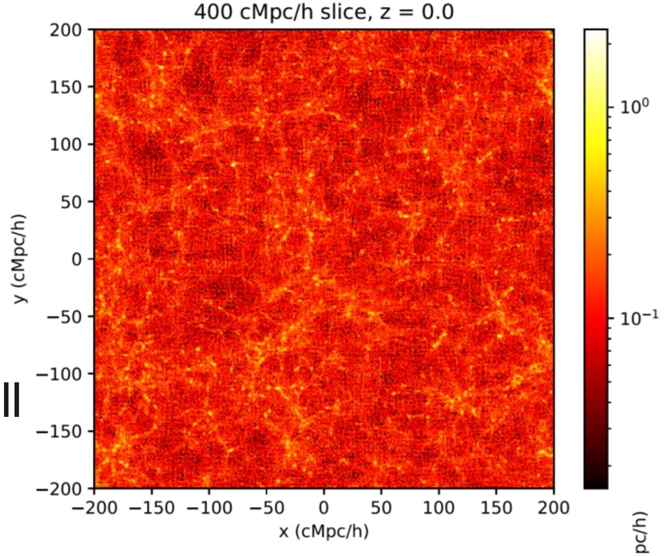
- η is half-mass radius ÷ tidal radius at pericentre, including EFE and tidal stress (estimate max. stable size: $G v_{MW} M_{dwarf}/r^2 = r\, g'_{MW}$)
- Analytic result (Zhao & Tian 2006) with critical threshold calibrated using numerical simulations (Asencio+ 2022, MNRAS, 515, 2981 on Fornax Cluster dwarfs)
- Critical η rises with eccentricity as less time spent at pericentre

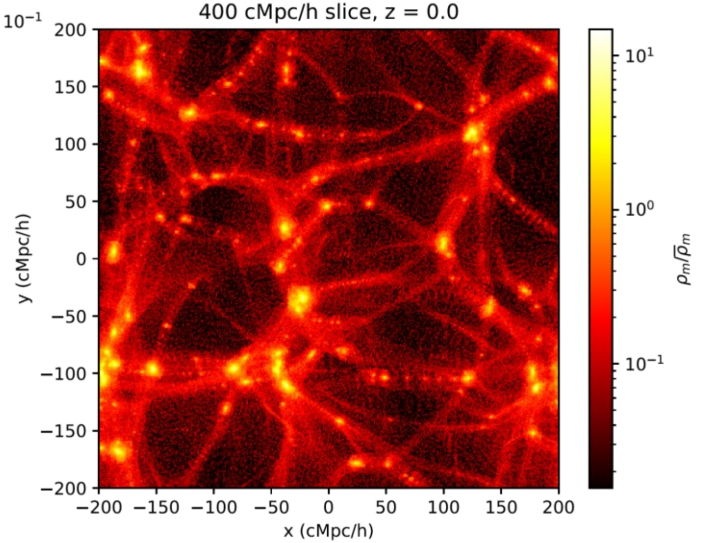
➢ Many ultrafaint satellites would be tidally unstable in MOND.



Large-scale structure simulations in MOND

- Need to include hot dark matter (HDM) component to match galaxy clusters (e.g., Bullet) and the CMB anisotropies ($g \approx 20\ a_0$, expansion history standard, free streaming effects small; [Haslbauer+ 2020](#))
- Power strongly suppressed on small scales compared to ΛCDM due to lack of cold dark matter
  - ➢ First galaxies at $z \approx 4$ (Wittenburg+)
  - ➢ Low-mass galaxy clusters not very common at present epoch
- Both are wrong observationally.

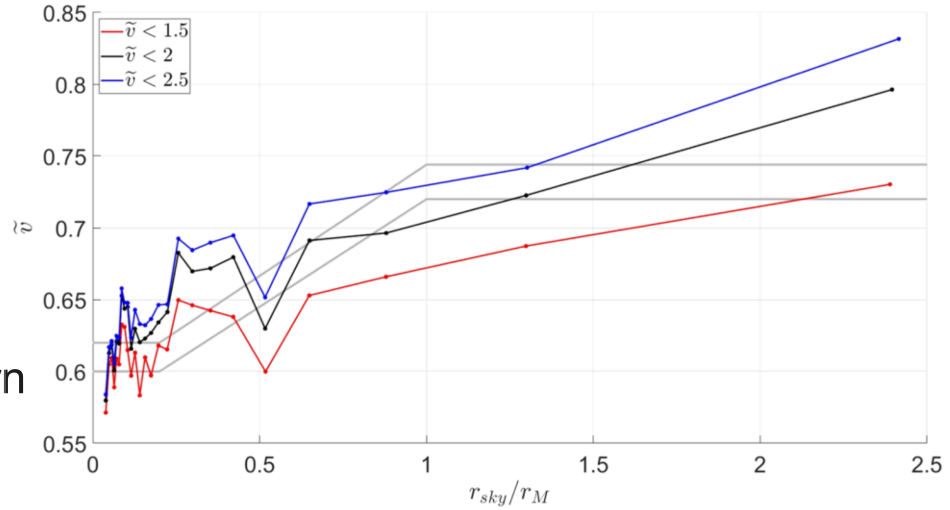




Russell+, in prep.
Left: ΛCDM, right: νHDM

Obtaining $\tilde{v}$ and its uncertainty

- Differences in quality cuts, e.g., allowing WBs at any Galactic latitude rather than requiring $|b| > 15°$
- Systematics tend to broaden the $\tilde{v}$ distribution as same velocity error implies larger $\tilde{v}$ error
- Rapidly rising median $\tilde{v}$ at large r/r_M sign of systematics
- MOND signal would look like the grey lines, but no return to Keplerian decline (quasi-Newtonian/EFE-dominated regime) seen far out
- Unlikely for systematic effects to hide the MOND signal and make result look like a flat line
- More generally, should forward model into space of observables (r_{sky}, $\tilde{v}$) rather than deduce theoretical quantities from the data: g not directly observed in WBs.



Analysis of data in Arxiv:2305.04613, Chae 2023