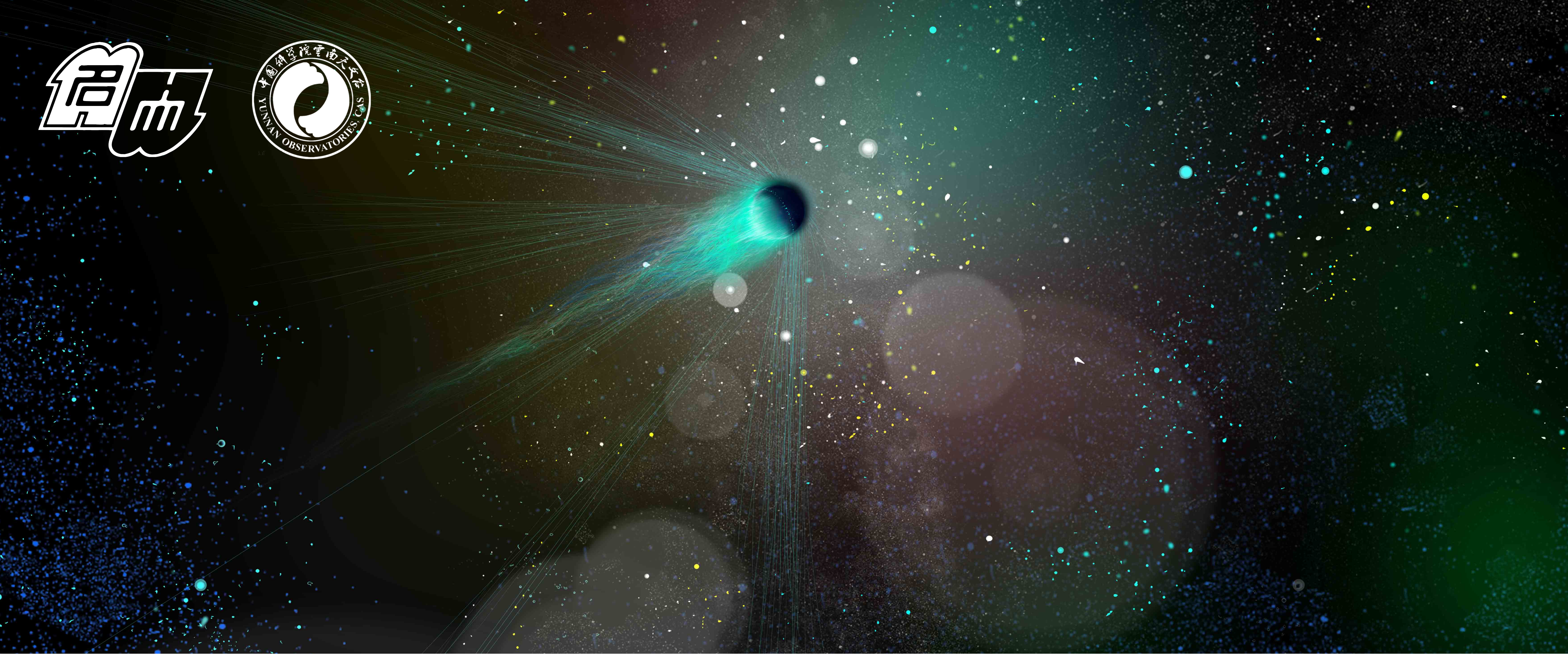


COUPLINGS BETWEEN DARK MATTER AND BARYONIC MATTER ON GALACTIC SCALES: THE RADIAL VS. VERTICAL

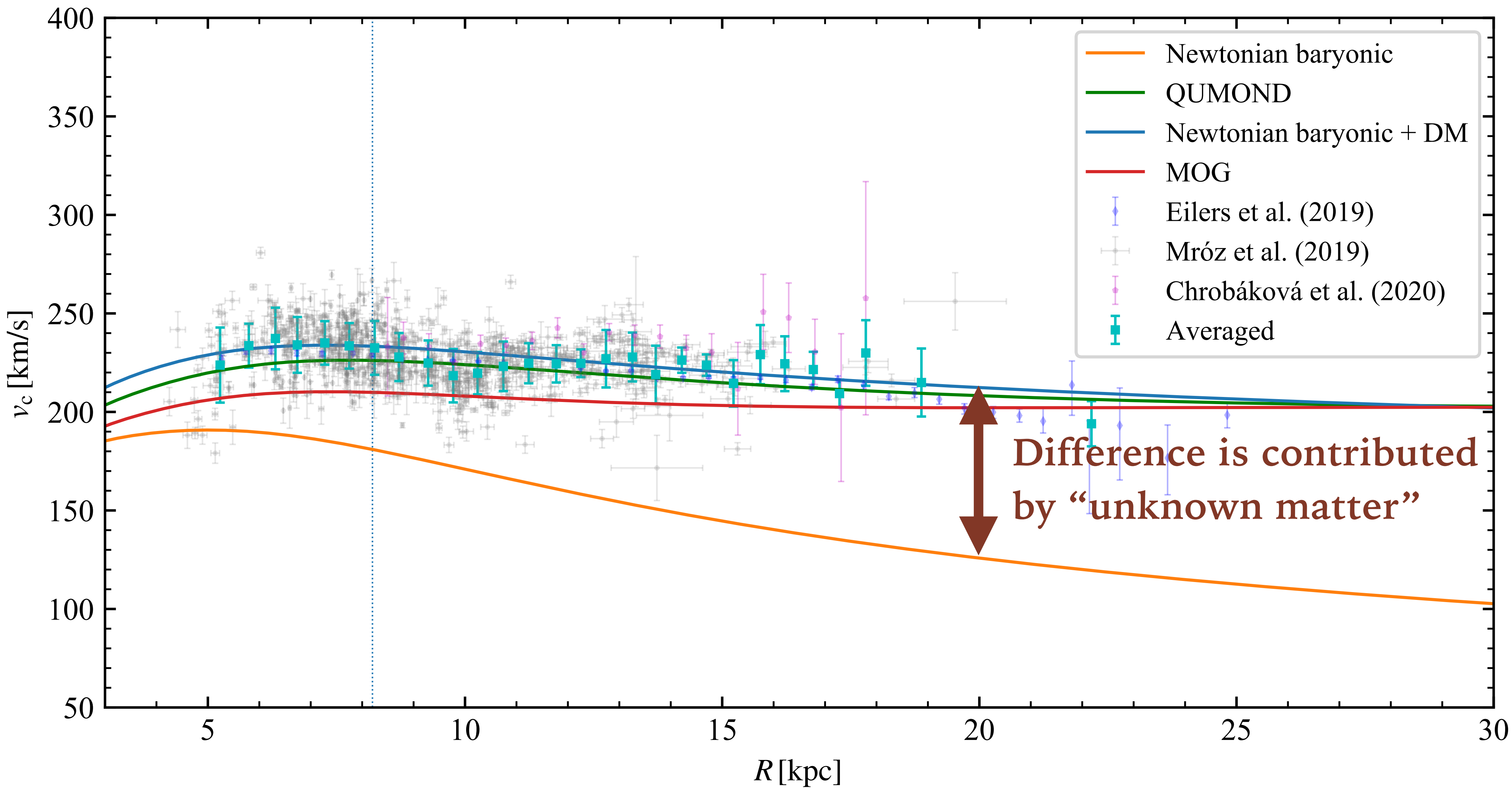
Hai-Xia Ma

Yongda Zhu, Xiao-Bo Dong, Yang Huang, and Tobias Mistele, et al.

Figure Credit: The Wandering Earth

MISSING MASS PROBLEM

- The dynamics of gas and stars in and around galaxies has been observed to be in **excess** of the Newtonian gravity of the total baryonic content of the galaxies, indicating the need for additional matter, for example, the **dark matter (DM)**.



ALTERNATIVE THEORIES

- Poisson Equation of Newtonian Dynamics

$$\nabla^2 \Phi^{\mathrm{N}} = 4\pi G\rho \qquad\qquad \nabla \cdot \mathbf{g}^{\mathrm{N}} = -\, 4\pi G\rho$$

- Modified Newtonian Dynamics (MOND: Milgrom 1983)

$$\boldsymbol{g} = \nu\left(g_N/a_0\right)\boldsymbol{g}^{\mathrm{N}}$$

$$\mu(x \gg 1) \approx 1, \quad \mu(x \ll 1) \approx x\,.$$

- Modified Gravity (MOG: Moffat & Rahvar 2013)

$$\Phi(\boldsymbol{x}) = -\, G_\infty \left[\int \frac{\rho\left(\boldsymbol{x}'\right)}{\left|\boldsymbol{x}-\boldsymbol{x}'\right|}\left(1 - \frac{G_\infty - G_{\mathrm{N}}}{G_\infty}\mathrm{e}^{-\mu\left|\boldsymbol{x}-\boldsymbol{x}'\right|}\right)\mathrm{d}^3\boldsymbol{x}'\right] \qquad\qquad G_\infty = (1+\alpha)G_N$$

ALTERNATIVE THEORIES

- Quasi-linear formulation of MOND (QUMOND: Milgrom 2010)

$$\nabla^2\Phi = \nabla\cdot\left[\nu\left(\frac{g^{\rm N}}{a_0}\right)\nabla\Phi^{\rm N}\right]$$

$$\nabla^2\Phi = \nabla\cdot\left[\nabla\Phi^{\rm N} + \tilde{\nu}\left(\frac{g^{\rm N}}{a_0}\right)\nabla\Phi^{\rm N}\right] = 4\pi G(\rho_{\rm b}+\rho_{\rm pdm})$$

$$\nabla^2\Phi^{\rm N} = 4\pi G\,\rho_{\rm b}$$

$$\tilde{\nu}(y) = \nu(y) - 1$$

where $\rho_{\rm pdm}$ is the "phantom matter density" which corresponds to the "phantom matter potential" :

$$\rho_{\rm pdm} = \frac{1}{4\pi G}\nabla\cdot\left[\tilde{\nu}\left(\frac{g^{\rm N}}{a_0}\right)\nabla\Phi^{\rm N}\right]$$

$$\nabla^2\Phi_{\rm pdm} = \nabla\cdot\left[\tilde{\nu}\left(\frac{g^{\rm N}}{a_0}\right)\nabla\Phi^{\rm N}\right] = 4\pi G\,\rho_{\rm pdm}$$

- The QUMOND potential then can be written as the sum of two scalar potentials: $\Phi = \Phi^{\rm N} + \Phi^{\rm pdm}$.

OBSERVED MASS DISCREPANCY - ACCELERATION RELATION

- **The mass discrepancy acceleration relation (MDAR)** describes the coupling between baryons and dark matter in galaxies: the ratio of total-to-baryonic mass at a given radius does not correlates with the acceleration due to baryons.

$$V^2_{\rm obs}(r)/V^2_{\rm bar}(r)$$

  For a spherical mass distribution:

$$V^2_{\rm obs}(r)/V^2_{\rm bar}(r) = M_{\rm tot}(r)/M_{\rm bar}(r)$$

- A very tight correlation was found between the mass discrepancy and the gravitational acceleration due to baryons $g_{\rm bar}$ (McGaugh 2004).
- The existence of the MDAR has been a challenge to the Λ cold dark matter (ΛCDM) galaxy formation model, while it can be explained by Modified Newtonian Dynamics.

JEANS EQUATION

- The Jeans equation is an important tool for understanding the dynamics of self-gravitating systems.

$$\rho \frac{\partial \bar{v}_j}{\partial t} + \rho \bar{v}_i \frac{\partial \bar{v}_j}{\partial x_i} = -\rho \frac{\partial \Phi}{\partial x_j} - \frac{\partial \left(\rho \sigma_{ij}^2 \right)}{\partial x_i}$$

- However, we have more unknown functions, but only Jeans equation and the Poisson's equation. **Jeans equation is not closed.**
- **Traditional wisdom -- (Over-)simplification for closure:**
  - A distribution function $f = f(H, L_z)$ for only two integrals of motion, namely the Hamiltonian of the system H and the z -direction angular momentum L_z.

SIMPLIFICATIONS OF JEANS EQUATIONS

- **Traditional wisdom -- (Over-)simplification for closure:**
  - A distribution function $f = f(H, L_z)$ for only two integrals of motion, namely the Hamiltonian of the system H and the z-direction angular momentum L_z.
- By doing so, one can have:

$$\sigma_{ij}^2 = \overline{(v_i - \overline{v_i})\left(v_j - \overline{v_j}\right)} = \overline{v_i v_j} - \overline{v_i}\,\overline{v_j}$$

  - The stellar velocity dispersion tensor having $\sigma_R = \sigma_z$ and $\sigma_{Rz} = 0$.
  - Thus the σ_* distribution in a meridional plane is isotropic, and the tilt angle of the velocity ellipsoid $\alpha = 0$.
  - Then the corresponding velocity-dispersion terms in Jeans equations are reduced or vanished accordingly, and the Jeans equations are closed

LIMITATIONS OF PREVIOUS SIMPLIFICATIONS

- However, the fact is that $\sigma_R \neq \sigma_z$ and $\sigma_{Rz} \neq 0$ in the observed disk galaxies, the tilt angle of the Milky Way's velocity ellipsoid is observed not zero.

- And there is more evidence supporting that the stellar orbits do respect a third integral of motion.

$$I_1 = v_R^2 + v_\theta^2 + v_z^2 - 2\Phi$$

$$I_2 = R v_\theta$$

$$I_3 = \left(R v_z - z v_R\right)^2 + z^2 v_\theta^2 + z_0^2 \left(v_z^2 - 2\Phi^*\right) \quad \textit{(Just for example)}$$

- Specifically, concerning Jeans-equations modeling of the MW, the necessity of incorporating the cross-dispersion term σ_{Rz} (i.e. tilt angle) in Jeans equations has been thoroughly analyzed.

THIS WORK: USING JES AS DISCRIMINATORS

- **Instead of trying to close Jeans equations (JEs), we use JEs as merit function (goodness-of-fit) to test gravitational models with Data!** For the over-determined system, it is not equation closure but **Data** to handle uniqueness (breaking degeneracy).
- The complete form of JEs is adopted, admitting three integrals of motion; DM, QUMOND, MOG models vs. kinematic data powered by the Gaia DR2.

$$\rho T_R \quad \boxed{\frac{\partial\left(\rho\sigma_R^2\right)}{\partial R} + \left(\frac{1-k_\theta}{R} + \frac{\partial\kappa}{\partial z}\right)\rho\sigma_R^2 + \kappa\frac{\partial\left(\rho\sigma_R^2\right)}{\partial z} - \rho\frac{V_\theta^2}{R}} = -\rho\frac{\partial\Phi}{\partial R}$$

$$\rho T_z \quad \boxed{\frac{\partial\left(\rho\sigma_z^2\right)}{\partial z} + \left(\frac{\xi}{R} + \frac{\partial\xi}{\partial R}\right)\rho\sigma_z^2 + \xi\frac{\partial\left(\rho\sigma_z^2\right)}{\partial R}} = -\rho\frac{\partial\Phi}{\partial z}$$

Mass models are axisymmetric: $\sigma_{R\theta} = \sigma_{\theta z} = 0$

$$\kappa = \frac{1}{2}\tan(2\alpha)(1-k_z)$$

$$\xi = \kappa/k_z$$

$$k_z = \sigma_z^2/\sigma_R^2$$

$$k_\theta = \sigma_\theta^2/\sigma_R^2$$

$$\Downarrow$$

$$T_R = \frac{\partial\Phi}{\partial R} \qquad T_z = \frac{\partial\Phi}{\partial z}$$

FIDUCIAL MODEL AND CONSTANTS

- The fiducial mass model we use is the one prescribed by Wang, Hammer & Yang (2022), which is built from Gaia DR2 and Gaia EDR3. It adopts the mass distribution profile formulae and basic structural parameter values from the best fit main model of McMillan (2017) for the bulge, stellar discs, and interstellar medium discs, and the Zhao's (Zhao 1996) profile for the DM halo.
- For galactic constants, we use the distance from the Sun to the Galactic Centre $R = 8.122\,\mathrm{kpc}$, and a nominal circular velocity $v = 229.0\,\mathrm{km\,s^{-1}}$at the radius of the Sun. The fiducial model of the Galactic mass distribution was built under the same constants.
- **Velocity dispersion data tracers: Red clump giant**
  - Huang et al. (2020) from Gaia DR2 and LAMOST of $\sim$ 137 000 red clump stars, with a good coverage of the Galactic disc of 4 ≤ R ≤ 16 kpc and |z| ≤ 4 kpc.
  - Sample consists of 116 000 geometric thin disk stars and 21 000 thick disk stars identified by their chemical properties.

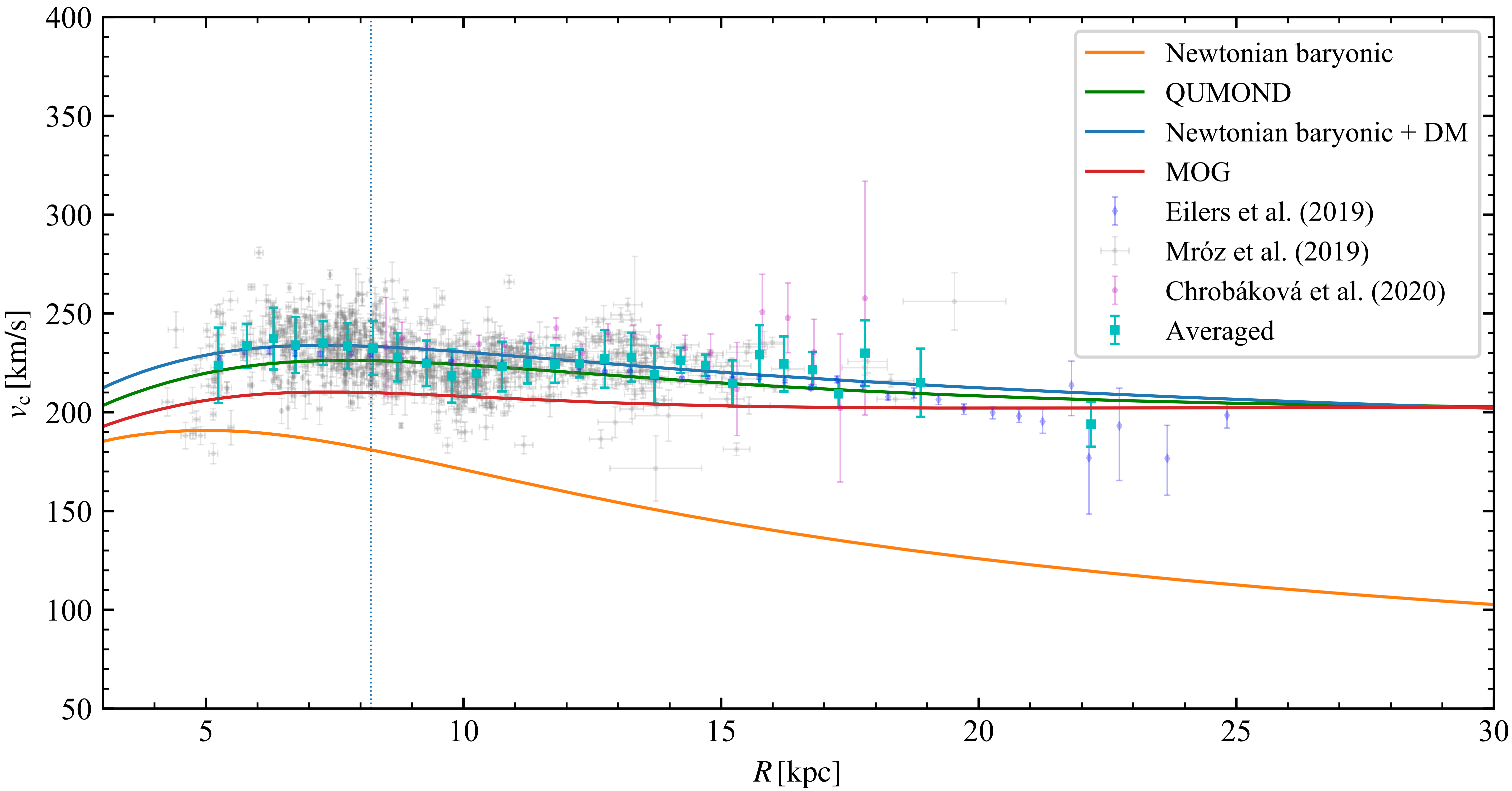
GRAVITATIONAL POTENTIAL MODELS

- **Baryon only**
- **Baryon + DM** (described by the Zhao's profile)
- **Moffat's MOG**
- **QUMOND**
  In this work, essentially we treat QUMOND as a gravitational potential model rather than a "modified gravity or dynamics" theory. We employ it in the fashion of $\rho_{\rm b} + \rho_{\rm pdm}$, with $\rho_{\rm pdm}$ as an alternative of popular DM haloes but reflecting the MOND effect.

$$\rho_{\rm pdm} = \frac{1}{4\pi G} \nabla \cdot \left[\tilde{\nu}\left(\frac{g^{\rm N}}{a_0}\right) \nabla \Phi^{\rm N} \right] \qquad \nabla^2 \Phi_{\rm pdm} = \nabla \cdot \left[\tilde{\nu}\left(\frac{g^{\rm N}}{a_0}\right) \nabla \Phi^{\rm N} \right] = 4\pi G \rho_{\rm pdm}$$

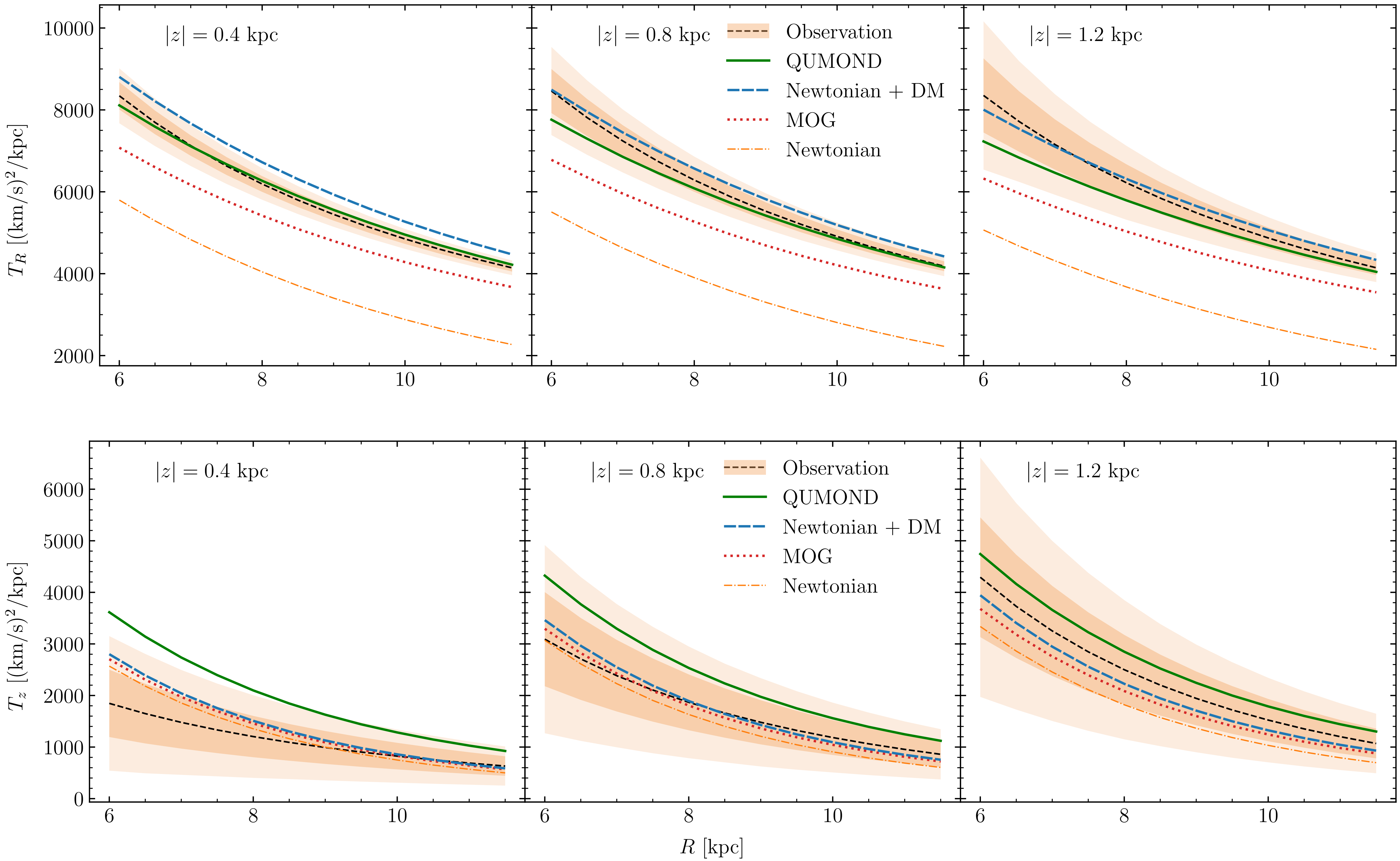
ROTATION CURVE

- As expected, the Newtonian baryon-only model under-predicts the rotation curve. DM and MOND match the data well within the 1σ errors of almost all the bins. The MOG model systematically smaller than most of the data points.



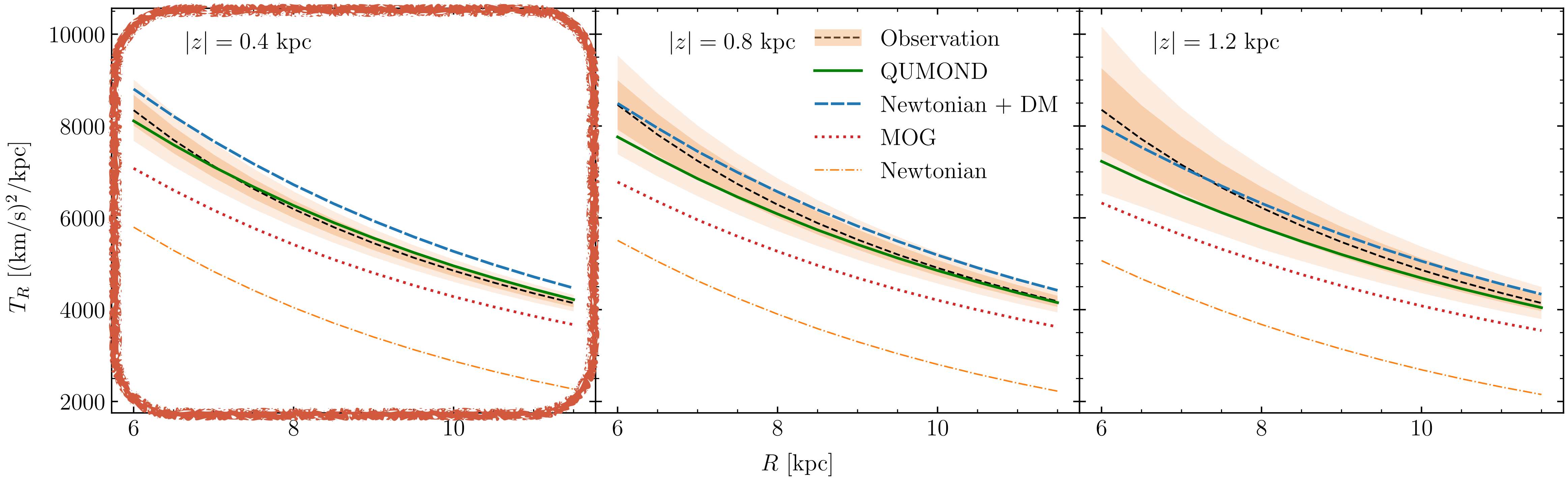
JEANS-EQUATIONS TESTS

- Radial Jeans-equation (TR) and vertical (Tz) tests



JEANS-EQUATIONS TESTS - RADIAL

- Radial Jeans-equation (TR) tests

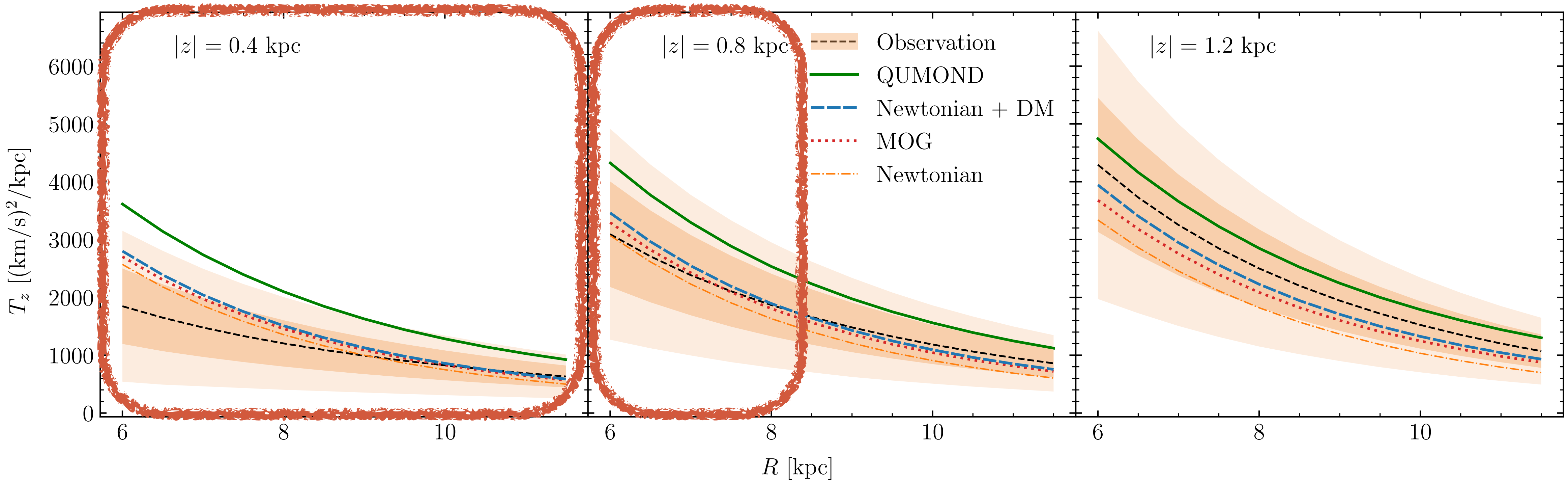


- The fiducial DM model basically lies within the 95 per cent confidence interval of the data for all the R range, but except for the case of |z| = 0.4 kpc, where the DM model goes outside the 95 per cent confidence interval for almost the entire R range.

- **The QUMOND model behaves best**: it lies within the 68 per cent (1σ) confidence interval for almost all (R, z) locations

JEANS-EQUATIONS TESTS - VERTICAL

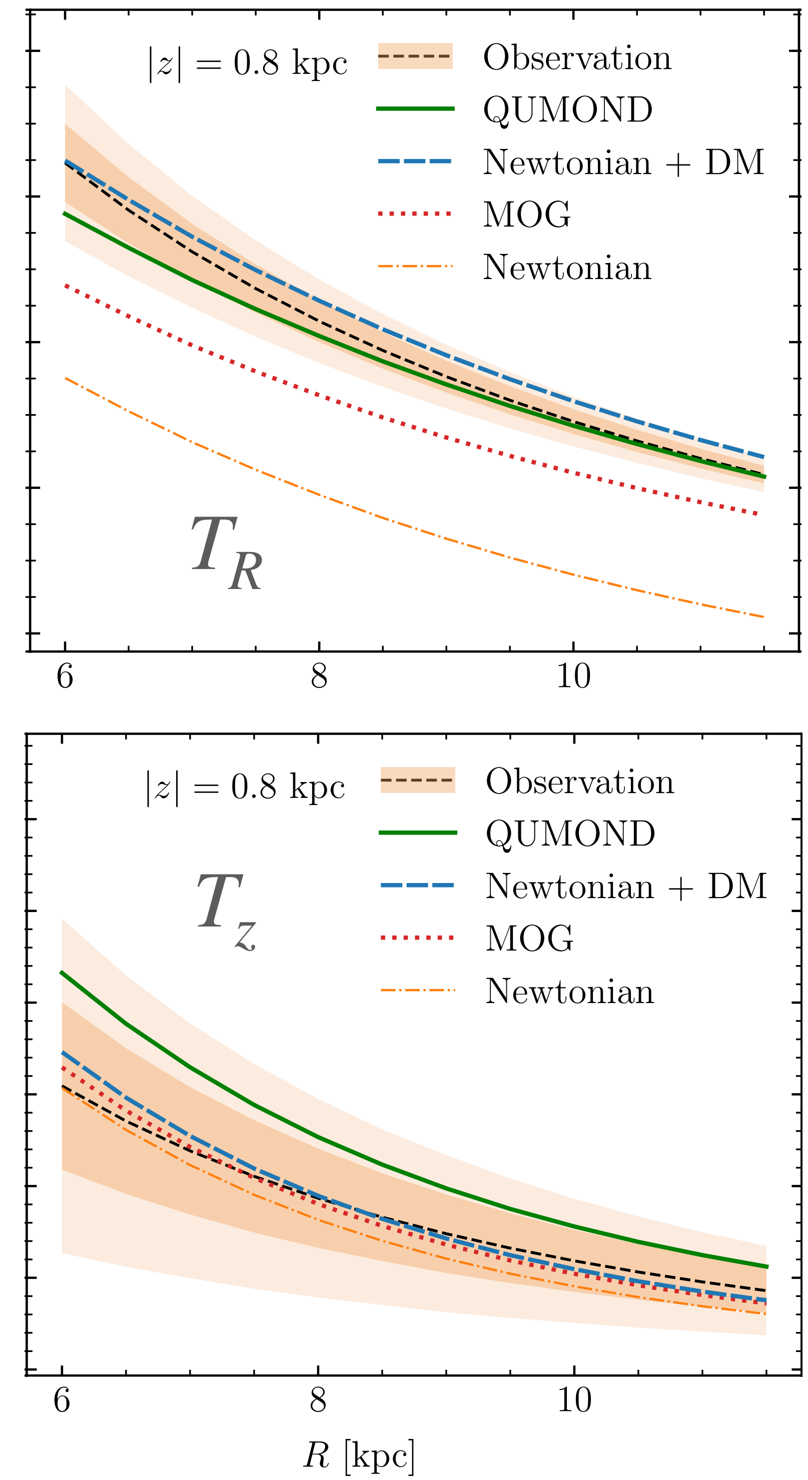
- Vertical Jeans-equation (Tz) tests



- All the four models are broadly consistent with the observations, while the Newtonian baryon-only, the fiducial DM, and MOG models lie close to each other, and are all within the 68 per cent confidence interval for almost all locations.
- However, for the locations at R < 8.5 kpc and |z| =0.4 kpc, QUMOND is outside the 68 per cent confidence.

JEANS-EQUATIONS TESTS

- Newtonian baryonic-only model and MOG obviously fail.
- Tz's discriminating power is weaker than TR due to the smaller vertical potential gradient in disc galaxies and larger relative errors in Tz. The observational reason is that the relative errors of σ_z is larger than that of σ_R by a factor of ~2, which are the dominating error terms of the observed vertical and radial accelerations Tz and TR. Thus the error bars and relative errors of Tz are significantly larger than those of TR at the same locations.
- Being conservative, yet we must note that the test depends on the tracer's density profile we adopt, and that at least DM and QUMOND cannot be discriminated for sure.



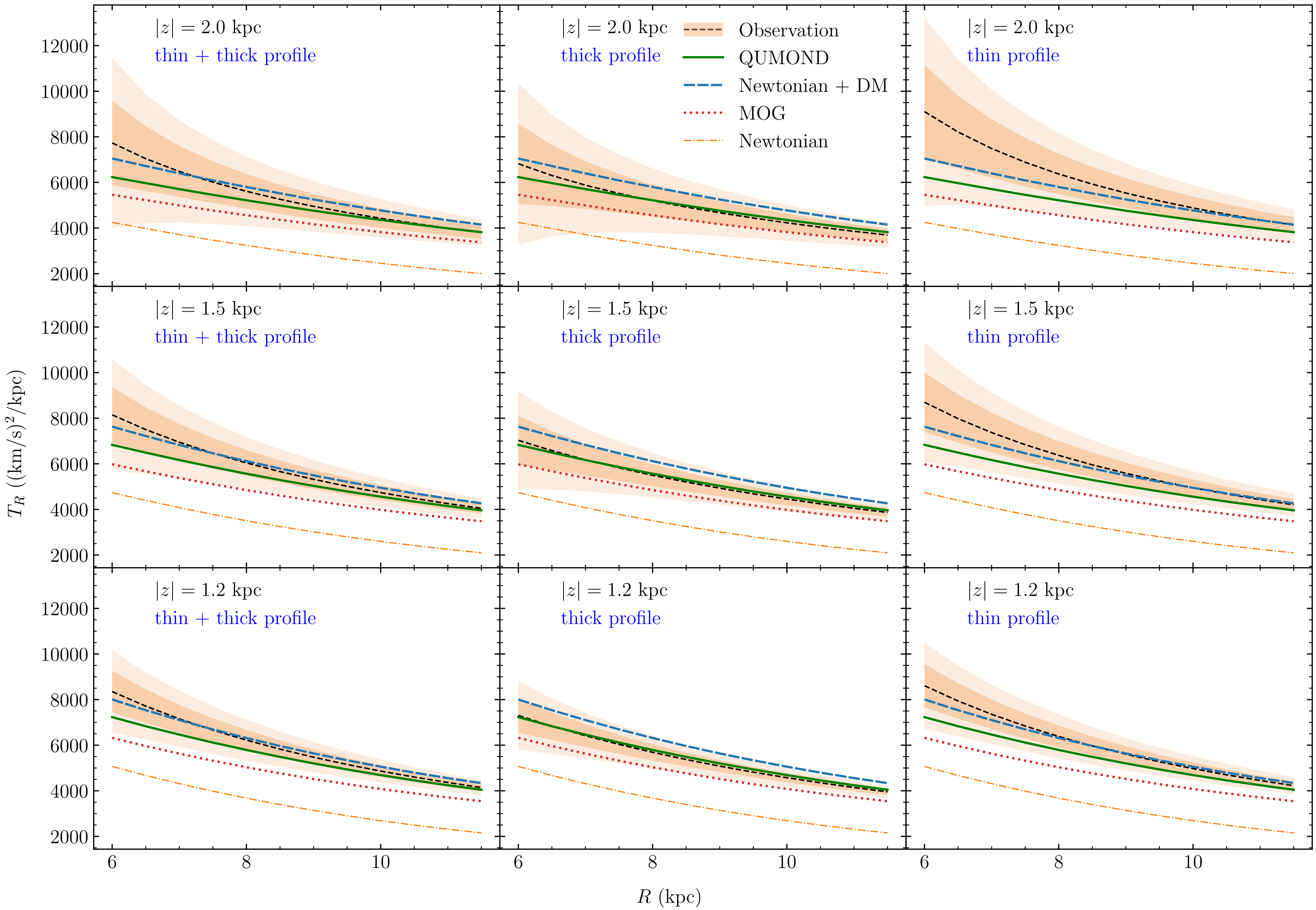
EFFECT OF TRACERS' DENSITY PROFILES

- In this work, since we lack the knowledge of the shape of the density profile of the tracer population, we have to represent it by using the profiles of general populations of the disc stars, such as the weighted thin+thick geometrical disc model.

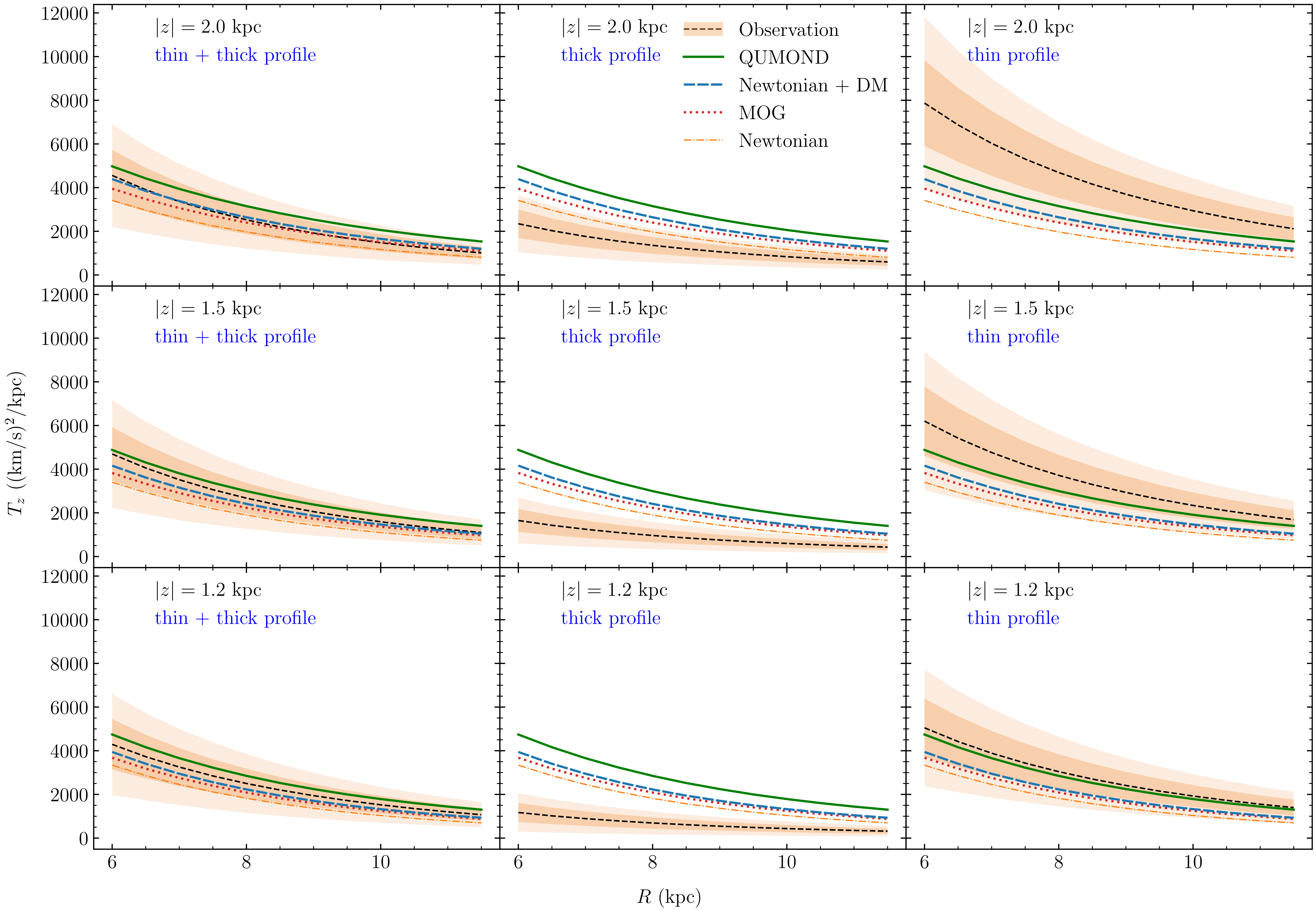
$$\rho(R, z) = 0.85 \times \rho_{\mathrm{d,thin}}(R, z) + 0.15 \times \rho_{\mathrm{d,thick}}(R, z)\,.$$

- The challenges behind this:
  - We are not sure if, and how well, the red clump stars follow the spatial distribution of general stars.
  - We are also not sure how well the chemically classified thin-disc red clump stars are consistent with the dynamically best-fit thin disc of Wang's model.
- Thus, we also use additional possible density profiles for the tracers to investigate the impact of the uncertainty in tracer's density profile to our Jeans-equations tests.

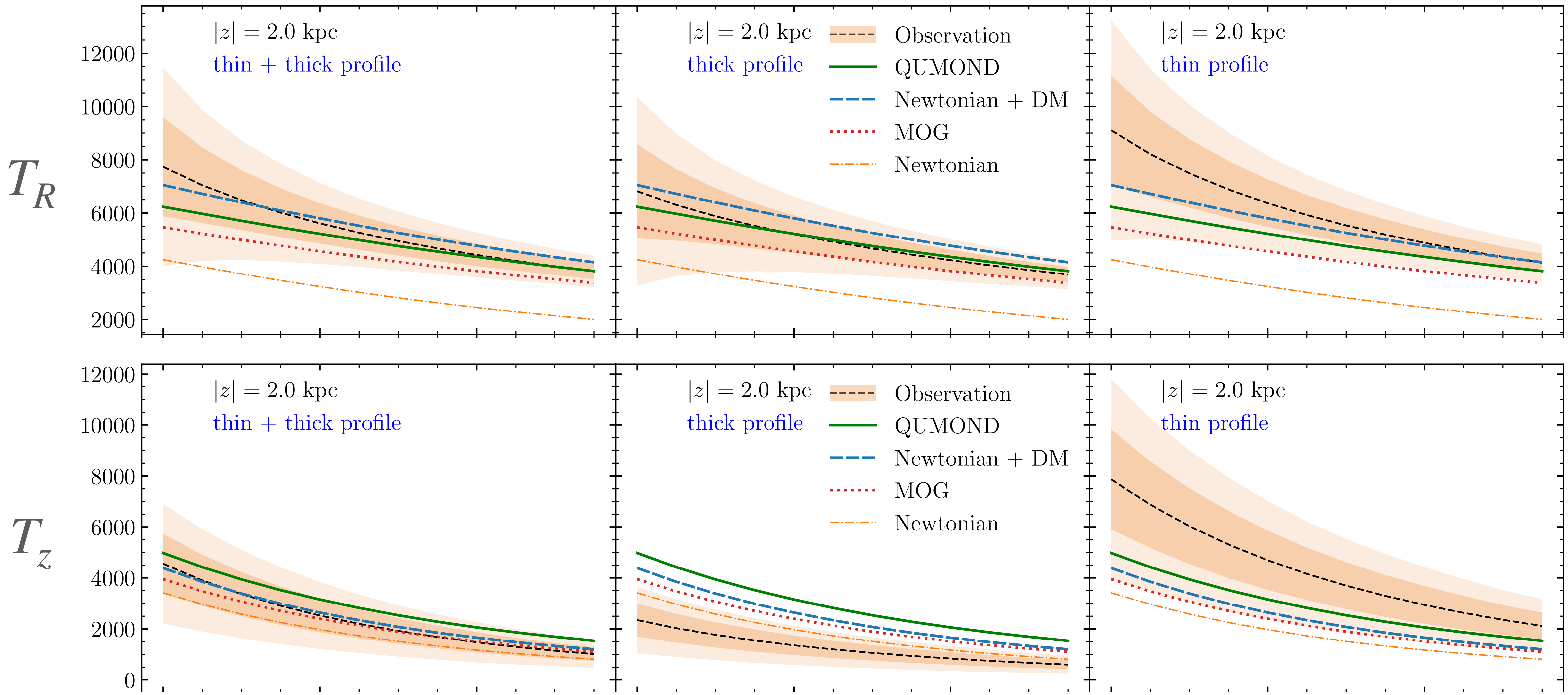
EFFECT OF TRACERS' DENSITY PROFILES



EFFECT OF TRACERS' DENSITY PROFILES

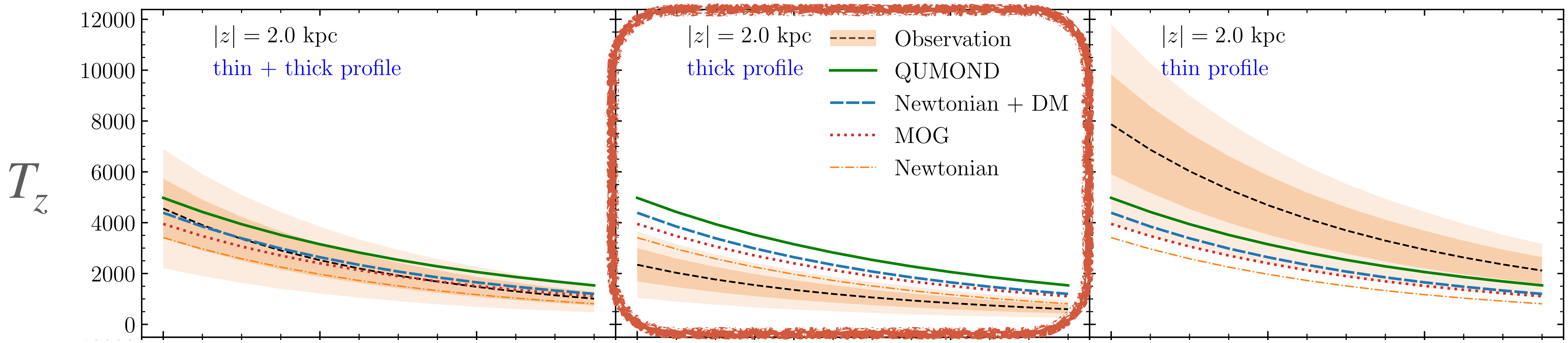


EFFECT OF TRACERS' DENSITY PROFILES



- While Tz changes dramatically in the 3 schemes, TR changes mildly. That is, **while the Tz test is sensitive to the tracer's density profile, the TR test is relatively insensitive and thus robust.**

EFFECT OF TRACERS' DENSITY PROFILES



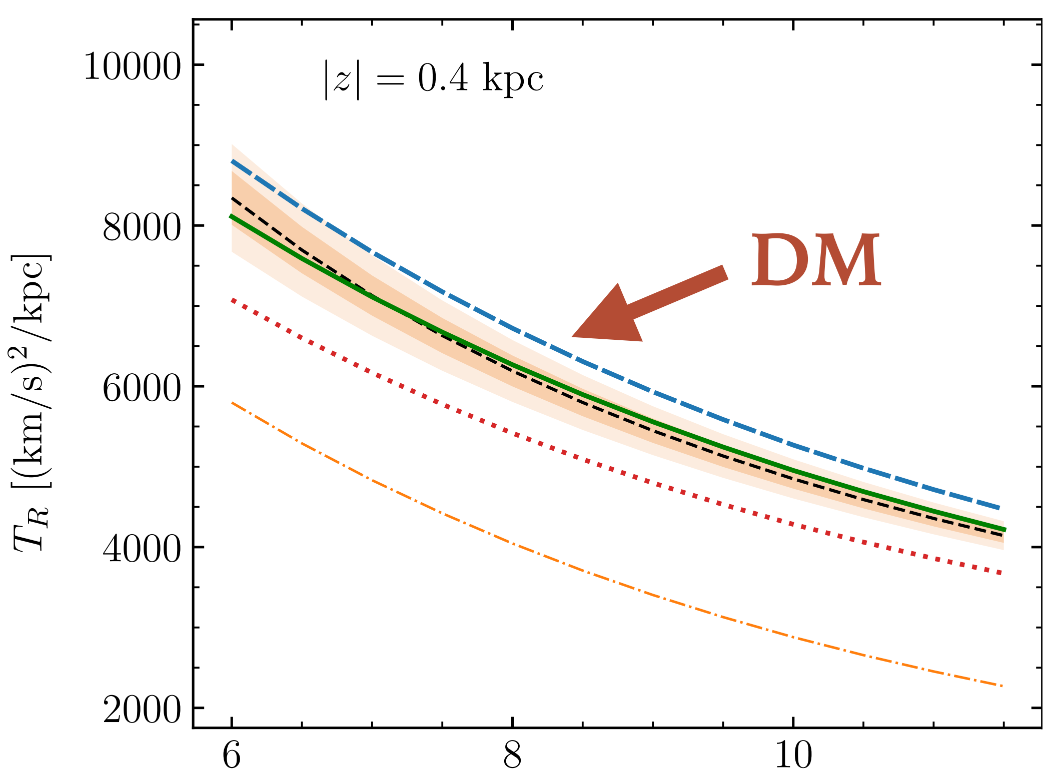
- The thick-disc profile scheme of the Tz tests yields that all the four gravitational models lie beyond the 95 per cent confidence intervals for almost all spatial locations. This fact indicates that the real density profile of the tracers, i.e. the red clump stars of Huang et al. (2020), is closer to the thin-disc profile than the thick-disc one.
- However, it is because that the sample of Huang et al. (2020) is mainly dominated by thin-disc red clump stars (116 000 of 137 000). Thus, the result for thick-disc profile does not means that this scheme rules out all the four gravitational models, but, again, means that **Tz test is sensitive to tracer's density profile.**

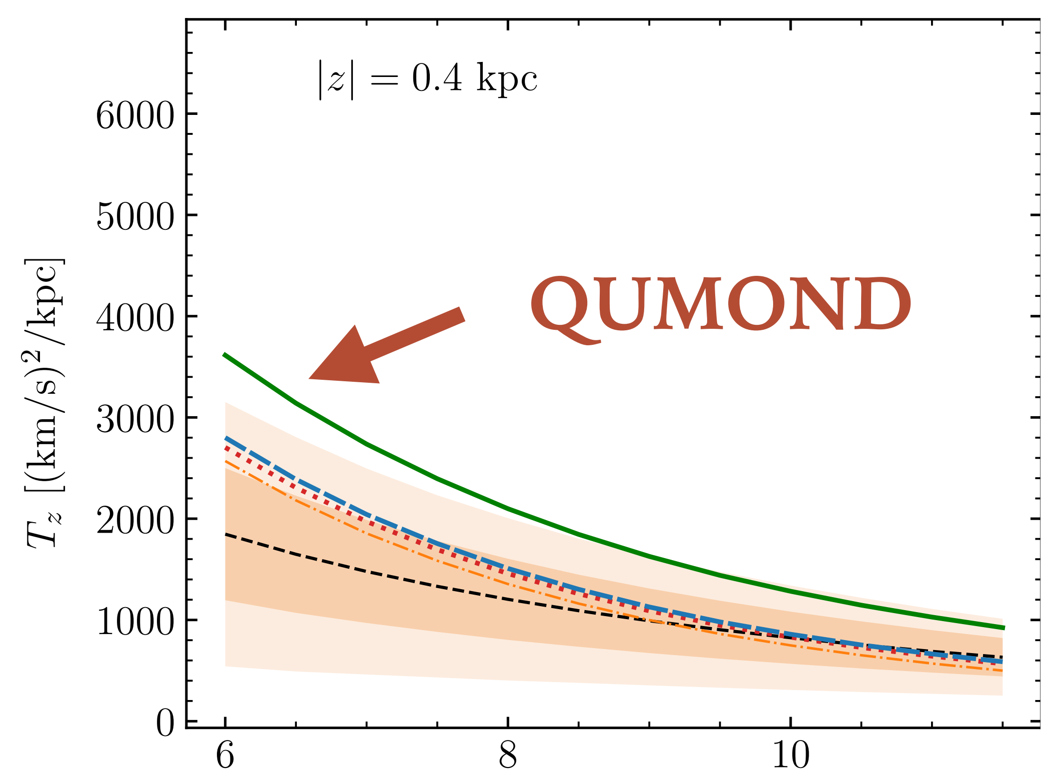
EFFECT OF TRACERS' DENSITY PROFILES

- Concerning the dependence of TR on tracer's density profile, the dependence is not negligible. Thus we would like to caution that if one use R-directional Jeans equation to calculate certain quantities, the uncertainty caused by tracer's density profile has to be accounted for.
- Concerning the sensitive dependence of Tz on tracer's density profile, if we can constrain the other quantities, i.e. gravitational potential and velocity dispersion, then we will be able to place tight constraints on the spatial distribution of a specific population of stars, by taking advantage of the sensitive Tz measure.
  - First, the Tz measure is employed to pick up plausible models for the tracer's density profile.
  - Then TR is used to discriminate various gravitational models with subtle discrepancies.
  - The two steps can be iterated to get both the best parameterized tracer's density profile and gravitational model.

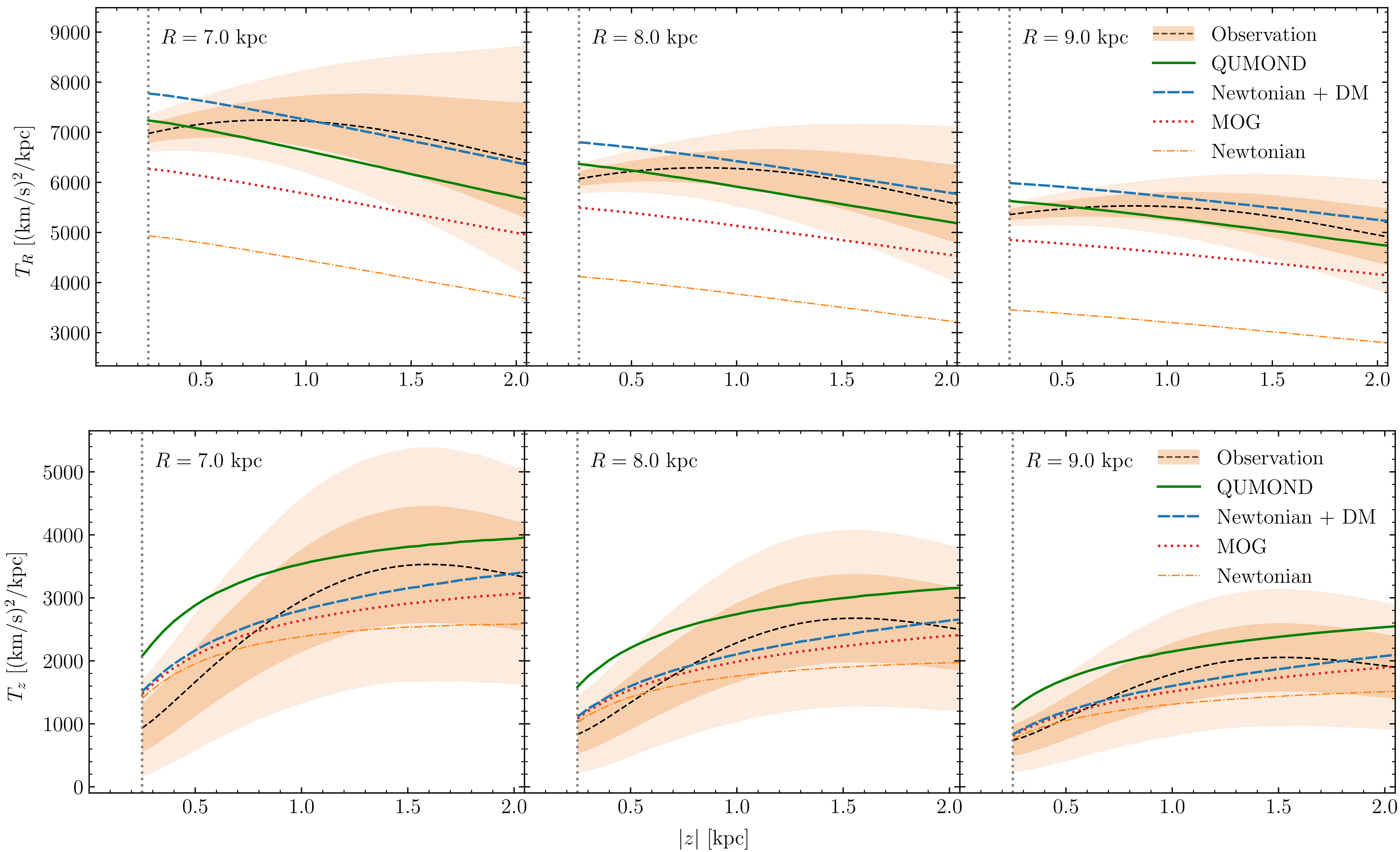
DYNAMICS AT LOW ALTITUDES

- The convergent result: the Newtonian baryon-only model and MOG are rejected, and the fiducial DM model and MOND are consistent with the TR and Tz data generally.
- However, there appear systematical trends at low-|z| locations discomforting for both DM and MOND.

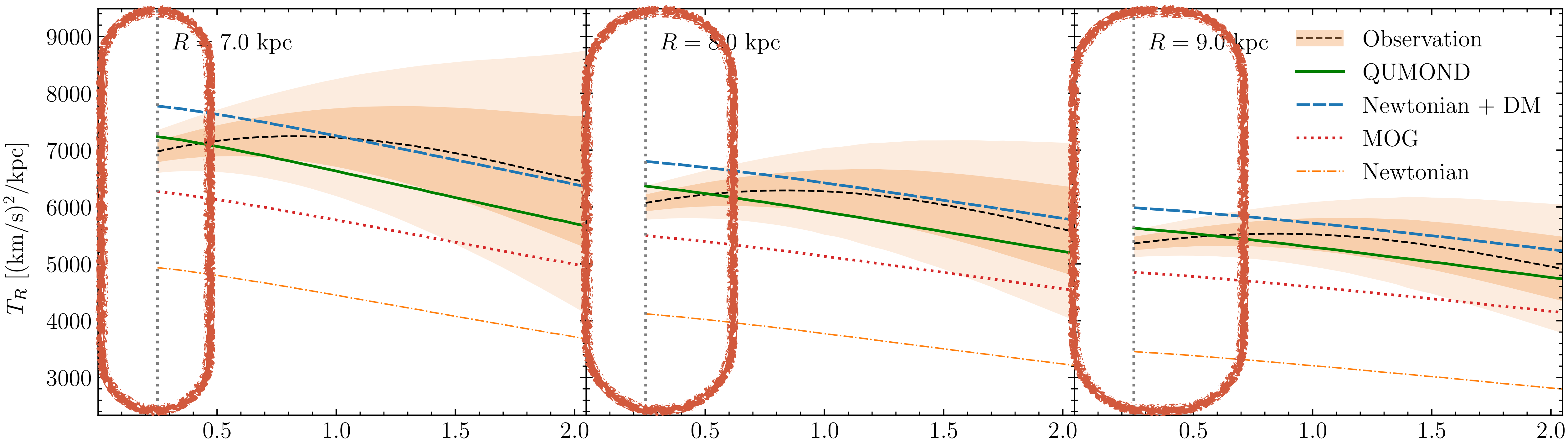




DYNAMICS AT LOW ALTITUDES



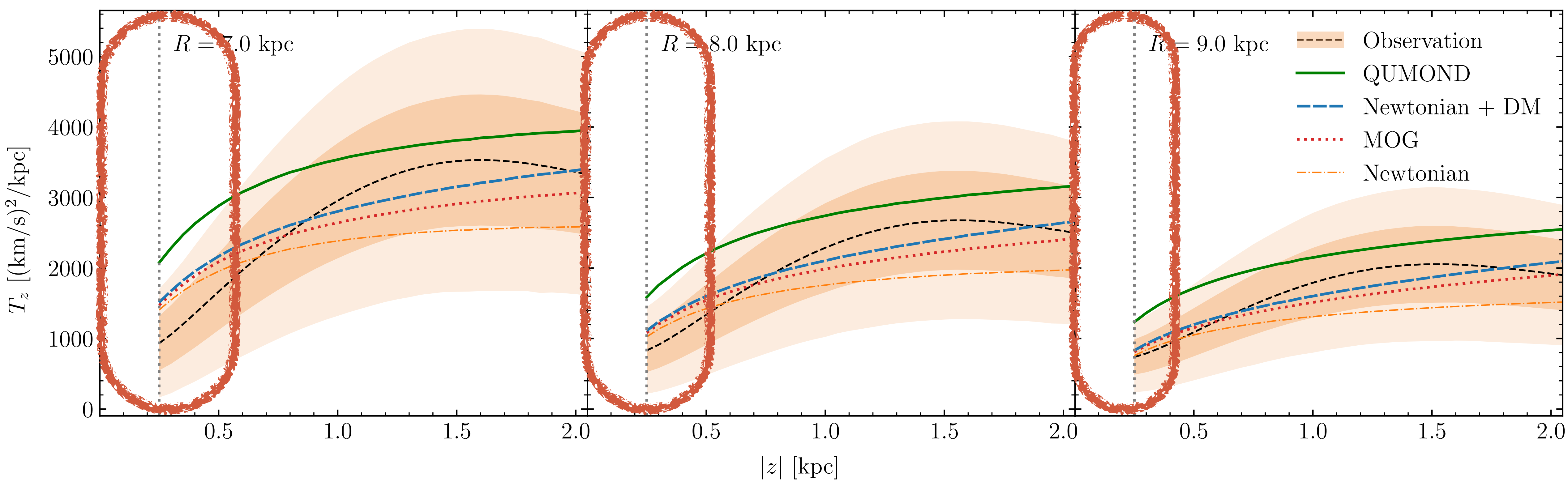
DYNAMICS AT LOW ALTITUDES - RADIAL



- In the TR–z plots, the radial field strength of the fiducial DM model lies outside the cent confidence intervals. This trend of inconsistency with the TR data gets somehow worse with R moving outwards.
- On the contrary, the radial field strength of QUMOND always lies within the 68 per cent confidence interval at every location.

DYNAMICS AT LOW ALTITUDES - VERTICAL

- In the Tz–z plots, the vertical field strength of the fiducial DM model always lies in the 68 per cent confidence interval of every locations.
- The vertical field strength of QUMOND lies slightly outside the confidence intervals at lower altitudes. This trend of inconsistency with the Tz data gets somehow alleviated with R moving outwards.



DYNAMICS AT LOW ALTITUDES

- In the literature, it is being hotly debated as to the shape of the Galactic DM halo is oblate, spherical, or prolate, with observational evidence both for and against an oblate shape of the inner Galactic gravitational potential.
- Our above analysis of the possible small-altitude problem show that there is a possibility that the real Galactic gravitational potential, particularly its inner part, is in between the fiducial DM model with a spherical DM halo and the QUMOND, i.e. in the DM language, the "*halo*" may be **oblate**.
- But still, as the exact $|z|$ range and the degree of DM and QUMOND deviating from the data depend on the tracer population and its density profile we use. And such results could also be biassed by the choice of tracers or samples, and the fitting process of the spatial distribution functions of velocity dispersion. **Thus at this point we leave this problem open.**

FURTHER EXPLORATION OF VERTICAL DYNAMICS

- In this work, when using the Huang et al. (2020) sample of red clump giants as tracers, we did not make a fine classification of the tracers (the work only divided the red cluster stars into thin and thick disk populations), and we did not fit the density profiles of the red cluster stars (we just borrowed the geometric thin and thick disk density profiles).
- To avoid the bias from tracer and tracers' density profiles, we are going to utilize the best data set of stars in the MW (Gaia DR3 + LAMOST DR8), and divide the red clump giants into sub-populations based on their chemical properties (metal abundance [Fe/H] and abundance ratio [α/Fe]). Then the functional form of the tracer vertical density profile will be carefully designed and tested, with each subsample having different density profile parameters. The optimization is then performed iteratively using alternating radial and vertical Jeans equations (i.e., T_R, T_z) to ensure the consistent distribution of the total galactic gravitational potential with each subsample.
- From the perspective of the DM paradigm instead, the vertical dynamics could be explain in the way that the shape of the MW's DM halo is oblate, yet not so extreme as the effective halo prescribed by QUMOND.

EXPLORING THE ‘EXTRA MASS/GRAVITY’

- **Excess/extra potential**: differences in gravitational potential predicted by the three models (denoted as model) compared with the Newtonian baryon-only case.

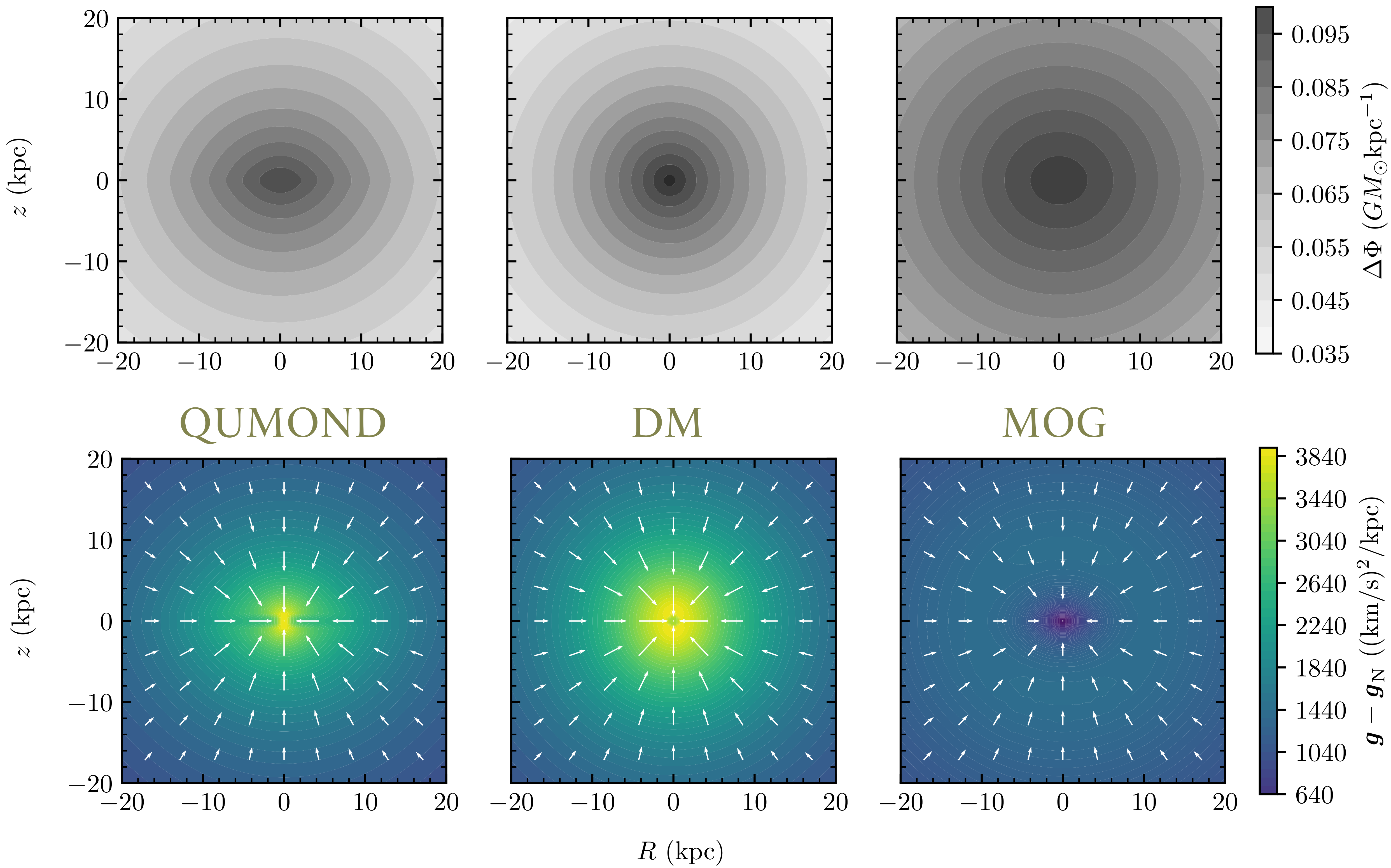
$$\Delta\Phi = \Phi_N - \Phi_{\mathrm{model}}$$

- **Excess/extra gravity**: corresponding gradients of the potential difference, namely the vector difference in field strength.

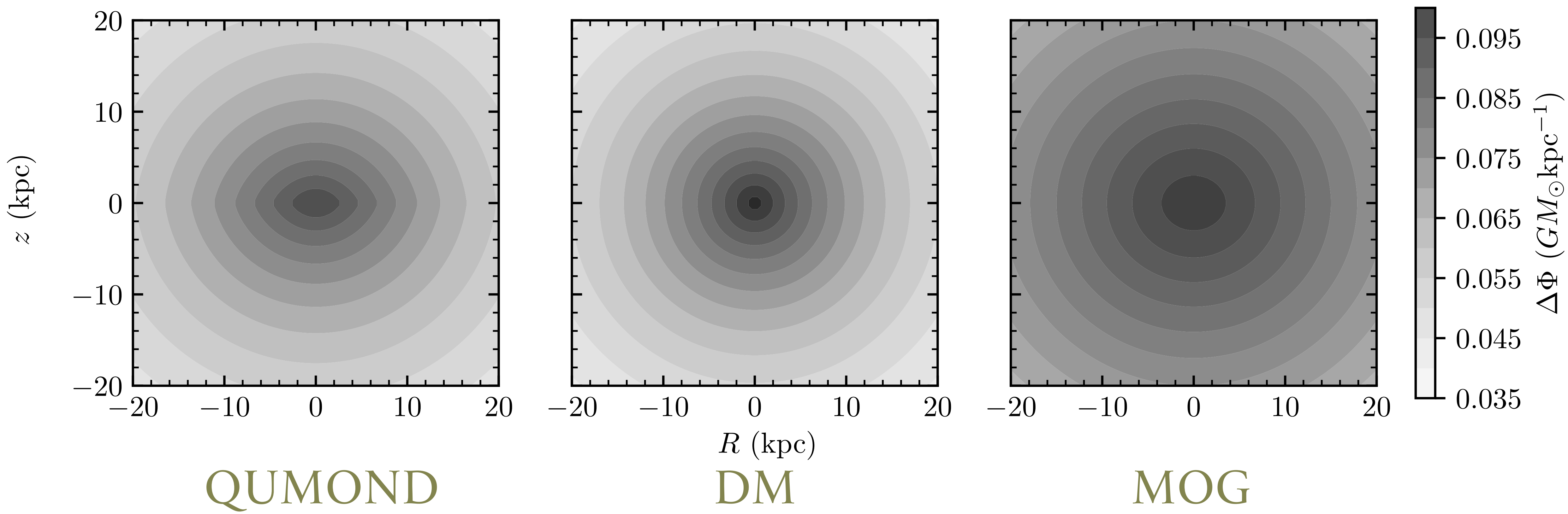
$$g_{\mathrm{model}} - g_{\mathrm{N}} = \nabla\left(\Phi_{\mathrm{N}} - \Phi_{\mathrm{model}}\right)$$

- The ‘**extra potential**’ can be translated into the **effective ‘extra mass’** in the Newtonian sense, simply using normal Poisson equation.
  In the case of the DM model, this translation is physical and exact, and the extra mass is just the DM halo.
  However, that such a translation is merely mathematical for any modified-gravity models, and the concept of ‘extra mass’ is even misleading. But in the case of QUMOND, interestingly, this translation is meaningful, and the ‘extra mass’ is the very concept of ‘phantom dark matter’.

EXPLORING THE 'EXTRA MASS/GRAVITY'



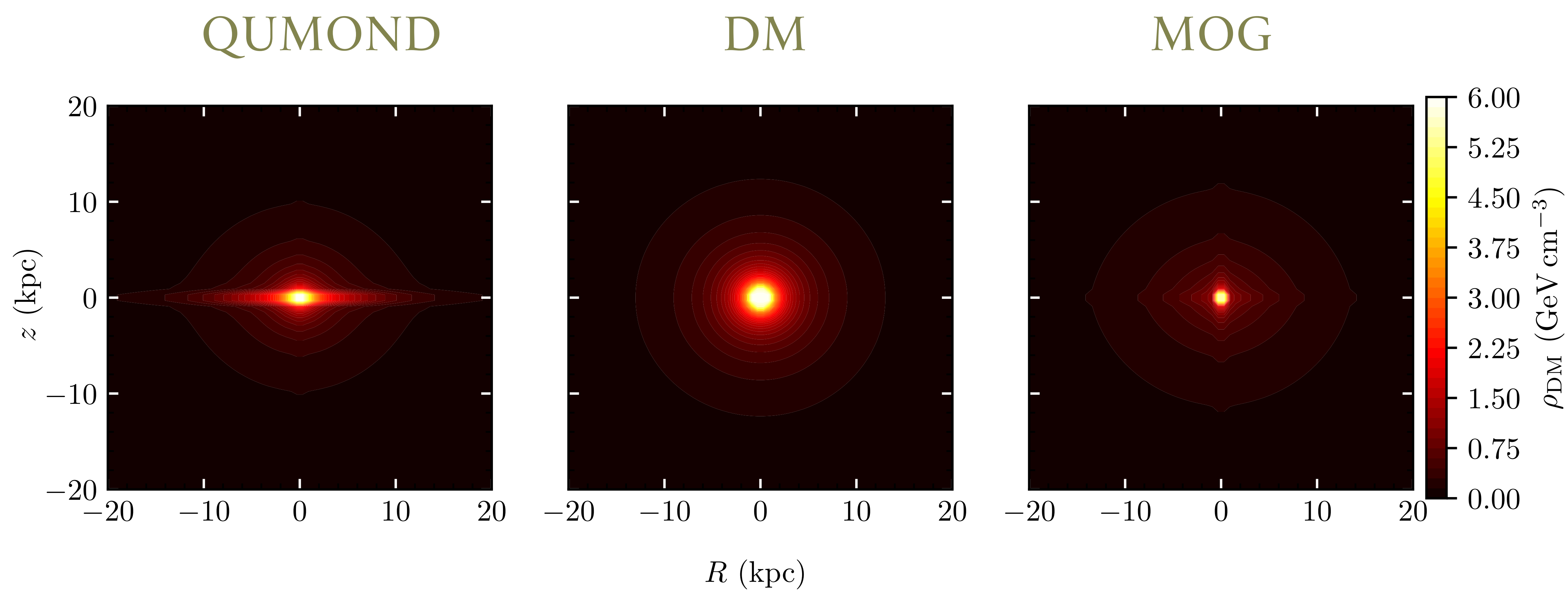
EXPLORING THE 'EXTRA MASS/GRAVITY'



- The extra potential of the fiducial model (namely the DM halo) is spherically symmetric as prescribed by the Zhao's profile.
- QUMOND (left-hand panel) gives a comparable extra potential in magnitude to the DM case, but the shape of the extra potential is fairly flatten in the z-direction (i.e. an oblate gravitational potential).
- MOG yields a slightly oblate extra potential (right-hand panel)

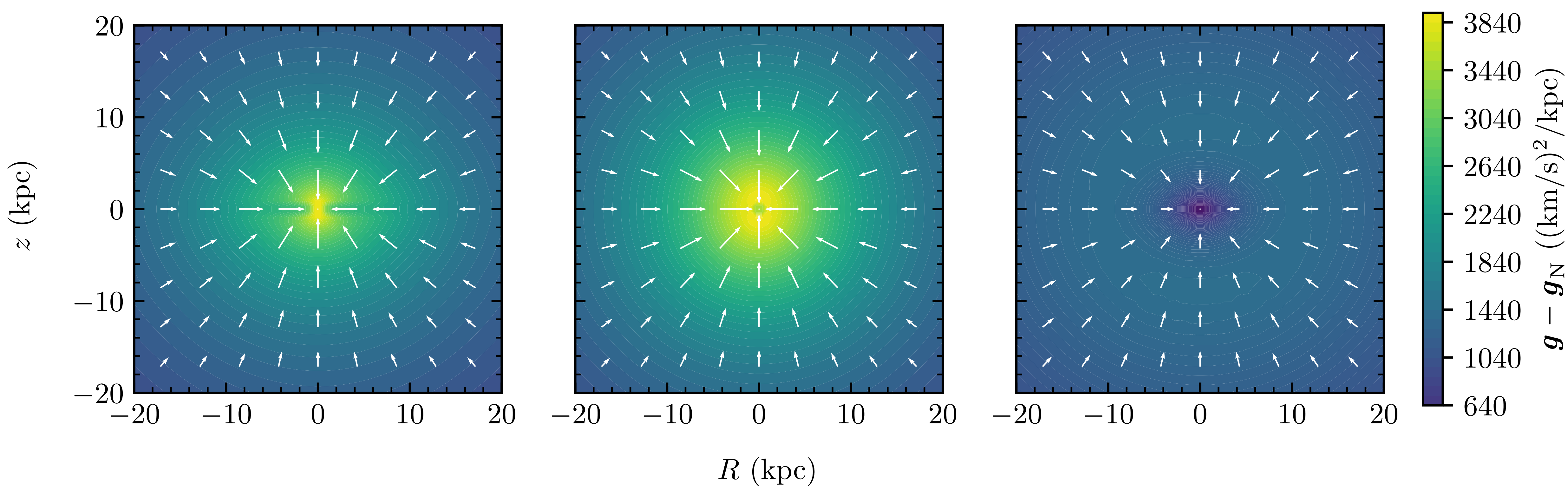
EXPLORING THE 'EXTRA MASS/GRAVITY'

- Compared with the density distribution of the DM halo of the fiducial mass model, the QUMOND PDM is morphologically closer to a traditional (quasi-)spherical DM halo plus a disc-shaped component.



VERTICAL DYNAMICS OF THE MW

- The PDM halo provides too much gravitational acceleration in the vertical direction (i.e., the acceleration $\mathbf{g}_{\mathrm{MDAR}}$ is larger than observation $\mathbf{g}_{\mathrm{obs}}$ in the vertical direction of the galaxy according to MDAR).
- This suggests that we can investigate the vertical dynamics of the MW, to seek for the truth whether MDAR (and thus the modified Newtonian dynamics -- MOND) performs well in the vertical direction of the MW.



SYNTHESIZING DM AND MOND: TOWARD AN EFFECTIVE THEORY?

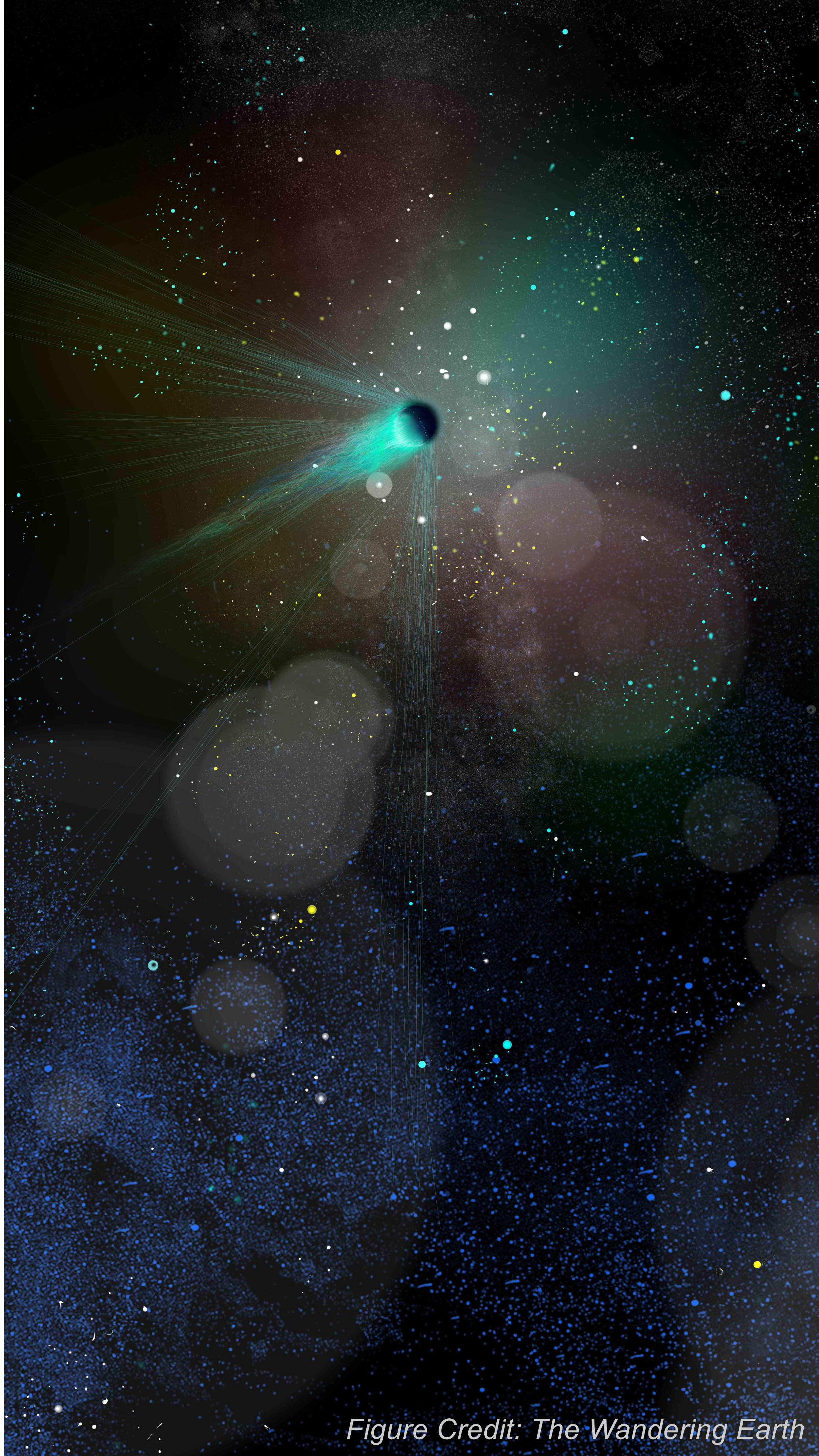
- This study reinforces the **effective equivalence** between MOND and Dark Matter at circum-galactic and galactic scales, known as 'CDM-MOND degeneracy'. Specifically, it examines the similarity between QUMOND's PDM and potential models of DM haloes.
- This effective equivalence might be calling forth a new synthesis reconciling and transcending both MOND and DM paradigms.
- Thinking practically, for any practical purposes for the study of the kinematics on galactic scales, people can safely use the QUMOND formula as an alternative of DM halo models. This approach will save the researchers from handling various prerequisites and fine tuning the cumbersome parameters of DM haloes.

SUMMARY

- The Newtonian baryon-only model fails both the rotation-curve test and the R-directional Jeans-equation test across all spatial (R, z) locations.
- Regarding models with additional mass or gravity (DM model with a spherical halo, MOND, and MOG), rotation-curve data alone can't definitively reject any of them.
- The key shared result among Jeans-equation tests is: both the DM model and MOND consistently fall within 95% confidence intervals in terms of both radial acceleration (T_R) and vertical acceleration (T_z) for nearly all locations with $|z|$ greater than a certain altitude ($|z| > 0.5$ kpc), while the MOG model deviates more.
- At low-$|z|$ locations, there may be problematic trends for MOND and the DM model: DM's radial field strength appears systematically larger than radial acceleration (T_R), and MOND's vertical field strength seems larger than vertical acceleration (T_z). Specifics depend on the tracer population and its density profile. The true Galactic gravitational potential, particularly its inner part, might be between the DM model and MOND, suggesting an oblate inner "*halo*" shape.

THANK YOU!

June 2023, at U. St Andrews



APPENDIX- SPATIAL DISTRIBUTION OF 3D KINEMATICS

- In order to reduce the impact of particular structures in the Galactic disc, we fit the velocity dispersions, $\sigma_R, \sigma_\theta,$ and σ_z, to the smooth analytic forms with respect to R and z (Binney et al. 2014).

$$\sigma(R, z) = \sigma_0 a_1 \exp\left[-a_2\left(R/R_\odot - 1\right)\right]\left[1 + \left(a_3 z/R\right)^2\right]^{a_4}$$