How "MOND-like" is Quasilinear MOND?

Investigating the Vertical Acceleration Field of the Milky Way

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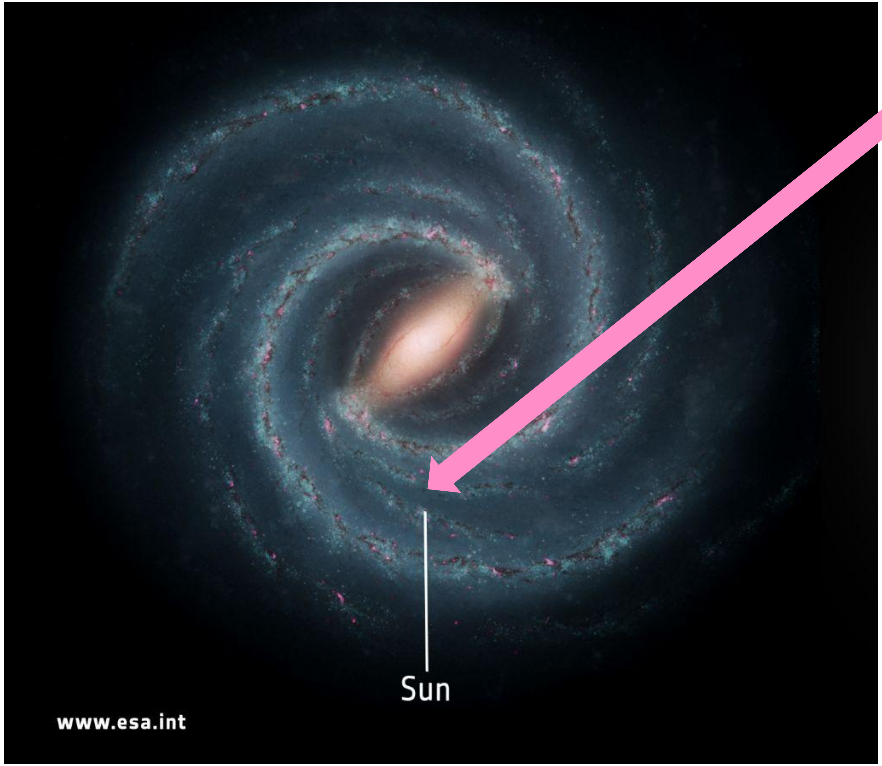
Discussions: Professor Stacy McGaugh, CWRU

40 years of MOND

June 2023 St. Andrews

Case Western Reserve University EST. 1826

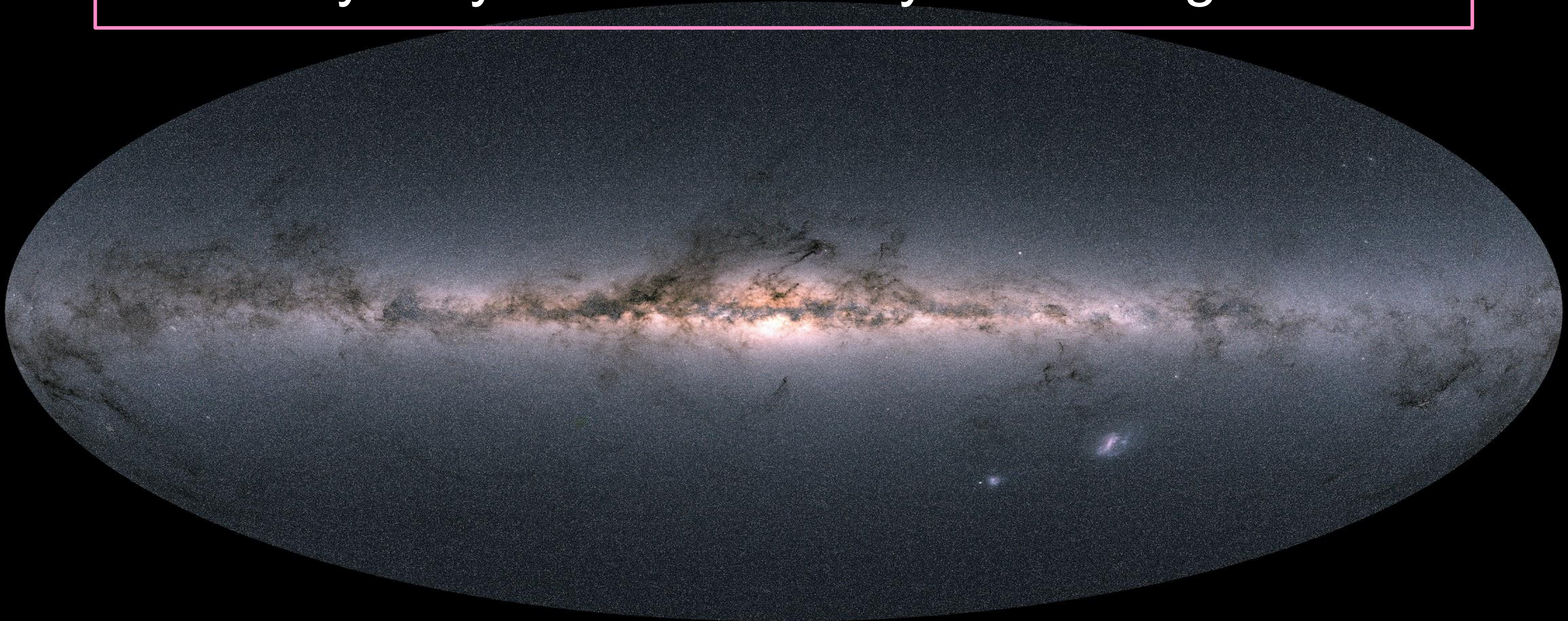
The Milky Way as a Laboratory for Testing MOND



The sun is around $2a_0$ (McGaugh 2016)

Moderate MOND behavior expected

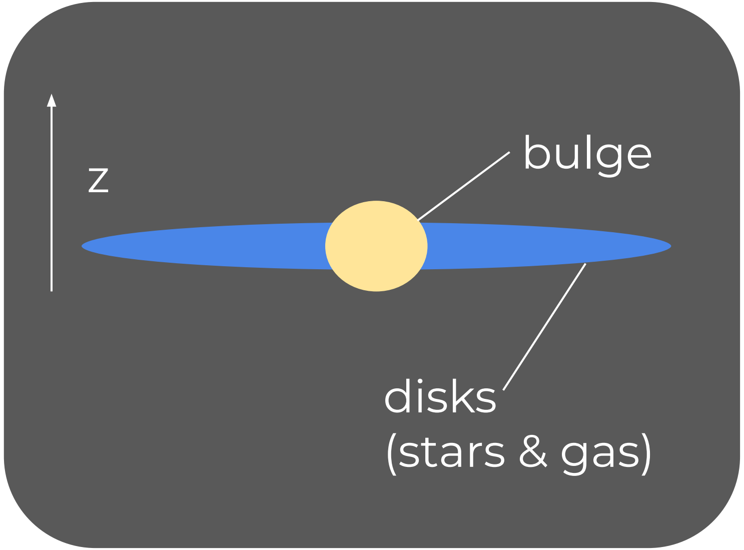
The Milky Way as a Laboratory for Testing MOND



Full astrometric solution (parallax, proper motion, etc.) for $>10^9$ stars in the Milky Way with unprecedented resolution (~.01 mas)

GAIA

Vertical Dynamics as a Test of MOND



Acceleration due to gravity **g**

$$g_{z,MOND} \neq g_{z,Newtonian^*}$$

Can use this as a test of MOND!

Nipoti et al. (2007)

*Newtonian ≡ Newtonian gravity + dark matter halo

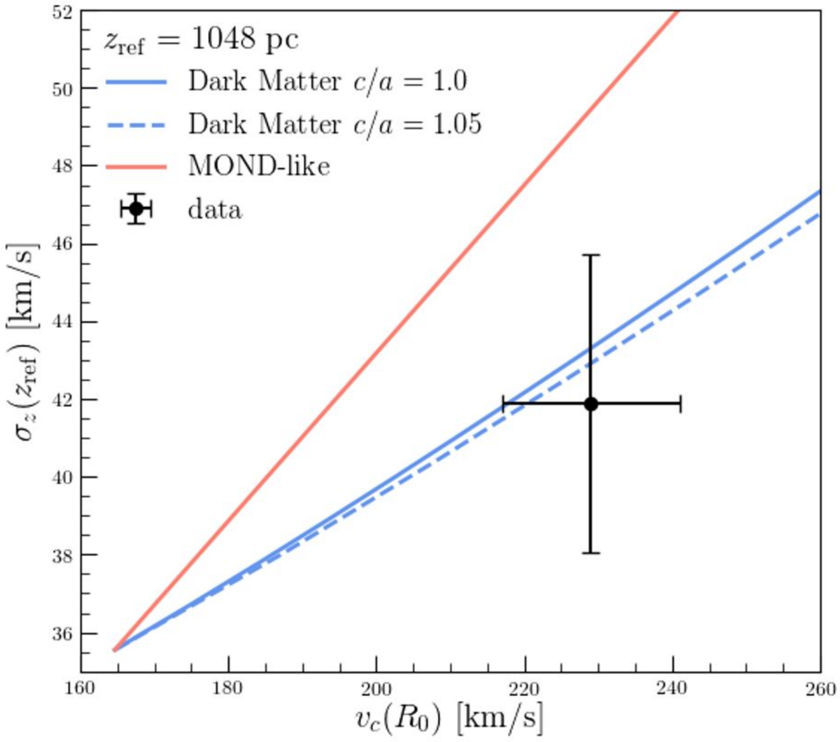
Vertical Dynamics as a Test of MOND: Previous Work

Nipoti et al. (2007)

- MOND vertical acceleration near the plane and vertical velocity dispersion larger than Newtonian gravity + DM

Lisanti et al. (2019)

- “MOND-like” models enhance radial & vertical accelerations equally → overpredicts vertical acceleration
- Anomalously large stellar bulge and/or anomalously small disk scale radius required to match observations



Lisanti et al. (2019)

The relevant flavors of MOND

Pristine MOND (Milgrom 1983)

- Algebraic interpolation of the Newtonian acceleration due to baryons

$$\mathbf{g}_P = \mathbf{g}_N \nu(\frac{g_N}{a_0})$$ Eq. 2 of Milgrom (1983)

- Not derivable from a Lagrangian, energy and momentum not conserved

Quasilinear MOND (Milgrom 2010)

- Derivable from a Lagrangian
- The Quasilinear MOND acceleration is the curl-free part of the Pristine MOND acceleration (Brown et al. 2018)

$$\nabla^2 \Phi = \nabla \cdot (\nu(\frac{\nabla \Phi_N}{a_0}) \nabla \Phi_N)$$

The relevant flavors of MOND

Pristine MOND

$$\mathbf{g}_P = \mathbf{g}_N \nu(\frac{g_N}{a_0})$$

Milgrom (1983) eq. 2

Good for: basic tenets of MOND, rotation curve analysis

Quasilinear MOND (QUMOND)

$$\vec{\nabla}^2 \Phi_Q = \vec{\nabla} \cdot (\nu(\frac{\vec{\nabla}\Phi_N}{a_0})\vec{\nabla}\Phi_N)$$

$$\Downarrow$$

$$\mathbf{g}_Q$$

Milgrom (2010)

Good for: non-test-particle motion

More computationally tractable than AQUAL/nonlinear MOND

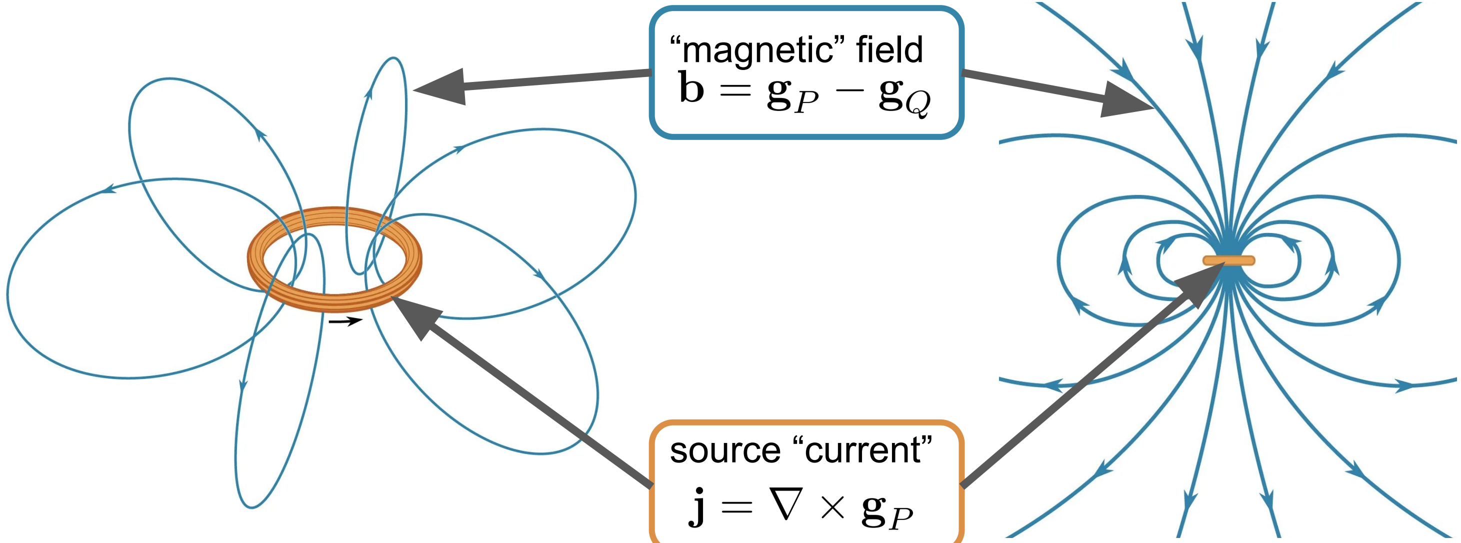
Magnetostatics Analogy for Quasilinear MOND

$\mathbf{g_P}$: Pristine MOND acceleration

$\mathbf{g_Q}$: Quasilinear MOND acceleration

$$\left.\begin{aligned} \nabla \cdot \mathbf{g}_Q &= \nabla \cdot \mathbf{g}_P \\ \nabla \times \mathbf{g}_Q &= 0 \end{aligned}\right\}$$

→ $\mathbf{g_Q}$ is the curl-free part of $\mathbf{g_P}$



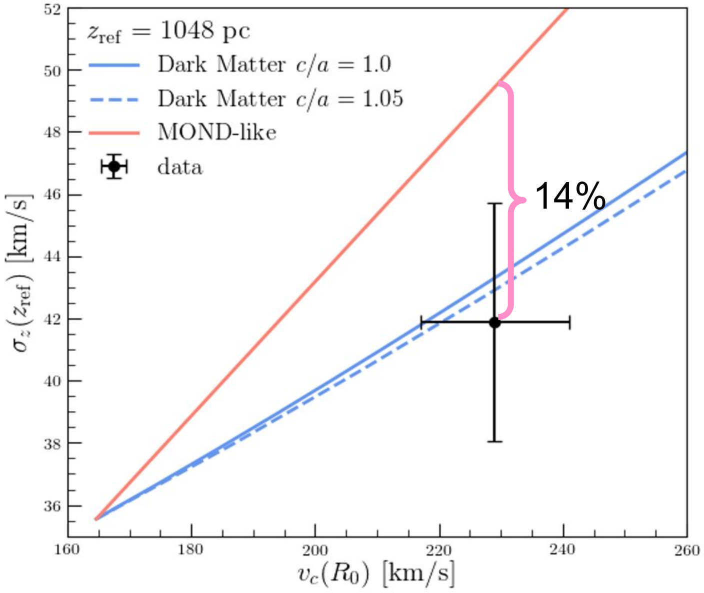
The difference between $\mathbf{g_P}$ and $\mathbf{g_Q}$ resembles a magnetic field sourced by the curl of $\mathbf{g_P}$

Brown et al. (2018)

Is MOND “MOND-like”?

Lisanti finds a small overprediction in the vertical acceleration **using Pristine MOND as a proxy for all “MOND-like theories”**

$$\mathbf{g}_P = \mathbf{g}_N \nu(\frac{g_N}{a_0})$$



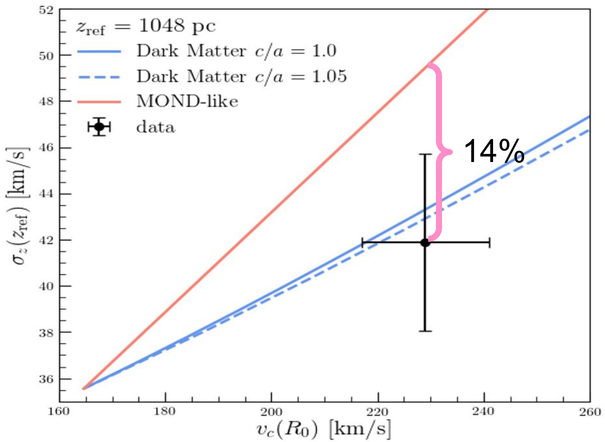
Lisanti et al. (2019)

Is MOND MOND-like?

Lisanti finds a small overprediction in the vertical acceleration using Pristine MOND as a proxy for all “MOND-like theories”

Central Question: Could this tension arise simply from the difference between Pristine MOND and Quasilinear MOND?

$$\mathbf{g}_P = \mathbf{g}_N \nu\left(\frac{g_N}{a_0}\right)$$



Lisanti et al. (2019)

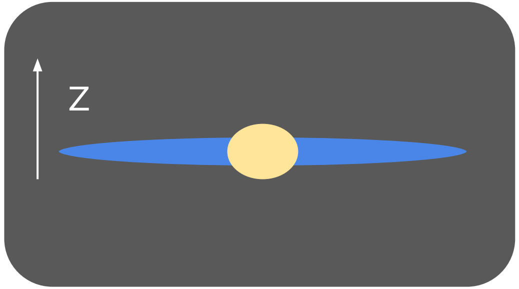
Is MOND MOND-like?

Lisanti finds a small overprediction in the vertical acceleration using Pristine MOND as a proxy for all "MOND-like theories"

Central Question: Could this tension arise simply from the difference between Pristine MOND and Quasilinear MOND?

$$g_{z,\mathrm{PMOND}} \overset{?}{\approx} g_{z,\mathrm{QUMOND}}$$

Quasilinear MOND Poisson Solver



$g_{z,PMOND}$ **&** $g_{z,QUMOND}$

QUMOND Poisson solver using Fourier methods and discrete differentiation:

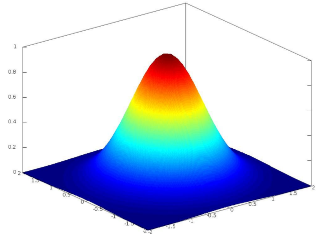
1. Solve Newtonian poisson eq to get Newtonian acceleration (in Fourier domain)
2. Interpolate to get Pristine MOND acceleration (in real domain)
3. Take curl free part to get Quasilinear MOND acceleration (in Fourier domain)

1. Solve the Newtonian Poisson eq. to get the Newtonian acceleration (in the Fourier domain)

The Brown method: Banishing Infinite Galaxies

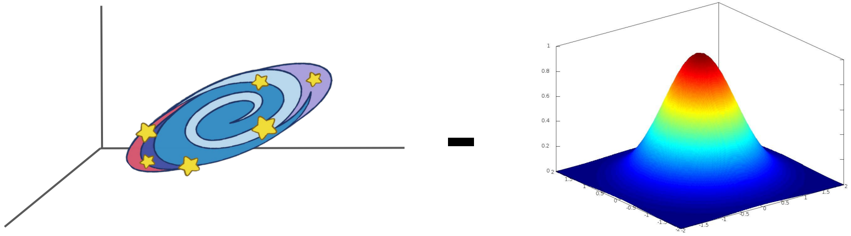
Gaussian subtraction (Brown et al. 2018)

1.



Construct ϱ_{gauss} with total mass equal to M_{gal}

2.



Solve the Newtonian Poisson equation for the difference between the two density distributions $\mathbf{\Phi}_{\text{diff}} = \mathbf{\Phi}_{\text{gal}} - \mathbf{\Phi}_{\text{gauss}}$

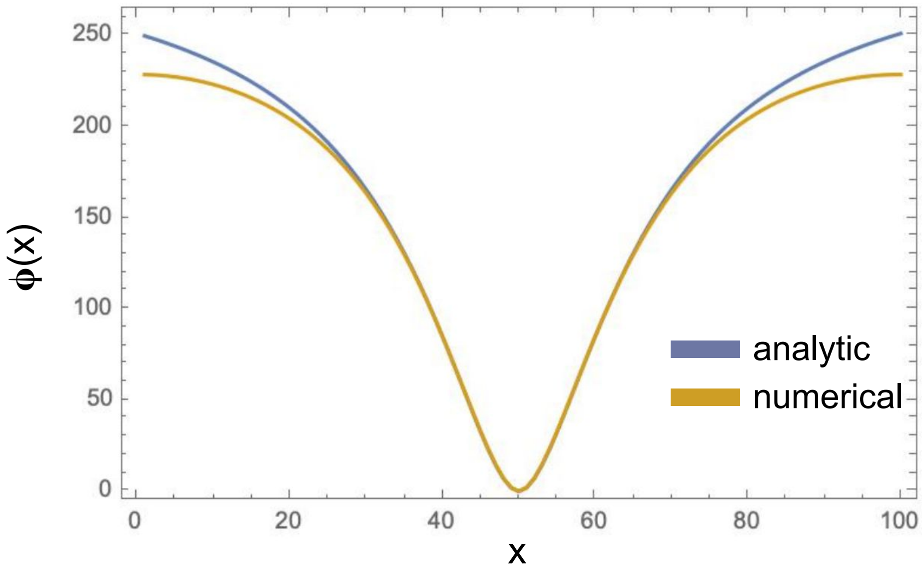
3. Differentiate to get $\mathbf{g}_{\text{diff}} = \mathbf{g}_{\text{gal}} - \mathbf{g}_{\text{gauss}}$

4. $\mathbf{g}_{\text{gal}} = \mathbf{g}_{\text{diff}} + \mathbf{g}_{\text{gauss}}$

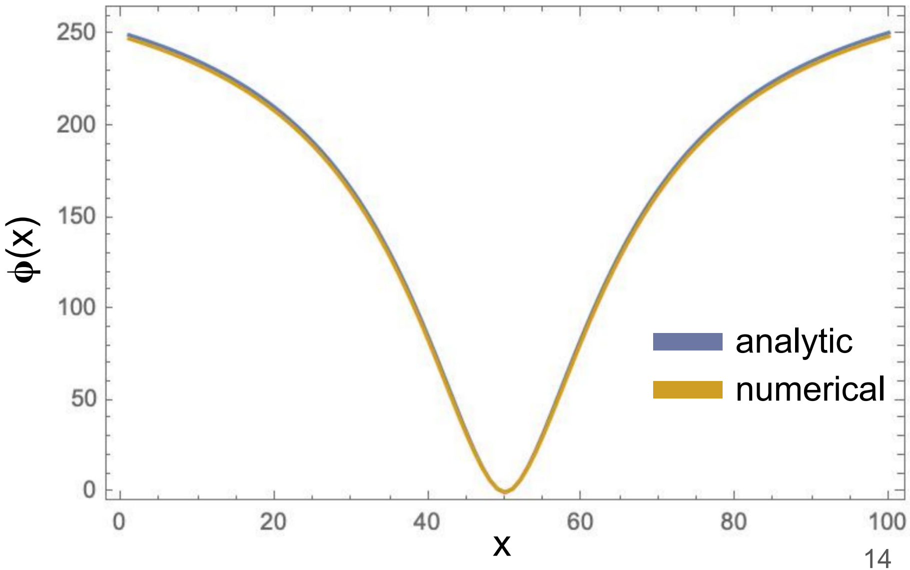
1. Solve the Newtonian Poisson eq. to get the Newtonian acceleration (in the Fourier domain)

The Brown method continued

Newtonian potential along the x-axis due to a spherical exponential galaxy $\rho = \alpha \exp\left(-\frac{r}{\lambda}\right)$ without accounting for periodic boundary conditions



Newtonian potential along the x-axis due to a spherical exponential galaxy $\rho = \alpha \exp\left(-\frac{r}{\lambda}\right)$ after applying Gaussian subtraction



1. Solve the Newtonian Poisson eq. to get the Newtonian acceleration (in the Fourier domain)

QUMOND Poisson Solver

$\mathbf{g}_{\text{Newtonian}}$

2. Interpolate to get Pristine MOND acceleration (in the real domain)

QUMOND Poisson Solver

$\mathbf{g}_{\text{Newtonian}} \rightarrow \mathbf{g}_{\text{Pristine MOND}}$

Interpolate

$$\mathbf{g}_P = \mathbf{g}_N \nu(\frac{g_N}{a_0})$$

$$\nu(x) = \frac{1}{1 - \exp(-\sqrt{x})}$$

McGaugh, Lelli, Schombert (2016)

3. Take the curl-free part to get Quasilinear MOND acceleration (in the Fourier domain)

QUMOND Poisson Solver

$$\mathbf{g}_P = \mathbf{g}_N \nu(\frac{g_N}{a_0})$$

$$\nu(x) = \frac{1}{1 - \exp(-\sqrt{x})}$$

McGaugh, Lelli, Schombert (2016)

3. Take the curl-free part to get Quasilinear MOND acceleration (in the Fourier domain)

QUMOND Poisson Solver

$\mathbf{g}_{\text{Newtonian}}$ → $\mathbf{g}_{\text{Pristine MOND}}$ → $\tilde{\mathbf{g}}_{\text{Pristine MOND}}$ → $\tilde{\mathbf{g}}_{\text{QUMOND}}$

Interpolate

$$\mathbf{g}_P = \mathbf{g}_N \nu\left(\frac{g_N}{a_0}\right)$$

$$\nu(x) = \frac{1}{1 - \exp(-\sqrt{x})}$$

Fourier transform

Take the curl free part

$$\tilde{\mathbf{g}}_Q = \frac{\mathbf{k}(\mathbf{k} \cdot \tilde{\mathbf{g}}_P)}{k^2}$$

$$\tilde{\mathbf{g}}_Q = \tilde{\mathbf{b}} + \tilde{\mathbf{g}}_P$$

↓

$\propto 1/r^4$

McGaugh, Lelli, Schombert (2016)

3. Take the curl-free part to get Quasilinear MOND acceleration (in the Fourier domain)

QUMOND Poisson Solver

$\mathbf{g}_{\text{Newtonian}} \rightarrow \mathbf{g}_{\text{Pristine MOND}} \rightarrow \tilde{\mathbf{g}}_{\text{Pristine MOND}} \rightarrow \tilde{\mathbf{g}}_{\text{QUMOND}} \rightarrow \mathbf{g}_{\text{QUMOND}}$

Interpolate

$$\mathbf{g}_P = \mathbf{g}_N \nu(\frac{g_N}{a_0})$$

$$\nu(x) = \frac{1}{1 - \exp(-\sqrt{x})}$$

Fourier transform

Take the curl free part

$$\tilde{\mathbf{g}}_Q = \frac{\mathbf{k}(\mathbf{k} \cdot \tilde{\mathbf{g}}_P)}{k^2}$$

$$\tilde{\mathbf{g}}_Q = \tilde{\mathbf{b}} + \tilde{\mathbf{g}}_P$$

$\downarrow$

$\propto 1/r^4$

Inverse Fourier transform

McGaugh, Lelli, Schombert (2016)

MOND Galactic Model of Lisanti et al.

Lisanti models:

- Stellar disk
- Gaseous disk

$$\rho_i(R, z) = \rho_{i,0}\exp(-R/h_{i,R} - |z|/h_{i,z})$$

$$i = *,\ g$$

- Stellar bulge $\rho_b(r) = \dfrac{M_* r_*}{2\pi r(r + r_*)^3}$

Fixed:

$$r_* = 600\ \mathtt{pc}$$

$$h_{g,z} = 130\ \mathtt{pc}$$

$$h_{*,z} = 300\ \mathtt{pc}$$

$$h_{g,R} = 2h_{*,R}$$

Remaining Parameters:

Results of Pristine MOND Bayesian likelihood analysis with local MW data (Lisanti et al. Table II)

$$\rho_{*,0} = 1.37\ M_\odot/\mathtt{pc}^3$$

$$h_{*,R} = 2410\ \mathtt{pc}$$

$$\rho_{g,0} = 0.25\ M_\odot/\mathtt{pc}^3$$

$$M_* = 4.29 * 10^{10}\ M_\odot$$

Our Galactic Model

h_R = 3210 pc
(Lisanti et al. RAR
fit for stellar disk)

$$\rho(R, z) = \rho_0 \exp(-R/h_R - |z|/h_z)$$

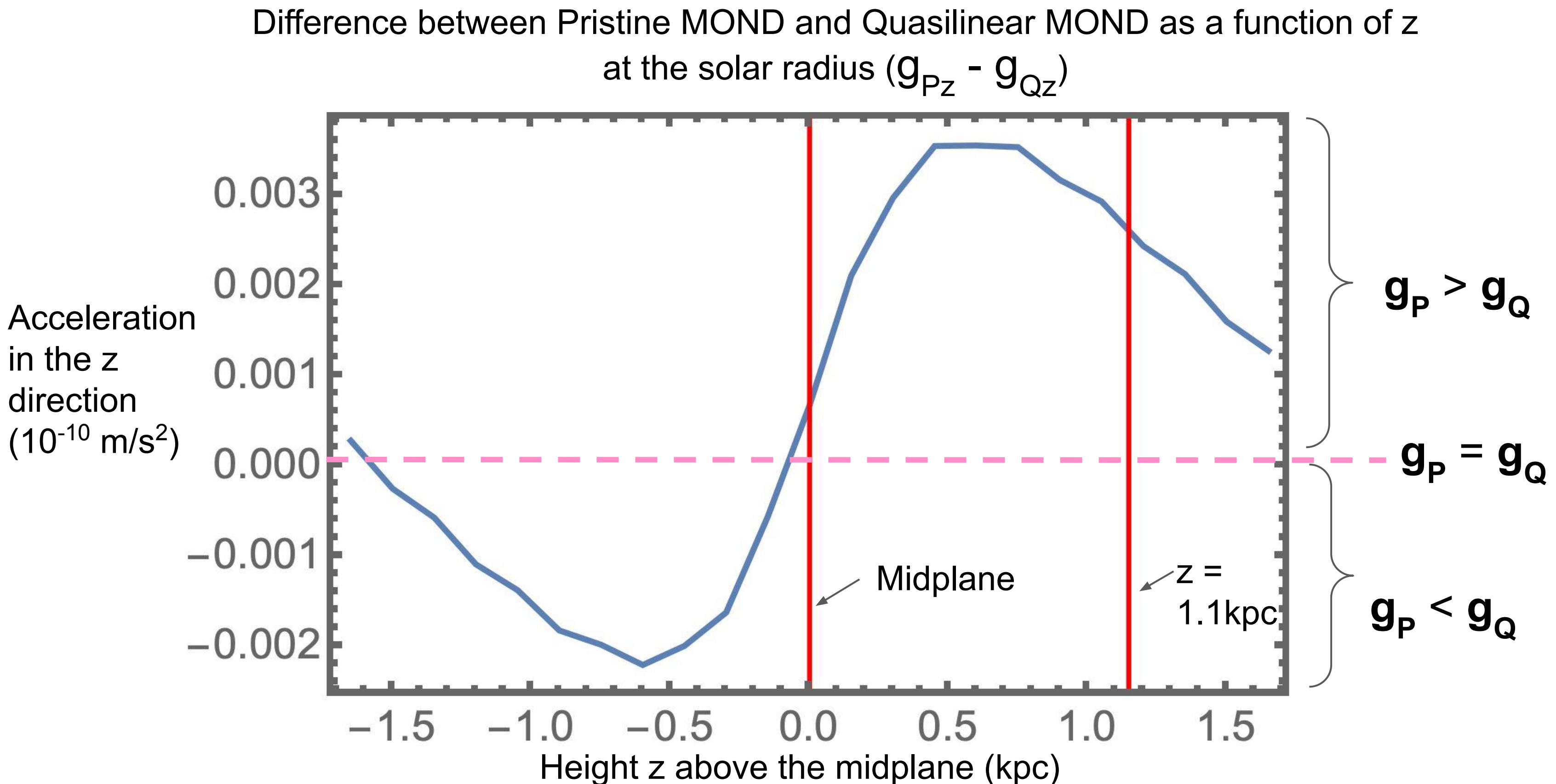
! Roughly chosen parameters for illustrative purposes !

Defined so that the radial acceleration at solar radius = 1.9 a_0

h_z = 300 pc
(From stellar counts, Bland-Hawthorn & Gerhard 2016)

Results - MOND is not "MOND-like"

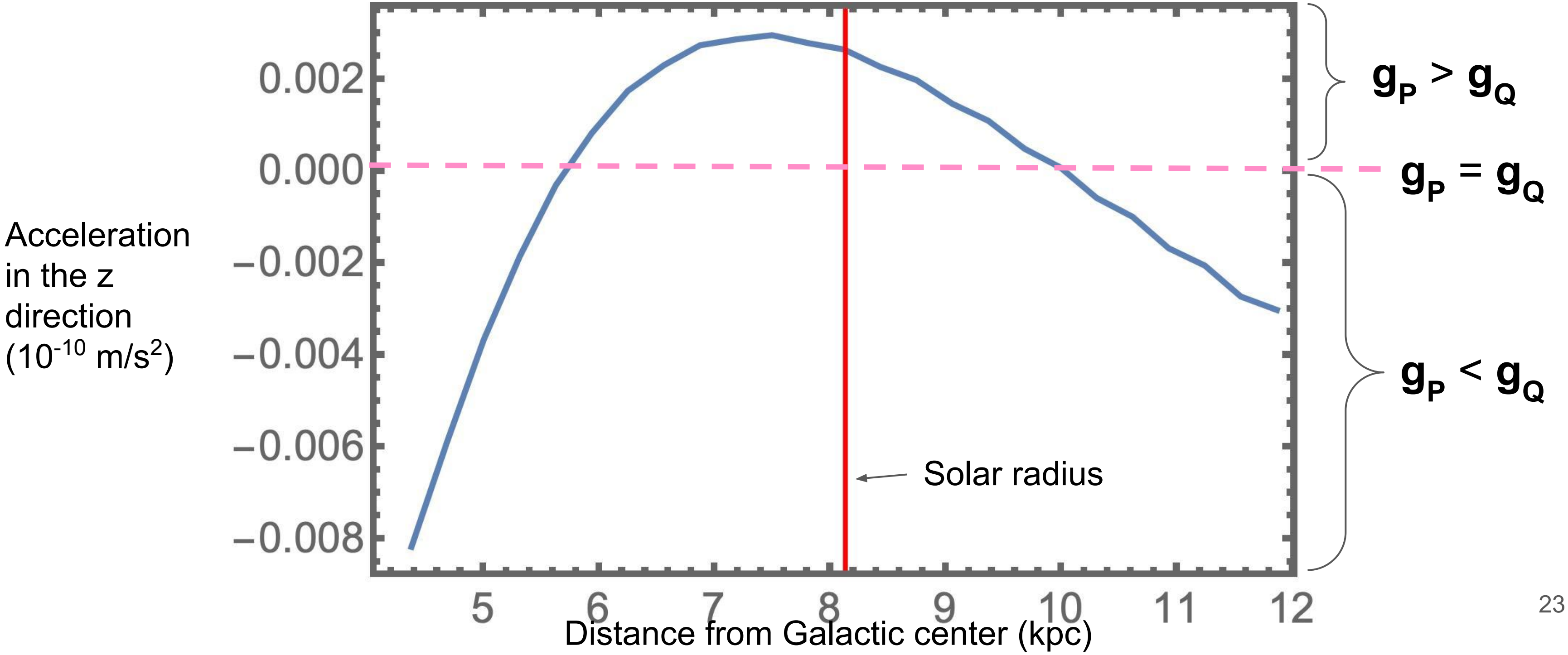
g_P: Pristine MOND acceleration
g_Q: Quasilinear MOND acceleration



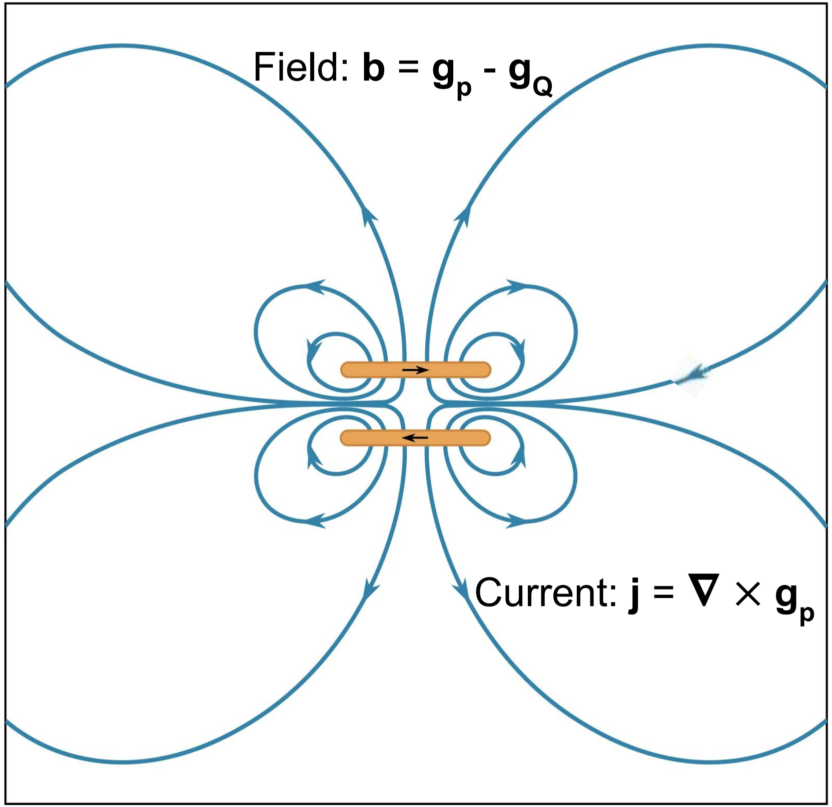
Results - MOND is not “MOND-like”

g_P: Pristine MOND acceleration
g_Q: Quasilinear MOND acceleration

Difference between Pristine MOND and Quasilinear MOND as a function of Galactic radius at z = 1.1kpc ($g_{Pz} - g_{Qz}$)



Why so small? Magnetostatics Interpretation



Small magnitude field due to *counterpropagating* coils nearly cancelling ⟷ Small difference between $\mathbf{g}_p$ & $\mathbf{g}_Q$

Brown et al. (2018)

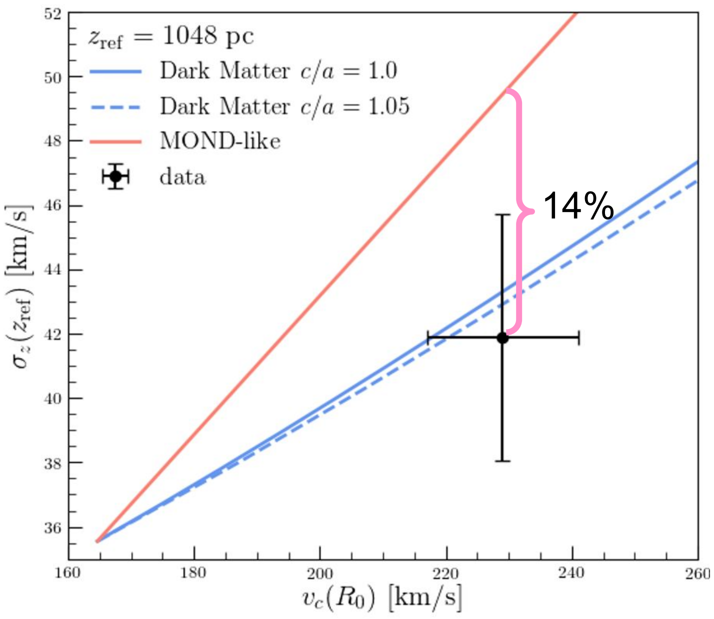
$\mathbf{g}_P$: Pristine MOND acceleration
$\mathbf{g}_Q$: Quasilinear MOND acceleration

Revisiting the Tension of Lisanti et al.

Is the 14% tension meaningful?

Lisanti's bulge mass is in tension with known data for both MOND *and* Dark Matter (Flynn 2006)

- Unrealistic prior on bulge mass (0 - 100 10^{10} $M_\odot$)
- Minimal amount of data used to constrain the parameters
  - Most sources eg. Binney & Tremaine Galactic Dynamics (2007) uses double the amount of constraints



Revisiting the Tension of Lisanti et al.

→ The Galactic model of Lisanti et al. may be in tension with most commonly used data

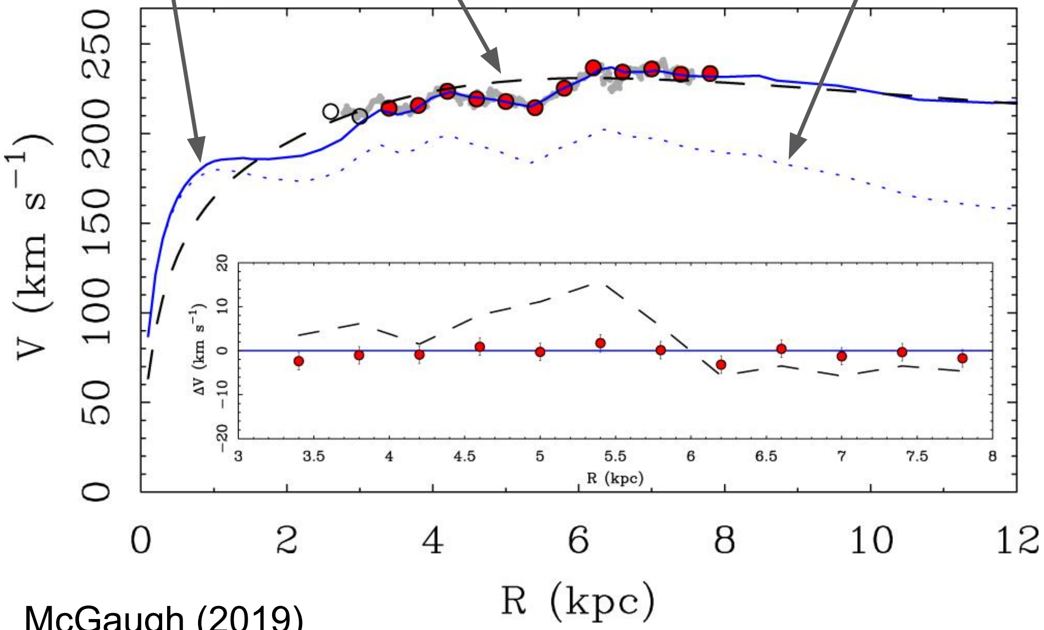
Ignoring the bulge in our single disk model is acceptable because $M_{bulge} << M_{disks}$

Limitations of Smooth Exponential Model

Blue solid: G4MB

Dashed: BR13

Blue dotted: baryons only



McGaugh (2019)

Table 1
Reduced χ^2_ν

	All data		Excluding $44 < \ell < 55°$	
Model	$V_c(R)$	$K_Z(R)$	$V_c(R)$	$K_Z(R)$
BR13	14.35	0.75	6.06	…
Q4MB	0.60	1.69	…	1.02

Note. BR13 from Bovy & Rix (2013); Q4MB from McGaugh (2016).

Limitations & Future Work

Limitations:

- Computational time
- Galactic models built on assumption of Newtonian gravity
- Vertical motions not in equilibrium (Haines et al. 2019)

Future work to study the success (or failure) of QUMOND in the solar neighborhood:

- Use for more detailed Galactic models
- Take full advantage of GAIA data & other constraints

Conclusions & Discussion

- MOND is **not** “MOND-like”
- It is not clear that there *is* a serious tension between QUMOND and local observables as stated in Lisanti et al.
- Further work remains to be done in order to evaluate the success of QUMOND in the vertical direction

Question for the audience:

- How much variation should we expect between MOND theories? Since the differences are detectable, should we be able to differentiate them?

Thank you!