AS 4022: Cosmology

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Online notes:

star-www.st-and.ac.uk/~hz4/cos/cos.html

take your own notes (including blackboard lectures

Look forward

Malcolm S. Longair's "Galaxy Formation" 2nd edition [Library]

Chpt 1-2,5-8: expanding metrics, energy density, curvature, distances

Chpt 4,11,15,20: DM, Structure growth, inflation

Chpt 9-10,13: Thermal History of Particle Reaction, Neutrinos, WIMPs

Text (intro): Andrew Liddle: Intro to Modern Cosmology
(advanced): John Peacock: Cosmological Physics
Web Lecture Notes: John Peacock, Ned Wright

Why Study Cosmology?

- **Fascinating questions:**
  - Birth, life, destiny of our Universe
  - Hot Big Bang --> (75% H, 25% He) observed in stars!
  - Formation of structure (galaxies …)

- **Technology -> much recent progress:**
  - Precision cosmology: uncertainties of 50% --> 2%

- **Deep mysteries remain:**
  - Dark Matter? Dark Energy? General Relativity wrong?

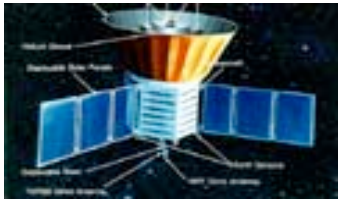
- **Stretches your mind:**
  - Curved expanding spaces, looking back in time, …

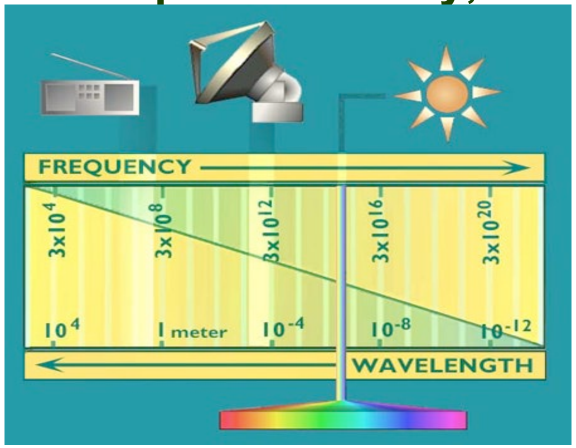
Observable Space-Time and Bands

See What is out there? In all Energy bands

Pupil → Galileo's Lens → 8m telescopes → square km arrays

Radio, Infrared ← optical → X-ray, Gamma-Ray (spectrum)





COBE satellites ← Ground → Underground DM detector

Know How were we created? XYZ & T ?

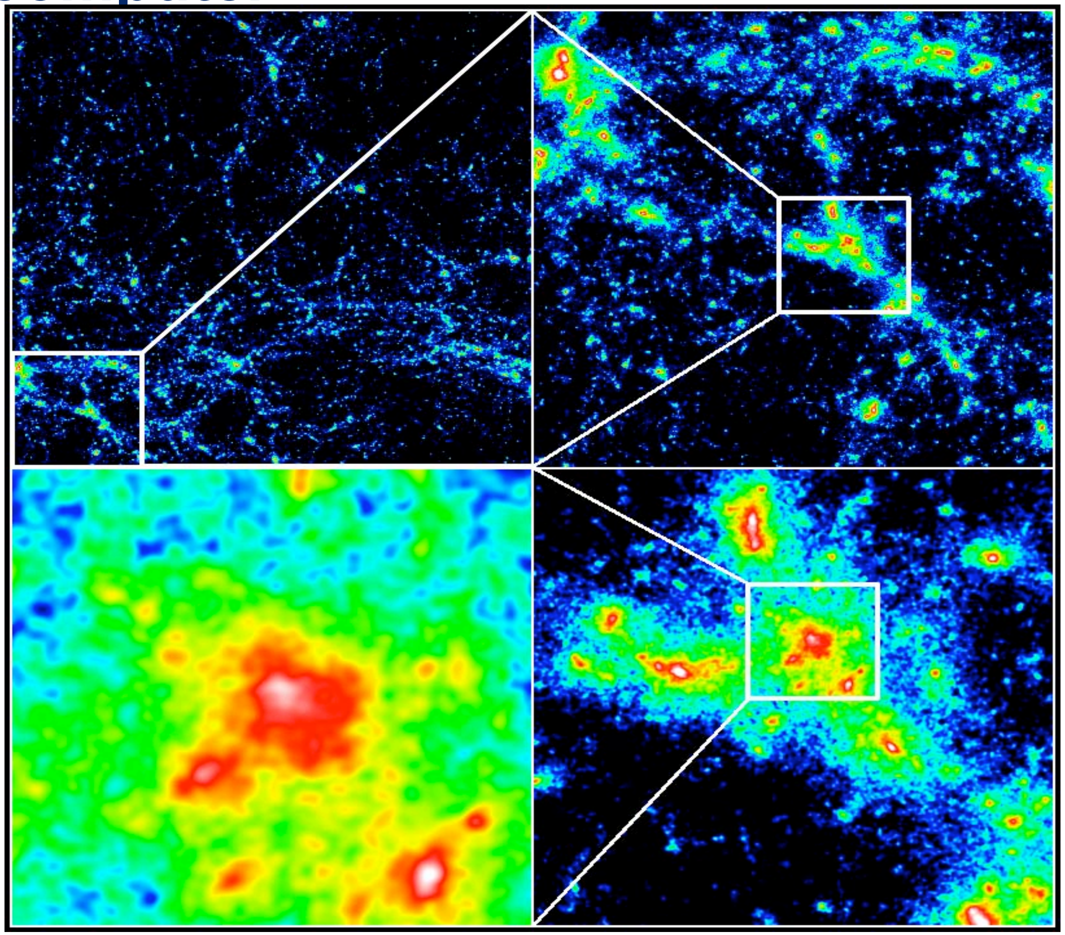
Us, CNO in Life, Sun, Milky Way, … further and further

→ first galaxy → first star → first Helium → first quark

Now → Billion years ago → first second → quantum origin

The Visible Cosmos:
a hierarchy of structure and motion

"Cosmos in a computer"



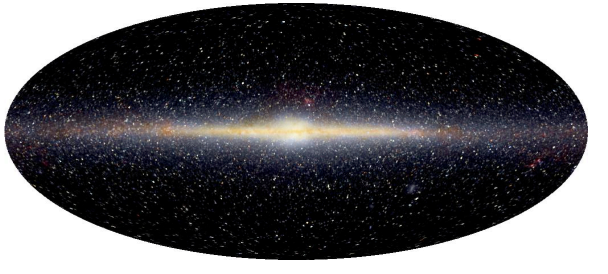
Observe A Hierarchical Universe

Planets

moving around stars;

Stars grouped together,

moving in a slow dance around the center of galaxies.



Cosmic Village

The Milky Way and Andromeda galaxies,

along with about fifteen or sixteen smaller galaxies,

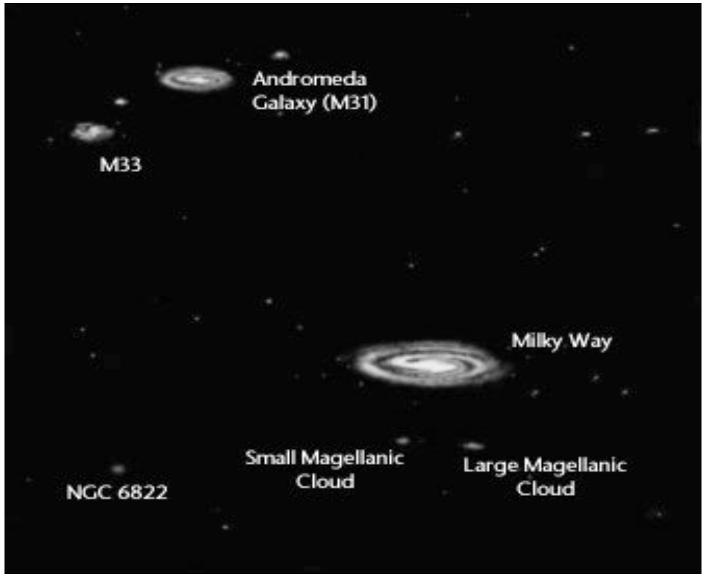
form what's known as the Local Group of galaxies.

The Local Group

sits near the outer edge of a supercluster, the Virgo cluster.

the Milky Way and Andromeda are moving toward each other,

the Local **e of the Virgo cluster, and**



the entire Virgo cluster itself,

is speeding toward a mass

known only as "The Great Attractor."

Hubble Deep Field:

At faint magnitudes, we see **thousands of Galaxies for every star** !

$\sim 10^{10}$ galaxies in the visible Universe

$\sim 10^{10}$ stars per galaxy

$\sim 10^{20}$ stars in the visible Universe

Galaxies themselves

some 100 billion of them in the observable universe—

form galaxy clusters bound by gravity as they journey through the void.

But the largest structures of all are superclusters,

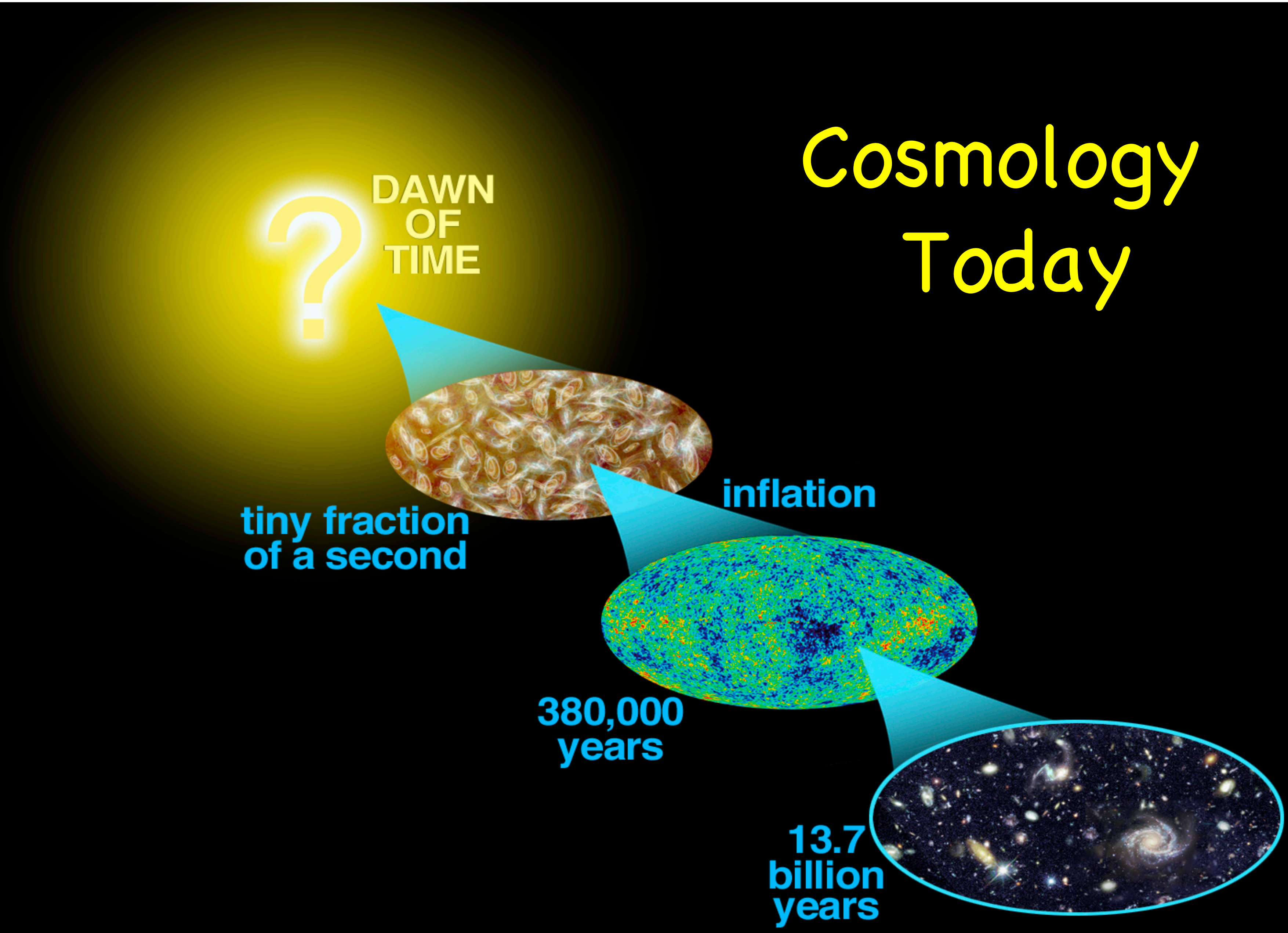
each containing thousands of galaxies

and stretching many hundreds of millions of light years.

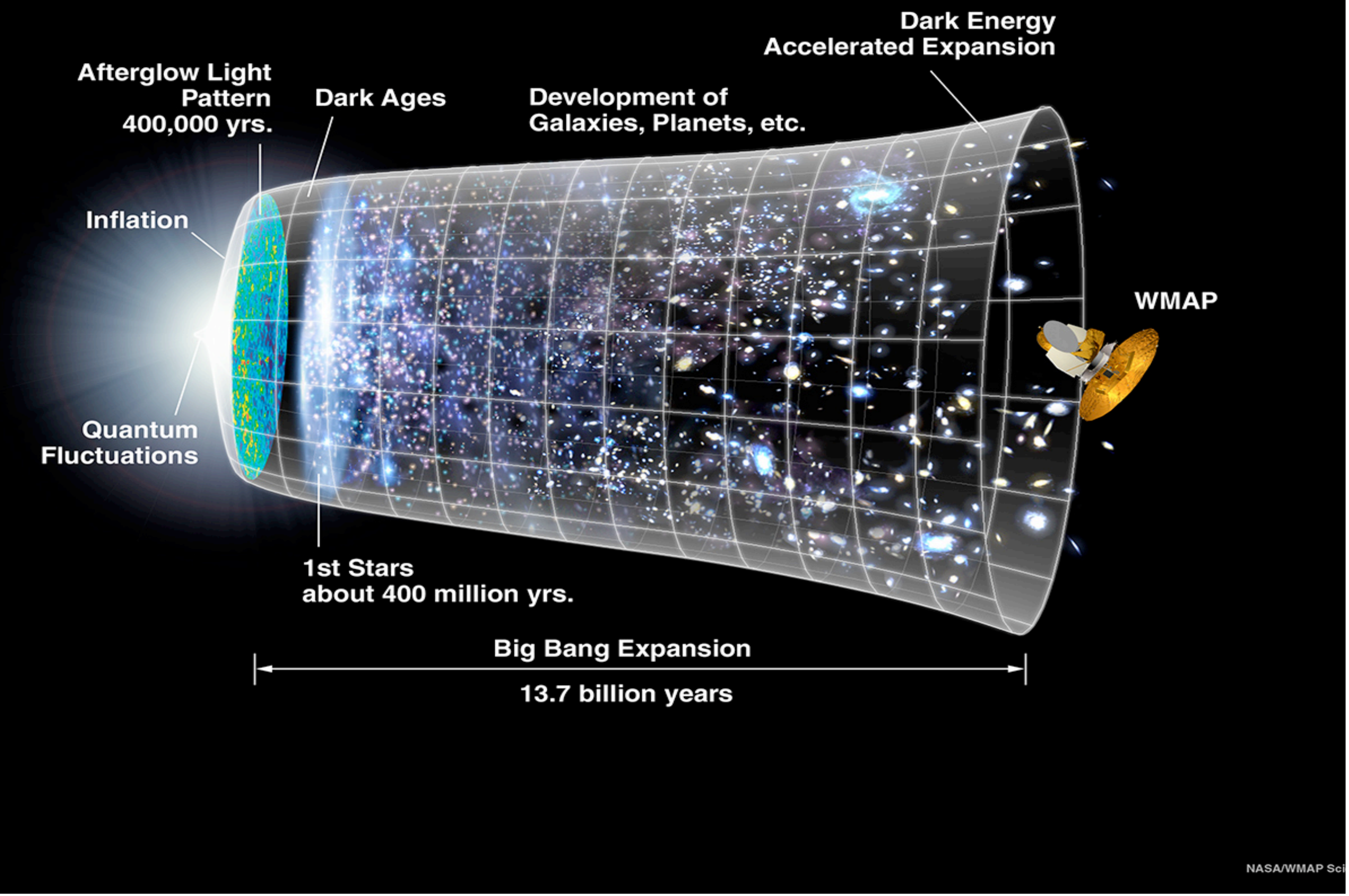
are arranged in filament or sheet-like structures,

between which are gigantic voids of seemingly empty space.

Cosmology Today



Accelerating/Decelerating Expansion



Introducing Gravity and DM (Key players)

These structures and their movements

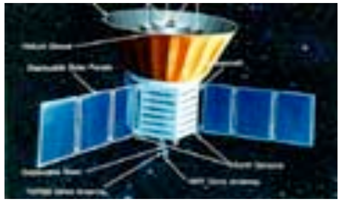
can't be explained purely by the expansion of the universe

must be guided by the gravitational pull of matter.

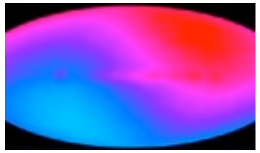
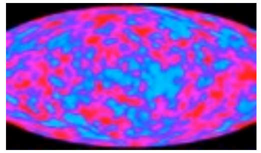
Visible matter is not enough

one more player into our hierarchical scenario:

dark matter.



Cosmologists hope to answer these questions:

How old is the universe? H_0

Why was it so smooth? P(k), inflation

How did structures emerge from smooth? N-body

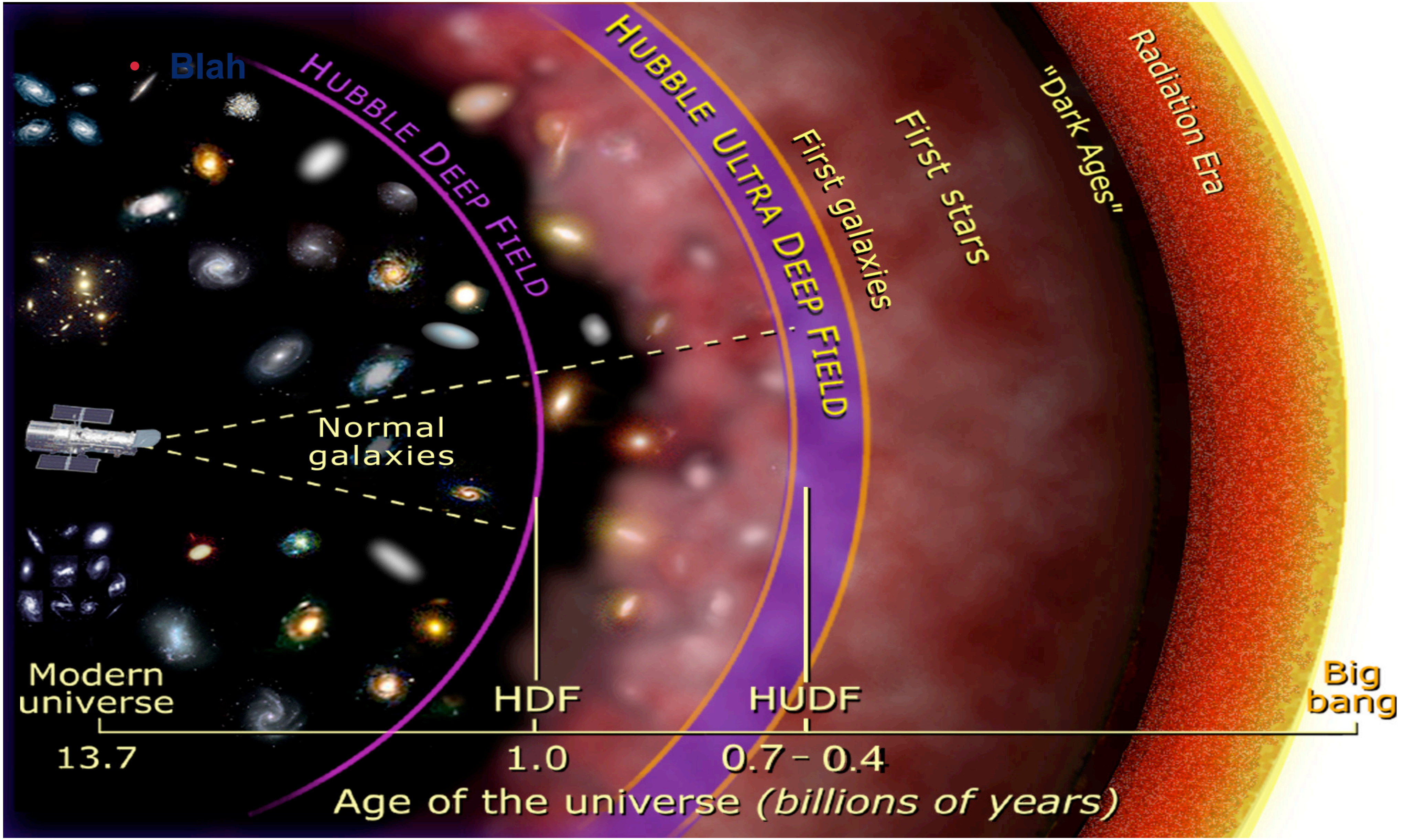
How did galaxies form? Hydro

Will the universe expand forever? Omega, Lamda

Or will it collapse upon itself like a bubble?

Looking Back in Time

- Blah



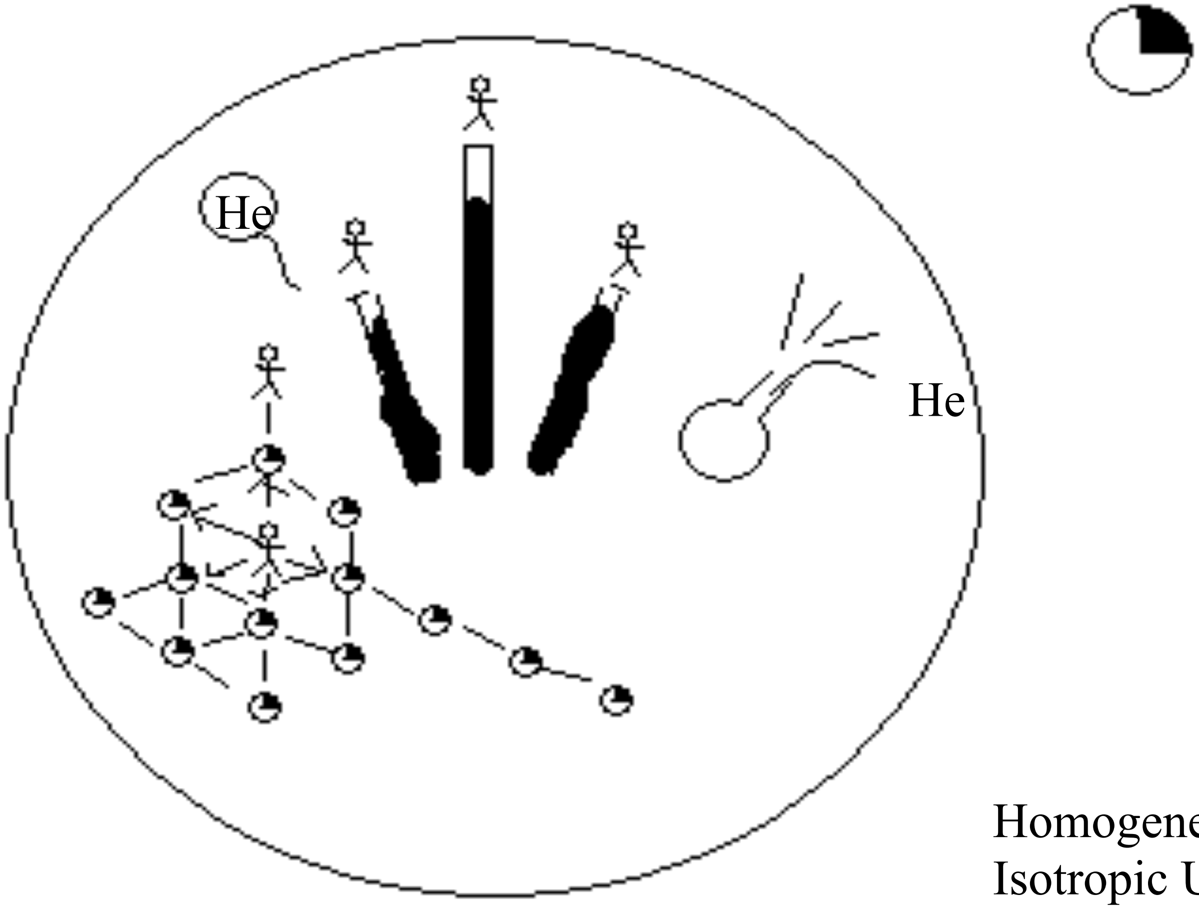
main concepts in cosmology

Expansion & Metric

Cosmological Redshift

Energy density

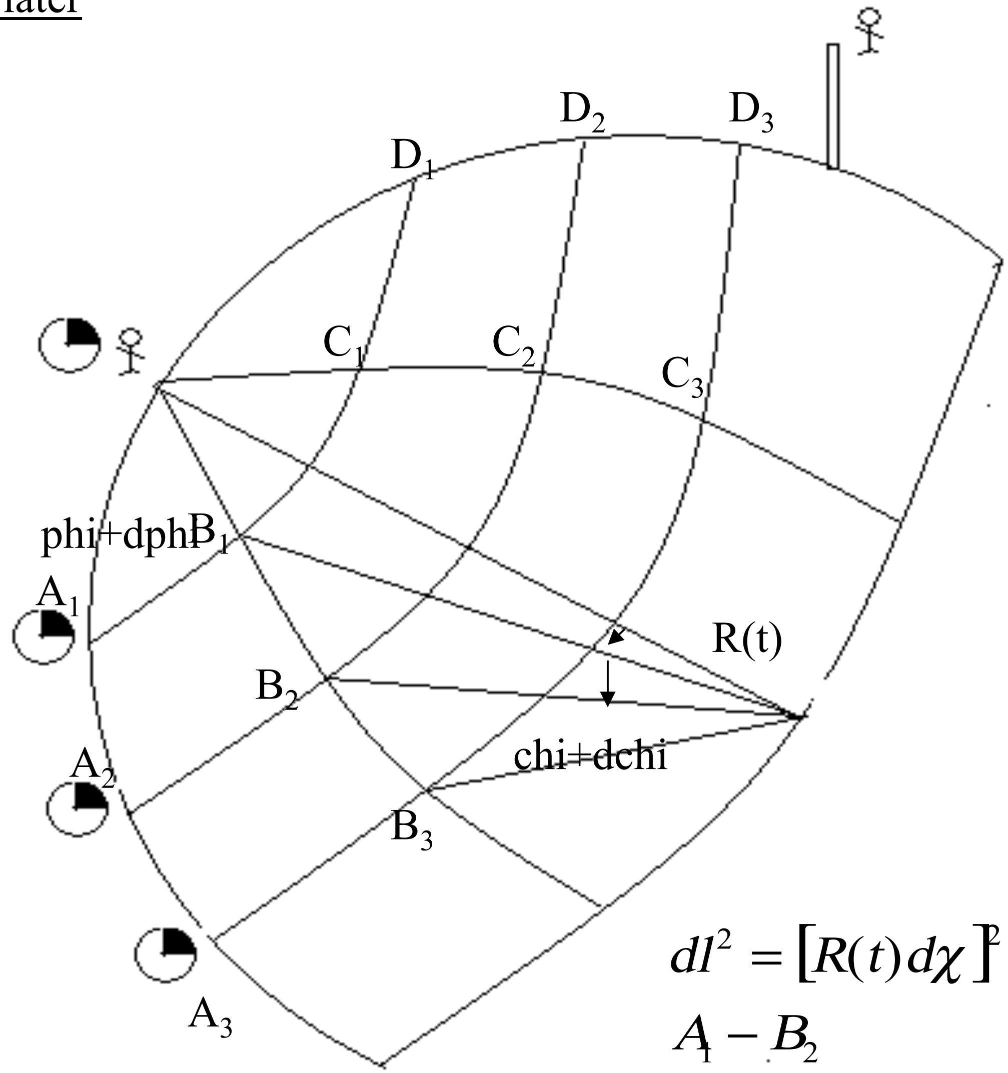
A few mins after New Year
Celebration at Trafalgar Square



Homogeneous
Isotropic Universe

$$Walking \leftrightarrow Elevating \leftrightarrow Earth\ \ Radius\ \ Stretching\ \ R(t)$$

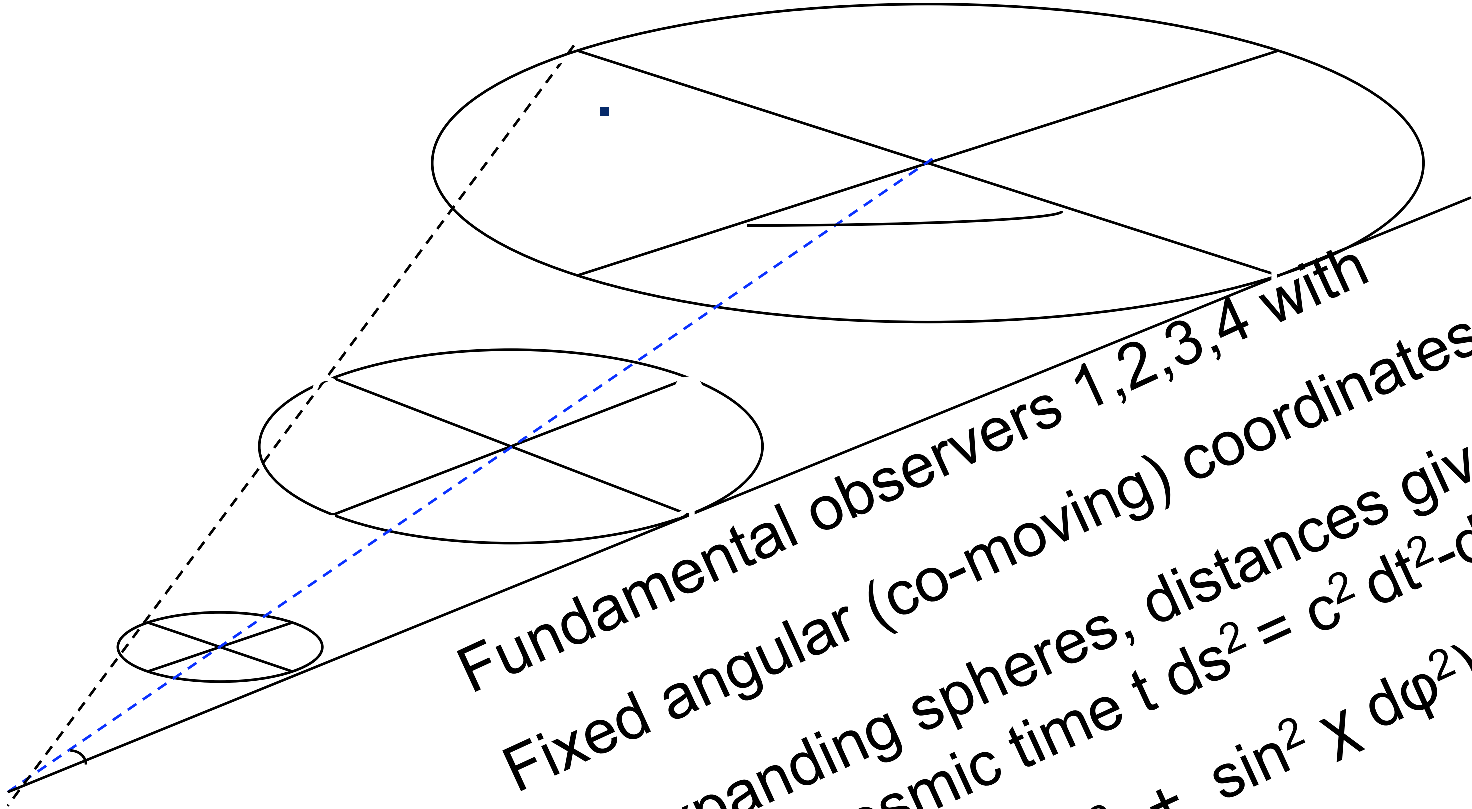
Feb 14   t=45 days later



$$dl^2 = \left[R(t)\,d\chi\right]^2 + \left[R(t)\sin\chi\,d\phi\right]^2$$

$$A_1 - B_2$$

1^{st} concept Metric: ant network on expanding sphere



Fundamental observers 1,2,3,4 with

Fixed angular (co-moving) coordinates (χ,φ)

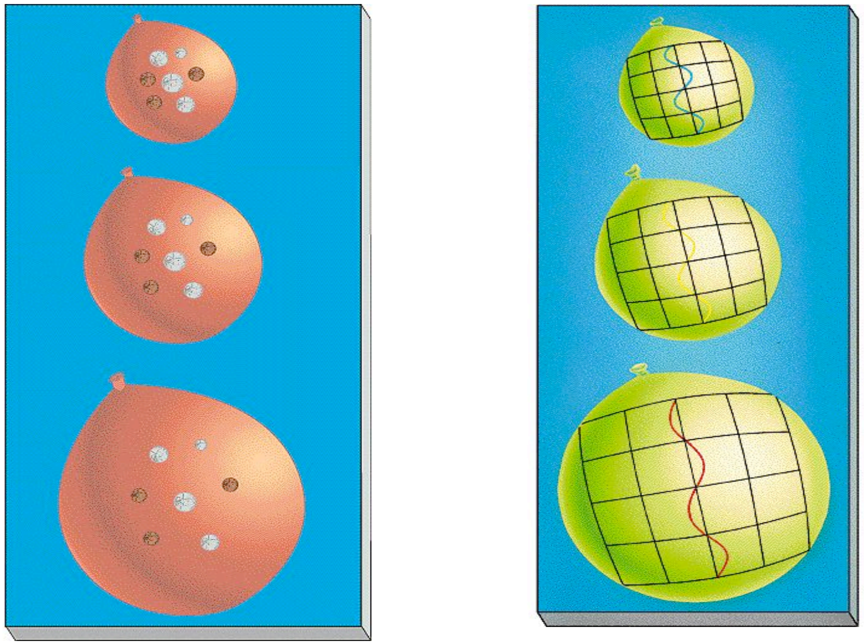
on expanding spheres, distances given by

Metric at cosmic time t $ds^2 = c^2\,dt^2 - dl^2$,

$dl^2 = R^2(t)\,(d\chi^2 + \sin^2\chi\,d\varphi^2)$

Stretch of photon wavelength in expanding space

Emitted with intrinsic wavelength λ_0 from Galaxy A at time $t<t_{now}$ in smaller universe $R(t) < R_{now}$

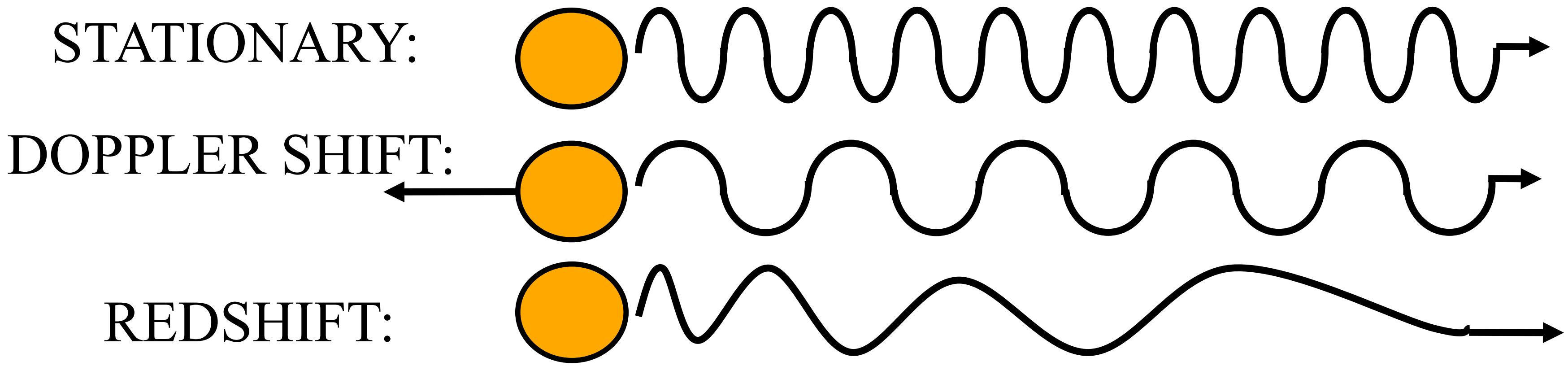


→ Received at Galaxy B now (t_{now}) with λ

$\lambda / \lambda_0 = R_{now} / R(t) = 1+z(t) > 1$

Redshift

- Expansion is a stretching of space.
- The more space there is between you and a galaxy, the faster it appears to be moving away.
- Expansion **stretches the wavelength of light**, causing a galaxy's spectrum to be REDSHIFTED:



REDSHIFT IS NOT THE SAME AS DOPPLER SHIFT

2^{nd} main concept: Cosmological Redshift

The space/universe is expanding,

Galaxies (pegs on grid points) are receding from each other

As a photon travels through space, its wavelength becomes stretched gradually with time.

Photons wave-packets are like links between grid points

This redshift is defined by:

$$z \equiv \frac{\lambda - \lambda_o}{\lambda_o}$$

$$\frac{\lambda}{\lambda_o} = 1 + z$$

Galaxy Redshift Surveys

Large Scale Structure:

Empty **voids**

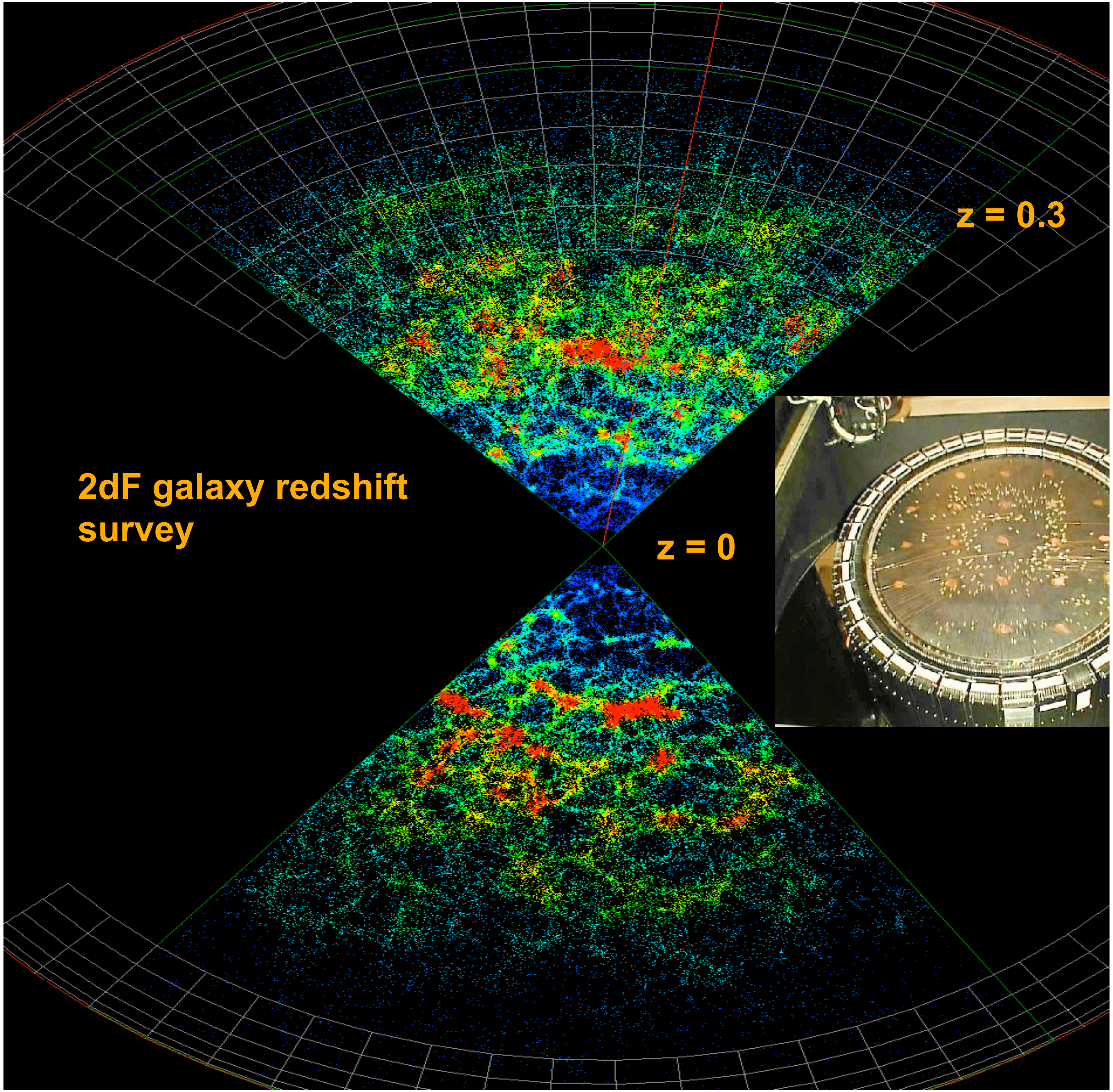
~50Mpc.

Galaxies are in
1. **Walls** between voids.

2. **Filaments** where walls intersect.

3. **Clusters** where filaments intersect.

Like Soap Bubbles !



E.g. Consider a quasar with redshift z=2. Since the time the light left the quasar the universe has expanded by a factor of 1+z=3. At the epoch when the light left the quasar,

the distance between us and Virgo (presently 15Mpc) was 5Mpc.

the CMB temperature then (presently 3K) was 9K.

the quasar appears receding with Doppler speed V=2c.

The quasar appears at a look-back "distance" d = [t(0)-t(z)] c, where the look-back time [t(0)-t(z)] ~ z/H_0 at small z<<1, H_0^{-1}=Taylor expansion coeff.

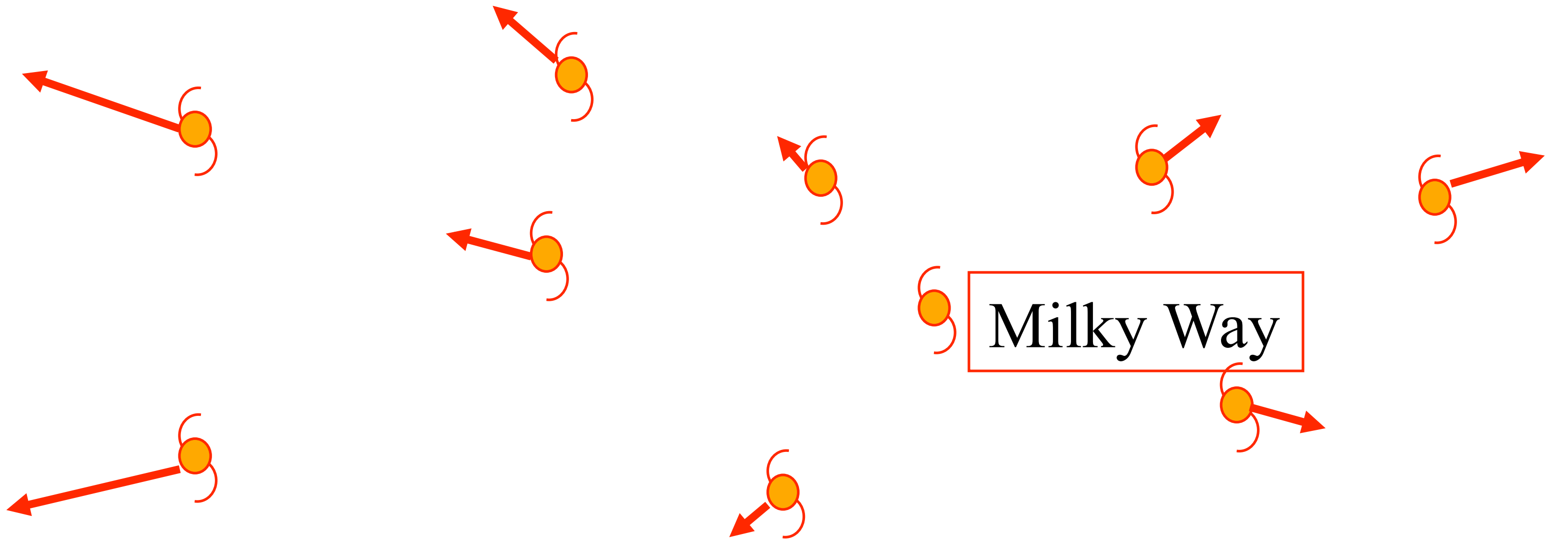
$$1+z = \frac{\lambda_{now}}{\lambda(t)} \quad \text{(wavelength)}$$

$$= \frac{R_{now}}{R(t)} \quad \text{(expansion factor)}$$

$$= \frac{T(t)}{T_{now}} \quad (\text{Photon Blackbody T } \propto 1/\lambda, \textit{why}?)$$

Universal Expansion

Hubble's law appears to violate The Copernican Principle. Are we at a special location?



Is everything moving away from us?

Universal Expansion

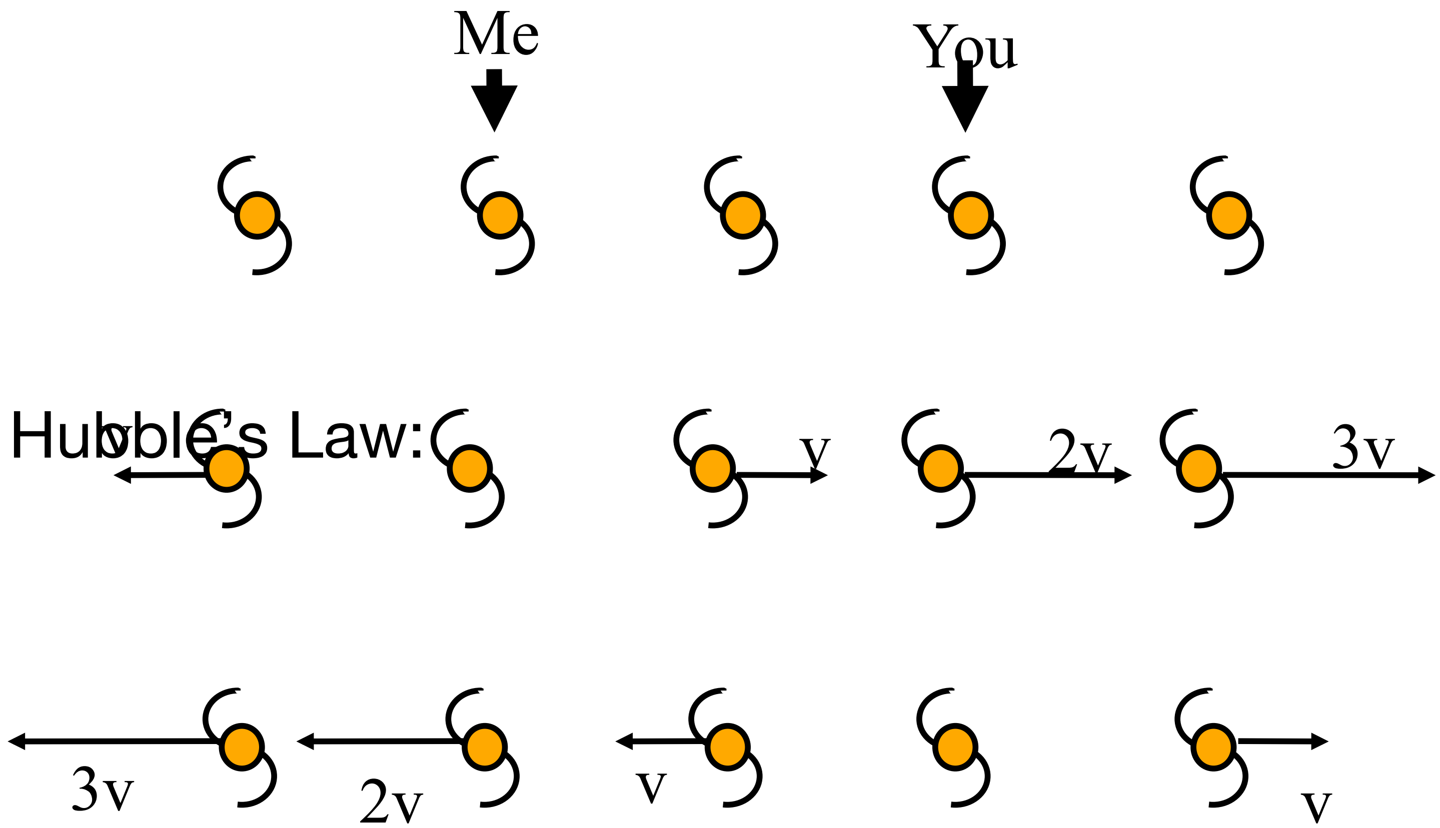
Q : What is so special about our location ?

A : Nothing !

According to Hubble's Law:

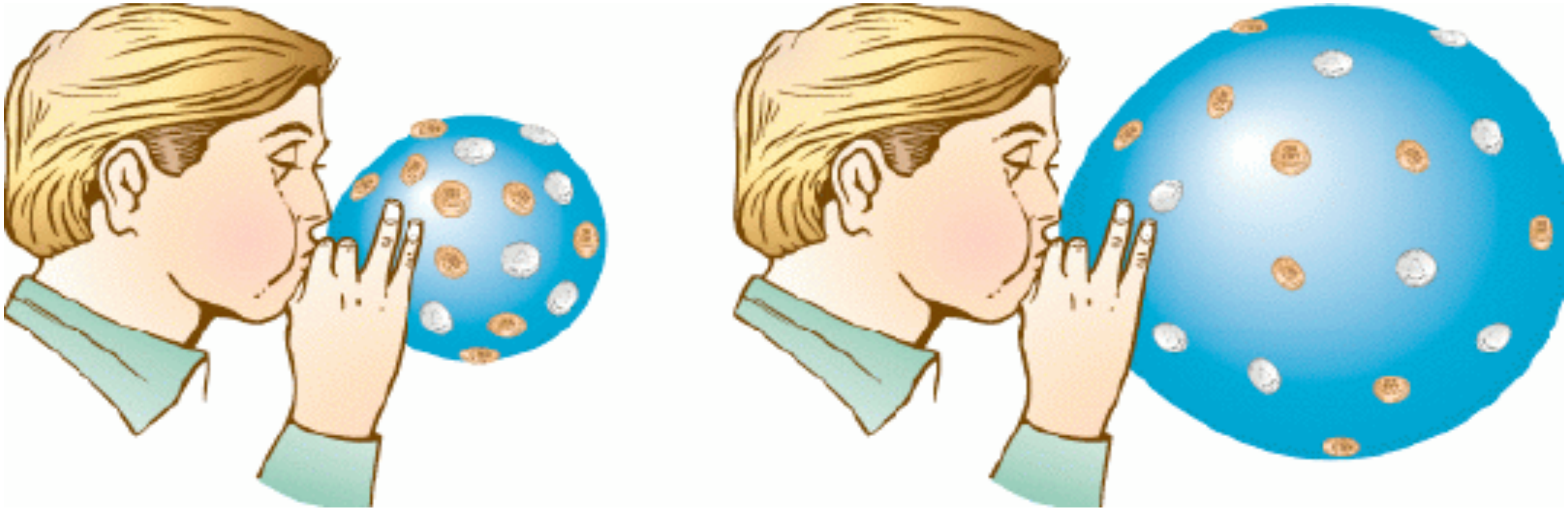
I see:

You see:



The Universal Expansion

- An observer in any galaxy sees all other galaxies moving away, with the same Hubble law.
- Expansion (or contraction) produces a centre-less but dynamic Universe.

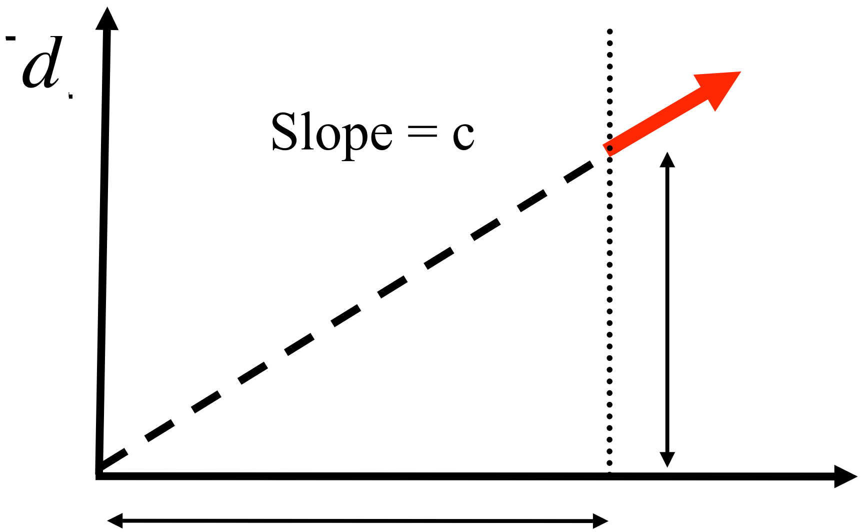


Hubble Law → typical age (at z=1)

$$V = H_0\, d$$

$$t_0 \approx \frac{d}{V} = \frac{1}{H_0} = \left(\frac{1\,\mathrm{Mpc}}{72\ \mathrm{km/s}}\right)\left(\frac{3\times10^{19}\mathrm{km}}{\mathrm{Mpc}}\right)\left(\frac{1\ \mathrm{yr}}{3\times10^{7}\mathrm{s}}\right)$$

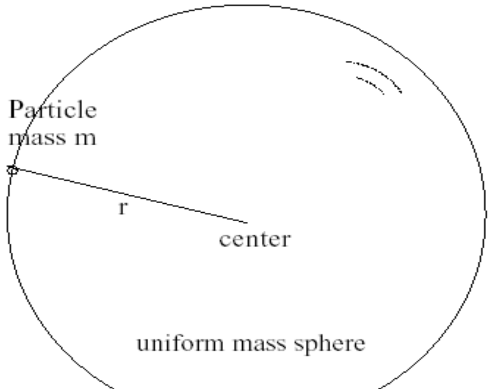
$$\approx 13\times10^{9}\,\mathrm{yr} = 13\ \mathrm{Gyr}.$$



Look back t_0-t

Convert H as a frequency Hertz, find an integer close to log10(H)? Multiple choices -30, -10, 0, 10, 30.

3^{rd} concept: The changing rate of expansion



Newtonian Analogy:

Consider a sphere of radius R(t) ,

➔ effective mass inside $M = 4\pi\rho R^3/3$

if energy density inside is $e=\rho(t) c^2$.

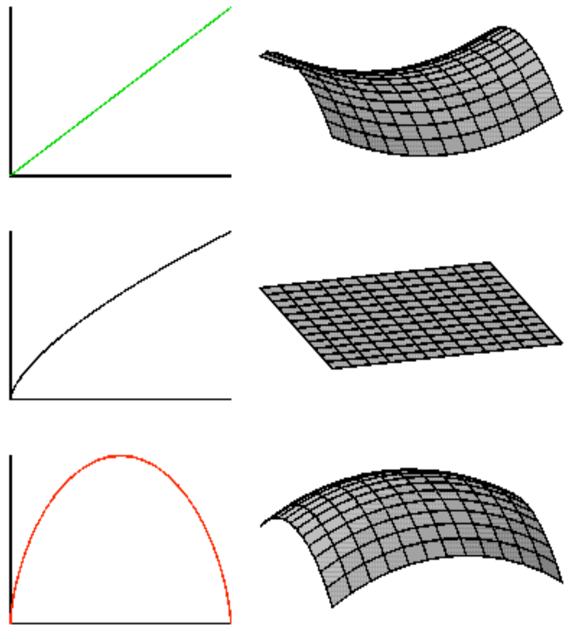
On this expanding sphere, a test m

Kin.E. + Pot.E. = const Energy

➔ $m (dR/dt)^2/2 - G m M/R = (-k/2)m c^2$

➔ $(dR/dt)^2 /(2c^2) - (4\pi G/3)\rho R^2/c^2 = -k/2$

Unitless cst k <0, =0, >0 → 3 geometries



Newtonian expansion satisfies

$H^2 = (dR/dt/R)^2 = (\rho + \rho_{cur}) (8\pi G/3)$

the cst k absorbed in "density"

$\rho_{cur} (t) = -k(cH_0^{-1}/R)^2 (3H_0^2/8\pi G)$

$\sim -k R^{-2} * cst$

Now $H = H_0$

4^{th} Concept: The Energy density of Universe

The Universe is made up of three things:

- **VACUUM**
- **MATTER**
- **PHOTONS (radiation fields)**

The total energy density of the universe is made up of the sum of the energy density of these three components.

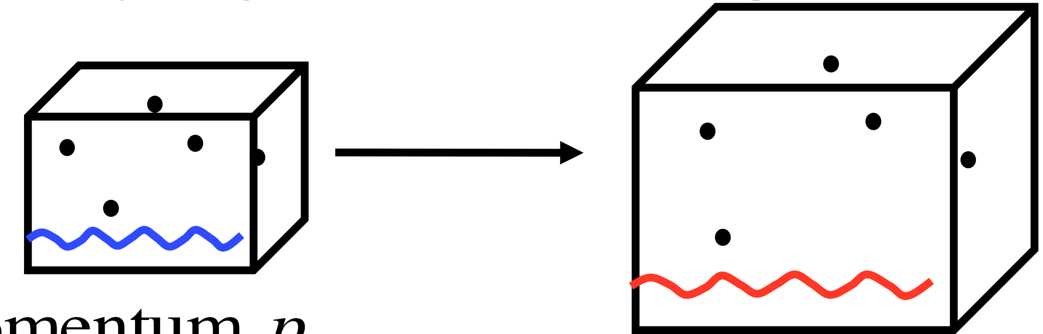
$$\varepsilon(t) = \varepsilon_{vac} + \varepsilon_{matter} + \varepsilon_{rad}$$

From t=0 to t=10^9 years the universe has expanded by R(t).

Energy Density of expanding box

volume R^3

N particles



particle mass m momentum p

energy $E = h\nu = \sqrt{m^2c^4 + p^2c^2} = m c^2 + \frac{p^2}{2m} + \ldots$

Cold gas or Cold DM $(p << mc)$

$$E \approx m c^2 = \text{const}$$

$$\varepsilon_M \approx \frac{N m c^2}{R^3} \propto R^{-3}$$

Radiation: $(m = 0)$

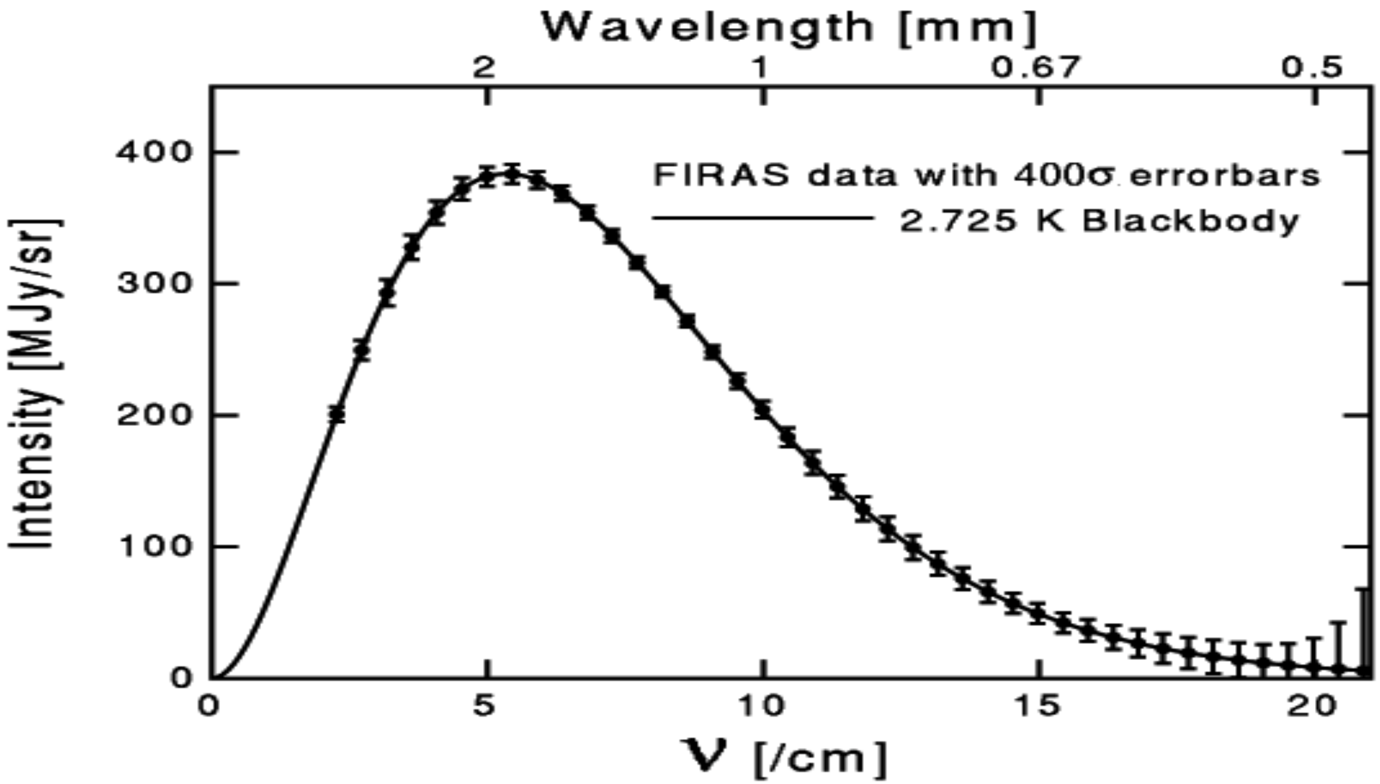
Hot neutrino: $(p >> mc > 0)$

$\lambda \propto R$ (wavelengths stretch) :

$$E = h \nu = \frac{h c}{\lambda} \propto R^{-1}$$

$$\varepsilon_R = \frac{N h \nu}{R^3} \propto R^{-4}$$

COBE spectrum of CMB



A perfect Blackbody !

No spectral lines -- strong test of Big Bang.
Expansion preserves the blackbody spectrum.

$$T(z) = T_0\,(1+z) \qquad T_0 \sim 3000\ K \quad z \sim 1100$$

Acronyms in Cosmology

- **Cosmic Background Radiation (CBR)**
  - **Or CMB (microwave because of present temperature 3K)**
    - Tutorial: Argue **about 10^5 photons fit in a 10cmx10cmx10cm microwave oven. [Hint: $3kT = h\, c / \lambda$]**

- **CDM/WIMPs: Cold Dark Matter, weakly-interact massive particles**
  - **If DM decoupled from photons at $kT \sim 10^{14}$K $\sim 0.04\, mc^2$**
    - Then **that dark particles were**
      - **non-relativistic ($v/c \ll 1$), hence "cold".**
      - **And massive ($m \gg m_{proton} = 1$ GeV)**

A general history of a massive particle

- Initially mass doesn't matter in hot universe
- relativistic, dense (comparable to photon number density $\sim R^{-3} \sim T^3$),
  - frequent collisions with other species to be in thermal equilibrium and cools with photon bath.
  - Photon numbers (approximately) conserved, so is the number of relativistic massive particles

Concept: Particle-Freeze-Out?

Freeze-out of equilibrium means NO LONGER in thermal equilibrium.

Freeze-out temperature means a species of particles have the SAME TEMPERATURE as radiation up to this point, then they bifurcate.

Decouple = switch off the reaction chain

= insulation = Freeze-out

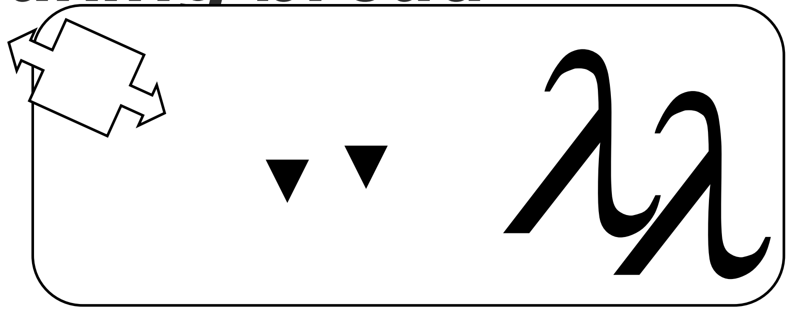
Neutrino: smallest cross-section and smallest mass

neutrinos (Hot DM) decouple from electrons (due to very weak interaction) while still hot (relativistic 0.5 Mev ~ $kT > mc^2$ ~ 0.02-2 eV)

Presently there are 3 x 113 neutrinos and 452 CMB photons per cm^3 . Details depend on

- **Neutrinos have 3 species of spin-1/2 fermions while photons are 1 species of spin-1 bosons**
- **Neutrinos are a wee bit colder, 1.95K vs. 2.7K for photons [during freeze-out of electron-positions, more photons created]**

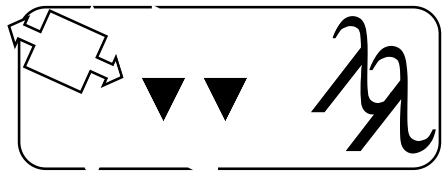
Eq. of State for Expansion & analogy of baking bread



Vacuum~air holes in bread

Matter ~nuts in bread

Photons ~words painted



Verify expansion doesn't change N_{hole}, N_{proton}, N_{photon}

No Change with rest energy of a proton, changes energy of a photon

$$\varepsilon(t) = \rho_{eff}(t)c^2$$

$$\frac{\varepsilon(t)}{c^2} = \rho_{eff}(t)$$

VACUUM ENERGY: $\rho = \text{constant} \quad \Rightarrow \; \mathrm{E}_{vac} \propto R^3$

MATTER:

$$\rho R^3 = \text{constant}, \;\; \Rightarrow \; m \approx \text{constant}$$

RADIATION: **number of photons N_{ph} = constant**

$$\text{Wavelength stretches}: \lambda \sim R$$

$$\Rightarrow n_{ph} \approx \frac{N_{ph}}{R^3}$$

$$\text{Photons:} \mathrm{E} = \mathrm{h}\upsilon = \frac{\mathrm{hc}}{\lambda} \sim \frac{1}{R}$$

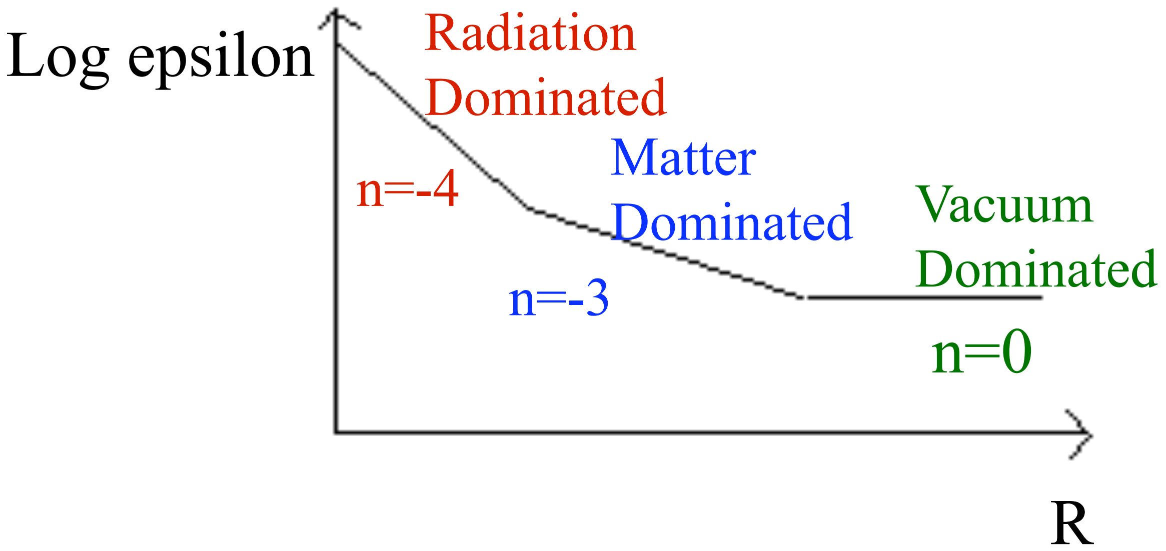
$$\Rightarrow \varepsilon_{ph} \sim n_{ph} \times \frac{hc}{\lambda} \sim \frac{1}{R^4}$$

Total Energy Density *rho* c^2 =epsilon

is given by:

$$\varepsilon \propto \varepsilon_{vac} + \varepsilon_{matter} + \varepsilon_{ph}$$

$$\propto R^0 \qquad \propto R^{-3} \qquad \propto R^{-4}$$



Tutorial: Typical scaling of expansion

$H^2=(dR/dt)^2/R^2=8\pi G\ (\rho_{cur}+ \rho_m + \rho_r + \rho_v\ )/3$

Assume domination by a component $\rho \sim R^{-n}$

Show Typical Solutions Are

$$\rho \propto R^{-n} \propto t^{-2}$$

$$n = 2(\text{curvature constant dominate})$$

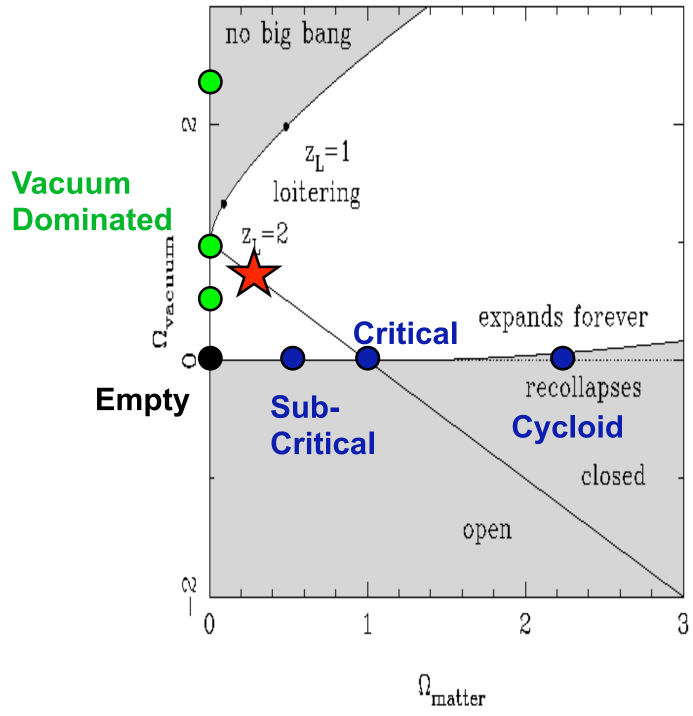
$$n = 3(\textit{matter}\ \text{dominate})$$

$$n = 4(\textit{radiation}\ \text{dominate})$$

$$n \sim 0(\text{vaccum dominate}) : \ln(R) \sim t$$

Argue also $H = (2/n)\ t^{-1} \sim t^{-1}$. Important thing is scaling!

Tutorial: Eternal Static (R=cst) and flat (k=0) Universe



- Einstein introduced Λ

to enable an eternal static universe.

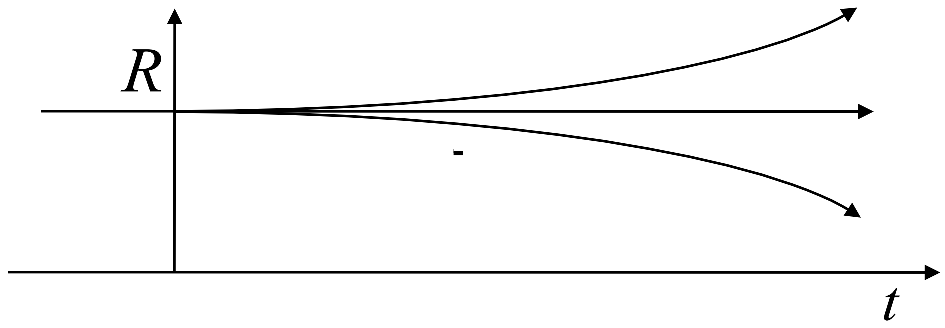
$$\dot{R}^2 = \left(\frac{8\pi G \rho + \Lambda}{3} \right) R^2 - k c^2$$

$$\ddot{R} = 0 \quad \rightarrow \quad \Lambda = \frac{3 k c^2}{R^2} - 8\pi G \rho$$

Einstein's biggest blunder. (Or, maybe not.)

Static models unstable.

Fine tuning.



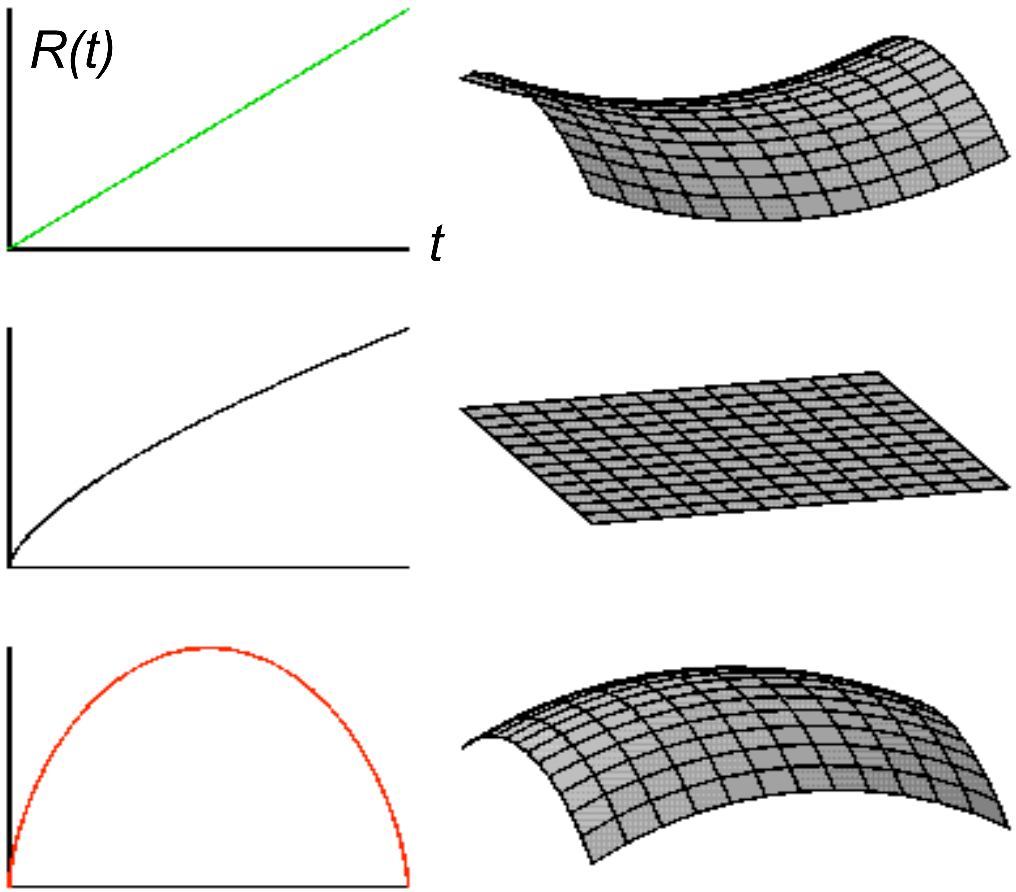
Density - Evolution - Geometry

$H^2 - (8\pi G/3)\,\rho = -k\,R^{-2}$, where $H=(dR/dt/R)$

$\rho < \rho_c$

$\rho = \rho_c$

$\rho > \rho_c$



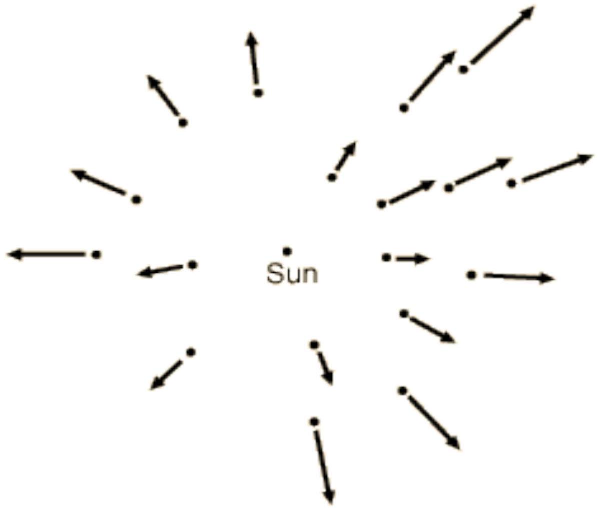
Open *k = -1*

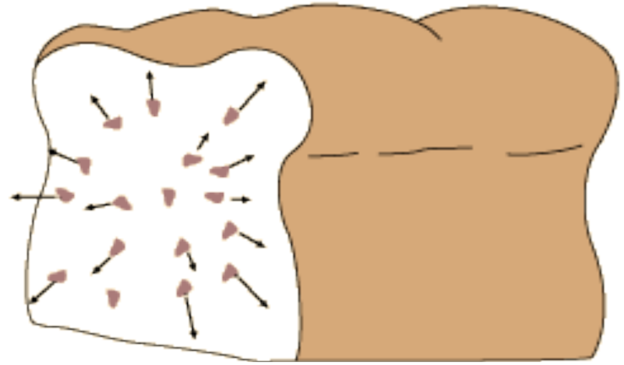
Flat *k = 0*

Closed *k = +1*

$$\rho_c \equiv \frac{3\,H_0^{\,2}}{8\pi\,G} \approx 10^{-26}\text{kg m}^{-3} \approx \frac{1.4\times10^{11}\,\text{Msun}}{(\text{Mpc})^3}$$

Isotropic Expansion



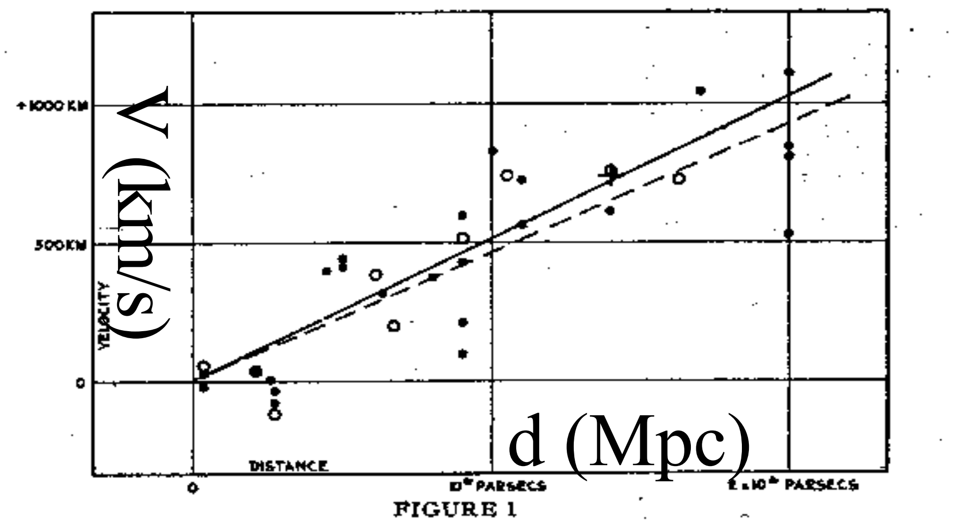


Hubble law :

$$V = H_0 \, d$$

Hubble "constant" :

$$H_0 \approx 500 \text{ km s}^{-1} \text{Mpc}^{-1}$$



Why WRONG? Extinction by interstellar dust was not then known, giving incorrect distances.

E.g.,: Empty Universe without vacuum

$$\dot{R}^2 = \left(\frac{8\pi G \rho + \Lambda}{3} \right) R^2 - k c^2$$

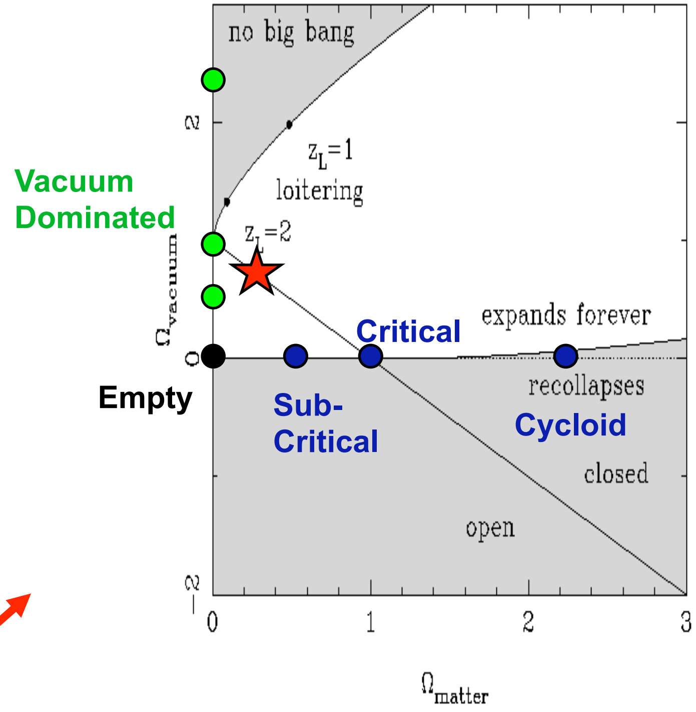
Set $\rho = 0, \quad \Lambda = 0.$ Then $\dot{R}^2 = -k c^2$

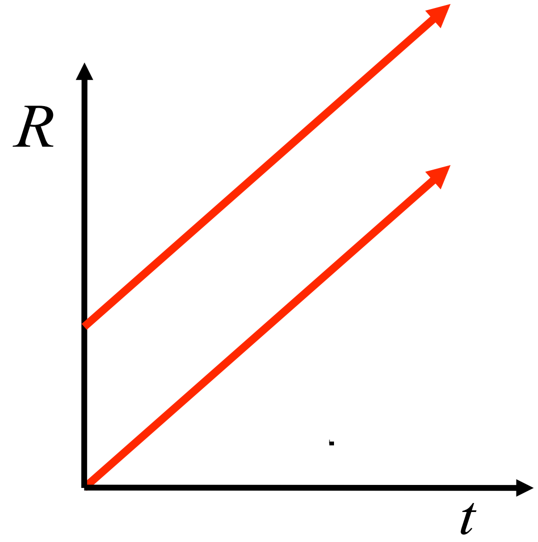
$\rightarrow \quad k = -1$ (negative curvature)

$$\dot{R} = c, \quad R = c\, t$$

$$H \equiv \frac{\dot{R}}{R} = \frac{1}{t}$$

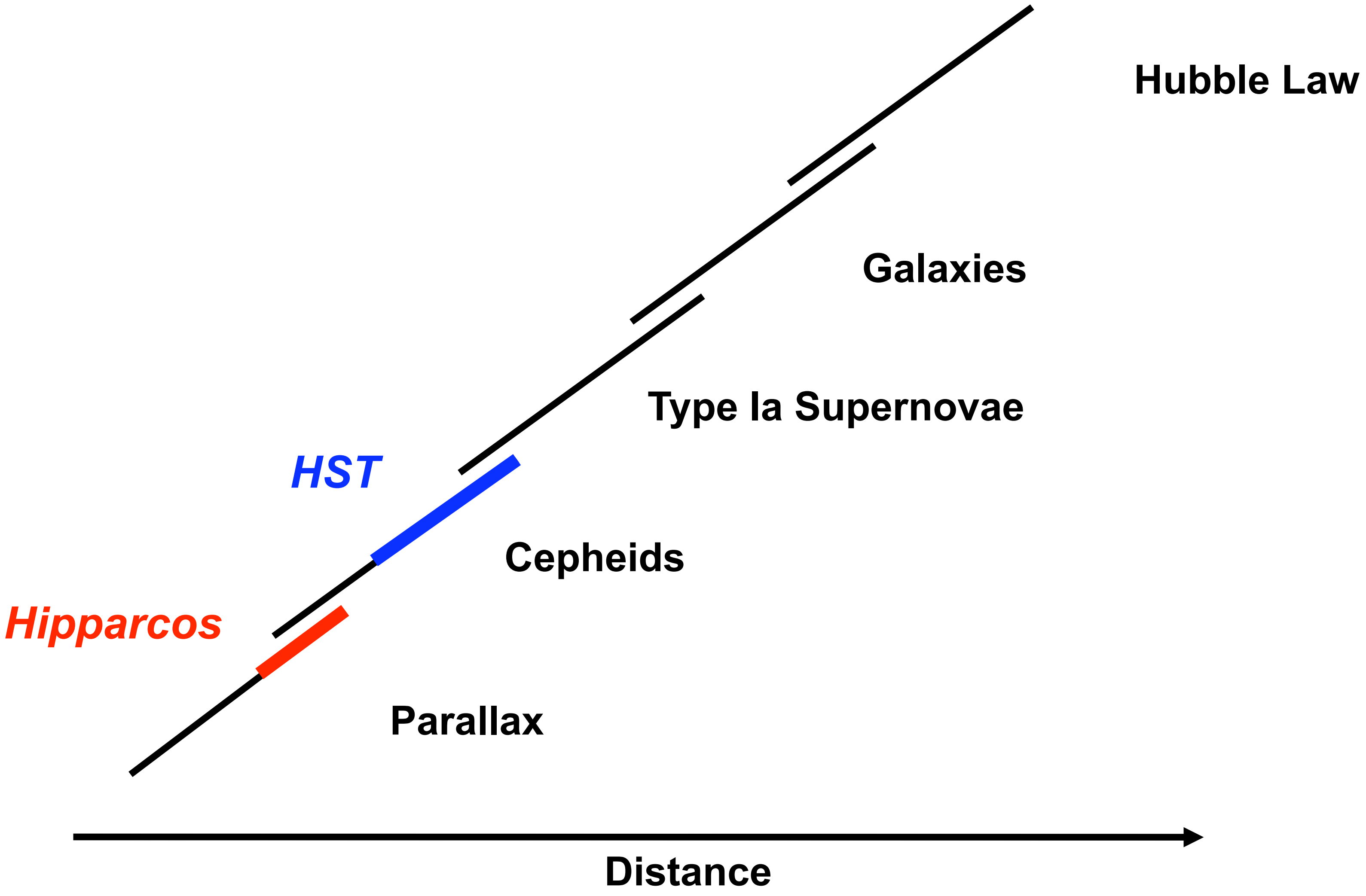
$$\text{age}: \quad t_0 = \frac{R_0}{c} = \frac{1}{H_0}$$





Negative curvature drives rapid expansion/flattening

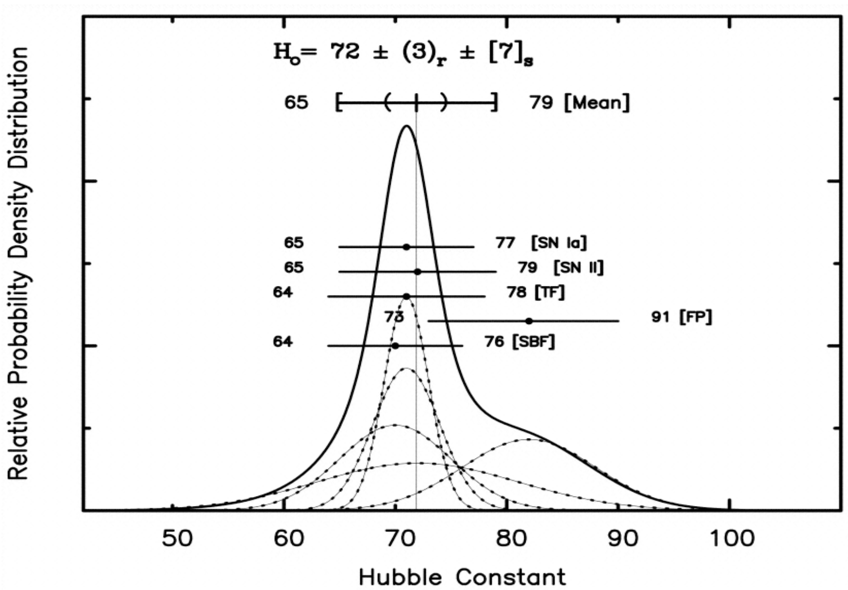
Cosmic Distance Ladder

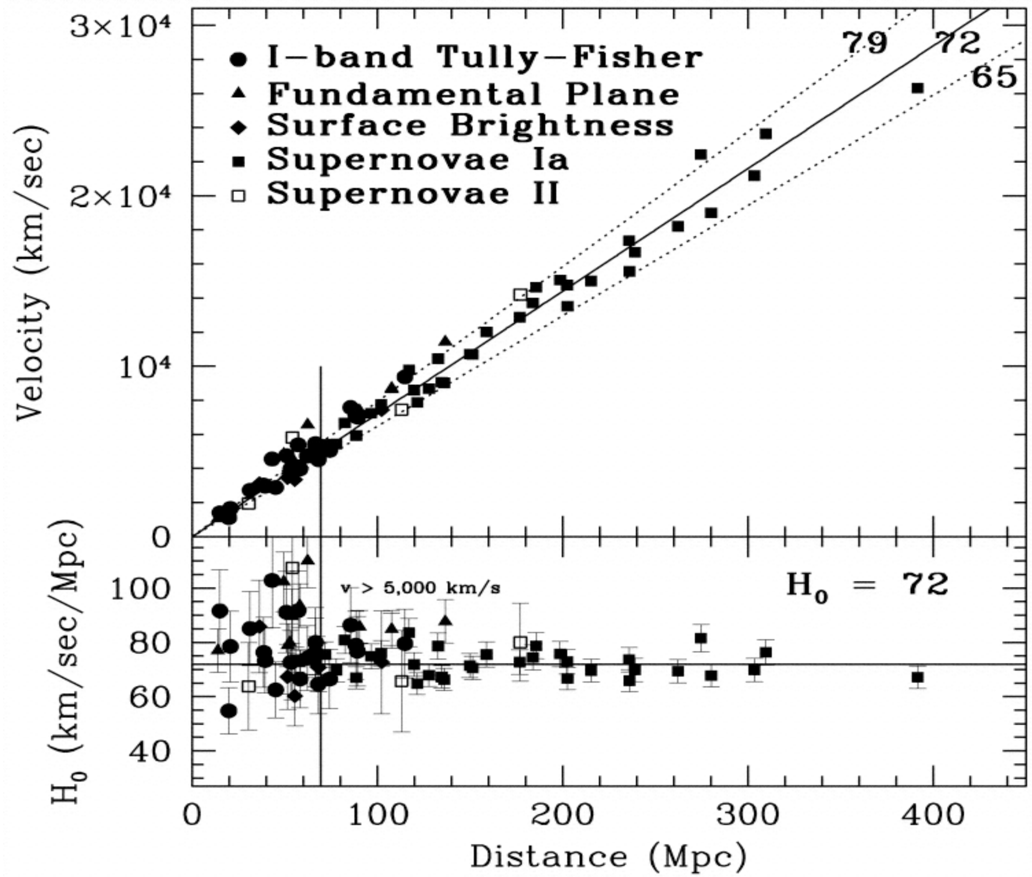


H_0 from the HST Key Project

$$H_0 \approx 72 \pm 3 \pm 7 \quad \mathrm{km\ s^{-1}\ Mpc^{-1}}$$

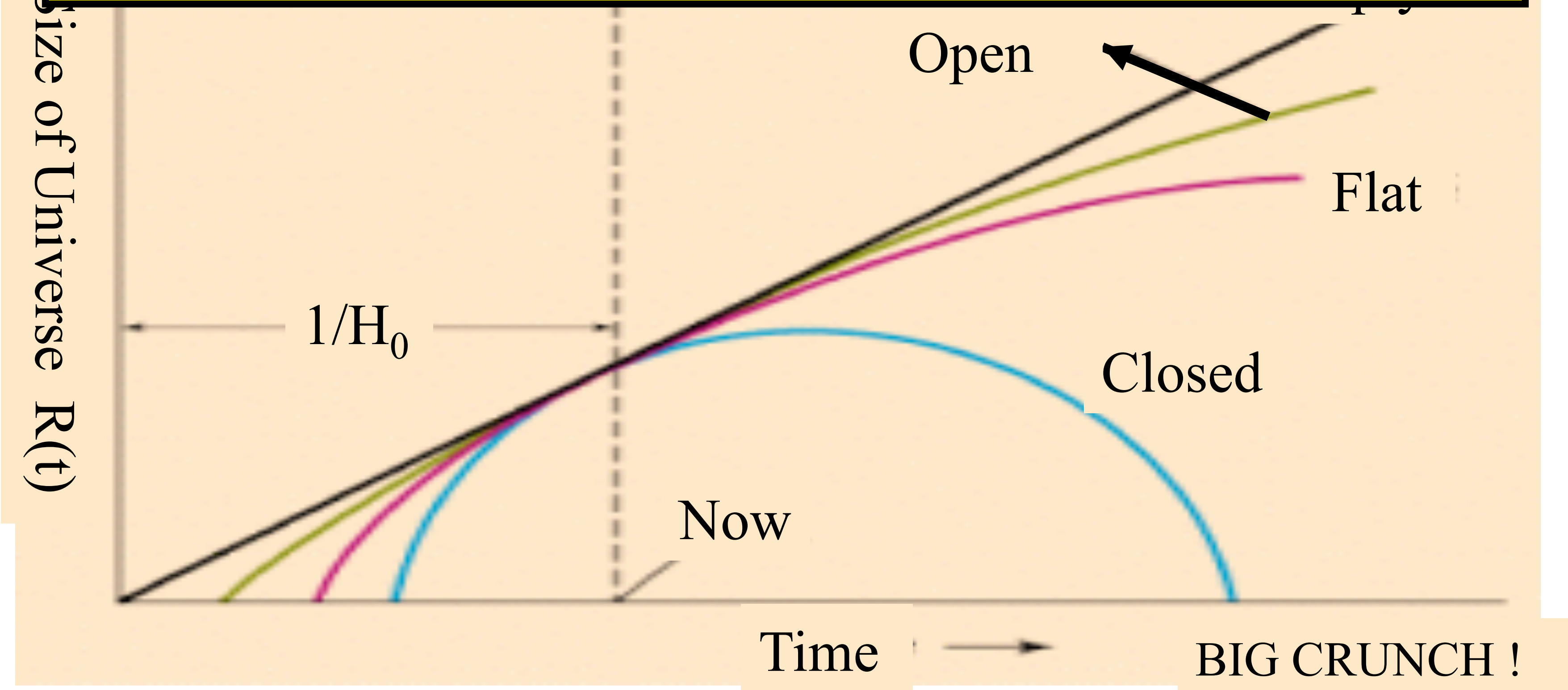
Freedman, et al.
2001 ApJ 553, 47.





Re-collapse or Eternal Expansion ?

Inflation => expect FLAT GEOMETRY
CRITICAL DENSITY



Hubble Parameter Evolution -- H(z)

$$H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k c^2}{R^2}$$

$$\frac{H^2}{H_0^{\;2}} = \Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda - \frac{k c^2}{H_0^{\;2} R_0^{\;2}} x^2$$

$$\text{evaluate at } x = 1 \quad \rightarrow \quad 1 = \Omega_0 - \frac{k c^2}{H_0^{\;2} R_0^{\;2}}$$

Dimensionless Friedmann Equation:

$$\frac{H^2}{H_0^{\;2}} = \Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda + (1 - \Omega_0)\, x^2$$

$$x = 1 + z = R_0 / R$$

$$\rho_c = \frac{3 H_0^{\;2}}{8\pi G}$$

$$\Omega_M \equiv \frac{\rho_M}{\rho_c}, \quad \Omega_R \equiv \frac{\rho_R}{\rho_c}$$

$$\Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_c} = \frac{\Lambda}{3 H_0^{\;2}}$$

$$\Omega_0 \equiv \Omega_M + \Omega_R + \Omega_\Lambda$$

Curvature Radius today:

$$R_0 = \frac{c}{H_0} \sqrt{\frac{k}{\Omega_0 - 1}} \quad \rightarrow \begin{cases} k = +1 & \Omega_0 > 1 \\ k = 0 & \Omega_0 = 1 \\ k = -1 & \Omega_0 < 1 \end{cases}$$

Density determines Geometry

Possible Universes

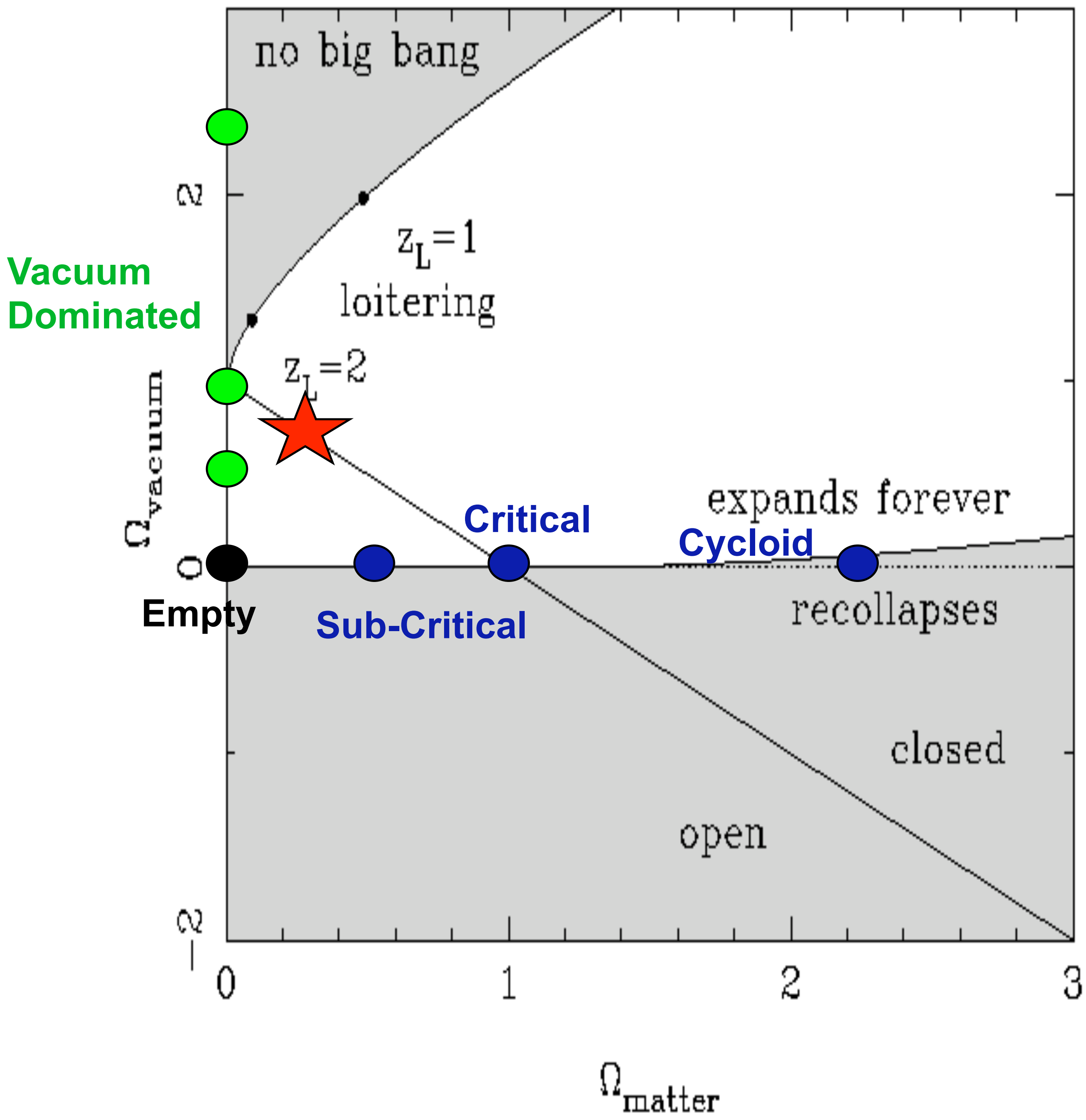
$$H_0 \approx 70 \frac{\text{km/s}}{\text{Mpc}}$$

$$\Omega_M \sim 0.3$$

$$\Omega_\Lambda \sim 0.7$$

$$\Omega_R \sim 8 \times 10^{-5}$$

$$\Omega = 1.0$$



Precision Cosmology

$h = 71 \pm 3$ expanding

$\Omega = 1.02 \pm 0.02$ flat

$\Omega_b = 0.044 \pm 0.004$ baryons

$\Omega_M = 0.27 \pm 0.04$ Dark Matter

$\Omega_\Lambda = 0.73 \pm 0.04$ Dark Energy

$t_0 = 13.7 \pm 0.2 \times 10^9$ yr now

$t_* = 180^{+220}_{-80} \times 10^6$ yr $z_* = 20^{+10}_{-5}$ reionisation

$t_R = 379 \pm 1 \times 10^3$ yr $z_R = 1090 \pm 1$ recombination

(From the WMAP 1-year data analysis)

Four Pillars of Hot Big Bang

Galaxies moving apart from each other

- Redshift or receding from each other
- Universe was smaller.

Helium production outside stars

- Universe was hot, at least 3×10^9K to fuse 4H → He, to overcome a potential barrier of 1MeV.

Nearly Uniform Radiation 3K Background (CMB)

- Universe has cooled, hence expanded by at least a factor 10^9. Photons (3K~10^{-4}eV) are only 10^{-3} of baryon energy density, so photon-to-proton number ratio ~ 10^{-3}(GeV/10^{-4}eV) ~ 10^8

Missing mass in galaxies and clusters (Cold DM)

- Cluster potential well is deeper than the potential due to baryons.
- CMB fluctuations: photons climb out of random potentials of DM.
- If 1/10 of the matter density in 1GeV protons, 9/10 in dark particles of e.g. 9GeV, then dark-to-proton number density ratio ~ 1

Cosmology Milestones

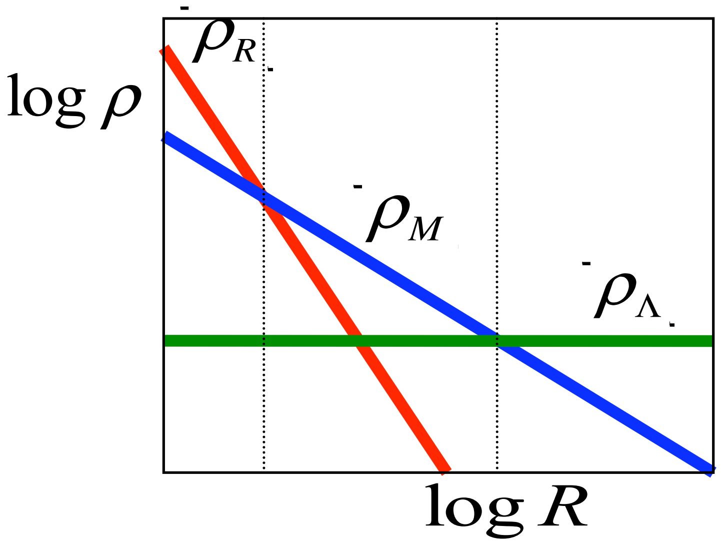
- 1925 Galaxy redshifts $\lambda = \lambda_0 \; (1 + z)$ $V = c\,z$
  - Isotropic expansion. (Hubble law $V = H_0 d$)
  - Finite age. ($t_0 = 13 \times 10^9$ yr)
- 1965 Cosmic Microwave Background (CMB)
  - Isotropic blackbody. $T_0 = 2.7$ K
  - Hot Big Bang $T = T_0 (1 + z)$
- 1925 General Relativity Cosmology Models :
  - Radiation era: $R \sim t^{1/2}$ $T \sim t^{-1/2}$
  - Matter era: $R \sim t^{2/3}$ $T \sim t^{-2/3}$
- 1975 Big Bang Nucleosynthesis (BBN)
  - light elements (^{1}H … ^{7}Li) $t \sim 3$ min $T \sim 10^9$ K
  - primordial abundances (75% H, 25% He) as observed!

Tutorial: 3 Eras: radiation-matter-vacuum

radiation : $\rho_R \propto R^{-4}$

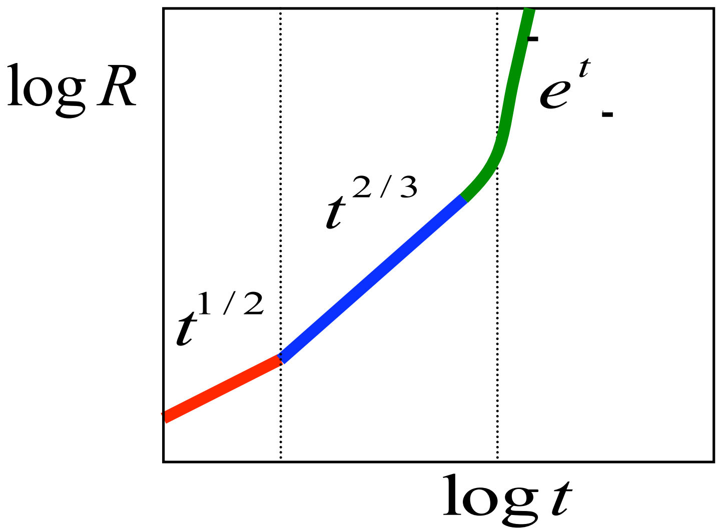
matter : $\rho_M \propto R^{-3}$

vacuum: $\rho_\Lambda = \text{const}$



$$a \equiv \frac{R}{R_0} = \frac{1}{1+z}$$

$$\rho = \frac{\rho_{R,0}}{a^4} + \frac{\rho_{M,0}}{a^3} + \rho_\Lambda$$



$\rho_R = \rho_M$ at $a \sim 10^{-4}$ $t \sim 10^4$ yr

$\rho_M = \rho_\Lambda$ at $a \sim 0.7$ $t \sim 10^{10}$ yr

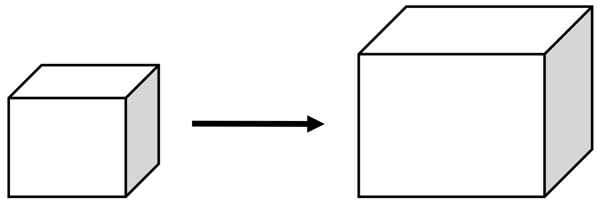
Presently vacuum is twice the density of matter.

5^{th} concept: Equation of State w

Equation of state :

$$\rho \propto R^{-n} \qquad n = 3(1+w)$$

$$w \equiv \frac{\text{pressure}}{\text{energy density}} = \frac{p}{\rho c^2} = \frac{n}{3} - 1$$



Radiation : $(n = 4, w = 1/3)$

$$p_R = \frac{1}{3}\rho_R \ c^2$$

Matter : $(n = 3, w = 0)$

$$p_M \sim \rho_M \ c_S^{\ 2} << \rho_M \ c^2$$

Vacuum : $(n = 0, w = -1)$

$$p_\Lambda = -\rho_\Lambda \ c^2$$

Negative Pressure ! ?

$$d[energy] = work$$

$$d[\rho c^2 R^3] = -p \ d[R^3]$$

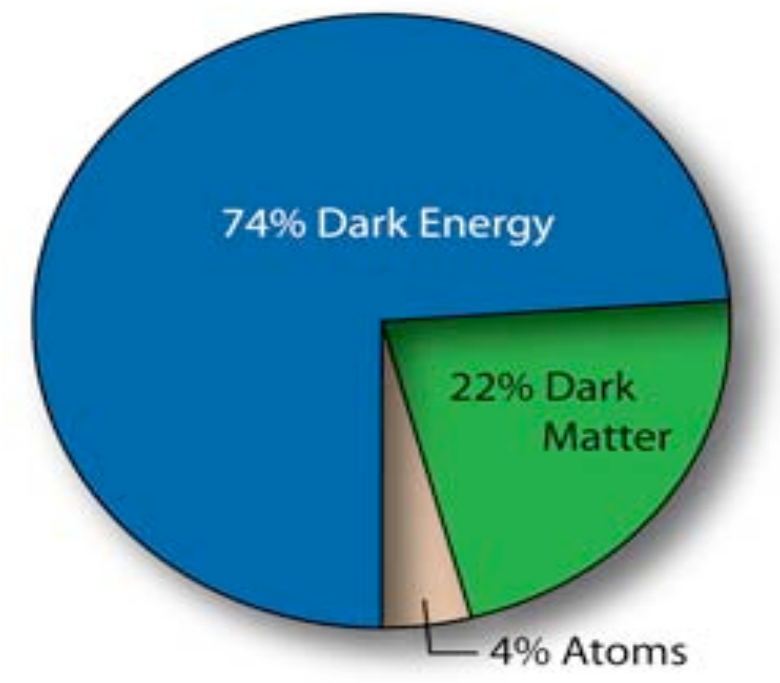
$$\rho c^2 (3R^2 dR) + R^3 c^2 d\rho = -p (3R^2 dR)$$

$$1 + \frac{R \, d\rho}{3\rho \, dR} = -\frac{p}{\rho c^2} \equiv -w$$

$$w = -\frac{1}{3}\frac{d[\ln \rho]}{d[\ln R]} - 1$$

$$w = \frac{n}{3} - 1$$

Current Mysteries from Observations



Dark Matter ?

Holds Galaxies together
Triggers Galaxy formation

Dark Energy ?

Drives Cosmic Acceleration
and negative w.

Modified Gravity ?

General Relativity wrong ?

Density Parameters

critical density :

$$\rho_c \equiv \frac{3\,H_0^{\,2}}{8\pi\,G}$$

density parameters (today) :

$$\Omega_R \equiv \frac{\rho_R}{\rho_c} \quad \Omega_M \equiv \frac{\rho_M}{\rho_c} \quad \Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_c} = \frac{\Lambda}{3\,H_0^{\,2}}$$

total density parameter today :

$$\Omega_0 \equiv \Omega_R + \Omega_M + \Omega_\Lambda$$

density at a past/future epoch in units of today's critical density :

$$\Omega \equiv \frac{\rho}{\rho_c} = \sum_w \Omega_w x^{3(1+w)} = \Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda \qquad x \equiv 1 + z = R_0 / R$$

in units of critical density at the past/future epoch :

$$\Omega(x) \equiv \frac{8\pi G \rho}{3H^2} = \frac{H_0^{\,2}}{H^2} \sum_w \Omega_w x^{3(1+w)} = \frac{\Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda}{\Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda + (1-\Omega_0)\,x^2}$$

Note: radiation dominates at high z, can be neglected at lower z.

Key Points

- Scaling Relation among
  - Redshift: z,
  - expansion factor: R
    - Distance between galaxies
  - Temperature of CMB: T
    - Wavelength of CMB photons: lambda
- Metric of an expanding 2D+time universe
  - Fundamental observers
    - Galaxies on grid points with fixed angular coordinates
- Energy density in
  - vacuum, matter, photon
  - How they evolve with R or z
- <u>If confused, recall the analogies of</u>
  - balloon, bread, a network on red giant star, microwave oven

Sample a wide range of topics Theoretical and Observational

Universe of uniform density

- Metrics ds, Scale R(t) and Redshift
- EoS for mix of vacuum, photon, matter, geometry, distances

Thermal history

- Freeze-out of particles,
- Neutrinos, CDM wimps
- Nucleo-synthesis He/D/H

Structure formation

- Inflation and origin of perturbations
- Growth of linear perturbation
- Relation to CMB peaks, sound horizon

Quest of H0 (obs.)

- Applications of expansion models
- Distances Ladders

Cosmic Background

- COBE/MAP/PLANCK etc.
- Parameters of cosmos

Quest for Omega (obs.)

- Galaxy and SNe surveys
- Luminosity Functions

(thanks to slides from K. Horne)

Hongsheng Zhao

6th concept: Distances in Non-Euclidean Curved Space

How Does Curvature affect Distance Measurements ?

Is the universe very curved?

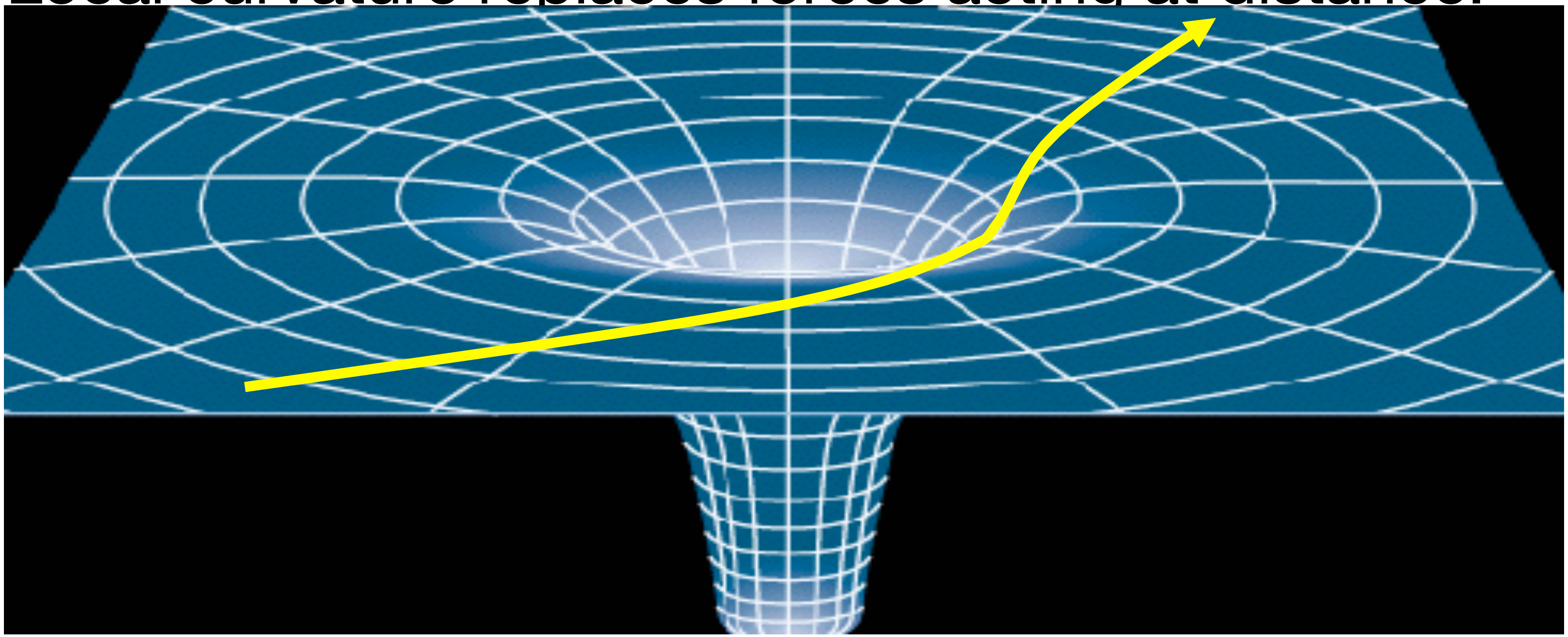
Geodesics

Gravity = curvature of space-time by matter/energy.

Freely-falling bodies follow **geodesic trajectories**.

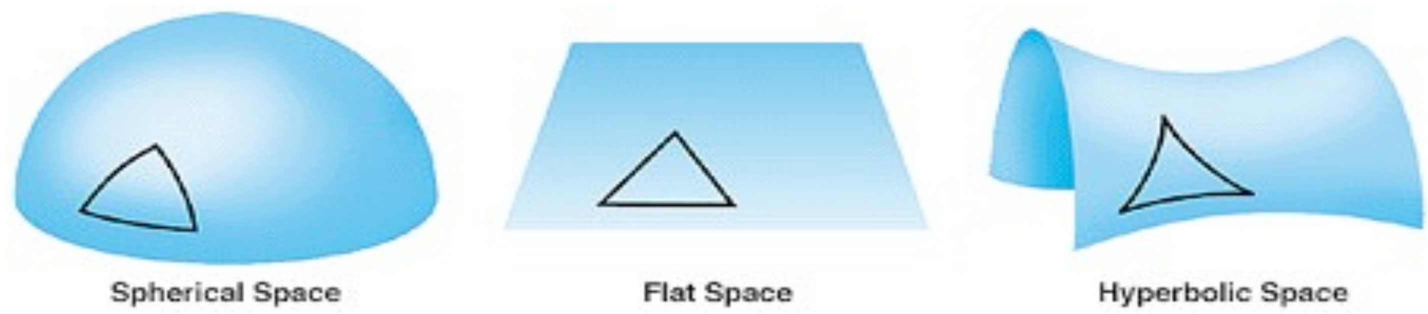
Shortest possible path in curved space-time.

Local curvature replaces forces acting at distance.



Is our Universe Curved?

	Closed	Flat	Open

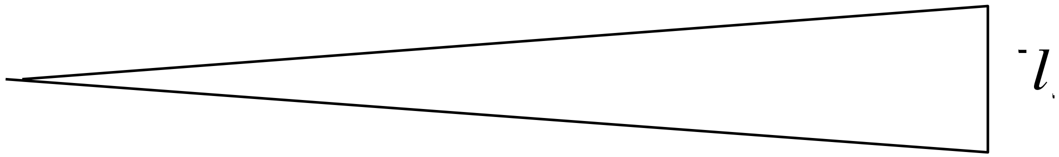


Curvature:	**+**	**0**	**--**
Sum of angles of triangle:	**> 180°**	**= 180°**	**< 180°**
Circumference of circle:	**$< 2\pi r$**	**$= 2\pi r$**	**$> 2\pi r$**
Parallel lines:	**converge**	**remain parallel**	**diverge**
Size:	**finite**	**infinite**	**infinite**
Edge:	**no**	**no**	**no**

Distance Methods

- **Standard Rulers ==> Angular Size Distances**

$$\theta = \frac{l}{D}$$

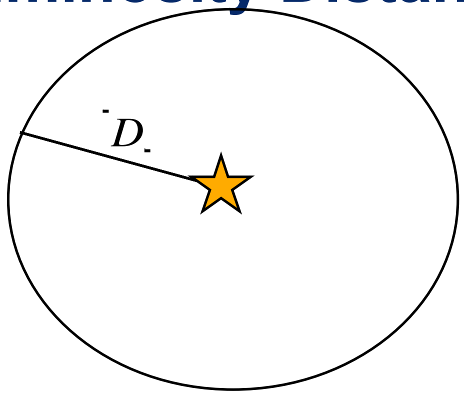


$$D_A = \frac{l}{\theta}$$

(for small angles << 1 radian)

- **Standard Candles ==> Luminosity Distances**

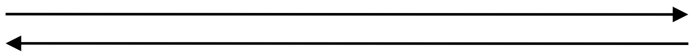
$$F = \frac{\text{energy/time}}{\text{area}} = \frac{L}{4\,\pi D^2}$$



$$D_L = \left(\frac{L}{4\,\pi\,F} \right)^{1/2}$$

- **Light Travel Time**

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{2\,D}{c}$$

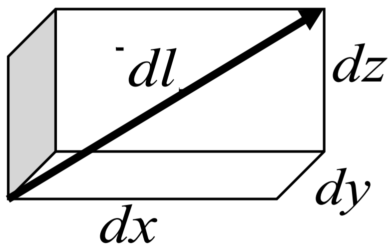


$$D_t = \frac{c}{2\,t}$$

(e.g. within solar system)

Flat Space: Euclidean Geometry

Cartesian coordinates :



$$1\,\mathrm{D}: \quad dl^2 = dx^2$$

$$2\,\mathrm{D}: \quad dl^2 = dx^2 + dy^2$$

$$3\,\mathrm{D}: \quad dl^2 = dx^2 + dy^2 + dz^2$$

$$4\,\mathrm{D}: \quad dl^2 = dw^2 + dx^2 + dy^2 + dz^2$$

Metric tensor : coordinates -> distance

$$dl^2 = (\,dx \quad dy \quad dz\,)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}$$

Orthogonal coordinates <--> diagonal metric

$$g_{xx} = g_{yy} = g_{zz} = 1$$

$$g_{xy} = g_{xz} = g_{yz} = 0$$

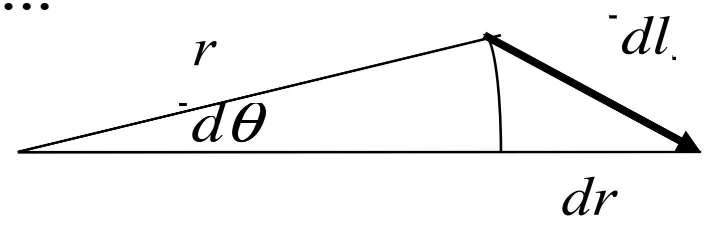
$$\text{symmetric : } g_{ij} = g_{ji}$$

Summation convention :

$$dl^2 = g_{ij}\, dx^i\, dx^j \equiv \sum_i \sum_j g_{ij}\, dx^i\, dx^j$$

Polar Coordinates

Radial coordinate r, angles $\phi, \theta, \alpha, ...$



1 D: $\quad dl^2 = dr^2$

2 D: $\quad dl^2 = dr^2 + r^2 \; d\theta^2$

3 D: $\quad dl^2 = dr^2 + r^2 \, (d\theta^2 + \sin^2\theta \; d\phi^2)$

4 D: $\quad dl^2 = dr^2 + r^2 \, [d\theta^2 + \sin^2\theta \, (d\phi^2 + \sin^2\phi \; d\alpha^2)]$

$$dl^2 = dr^2 + r^2 \; d\psi^2 \qquad \text{generic angle :} \quad d\psi^2 = d\theta^2 + \sin^2\theta \, d\phi^2 + ...$$

$$dl^2 = (dr \quad d\theta \quad d\phi) \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2\theta \end{pmatrix} \begin{pmatrix} dr \\ d\theta \\ d\phi \end{pmatrix}$$

$g_{rr} = ? \quad g_{r\theta} = ?$

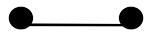
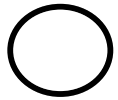
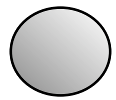
$g_{\theta\theta} = ?$

$g_{\phi\phi} = ?$

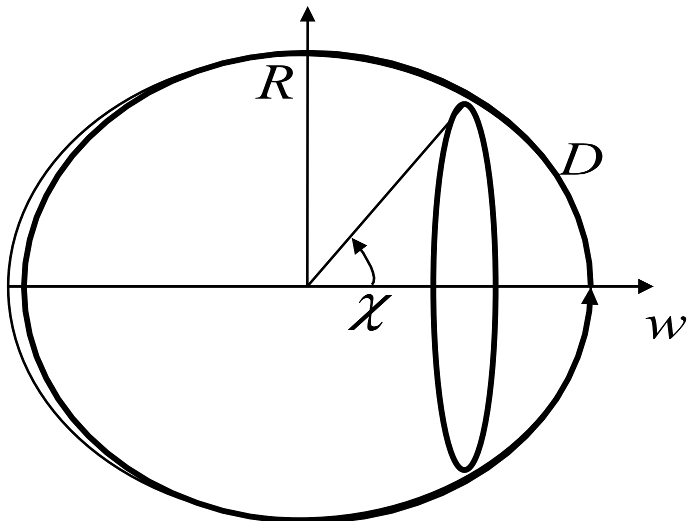
$g_{\alpha\alpha} = ?$

reading: Embedded Spheres

R = radius of curvature

1 − D :	$R^2 = x^2$	0 - D 2 points	
2 − D :	$R^2 = x^2 + y^2$	1 - D circle	
3 − D :	$R^2 = x^2 + y^2 + z^2$	2 - D surface of 3 - sphere	
4 − D :	$R^2 = x^2 + y^2 + z^2 + w^2$	3 - D surface of 4 - sphere	

?



Reading: Non-Euclidean Metrics

$k = -1, 0, +1 \quad (\text{open, flat, closed})$

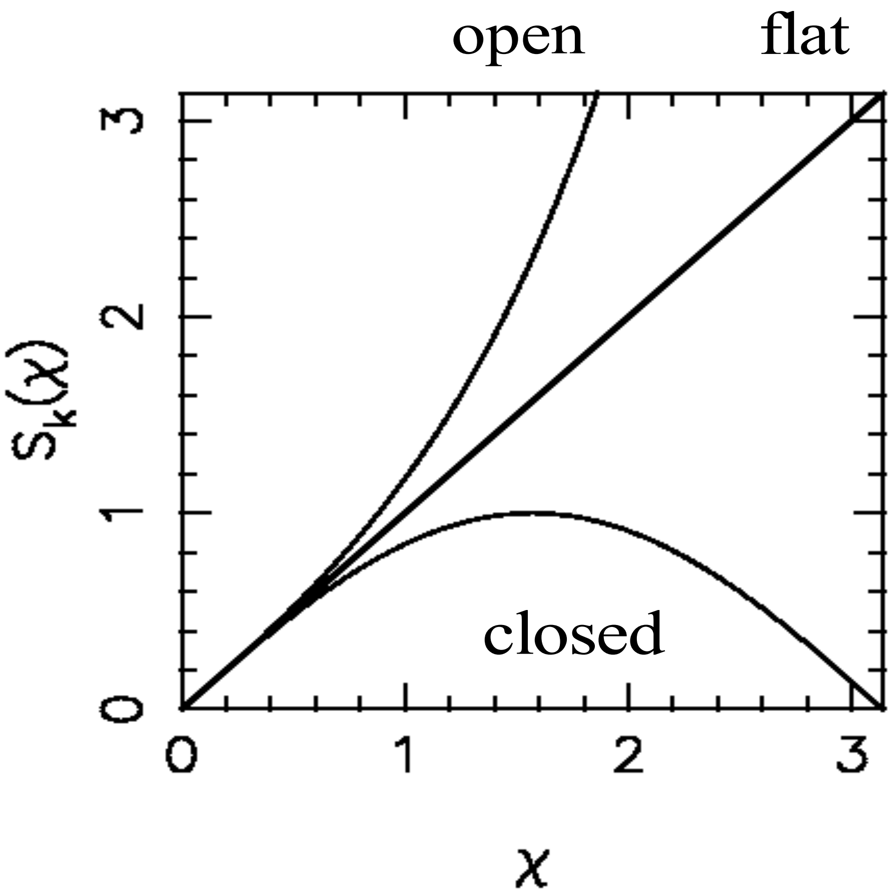
$$dl^2 = \frac{dr^2}{1 - k\,(r/R)^2} + r^2 d\psi^2$$

dimensionless radial coordinates :

$$u = r/R = S_k(\chi)$$

$$dl^2 = R^2 \left(\frac{du^2}{1 - k\,u^2} + u^2 d\psi^2 \right)$$

$$= R^2\,(\,d\chi^2 + S_k^{\,2}(\chi)\,d\psi^2)$$



$$S_{-1}(\chi) \equiv \sinh(\chi)\ , \quad S_0(\chi) \equiv \chi\ , \quad S_{+1}(\chi) \equiv \sin(\chi)$$

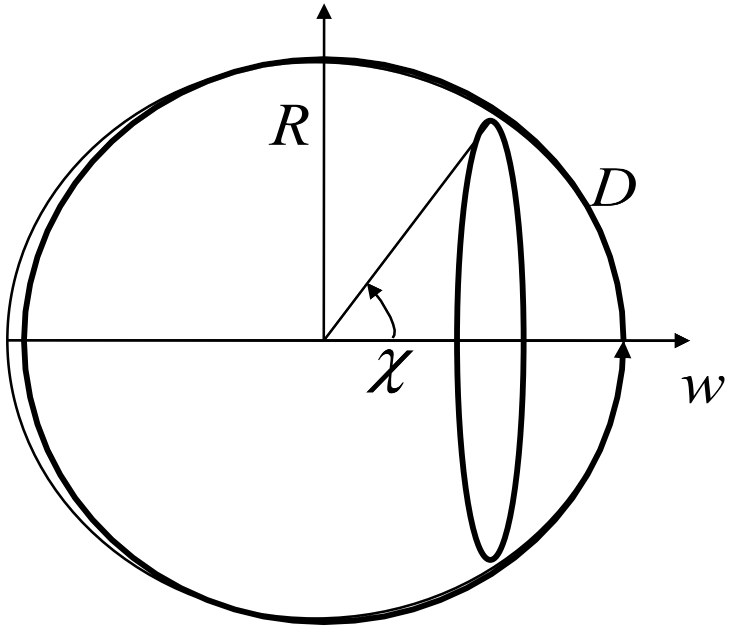
Reading: Circumference

metric :

$$dl^2 = \frac{dr^2}{1 - k\,(r/R)^2} + r^2\, d\theta^2$$

radial distance (for $k = +1$):

$$D = \int_0^r \frac{dr}{\sqrt{1 - k\,(r/R)^2}} = R \sin^{-1}(r/R)$$



circumference :

$$C = \int_0^{2\pi} r\, d\theta = 2\pi\, r$$

"circumferencial" distance : $r \equiv \dfrac{C}{2\pi} = R\, S_k(D/R) = R\, S_k(\chi)$

If $k = +1$, coordinate r breaks down for $r > R$

Reading: Circumference

metric :

$$dl^2 = R^2 \left(d\chi^2 + S_k^2(\chi)\, d\theta^2 \right)$$

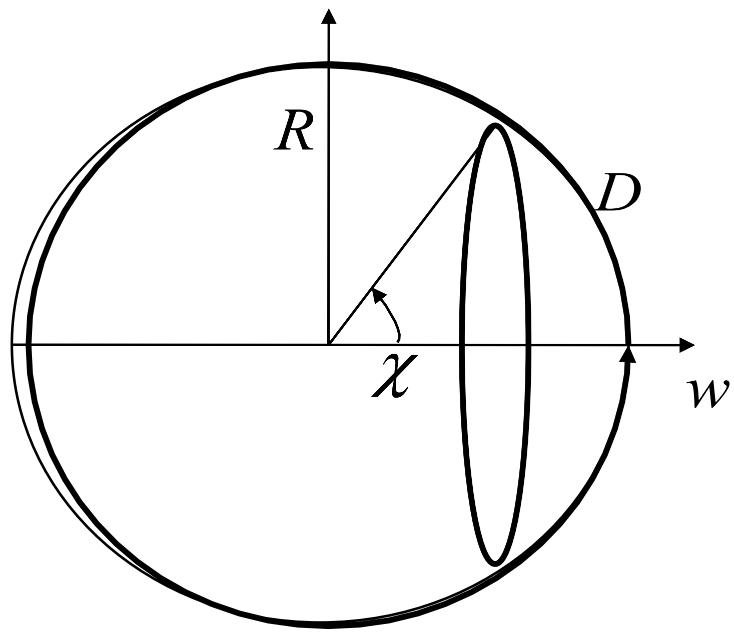
radial distance :

$$D = \int \sqrt{g_{\chi\chi}}\, d\chi = \int_0^{\chi} R\, d\chi = R\,\chi$$

circumference :

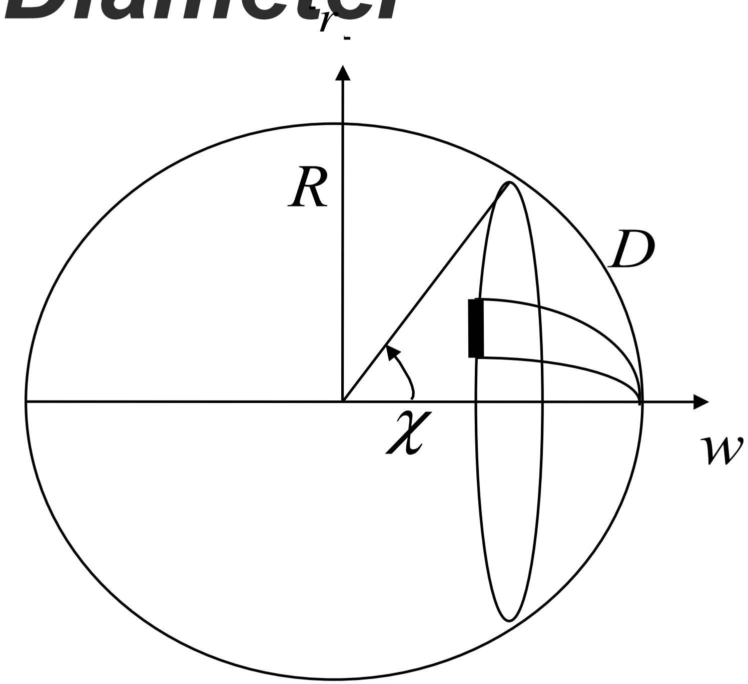
$$C = \oint \sqrt{g_{\theta\theta}}\, d\theta = \int_0^{2\pi} R\, S_k(\chi)\, d\theta = 2\pi\, R\, S_k(\chi)$$

$$= 2\pi\, D\, \frac{S_k(\chi)}{\chi}$$



Same result for any choice of coordinates.

Reading: Angular Diameter



metric :

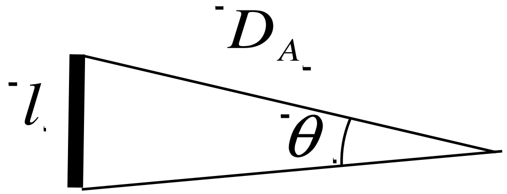
$$dl^2 = R^2 \,(\, d\chi^2 + S_k^2(\chi)\, d\theta^2 \,)$$

radial distance :

$$D = \int \sqrt{g_{\chi\chi}}\, d\chi = \int_0^\chi R \, d\chi = R \, \chi$$

linear size : $(\, l << D \,)$

$$l = \int \sqrt{g_{\theta\theta}}\, d\theta = R \, S_k(\chi)\, \theta$$



angular size :

$$\theta = \frac{l}{D_A}$$

$D = R \;\chi =$ Radial Distance

$D_A = R \, S_k(\chi) =$ Angular Diameter Distance

Reading: Area of Spherical Shell

radial coordinate χ, angles θ , ϕ :

$$dl^2 = R^2 \, [\, d\chi^2 + S_k^2(\chi) \, (d\theta^2 + \sin^2\theta \, d\phi^2) \,]$$

area of shell :

$$A = \int \sqrt{g_{\theta\theta}}\, d\theta \sqrt{g_{\phi\phi}}\, d\phi$$

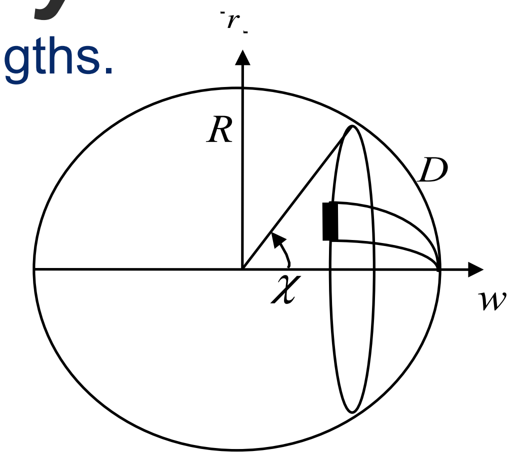
$$= R^2 \, S_k^2(\chi) \int_0^{\pi} d\theta \, \sin\theta \int_0^{2\pi} d\phi$$

$$= 4\pi \, R^2 \, S_k^2(\chi)$$

flux :

$$F = \frac{L}{A} = \frac{L}{4\pi \, D_L^2} \qquad D_L = R \, S_k(\chi) = \text{ Luminosity Distance}$$

[we will work with flats only]
Curved Space Summary



- The **metric** converts coordinate steps (grids) to physical lengths.
- Use the metric to compute lengths, areas, volumes, …
- Radial distance:

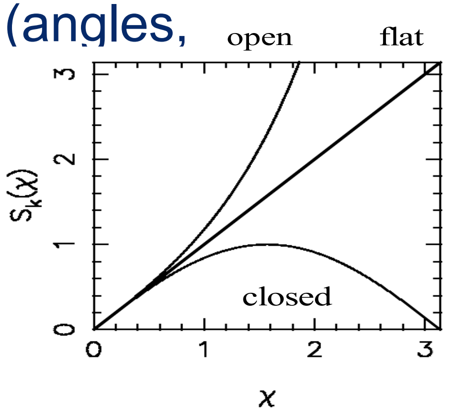
$$D \equiv \int \sqrt{g_{rr}}\, dr = R\,\chi$$

- “Circumferencial” distance

$$r \equiv \frac{C}{2\pi} = \left(\frac{A}{4\pi}\right)^{1/2} = \frac{1}{2\pi}\int_0^{2\pi} \sqrt{g_{\phi\phi}}\, d\phi = R\,S_k(\chi) = R\,S_k(D/R)$$

- “Observable” distances, defined in terms of local observables (angles, fluxes), give r, not D.

$$D_A \equiv \frac{l}{\theta} = r \qquad D_L \equiv \left(\frac{L}{4\pi F}\right)^{1/2} = r$$



- r < D (positive curvature, S_{+1} (x)=sin x) (negative, S_{-1} (x)=sinh x) or r=D (flat, S_0(x)=x)

Olber's Paradox

Why is the sky dark at night ?

Flux from all stars in the sky :

$$F = \int n_* \, F_* \, \mathrm{d}(\mathrm{Vol}) = \int_0^{\chi_{\max}} n_* \left(\frac{L_*}{A(\chi)} \right) (A(\chi) \, R \, d\chi)$$

$$= n_* \, L_* \, R \, \chi_{\max}$$

$$\Rightarrow \infty \quad \text{for flat space}, \; R \to \infty.$$

A dark sky may imply :

(1) an edge (we don't observe one)

(2) a curved space (finite size)

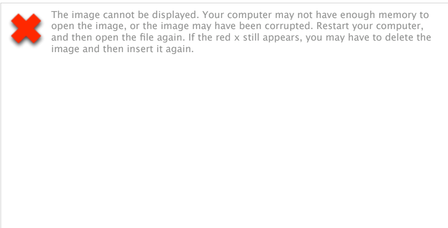
(3) expansion ($R(t)$ => finite age, redshift)

Minkowski Spacetime Metric

$$ds^2 = -c^2 dt^2 + dl^2$$

$$d\tau^2 = dt^2 - \frac{dl^2}{c^2} = dt^2\left(1 - \frac{1}{c^2}\left(\frac{dl}{dt}\right)^2\right)$$

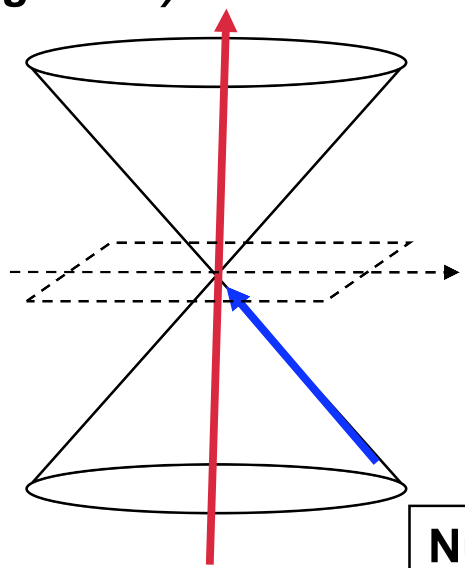
***Proper time** (moving clock):*



Time-like intervals:
ds² < 0, dτ² > 0
Inside light cone.
Causally connected.

Space-like intervals:
ds² > 0 , dτ² < 0

Outside light cone.
Causally disconnected.

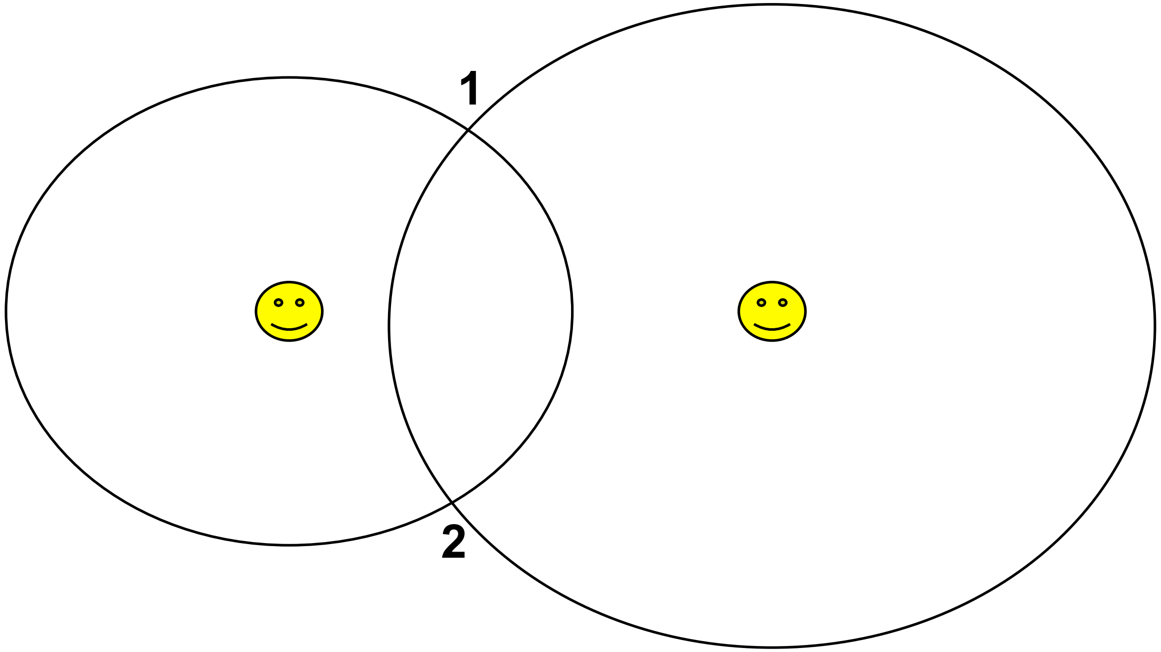


World line of massive particle at rest.

Null intervals light cone:
v = c, *ds² = 0*

Photons arrive from our past light cone.

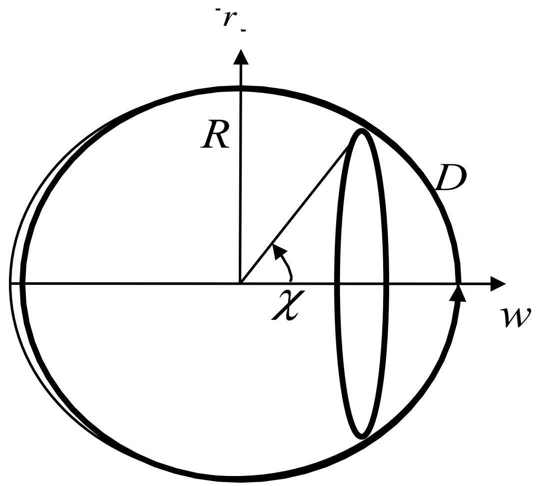
Cosmological Principle (assumed) + Isotropy (observed) => Homogeneity



$$\rho_1 = \rho_2 \quad \text{otherise not isotropic for equidistant fidos}$$

7^{th} Concept: Robertson-Walker metric
uniformly curved, evolving spacetime

$$ds^2 = -c^2dt^2 + R^2(t)\,(d\chi^2 + S_k^2(\chi)\,d\psi^2)$$
$$= -c^2dt^2 + R^2(t)\left(\frac{du^2}{1-k\,u^2} + u^2\,d\psi^2\right)$$
$$= -c^2dt^2 + a^2(t)\left(\frac{dr^2}{1-k\,(r/R_0)^2} + r^2\,d\psi^2\right)$$



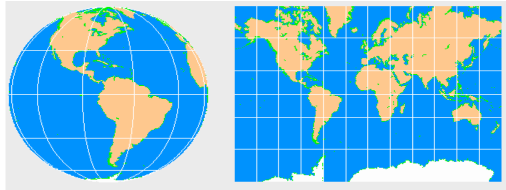
$$S_k(\chi) = \begin{cases} \sin\chi & (k=+1) & \text{closed} \\ \chi & (k=0) & \text{flat} \\ \sinh\chi & (k=-1) & \text{open} \end{cases}$$

$$d\psi^2 \equiv d\theta^2 + \sin^2\theta\,d\phi^2$$
$$a(t) \equiv R(t)/R_0$$
$$R_0 \equiv R(t_0)$$

radial distance $= D(t) = R(t)\,\chi$

circumference $= 2\pi\,r(t)$ $\qquad r(t) = a(t)\,r = R(t)\,u = R(t)\,S_k(\chi)$

coordinate systems



Distance varies in time:

$$D(t)$$

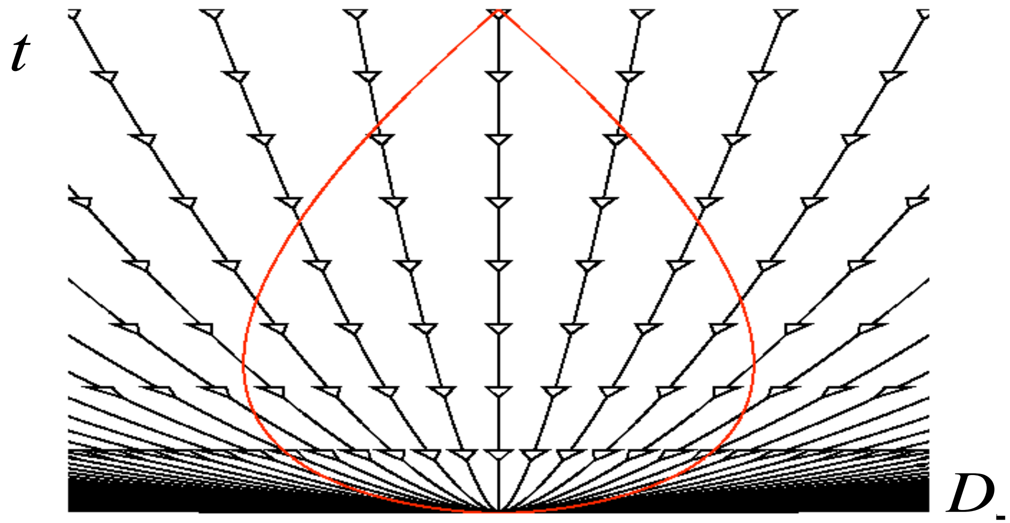
"Fiducial observers" (Fidos)

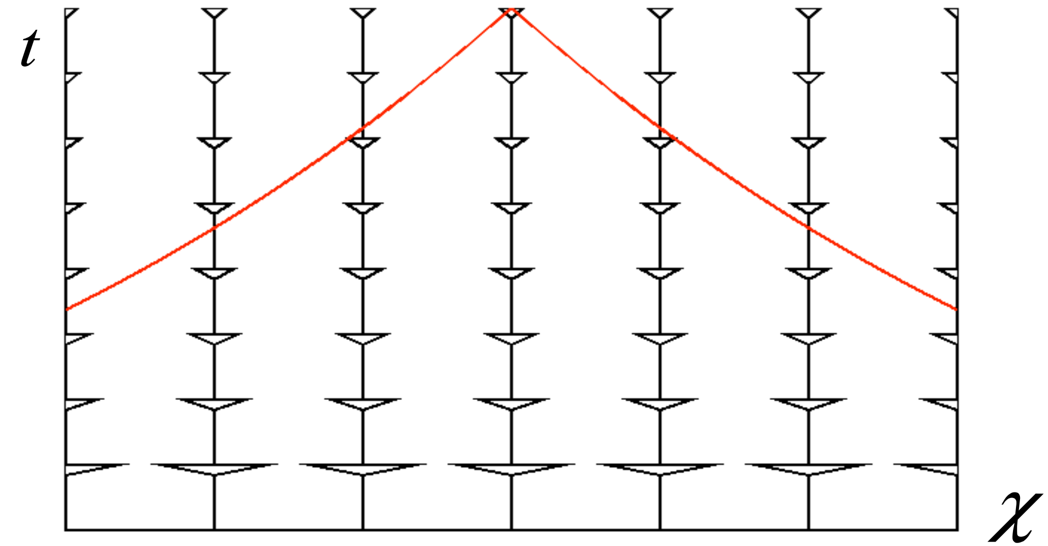
$$D(t) = R(t)\,\chi$$

"Co-moving" coordinates

$$\chi \quad \text{or} \quad D_0 \equiv R_0\,\chi$$

Labels the Fidos





Distances-Redshift relation

- We observe the **redshift :**

$$z \equiv \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\lambda}{\lambda_0} - 1 \quad \lambda = \text{observed}, \quad \lambda_0 = \text{emitted (rest)}$$

- Hence we know the **expansion factor:**

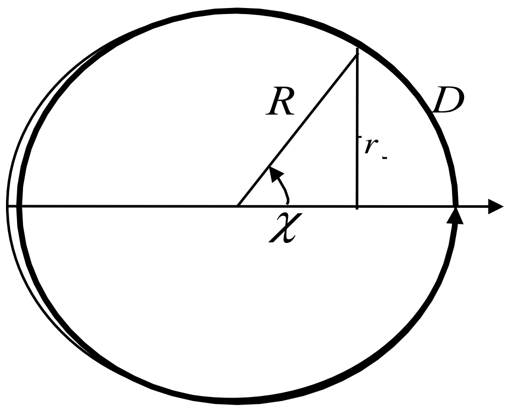
$$x \equiv 1 + z = \frac{\lambda}{\lambda_0} = \frac{\lambda(t_0)}{\lambda(t)} = \frac{R(t_0)}{R(t)} = \frac{R_0}{R(t)}$$

- Need the time of light emitted
- Need coordinate of the source
- Need them as functions of
- Distances

$$t(z) = ?$$

$$\chi(z) = ?$$

$$H_0 \quad \Omega_M \quad \Omega_\Lambda$$



$$D(t,\chi) = R(t)\,\chi \qquad D_A = r_0(\chi) / (1+z)$$

$$r(t,\chi) = R(t)\,S_k(\chi) \qquad D_L = r_0(\chi)(1+z)$$

- E.g. **D_L is 4 x D_A for an object at z=1.**

Tutorial: Time -- Redshift relation

$$x = 1 + z = \frac{R_0}{R}$$

$$\frac{dx}{dt} = -\frac{R_0}{R^2}\frac{dR}{dt}$$

$$= -\frac{R_0}{R}\frac{\dot{R}}{R}$$

$$= -x\ H(x)$$

$$\therefore\ dt = \frac{-dx}{x\ H(x)} = \frac{-dz}{(1+z)\ H(z)}$$

Memorise this derivation!

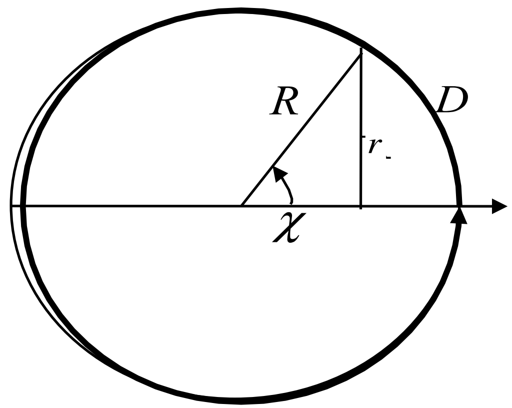
Hubble parameter : $H \equiv \frac{\dot{R}}{R}$

Tutorial:
Time and Distance vs Redshift

$$\frac{d}{dt}\left(x = 1 + z = \frac{R_0}{R} \right) \rightarrow \quad dt = \frac{-dx}{x\,H(x)}$$

Look - back time :

$$t(z) = \int_t^{t_0} dt = \int_{1+z}^{1} \frac{-dx}{x\,H(x)} = \int_1^{1+z} \frac{dx}{x\,H(x)}$$



Age: $t_0 = t(z \rightarrow \infty)$

Distance : $D = R\,\chi \qquad r = R\,S_k(\chi)$

$$\chi(z) = \int d\chi = \int_t^{t_0} \frac{c\,dt}{R(t)} = \frac{c}{R_0}\int_1^{1+z} \frac{R_0}{R(t)} \frac{dx}{x\,H(x)} = \frac{c}{R_0}\int_1^{1+z} \frac{dx}{H(x)}$$

Horizon : $\chi_H = \chi(z \rightarrow \infty)$

Need to know *R(t)*, or R_0 and *H(x)*.

Einstein's General Relativity

- 1. ***Spacetime geometry tells matter how to move***
  - gravity = effect of curved spacetime
  - free particles follow geodesic trajectories
    - $ds^2 < 0$ v < c time-like massive particles
    - $ds^2 = 0$ v = c null massless particles (photons)
    - $ds^2 > 0$ v > c space-like tachyons (not observed)

- 2. ***Matter (+energy) tells spacetime how to curve***
  - Einstein field equations
    - nonlinear
    - second-order derivatives of metric

with respect to space/time coordinates

8^{th} concept: Einstein Field Equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R^{\alpha}{}_{\alpha}\, g_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu} - \Lambda\, g_{\mu\nu}$$

$g_{\mu\nu} =$ spacetime metric ($ds^2 = g_{\mu\nu}\, dx^{\mu}\, dx^{\nu}$)

$G_{\mu\nu} =$ Einstein tensor (spacetime curvature)

$R_{\mu\nu} =$ Ricci curvature tensor

$R^{\alpha}{}_{\alpha} =$ Ricci curvature scalar

$G =$ Netwon's gravitational constant

$T_{\mu\nu} =$ energy - momentum tensor

$\Lambda =$ cosmological constant

Homogeneous perfect fluid

density ρ pressure p

Einstein field equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^2} \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} - \Lambda \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

---> Friedmann equations :

$$\dot{R}^2 = \left(\frac{8\pi G \rho + \Lambda}{3} \right) R^2 - k\,c^2 \qquad \textbf{energy}$$

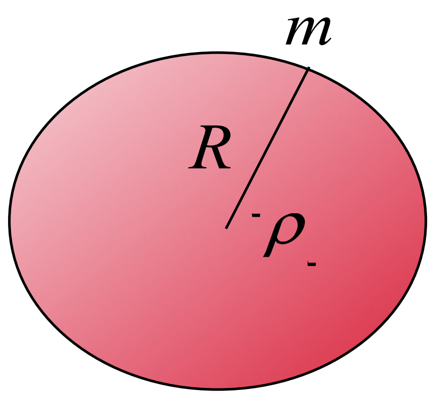
$$\ddot{R} = -\frac{4\pi G}{3c^2} (\rho c^2 + 3p)\, R + \frac{\Lambda}{3} R \qquad \textbf{momentum}$$

Note: energy density and pressure decelerate, Λ accelerates.

Reading: Newtonian Analogy

$$E = \frac{m}{2}\dot{R}^2 - \frac{G\,M\,m}{R} \qquad M = \frac{4\pi}{3} R^3 \rho$$

$$\dot{R}^2 = \frac{8\pi\,G}{3}\rho\,R^2 + \frac{2E}{m}$$

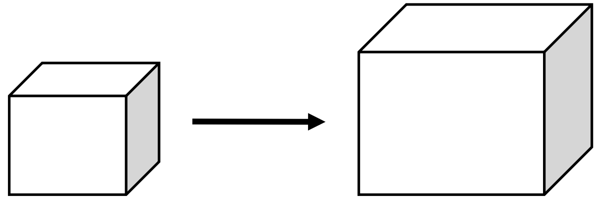


Friedmann equation:

$$\dot{R}^2 = \left(\frac{8\pi\,G\,\rho + \Lambda}{3}\right) R^2 - k\,c^2$$

same equation if $\rho \rightarrow \rho + \frac{\Lambda}{8\pi\,G}, \quad \frac{2E}{m} \rightarrow -k\,c^2$

Reading: Local Conservation of Energy



$$d[energy] = work$$

$$d[\rho\, c^2 R^3] = -p\, d[R^3]$$

$$\dot\rho\, c^2 R^3 + \rho\, c^2\,(3\,R^2 \dot R) = -p\,(3\,R^2 \dot R)$$

$$\dot\rho = -3\left(\rho + \frac{p}{c^2}\right)\frac{\dot R}{R} \qquad p = p(\rho) = \text{equation of state}$$

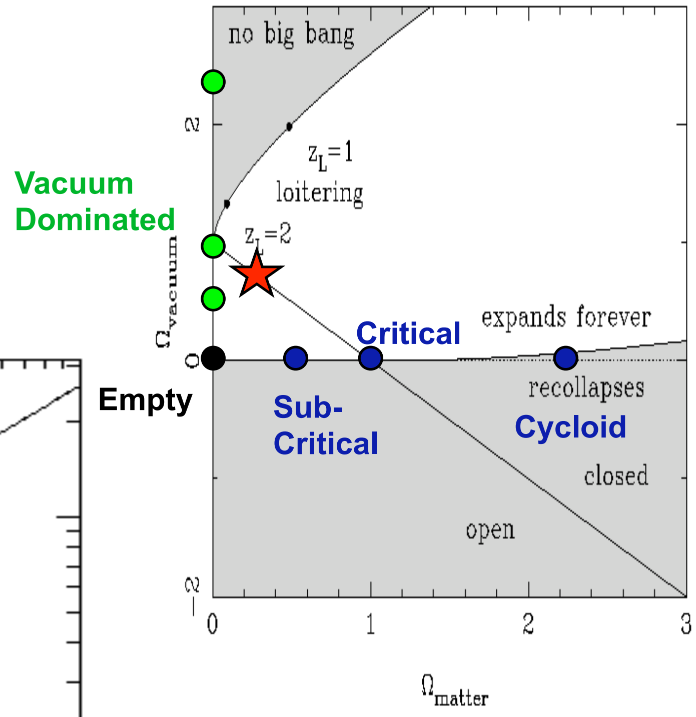
$$\text{Friedmann 1}: \dot R^2 = \frac{8\pi G}{3}\rho\, R^2 + \frac{\Lambda}{3}R^2 - k\,c^2$$

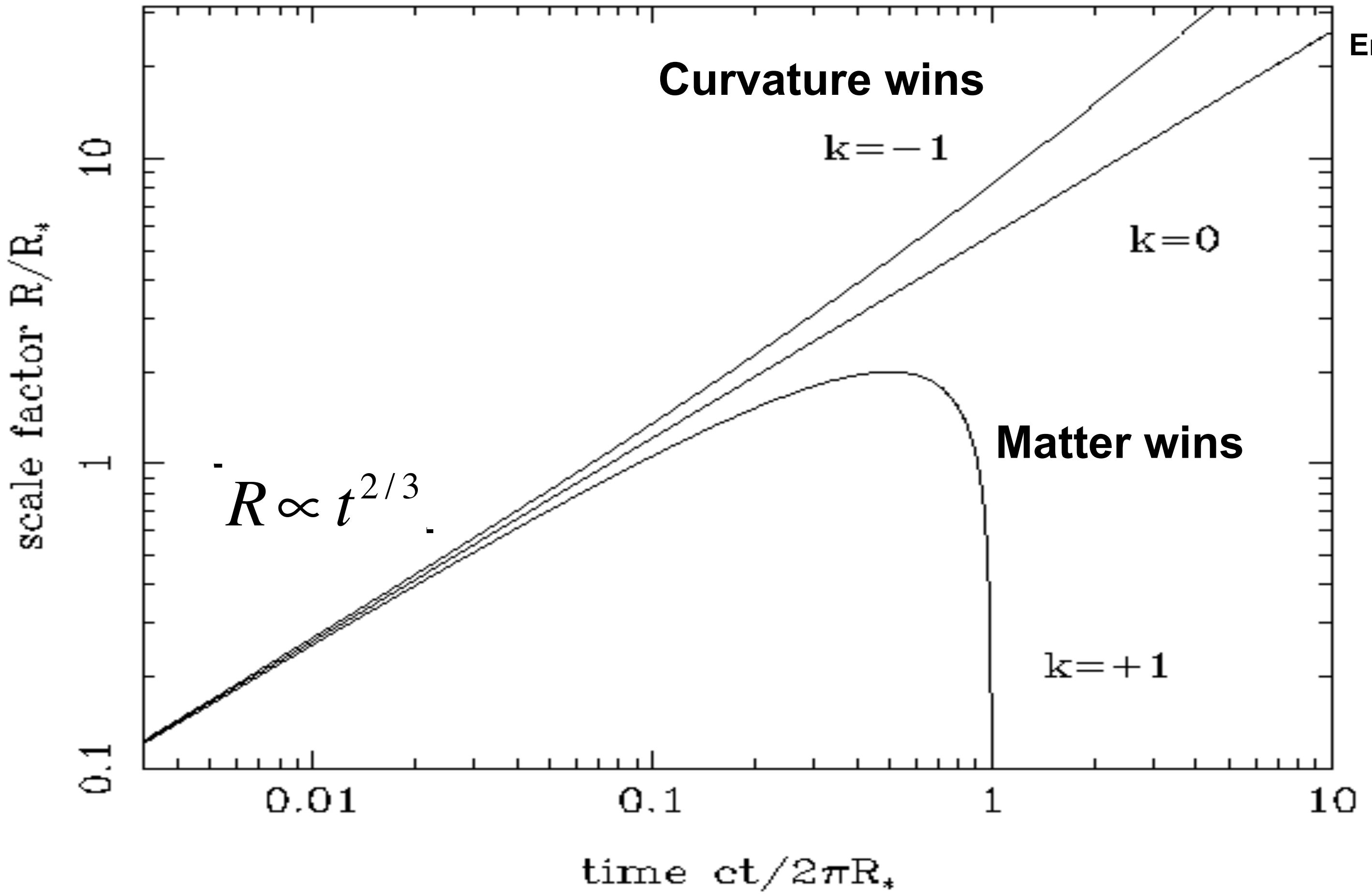
$$(2\,\dot R\,\ddot R) = \frac{8\pi G}{3}(\dot\rho\, R^2 + 2\,R\,\dot R\,\rho\,) + \frac{\Lambda}{3}(2\,R\,\dot R)$$

$$\ddot R = \frac{8\pi G}{3}\left(\frac{\dot\rho\, R^2}{2\,\dot R} + R\,\rho\right) + \frac{\Lambda}{3}R$$

$$\ddot R = -\frac{4\pi G}{3}\left(\rho + \frac{3\,p}{c^2}\right)R + \frac{\Lambda}{3}R = \text{Friedmann 2}$$

Reading: Matter-dominated Universes





All evolve as
$R \sim t^{2/3}$
at early times.

Critical Universe
(Einstein - de Sitter)

$$\Omega_M \equiv \frac{\rho_M}{\rho_c} = 1$$

$$\Omega_R = \Omega_\Lambda = 0 \quad \rightarrow \quad k = 0 \ \ (\text{ flat })$$

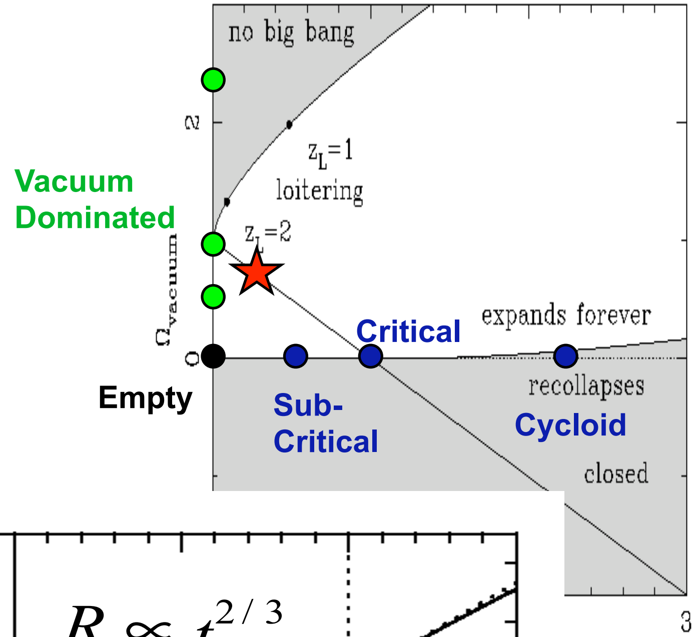
$$\rho = \frac{3\,H_0^{\,2}}{8\pi\,G}\left(\frac{R_0}{R}\right)^3$$

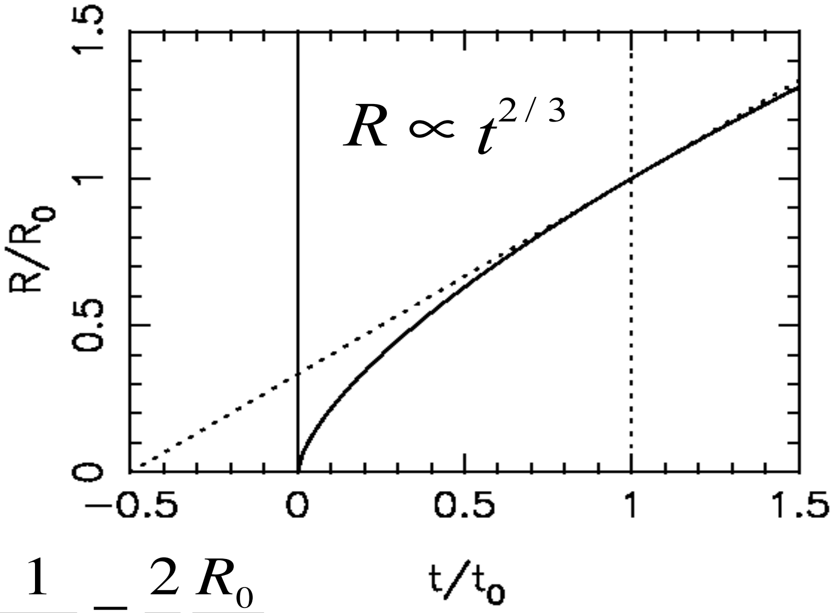
$$\dot{R}^2 = \frac{8\pi\,G}{3}\rho\,R^2 = \frac{H_0^{\,2}\,R_0^{\,3}}{R}$$

$$dR\;R^{1/2} = H_0\;R_0^{\,3/2}\;dt$$

$$\frac{2}{3}R^{3/2} = H_0\;R_0^{\,3/2}\;t$$

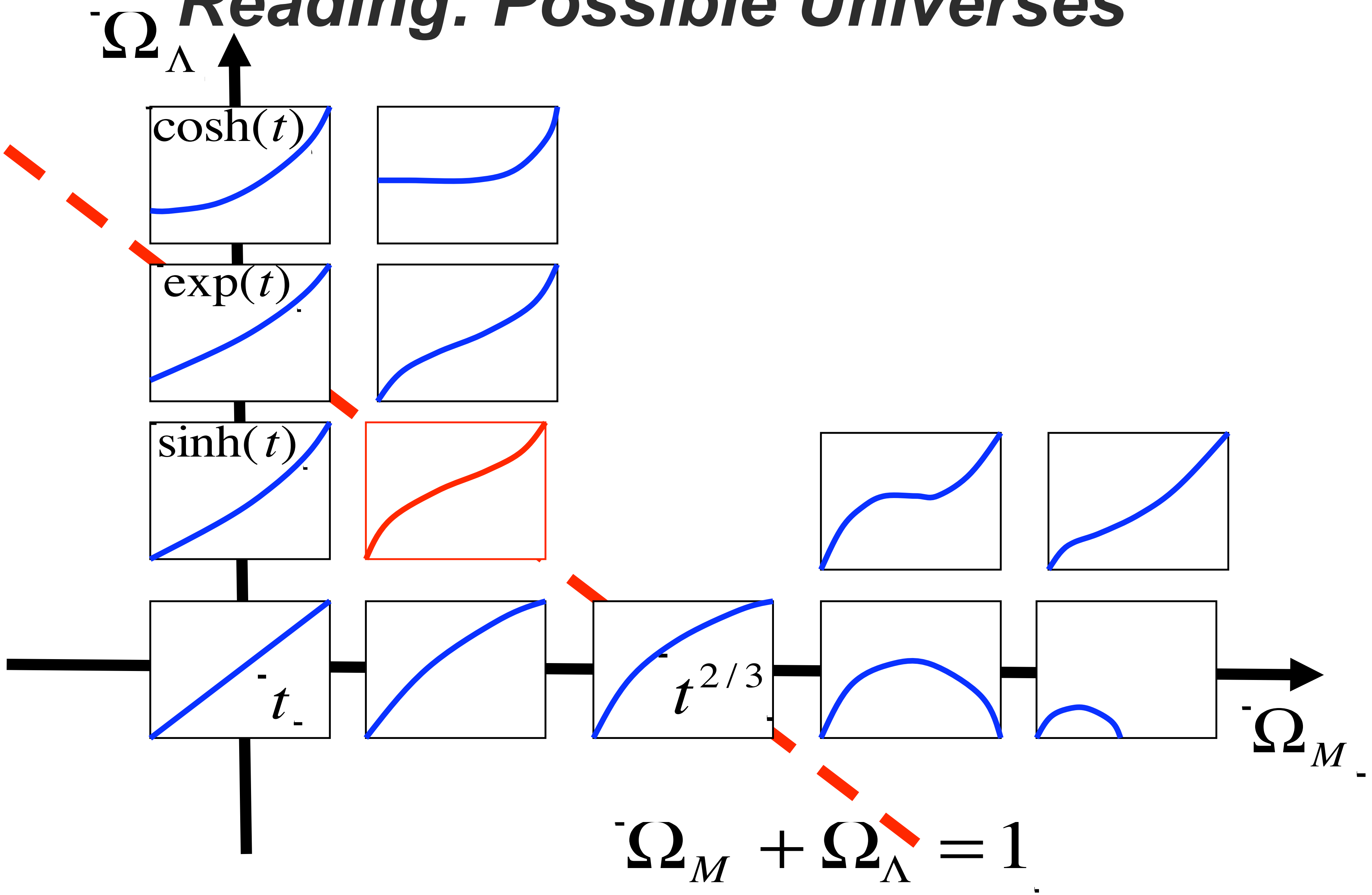
$$\frac{R}{R_0} = \left(\frac{t}{t_0}\right)^{2/3}, \qquad \text{age :} \quad t_0 = \frac{2}{3}\frac{1}{H_0} = \frac{2}{3}\frac{R_0}{c}$$





Matter decelerates expansion.

Reading: Possible Universes



Reading: Rad. => Matter =>Vacu.

$$\Omega(z) = \Omega_R\, x^4 + \Omega_M\, x^3 + \Omega_\Lambda = \sum_i \Omega_i\, x^{3(w_i+1)}$$

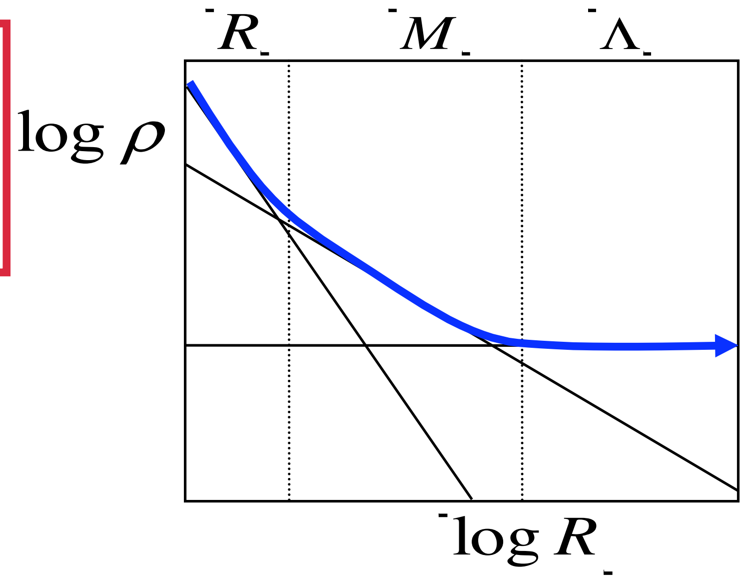
$$x \equiv 1 + z = R_0 / R \equiv a^{-1} \qquad w \equiv p / \rho c^2$$

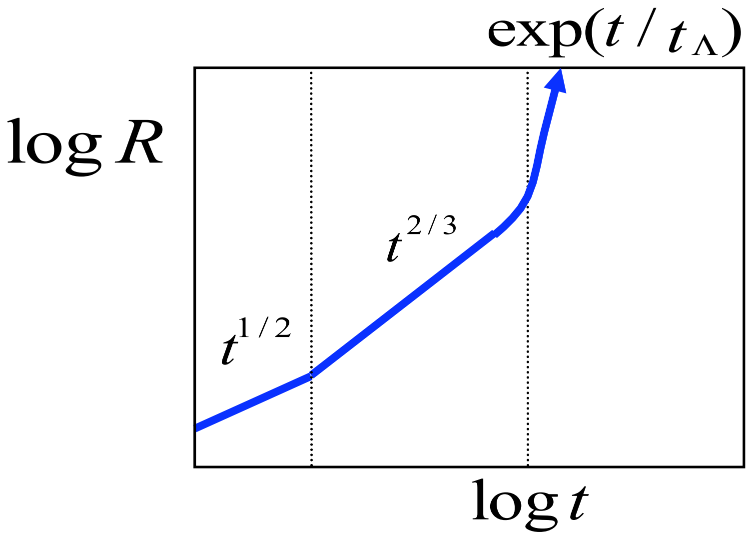
$$\Omega_R x^4 = \Omega_M x^3 \quad \rightarrow \quad x = \Omega_M / \Omega_R$$

$$\rightarrow \quad z = \left(\frac{\Omega_M}{\Omega_R}\right) - 1 = \frac{0.3}{8.4\times10^{-5}} \approx 3600$$

$$\Omega_M x^3 = \Omega_\Lambda \rightarrow \quad x^3 = \Omega_\Lambda / \Omega_M$$

$$\rightarrow \quad z = \left(\frac{\Omega_\Lambda}{\Omega_M}\right)^{\frac{1}{3}} - 1 = \left(\frac{0.7}{0.3}\right)^{\frac{1}{3}} - 1 \approx 0.33$$





Tutorial: What observations justify the "Concordance" Parameters?

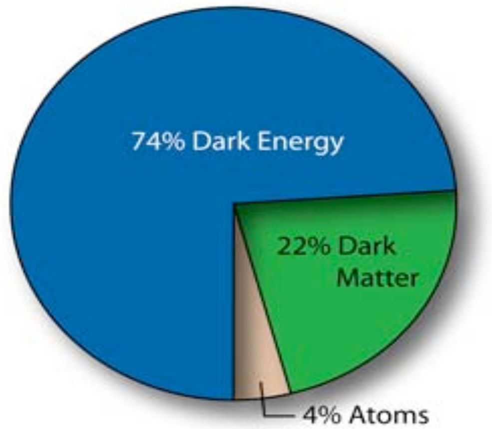
$$H_0 \equiv 100\ h\ \frac{\text{km/s}}{\text{Mpc}} \approx 70\frac{\text{km/s}}{\text{Mpc}} \qquad h \approx 0.7$$

$$\Omega_R \approx 4.2\times10^{-5}\,h^{-2} \approx 8.4\times10^{-5}\ (CMB\ \ photons + neutrinos)$$

$$\Omega_B \sim 0.02\,h^{-2} \sim 0.04 \quad (baryons)$$

$$\Omega_M \sim 0.3 \quad (Dark\ Matter)$$

$$\Omega_\Lambda \sim 0.7 \quad (Dark\ Energy)$$



$$\Omega_0 \equiv \Omega_R + \Omega_M + \Omega_\Lambda = 1.0 \quad \rightarrow \quad Flat\ Geometry$$

9^{th} Concept: "Concordance" Model

Three main constraints:

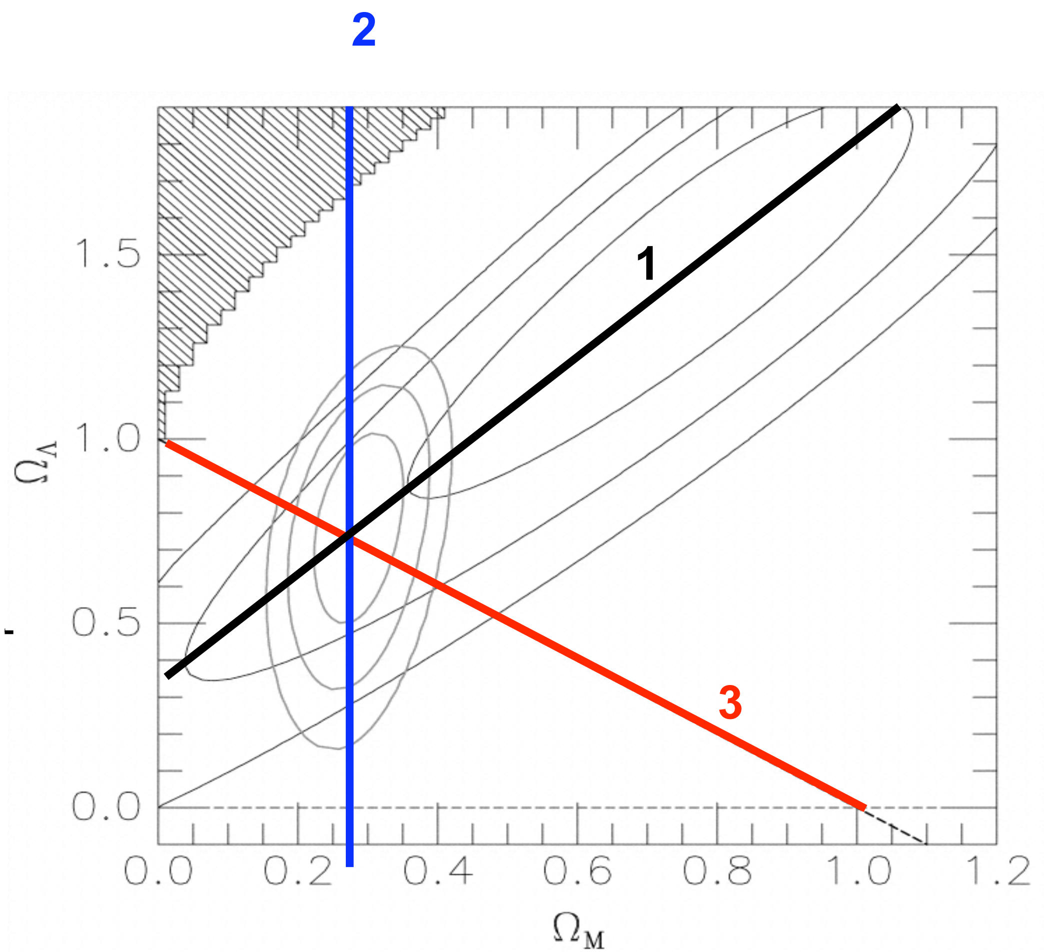
1. Supernova Hubble Diagram
2. Galaxy Counts + M/L ratios

   $\Omega_M \sim 0.3$

3. Flat Geometry

   (inflation, CMB fluctuations)

   $\Omega_0 = \Omega_M + \Omega_\Lambda = 1$



concordance model

$$H_0 \approx 72 \quad \Omega_M \approx 0.3 \quad \Omega_\Lambda \approx 0.7$$

Beyond H_0

- Globular cluster ages: $t < t_0$ --> acceleration
- Radio jet lengths: $D_A(z)$ --> deceleration
- Hi-Redshift Supernovae: $D_L(z)$ --> acceleration
- Dark Matter estimates ---> $\Omega_M \sim 0.3$
- Inflation ---> Flat Geometry $\Omega_0 \approx 1.0$
- CMB power spectra
- "Concordance Model" $\Omega_M \sim 0.3 \quad \Omega_\Lambda \sim 0.7$

$$\Omega_0 = \Omega_M + \Omega_\Lambda \approx 1.0$$

Deceleration parameter

$$a(t) \equiv \frac{R(t)}{R_0} = 1 + H_0\,(t - t_0) - \frac{q_0}{2} H_0^{\,2}\,(t - t_0)^2 + \ldots$$

Friedmann momentum equation :

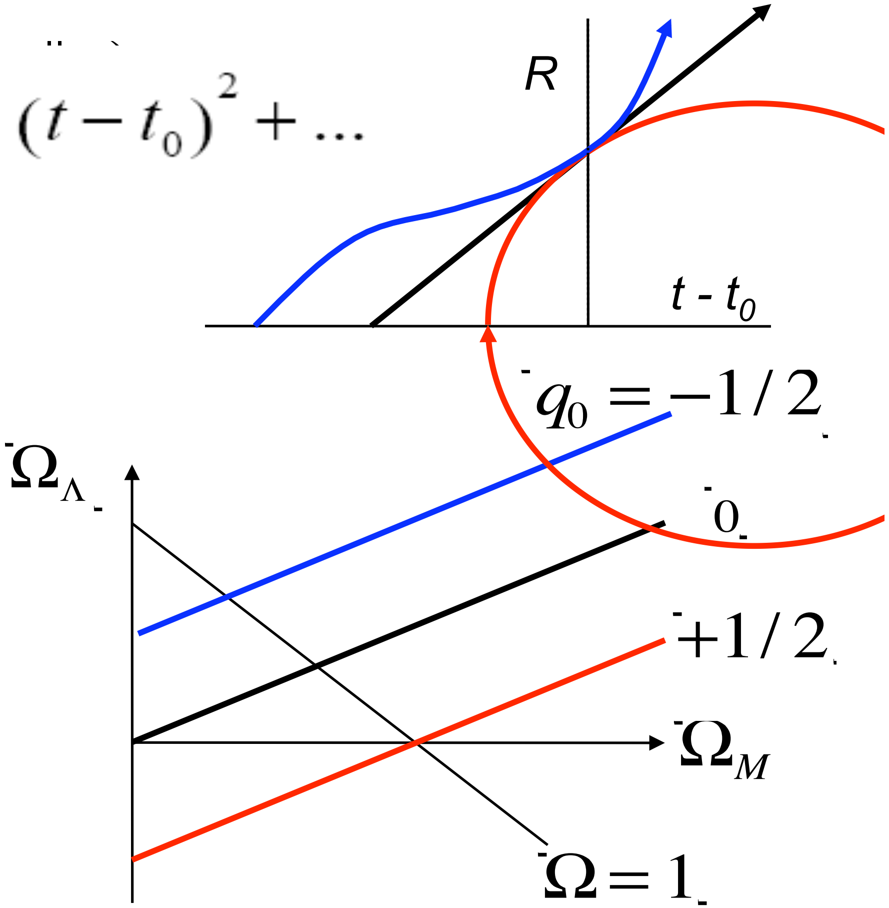
$$\ddot{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) R + \frac{\Lambda}{3} R$$

$$\frac{\ddot{R}}{H_0^{\,2} R} = -\frac{4\pi G}{3 H_0^{\,2}} \rho (1 + 3w) + \frac{\Lambda}{3 H_0^{\,2}}$$

$\rho, p > 0$ decelerate, $\Lambda > 0$ accelerates

Equation of state : $p = \sum_i w_i\, \rho_i\, c^2$

$$w_R = \frac{1}{3} \quad w_M = 0 \quad w_\Lambda = -1$$



$$q_0 = -\left(\frac{\ddot{R}}{R\,H^2}\right)_0 = \sum_i \left(\frac{1 + 3\,w_i}{2}\right) \Omega_i = \Omega_R + \frac{\Omega_M}{2} - \Omega_\Lambda$$

- $q_0 > 0$ => **deceleration**
- $q_0 = 0$ => *constant velocity*
- $q_0 < 0$ => **acceleration**

Deceleration Parameter

$$q_0 \equiv -\left(\frac{\ddot{R}R}{\dot{R}^2}\right)_0 = \frac{\Omega_M}{2} - \Omega_\Lambda$$

Matter decelerates

Vacuum (Dark) Energy accelerates

Measure q_0 via :

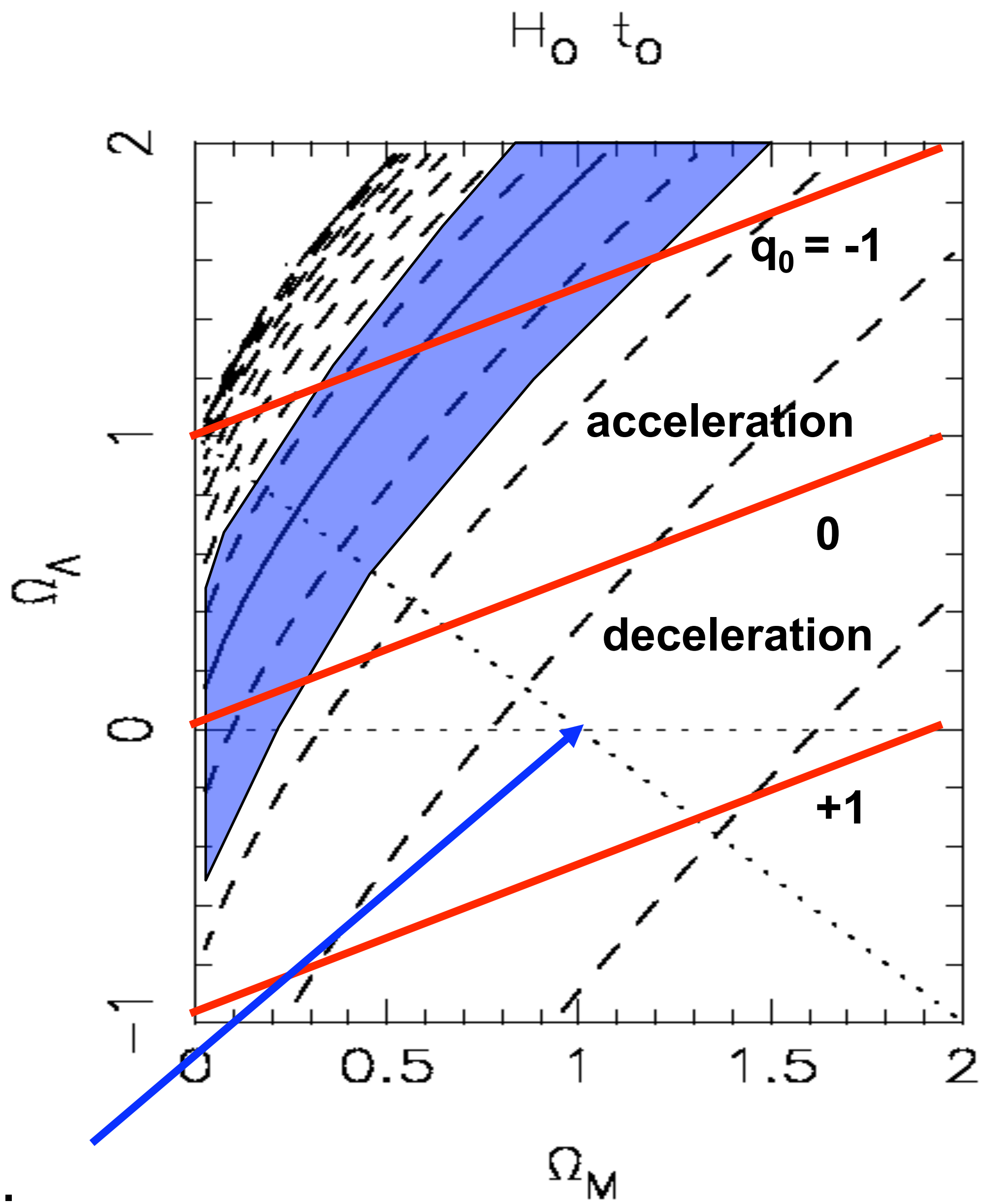
1 . $D_A(z)$

(e.g. radio jet lengths)

2. $D_L(z)$

(curvature of Hubble Diagram)

Critical density matter-only --> q_0=1/2.



Tutorial: Observable Distances

angular diameter distance :

$$\theta = \frac{l}{D_A} \qquad D_A = \frac{r_0}{(1+z)} = \frac{c\,z}{H_0}\left(1 - \frac{q_0+3}{2}\,z + \ldots\right)$$

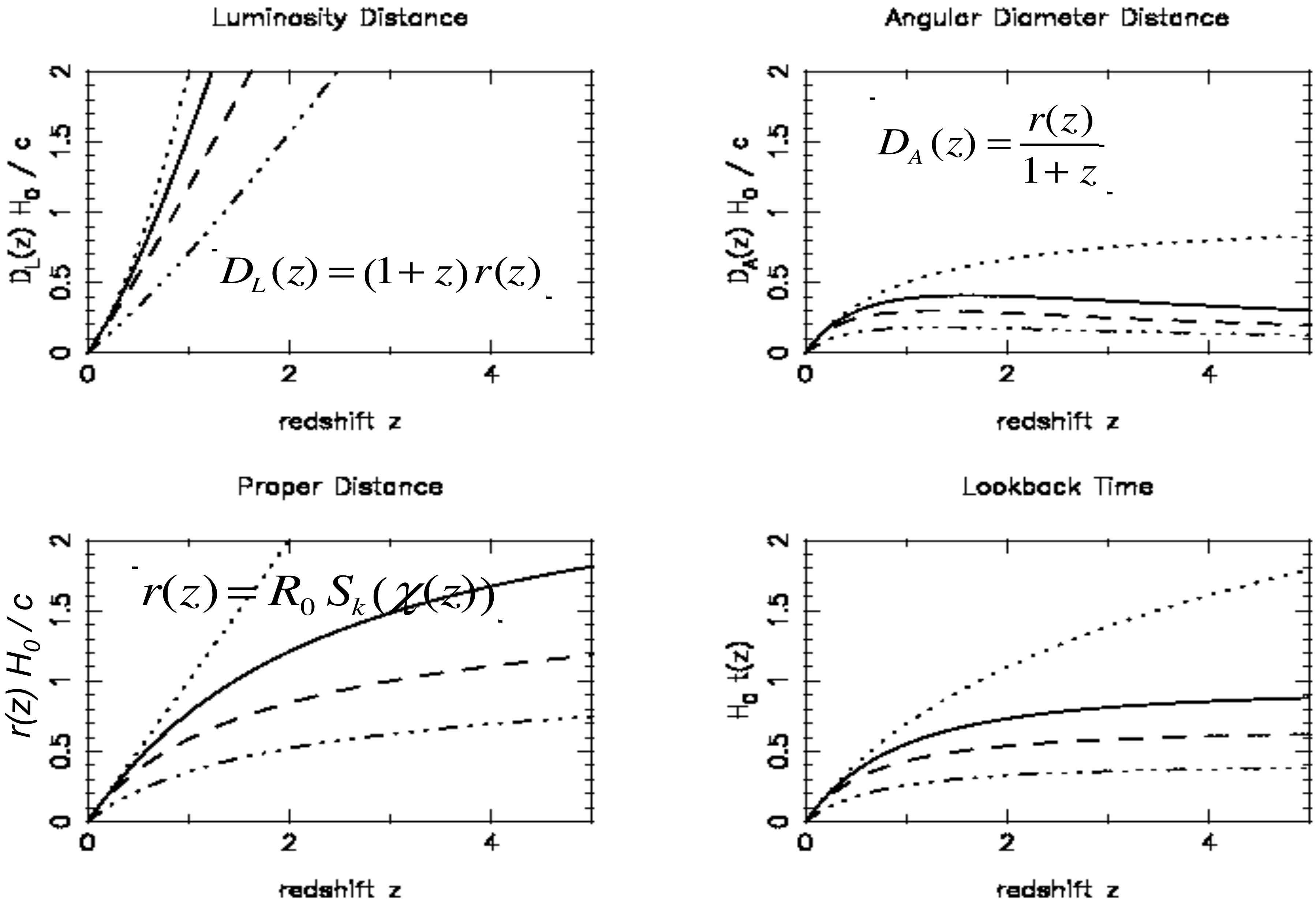
luminosity distance :

$$F = \frac{L}{4\pi\,D_L^{\;2}} \qquad D_L = r_0\,(1+z) = \frac{c\,z}{H_0}\left(1 + \frac{1-q_0}{2}\,z + \ldots\right)$$

deceleration parameter :

$$q_0 = \frac{\Omega_M}{2} - \Omega_\Lambda$$

Verify these low-z expansions.


Luminosity Distance
$D_L(z)\,H_0\,/\,c$
$D_L(z) = (1+z)\,r(z)$
redshift z
Angular Diameter Distance
$D_A(z)\,H_0\,/\,c$
$D_A(z) = \frac{r(z)}{1+z}$
redshift z
Proper Distance
$r(z)\,H_0\,/\,c$
$r(z) = R_0\,S_k(\chi(z))$
redshift z
Lookback Time
$H_0\,t(z)$
redshift z

Reading: Hubble Diagram

$$m = M + 5\log\left(\frac{D_L(z)}{\mathrm{Mpc}}\right) + 25 + A + K(z)$$

$m =$ apparent mag

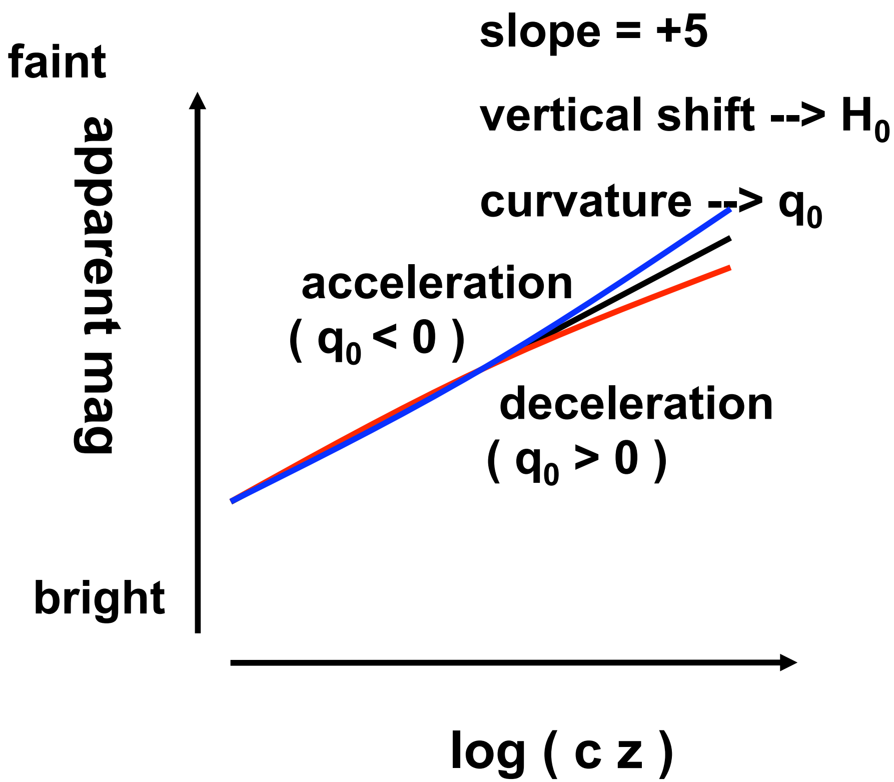
$M =$ absolute mag

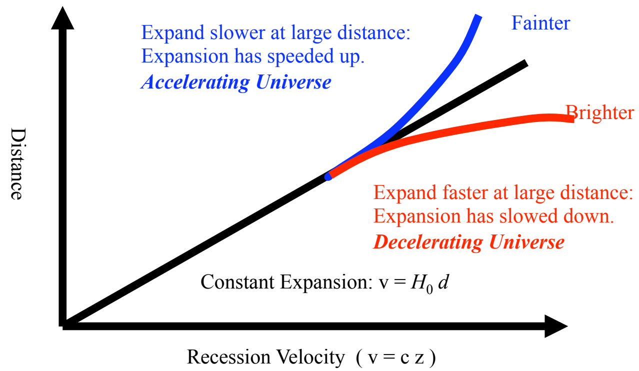
$A =$ extinction (dust in galaxies)

$K(z) =$ K correction

(accounts for redshift of spectra relative to observed bandpass)

$$D_L(z) = \frac{c\,z}{H_0}\left(1 + \frac{1-q_0}{2}z + \ldots\right)$$





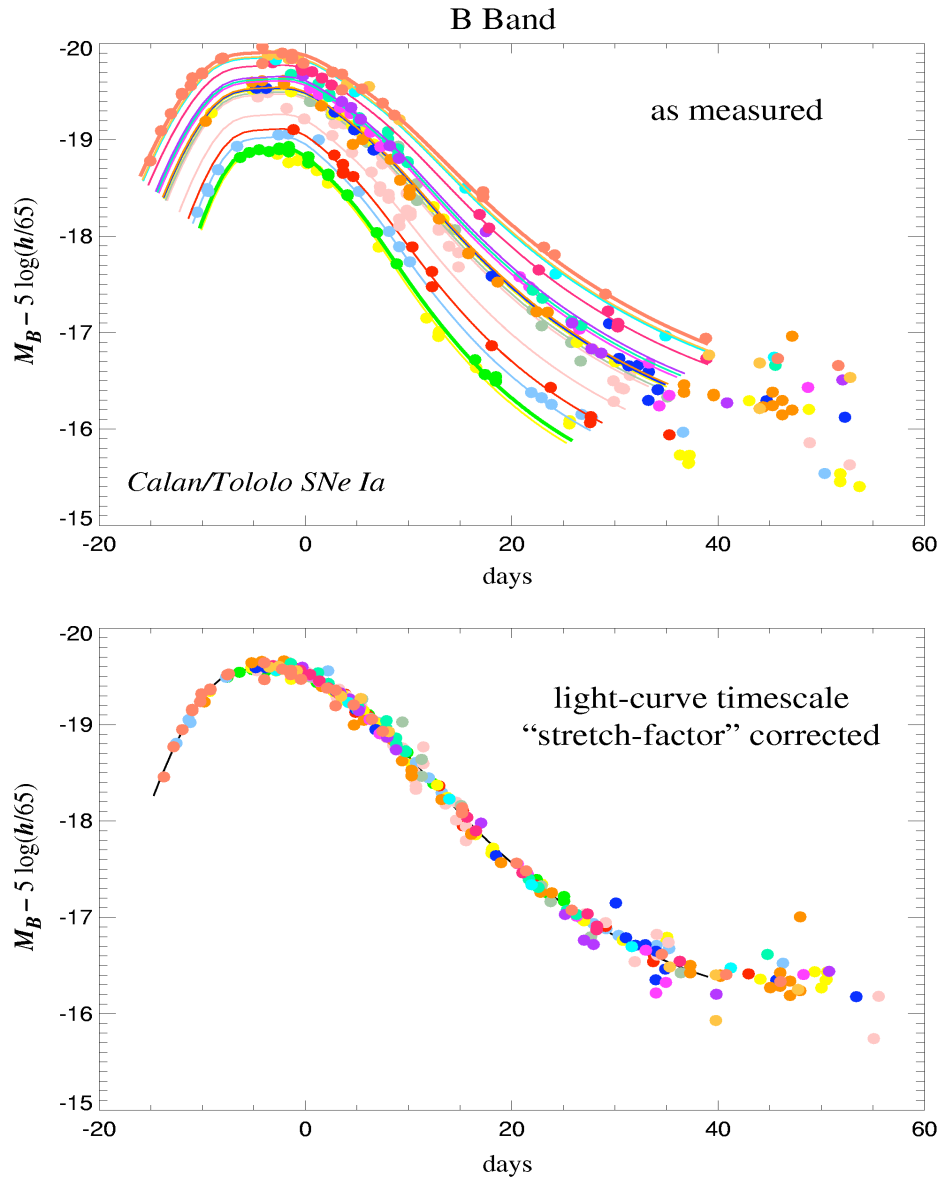
Calibrating "Standard Bombs"

1. Brighter ones decline more slowly.

2. Time runs slower by factor (1+z).

AFTER correcting:
Constant peak brightness
M_B = -19.7

Observed peak magnitude: m = M + 5 log (d/Mpc) + 25
gives the distance!



Kim, *et al.* (1997)

Absolute magnitude M_B ==>

Time ==>

SN Ia at z ~ 0.8 are ~25% fainter than expected

Acceleration (! ?)

1. Bad Observations?

-- 2 independent teams agree

1. Dust ?

-- corrected using reddening

2. Stellar populations ?

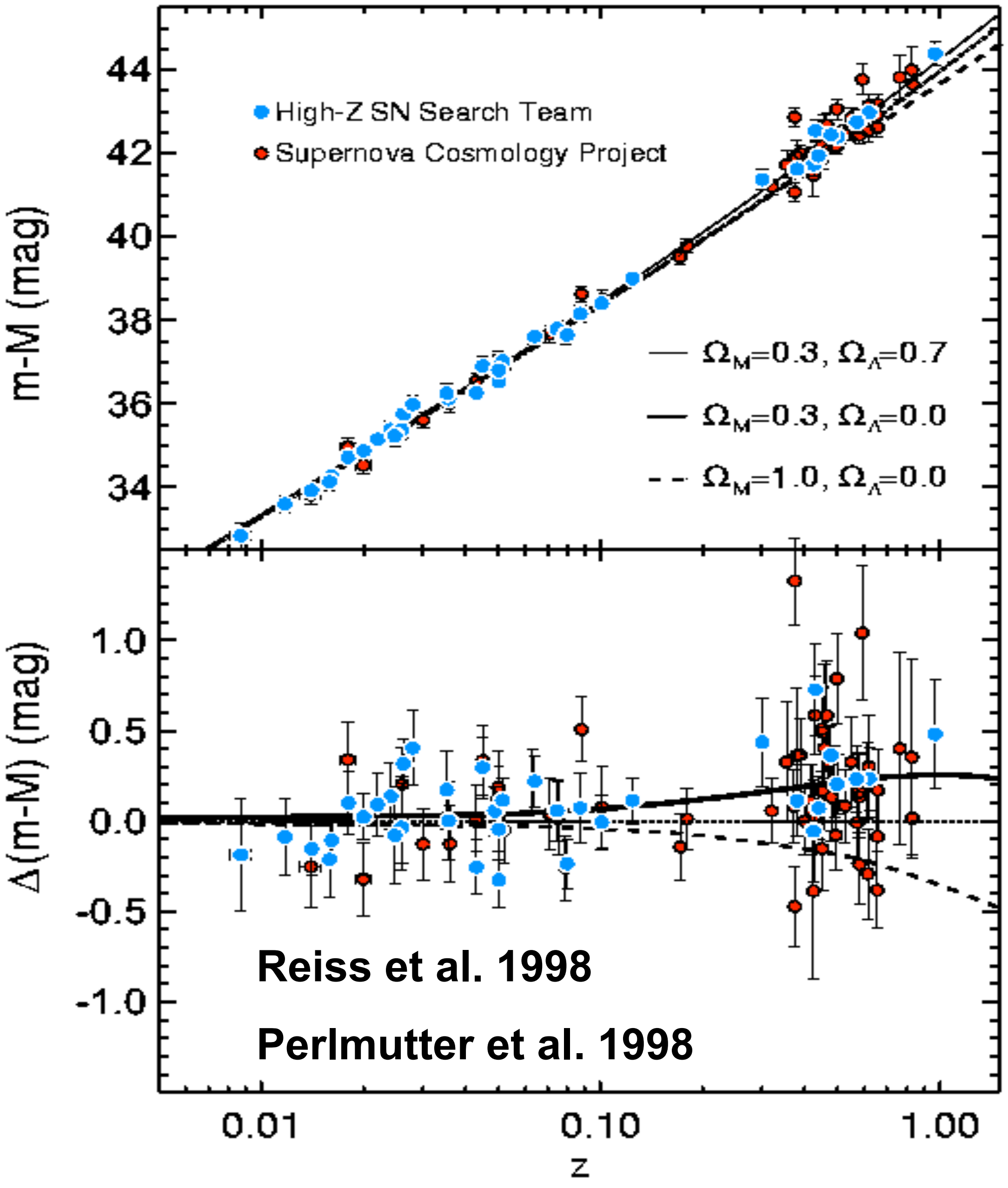
-- earlier generation of stars

-- lower metalicity

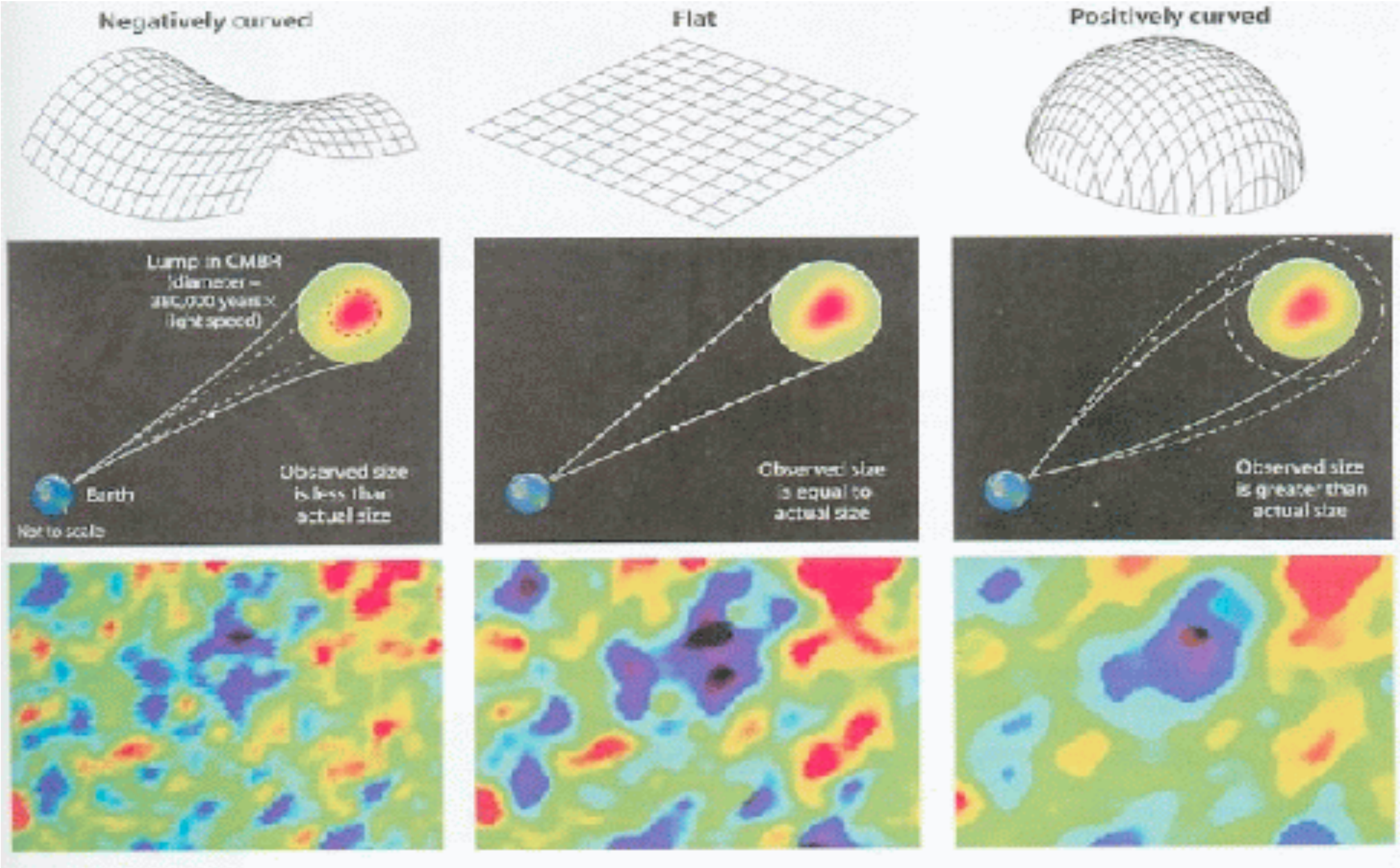
3. Lensing?

-- some brighter, some fainter

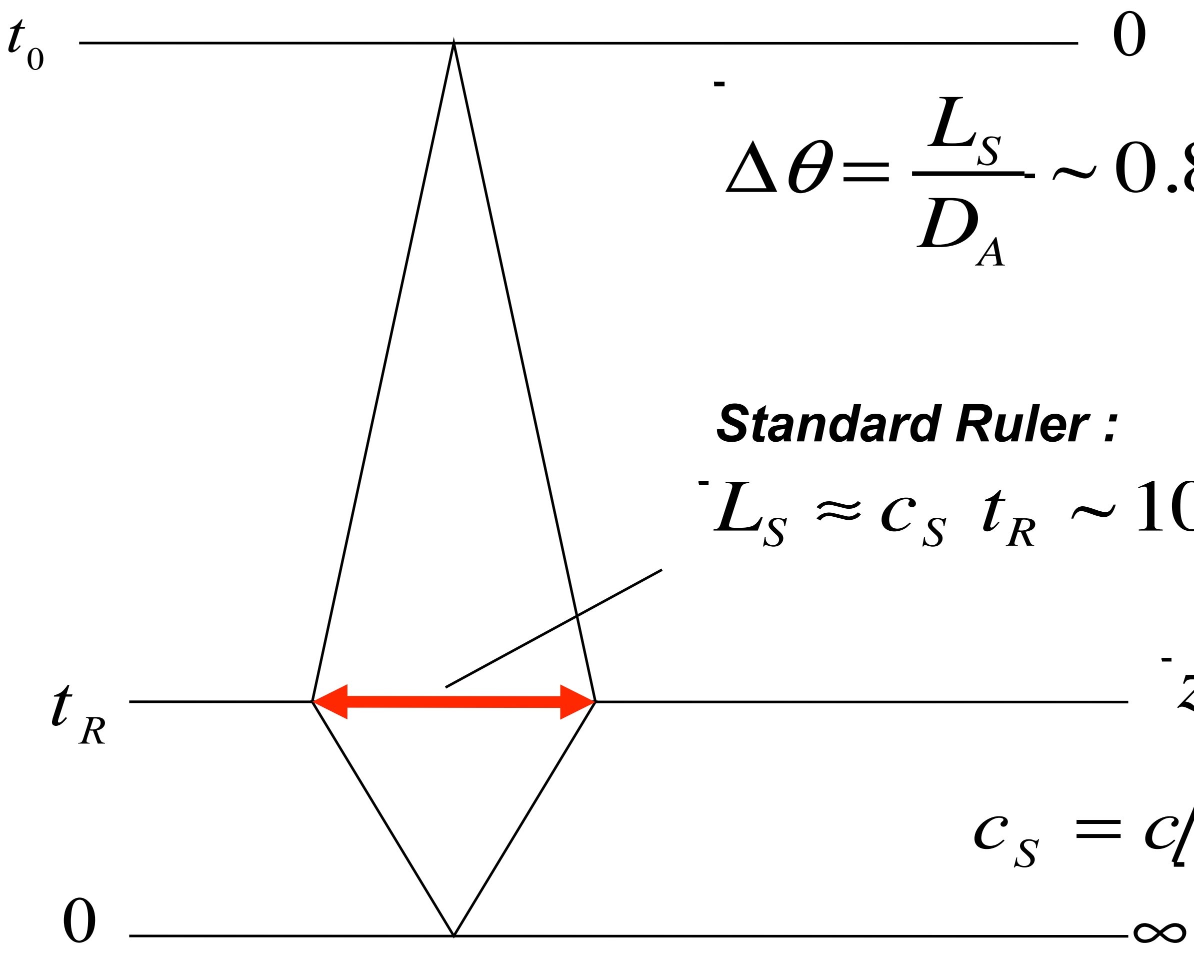
-- effect small at z ~ 0.8



CMB Angular scale --> Geometry



10^{th} Concept: Sound Horizon at z = 1100



$$\Delta\theta = \frac{L_S}{D_A} \sim 0.8^\circ$$

Standard Ruler :

$$L_S \approx c_S \; t_R \sim 100 \text{ kpc}$$

$$z_R \approx 1100$$

$$c_S = c/\sqrt{3}$$

Tutorial: Sound Horizon at z=1100

distance travelled by a sound wave

$$c_S \, dt$$

expand each step by factor $R(t_R)/R(t)$:

$$L_S(t_R) = R(t_R) \int_0^{t_R} \frac{c_S \, dt}{R(t)}$$

$$= \frac{R_0}{1+z} \int_{1+z}^{\infty} \frac{x}{R_0} \frac{c_S \, dx}{x \, H(x)}$$

$$= \frac{c_S}{(1+z)} \int_{1+z}^{\infty} \frac{dx}{H(x)}$$

$$= \frac{c_S}{(1+z) \, H_0} \int_{1+z}^{\infty} \frac{dx}{\sqrt{x^4 \, \Omega_R + x^3 \, \Omega_M + \Omega_\Lambda + (1-\Omega_0) \, x^2}}$$

$$\approx \frac{c_S}{(1+z) \, H_0} \int_{1+z}^{\infty} \frac{dx}{\sqrt{x^4 \, \Omega_R + x^3 \, \Omega_M}}$$

recombination
at ***z = 1100***

dt = - dx / x H(x)
R(t) = R₀ / x

***H(x)* from Friedmann Eqn.**

keep 2 largest terms.

$$x \equiv 1 + z = \frac{R_0}{R(t)}$$

$$dt = \frac{-dx}{x \, H(x)}$$

sound speed

$$c_S \approx \frac{c}{\sqrt{3}}$$

Sound Horizon at z = 1100

$$L_S(t_R) = \frac{c_S}{(1+z)} \int_{1+z}^{\infty} \frac{dx}{H(x)} \approx \frac{c_S}{(1+z)\,H_0} \int_{1+z}^{\infty} \frac{dx}{\sqrt{x^4\,\Omega_R + x^3\,\Omega_M}}$$

$$= \frac{c_S}{(1+z)\,H_0\sqrt{\Omega_R}} \int_{1+z}^{\infty} \frac{dx}{\sqrt{x^3(x+x_0)}} \qquad x_0 \equiv \frac{\Omega_M}{\Omega_R} \approx 3500\left(\frac{\Omega_M}{0.3}\right)$$

$$= \frac{c_S}{(1+z)\,H_0\sqrt{\Omega_R}} \left(-\frac{2}{x_0}\sqrt{1+\frac{x_0}{x}}\right)_{1+z}^{\infty}$$

$$= \frac{2c_S}{(1+z)\,H_0\sqrt{\Omega_M\,x_0}} \left(\sqrt{1+\frac{x_0}{1+z}} - 1\right) \qquad c_S = \frac{c}{\sqrt{3}}$$

$$= \frac{c}{H_0} \frac{2\left(\sqrt{4.6}-1\right)}{1100\sqrt{3\times 0.3\times 3500}}$$

$$= 3.4\times 10^{-5}\,\frac{c}{H_0} \approx 110\,\left(\frac{0.7}{h}\right)\left(\frac{0.3}{\Omega_M}\right)^{1/2}\ \text{kpc}$$

Expands by factor 1 + z = 1100 to ~120 Mpc today.

Reading: Angular scale → W₀

sound horizon :

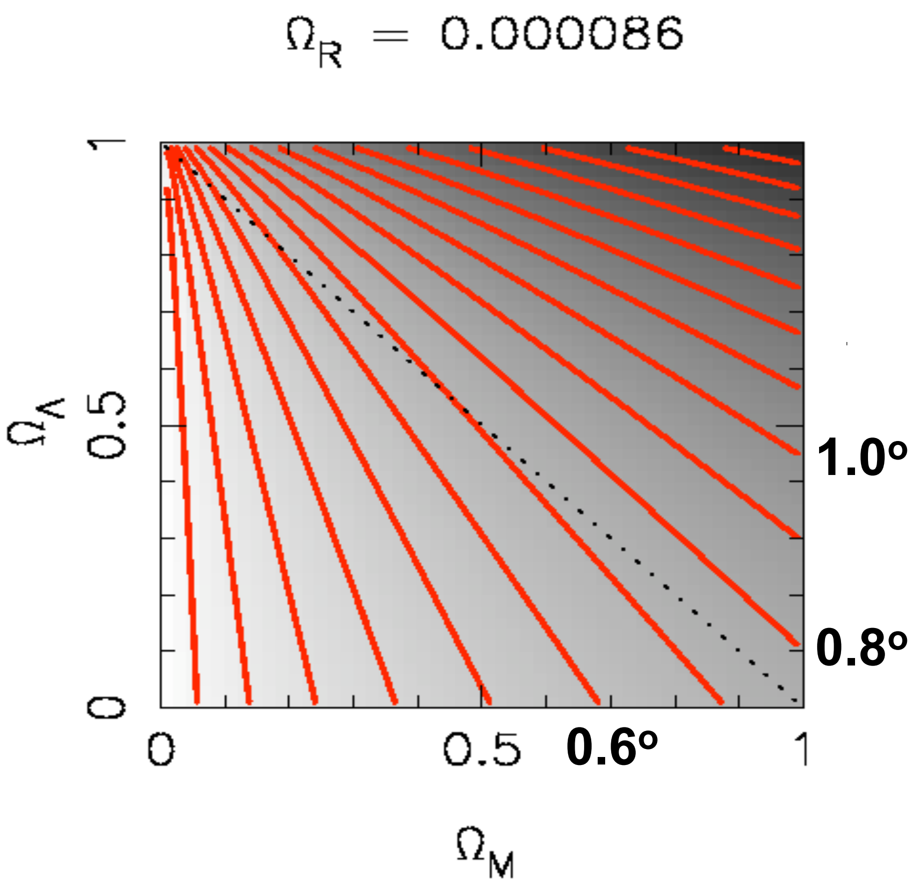
$$L_S(z) = \frac{1}{1+z} \int_{1+z}^{\infty} \frac{c_S \, dx}{H(x)}$$

angular diameter distance :

$$D_A(z) = \frac{R_0 \; S_K(\chi)}{1+z} \qquad \chi = \int_t^{t_0} \frac{c \, dt}{R(t)} = \frac{c}{R_0} \int_1^{1+z} \frac{dx}{H(x)}$$

angular scale

$$\theta = \frac{L_S(z)}{D_A(z)} = \frac{\displaystyle\int_{1+z}^{\infty} \frac{c_S \, dx}{H(x)}}{R_0 \; S_k\left(\dfrac{c}{R_0} \displaystyle\int_1^{1+z} \frac{dx}{H(x)} \right)}$$

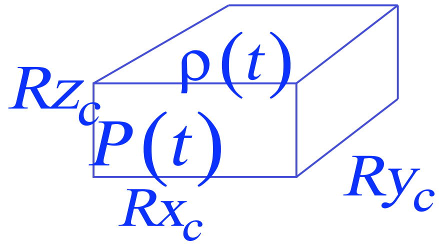


Angular scale depends mainly on the curvature.

Gives theta~ 0.8° for flat geometry,
***Omega* =1**

Evolution of Sound Speed

Expand a box of fluid



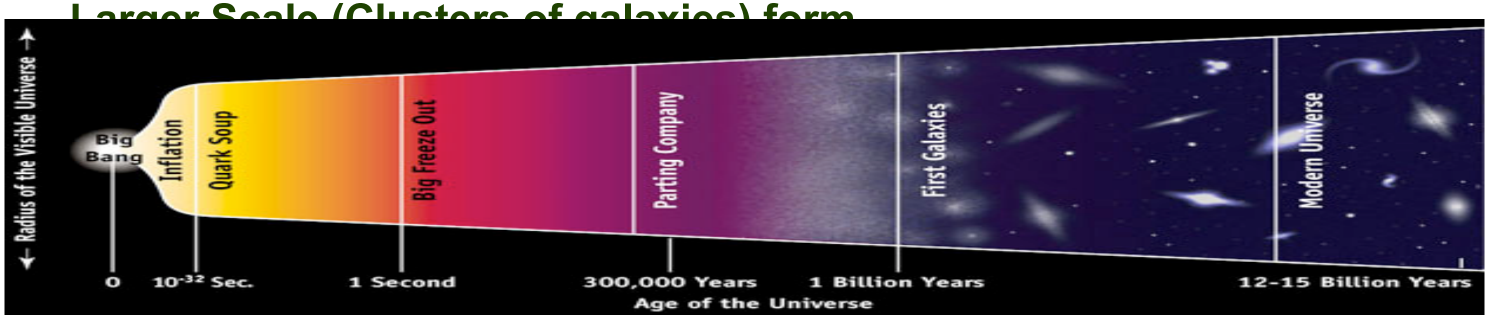
Sound Speed

$$C_s^2 \equiv \frac{\partial P / \partial (vol)}{\partial \rho / \partial (vol)}, = \frac{\partial P / \partial R}{\partial \rho / \partial R}$$

$$Vol = R^3(t) \cdot x_c y_c z_c \propto R^3(t)$$

Brief History of Universe

- **Inflation**
  - Quantum fluctuations of a tiny region
  - Expanded exponentially
- **Radiation cools with expansion T ~ 1/R ~$t^{-2/n}$**
  - He and D are produced (lower energy than H)
  - Ionized H turns neutral (recombination)
  - Photon decouple (path no longer scattered by electrons)
- **Dark Matter Era**
  - Slight overdensity in Matter can collapse/cool.
  - Neutral transparent gas
- **Lighthouses (Galaxies and Quasars) form**
  - UV photons re-ionize H
  - Larger Scale (Clusters of galaxies) form



Coupled radiation-baryon relativistic fluid

	Radiation	Matter
Where fluid density $\rho(t) =$	ρ_r	ρ_m
Fluid pressure $P(t) =$	$\frac{c^2}{3}\rho_r$	$\frac{\rho_m}{\mu} \cdot KT_m$

Matter number density

Random motion energy
Non-Relativistic
IDEAL GAS

Note $\rho_r \propto R^{-4}$

$$\rho_m \propto R^{-3}$$

Neglect $\frac{1}{\mu} KT_m << c^2$

Tutorial: Show $C^2_s = c^2/3 \,/(1+Q)$, $Q = (3\,\rho_m)/(4\,\rho_r)$, ➔ C_s drops

from c/sqrt(3) at radiation-dominated era

to c/sqrt(Y) at dark-matter-radiation equality

Reading: Sound Speed & Gas Temperature: $Cs^2 \sim T \sim T_{ph} \sim 1500K*(z/500)$ before decouple.

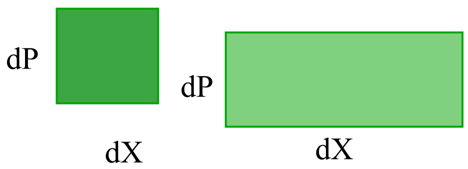
But After decoupling ($z<500$),
$Cs \sim 6\,(1+z)$ m/s because

$\underline{d}^3\underline{P}\ \underline{d}_x^3$ invarient phase space volume

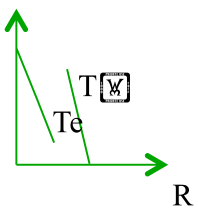
So: $P \propto x^{-1} \propto R^{-1}$

$$\frac{3}{2}\times T_e = m\upsilon^2/2 \propto R^{-2}$$

$$Te \sim 1500\,K?\left(\frac{1+z}{500}\right)^2$$



$$T_e \infty C_s^2 \infty R^{-2}$$



Except reionization $z \sim 10$ by stars quasars

What have we learned?

What determines the patterns of CMB at last scattering

- Analogy as patterns of fine sands on a drum at last hit.

Heading to: origins of perturbations:

- inflation brings patches in contact
- Creates a flat universe naturally.

11^{th} Concept: Inflation in Early Universe

Consider universe goes through a phase with

$\rho(t) \sim R(t)^{-n}$

$R(t) \sim t^q$ *where* q=2/n

Problems with normal expansion theory (n=2,3,4):

What is the state of the universe at t→0? Pure E&M field (radiation) or exotic scalar field?

Why is the initial universe so precisely flat?

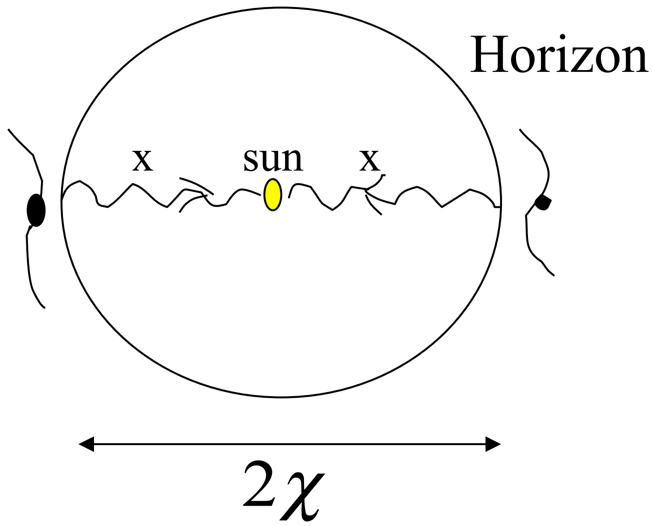
What makes the universe homogeneous/similar in opposite directions of horizon?

Solutions: Inflation, i.e., n=0 or n<2

Maybe the horizon can be pushed to infinity?

Maybe there is no horizon?

Maybe everything was in Causal contact at early times?



Why are these two galaxies so similar without communicating yet?

$$\frac{\varepsilon_K(z)}{\varepsilon\ (z)} = \frac{\varepsilon_K(0)\times R^{-2}}{\varepsilon(0)\times R^{-n}} \sim R^{n-2} \sim 0 \text{ at } t=0$$

Why is the curvature term so small (universe so flat) at early universe if radiation dominates n=4 >2?

Inflation broadens Horizon

Light signal travelling with speed c on an expanding sphere R(t), e.g., a fake universe R(t)=1 lightyr (t/1yr)^q

Emitted from time t_i

By time t=1yr will spread across (co-moving coordinate) angle $\mathbf{c}_c$

Horizon in co-moving coordinates

$$x_c = \int_{t_i}^{1} \frac{\mathrm{cdt}}{\mathrm{R(t)}} = \int_{t_i}^{1} \frac{\mathrm{cdt}}{\mathrm{t}^{q}} = \frac{(1^{1-q} - t_i^{1-q})}{(1-q)}$$

Normally $x_c < \frac{1}{(1-q)}$ is finite if q=2/n<1

(e.g., n=3 matter-dominate or n=4 photon-dominate)

INFLATION phase $x_c = \frac{(t_i^{1-q} - 1)}{(q-1)}$ can be very large for very small t_i if q=2/n>1

(e.g., $t_i = 0.01, q = 2, x_c = 99 >> \pi$, Inflation allows we see everywhere)

Tutorial: Inflation dilutes any initial curvature of a quantum universe

$$\frac{\varepsilon_K(R)}{\varepsilon\ (R)} = \frac{\varepsilon_K(R_i)}{\varepsilon(R_i)}\left(\frac{R}{R_i}\right)^{n-2}$$ ~ 0 (for n<2) sometime after R>>R_i

even if initially the universe is curvature-dominated $\frac{\varepsilon_K(R_i)}{\varepsilon(R_i)} = 1$

E.g.

If a toy universe starts with $\frac{\varepsilon_K(R_i)}{\varepsilon(R_i)} = 0.1$ inflates from $t_i=10^{-40}$sec to $t_f=1$sec with n=1,

and then expand normally with n=4 to t=1 year,

SHOW at this time the universe is far from curvature-dominated.

Exotic Pressure drives Inflation

$$P = -\frac{d(\rho c^2 R^3)}{d(R^3)}$$

=>

$$\frac{\rho}{3} + \frac{P}{c^2} = -\frac{d(\rho R^2)}{3RdR} = \frac{n-2}{3}\rho \quad \text{if } \rho \sim R^{-n}$$

=>

$P/\rho c^2 = (n-3)/3$

Inflation $n < 2$ requires exotic (negative) pressure,

define $w = P/\rho c^2$, then $w = (n-3)/3 < 0$,

Verify negligble pressure for cosmic dust (matter),

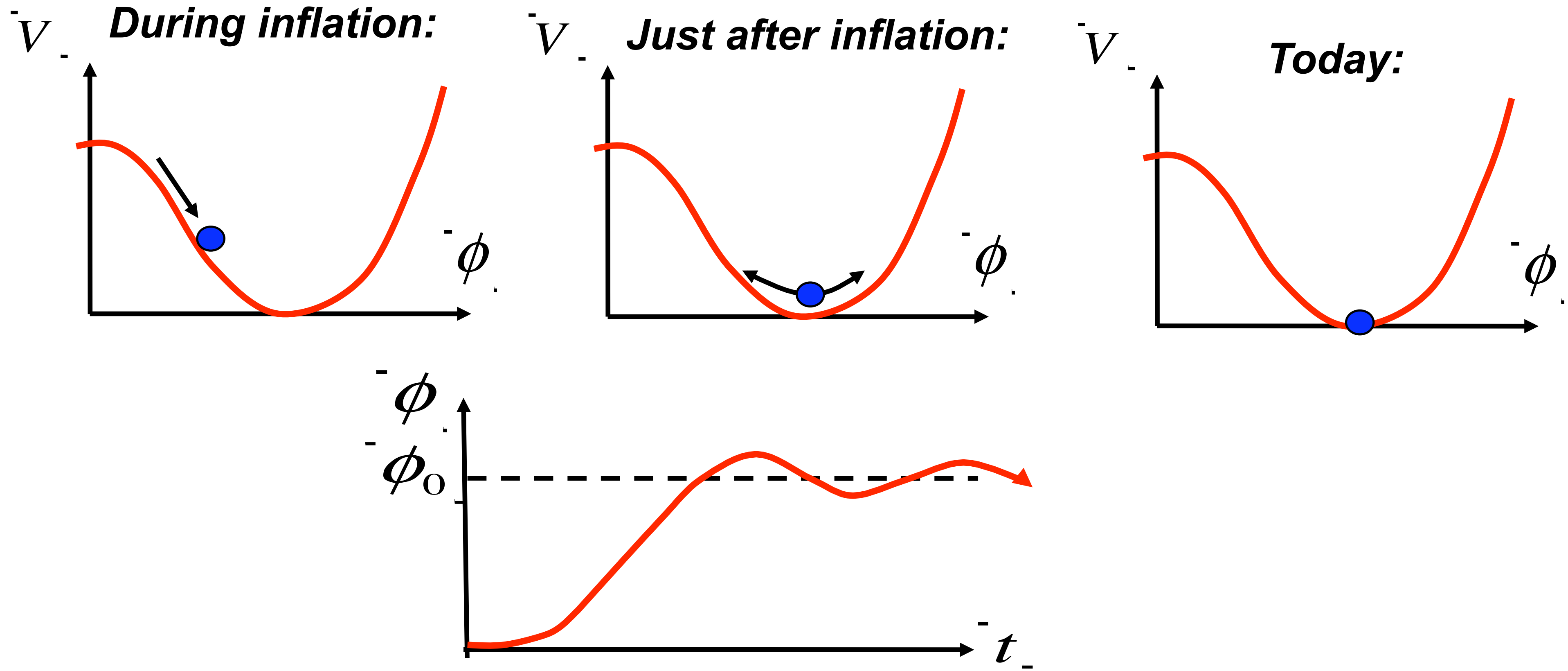
Verify for radiation $P = \rho c^2 / 3$

Verify for vaccum $P = -\rho c^2$

Reading: 1980: Inflation (Alan Guth)

- Universe born from "**nothing**" ?
- A **quantum fluctuation** produces a tiny bubble of "**False Vacuum".**
- High vacuum energy drives **exponential expansion**, also known as **"inflation."**
- Universe expands by huge factor in tiny fraction of second, as false vacuum returns to true vacuum.
- Expansion so fast that **virtual particle-antiparticle pairs** get separated to become **real particles and anti-particles**.
- Stretches out all structures, giving a **flat geometry** and uniform T and r, with **tiny ripples**.
- Inflation launches the **Hot Big Bang**!

Reading: Scalar Field Dynamics



Kinetic energy of the oscillations is damped.
Re-heats the Universe, creating all types of particle-antiparticle pairs, launching the Hot Big Bang.

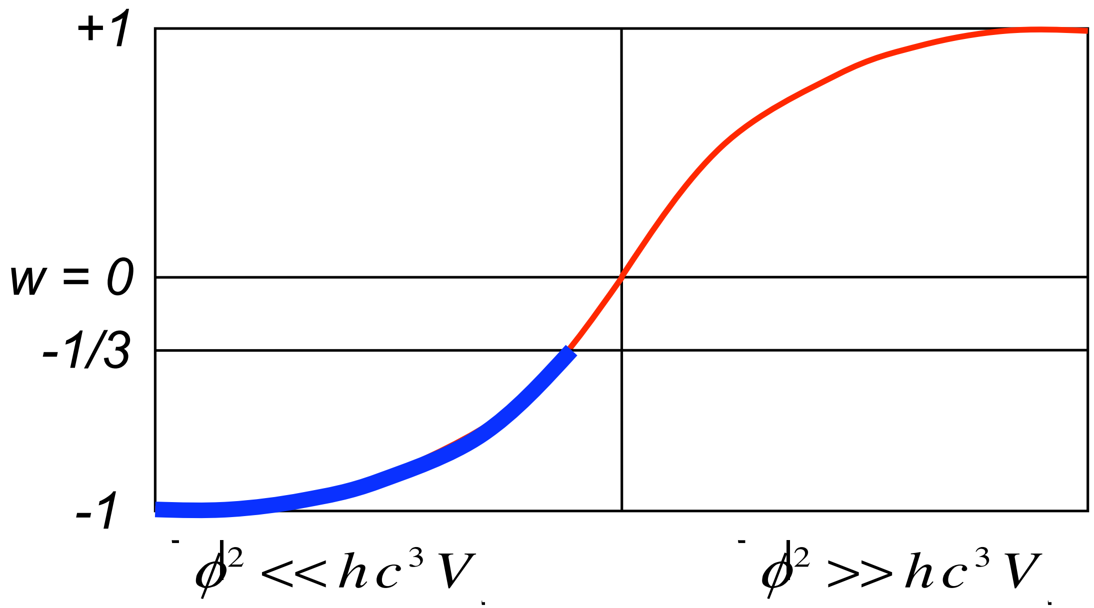
Reading: Scalar Field Equation of State

Equation of State: $w = p / \varepsilon$

$$p_\phi = \frac{1}{2}\frac{1}{\hbar c^3}\left(\dot\phi^2 + c^2 \nabla \varphi^2\right) - V(\phi)$$

$$\varepsilon_\phi = \frac{1}{2}\frac{1}{\hbar c^3}\left(\dot\phi^2 + c^2 \nabla \varphi^2\right) + V(\phi)$$

Uniform field: $c^2 (\nabla\phi)^2 \ll \dot\phi^2$



Required for inflation:

$$w = \frac{p}{\varepsilon} = \frac{\dot\phi^2 - 2\hbar c^3 V}{\dot\phi^2 + 2\hbar c^3 V} < -\frac{1}{3}$$

$$\varepsilon + 3p = 4\left(\dot\phi^2 - \hbar c^3 V\right) < 0$$

Terminal velocity:

$$\dot\phi \Rightarrow \frac{\hbar c^3}{3H}\frac{\partial V}{\partial \phi}$$

$$\left(\frac{\partial V}{\partial \phi}\right)^2 < \frac{9H^2}{\hbar c^3} V$$

Need a long slow roll over a nearly flat potential.

Tutorial: ants on a sphere

Consider a micro-cosmos of N-ants inhabiting an expanding spherical surface of radius $R=R_0\ (t/t_0)^a$, where presently we are at $t=t_0$ =1min, $R=R_0$ =1lightsecond. Let a=1/2, N=100. What is the present rate of expansion dR/dt/R = in units of 1/min? How does the ant surface density change as function of cosmic time? [due 6Mar]

Light emitted by ant-B travels a half circle and reaches ant-A now, what redshift was the light emitted? What is the angular diameter distance to the emission redshift? [due 10Apr]

Let each ant conserve its random angular momentum per unit mass J=(1lightsecond)*(1m/min) with respect to the centre of the sphere. Estimate the age of universe when the ants were moving relativistic. How far has ant B travelled since the emission and since the beginning of the universe?

What have we learned?

Where are we heading?

Inflation as origin of perturbation, flatness, horizon.

Sound speed of gas before/after decoupling, and sound horizon.

Topics Next:

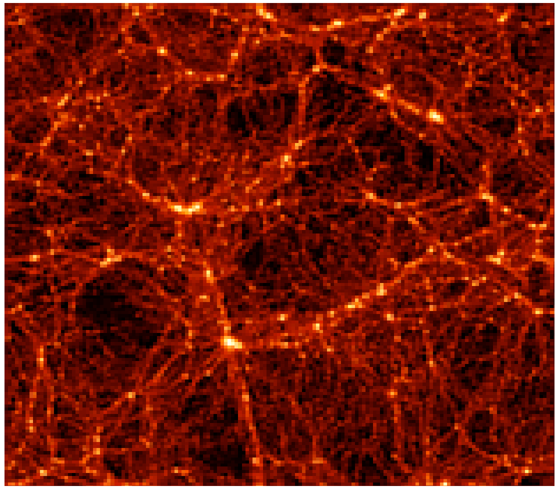
Growth of [bankruptcy of uniform universe]

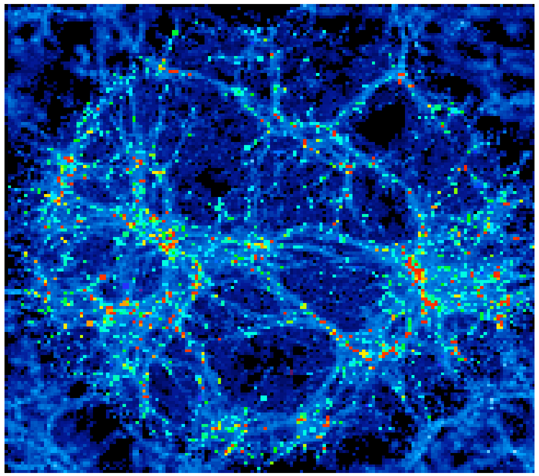
Density Perturbations (how galaxies form by N-body simulations)

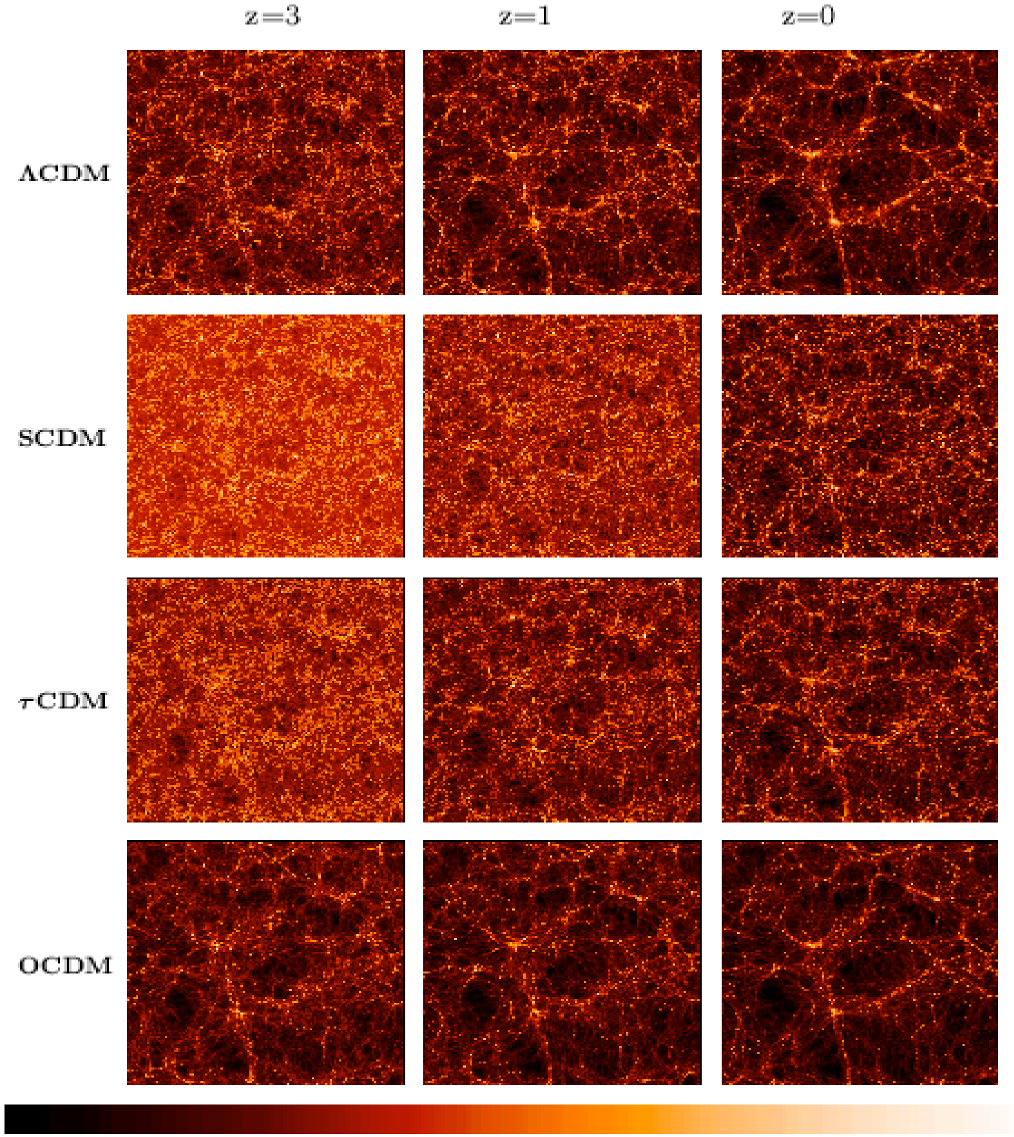
peculiar velocity (how galaxies move and merge)

CMB fluctuations (temperature variation in CMB)

Reading: Supercomputer Simulations



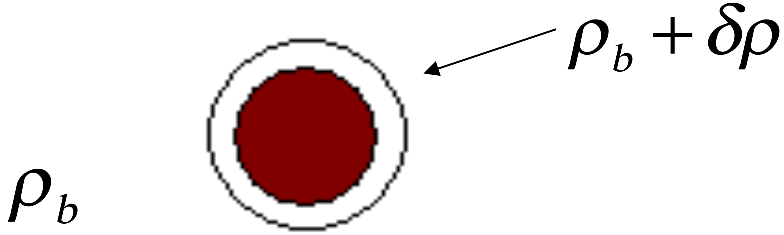


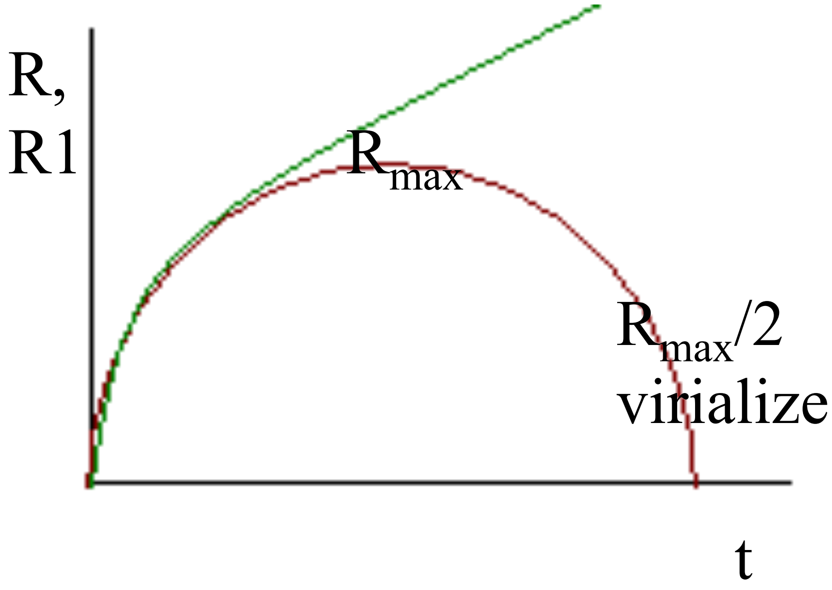


Non-linear Collapse of an Overdense Sphere

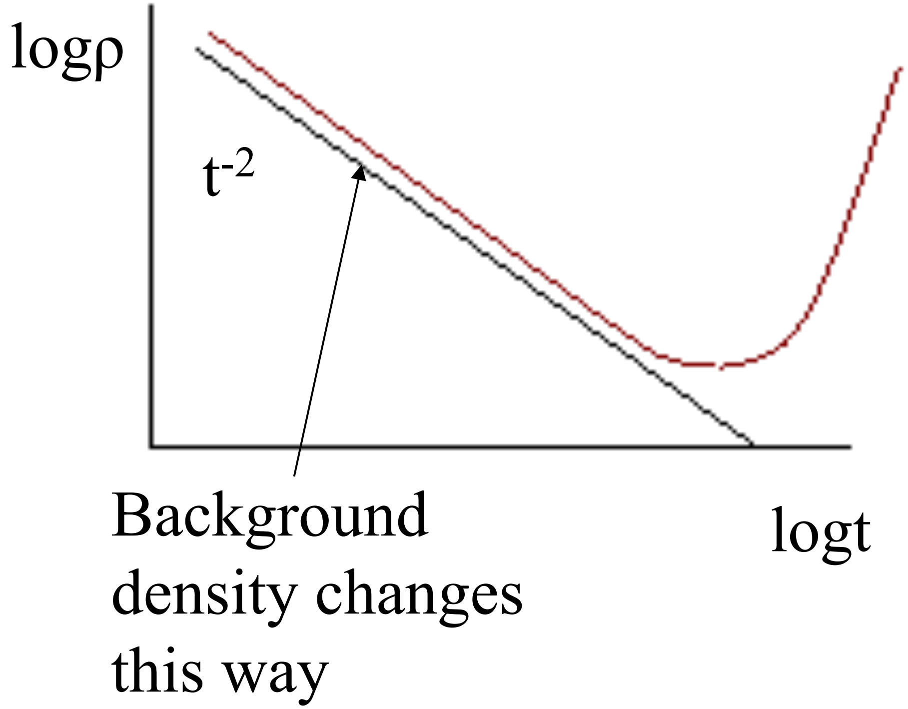
An overdense sphere is a very useful non linear model as it behaves in exactly the same way as a closed sub-universe.

Any spherically symmetric perturbation will clearly evolve at a given radius in the same way as a uniform sphere containing the same amount of mass.





$$\rho_b = \frac{1}{6\pi G t^2}$$



reading: Gradual Growth of perturbation

$$\rightarrow \frac{\delta\rho}{\rho} = \frac{3c^2}{8\pi G}\frac{1}{\rho R^2} \propto \begin{cases} R^2 & (\text{mainly radiation } \rho \propto R^{-4}) \\ R & (\text{mainly matter } \rho \propto R^{-3}) \end{cases}$$

Perturbations Grow!

E.g., δ changes by a factor of 10 between z=10 and z=100 (matter era), and a factor of 100 between z=10^5 and z=10^6 (Radiation Era)

reading: Peculiar Motion

The motion of a galaxy has two parts:

$$\vec{v} = \frac{d}{dt}\left[R(t)\theta(t)\right]$$

Proper length vector

$$= \underline{\underline{\dot{R}(t).\theta}} + \underline{\underline{R(t)\dot{\theta}(t)}}$$

Uniform expansion v_o

Peculiar motion v

reading: Damping of peculiar motion (in the absence of overdensity)

Generally peculiar velocity drops with expansion.

$$R^2\dot{\theta} = R * (R\dot{\theta}) = \text{constant} \sim \text{"Angular Momentum"}$$

Similar to the drop of (non-relativistic) sound speed with expansion

$$\delta v = R(t)\,\dot{x}_c = \frac{\text{constant}}{\text{R(t)}}$$

Reading: Equations governing Fluid Motion

$$\nabla^2 \phi = 4\pi G \rho \qquad \text{(Poissons Equation)}$$

$$\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{dt}} = \frac{d \ln \rho}{dt} = -\vec{\nabla}.\vec{v} \qquad \text{(Mass Conservation)}$$

$$\frac{\mathrm{dv}}{\mathrm{dt}} = -\vec{\nabla}\phi - \underline{\underline{c_s^2 \vec{\nabla} \ln \rho}} \qquad \text{(Equation of motion)}$$

$$\rightarrow \frac{\nabla P}{\rho} \quad \text{since } \partial P = c_s^2 \partial \rho$$

Reading: Decompose into unperturbed + perturbed

Let

$$\rho = \rho_o + \delta\rho$$

$$v = v_o + \delta v = \dot{R}\chi_c + R\dot{\chi}_c$$

$$\phi = \phi_o + \delta\phi$$

We define the Fractional Density Perturbation:

$$\delta = \frac{\delta\rho}{\rho_o} = \delta(t)\exp(-i\vec{k}\bullet\vec{x}),$$

$$|\vec{k}| = 2\pi/\lambda, \quad \text{where } \lambda = R(t)\lambda_c$$

$$\vec{k}\bullet\vec{x} = \vec{k}_c\bullet\vec{x}_c \qquad x(t) = R(t)\chi_c$$

Reading: Motion driven by gravity: $$\vec{g}_o(t) + \vec{g}_1(\theta, t)$$

due to an overdensity: $$\rho(t) = \rho_o(1 + \delta(\theta, t))$$

Gravity and overdensity by Poisson's equation:

$$-\vec{\nabla} \bullet g_1 = 4\pi G \rho_o \delta$$

Continuity equation:

$$-\vec{\nabla} \bullet \delta \vec{v} = \frac{d}{dt}\left(\delta(\theta, t)\right)$$

← The over density will rise if there is an inflow of matter

Peculiar motion δv and peculiar gravity g_1 both scale with δ and are in the same direction.

12th Concept: THE equation for structure formation

In matter domination

Equation becomes

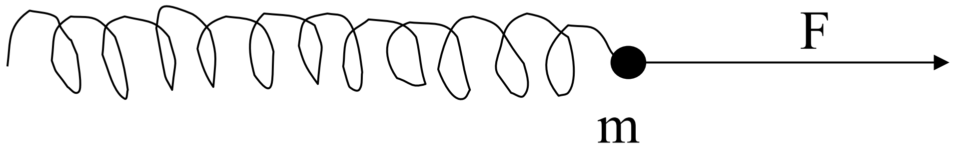
$$-c_s^2 k^2$$

$$\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{R}}{R}\frac{\partial \delta}{\partial t} = (4\pi G\rho_o + c_s^2 \nabla^2)\delta$$

Gravity has the tendency to make the density perturbation grow exponentially on large scale.

Pressure makes it oscillate on small scale.

Each eq. is similar to a forced spring



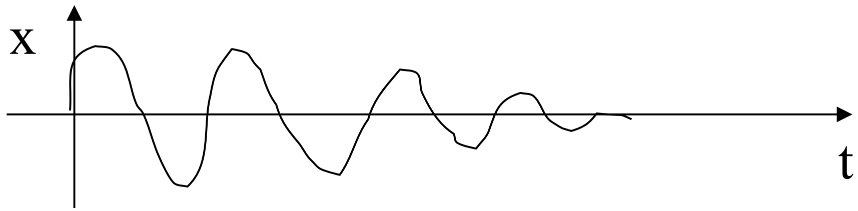
Restoring

$$\frac{d^2 x}{dt^2} = \frac{F}{m} - \omega^2 x - \mu \frac{dx}{dt}$$

Term due to friction

$$\frac{d^2 x}{dt^2} + \mu \frac{dx}{dt} + \omega^2 x = \frac{F(t)}{m}$$

(Displacement for Harmonic Oscillator)



e.g., Nearly Empty Pressure-less Universe

$$\rho \sim 0$$

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{2}{t}\frac{\partial \delta}{\partial t} = 0, \quad H = \frac{\dot{R}}{R} = \frac{1}{t} \quad (R \propto t)$$

$$\delta \propto t^0 = \text{constant}$$

$\rightarrow$ no growth

Reading: Jeans Instability (no expansion)

Case 1- no expansion

$$\dot{R} = 0$$

- the density contrast [illegible] has a wave-like form

for the harmonic oscillator equation

$$\delta = \delta_o \exp(i\vec{k}.\vec{r} - i\omega t)$$

where we have the dispersion relation

$$\frac{\partial^2 \delta}{\partial t^2} + 2 * 0 * \frac{\partial \delta}{\partial t} = -\omega^2 \delta$$

$$\omega^2 \equiv \underline{\underline{c_s^2 k^2}} - \underline{\underline{4\pi G \rho}}$$

Pressure support

gravity

Reading: At the (proper) JEANS LENGTH scale we switch from

Oscillations for shorter wavelength modes to

the exponential growth of perturbations for longer wavelength

$$\lambda_J = c_s \tau, \quad \text{where timescale } \tau = \sqrt{\frac{\pi}{G\rho}}$$

$□ < □_J$, $□^2 > 0$ → oscillation of the perturbation.

$□□□_J$, $□^2□0$ → exponential growth/decay

$$\delta \propto \exp(\pm \Gamma t) \text{ where } \Gamma = \sqrt{-\omega^2}$$

reading: Jeans Instability

Case 2: on very large scale $\lambda \gg \lambda_J = c_s\, t$ of an Expanding universe

Neglect Pressure (restoring force) term

Grow as delta ~ R ~ $t^{2/3}$ for long wavelength mode if Omega_m=1 universe.

$$c_s^2 k^2 << 4\pi G\rho = c_s^2 k_J^2$$

$$\frac{\partial^2 \delta}{\partial t^2} + 2H\frac{\partial \delta}{\partial t} = \underline{\underline{4\pi G\rho_m \delta}}$$

$$2H \rightarrow 2/(3t)$$

$$4\pi G\rho_m \rightarrow 2/(3t^2)$$

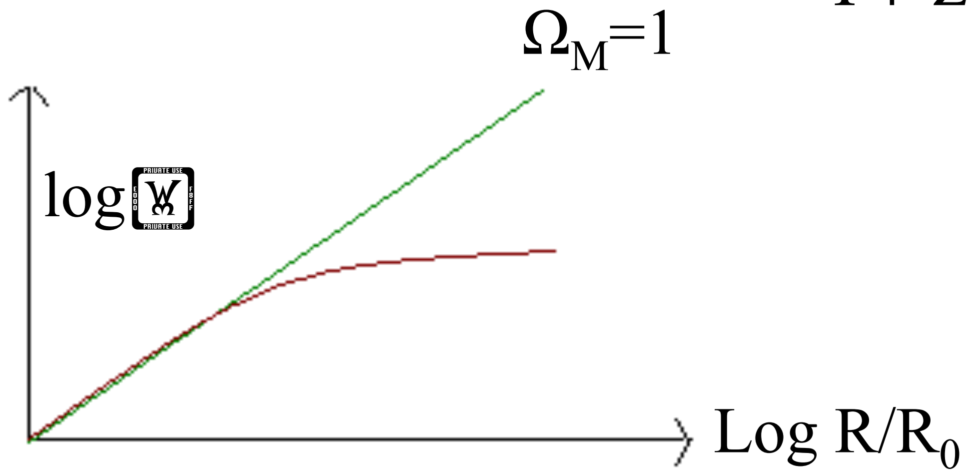
Reading: E.g.,

Einstein de Sitter Universe

$$\Omega_M = 1, H = \frac{\dot{R}}{R} = \frac{2}{3t}$$

Verify Growth Solution $\delta \propto R \propto t^{\frac{2}{3}} \propto \frac{1}{1+z}$

Generally



reading: Case III: Relativistic (photon) Fluid

equation governing the growth of perturbations being:

1/t $1/t^2$

$$\Rightarrow \frac{d^2\delta}{dt^2} + 2H\frac{d\delta}{dt} = \delta.\left(\frac{32\pi G\rho}{3} - k^2 c_s^2\right)$$

Oscillation solution happens on small scale $2\pi/k = \lambda < \lambda_J$

On larger scale, growth as

$$\Rightarrow \delta \propto t \propto R^2 \quad \text{for length scale } \lambda >> \lambda_J \sim c_s t$$

Where are we heading?

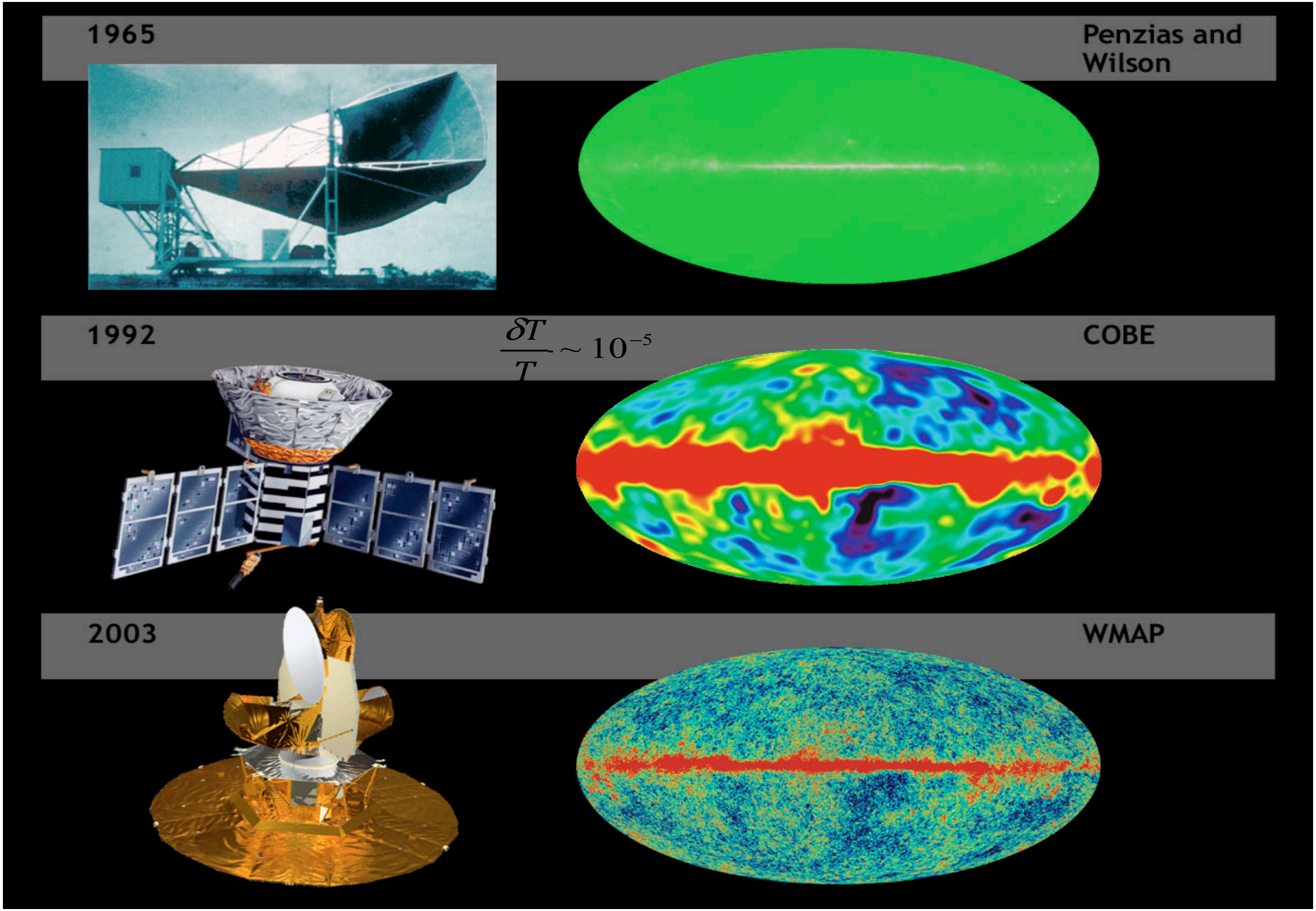
Large scale OverDensity grows as

R (matter dominated) or R^2 (radiation dominated)

Peculiar velocity points towards overdensity.

*Small scale oscillation if baryon-radiation dominated.

Topics Next: CMB equations


1965
Penzias and Wilson
1992
$\frac{\delta T}{T} \sim 10^{-5}$
COBE
2003
WMAP

Recombination Epoch (z~1100)

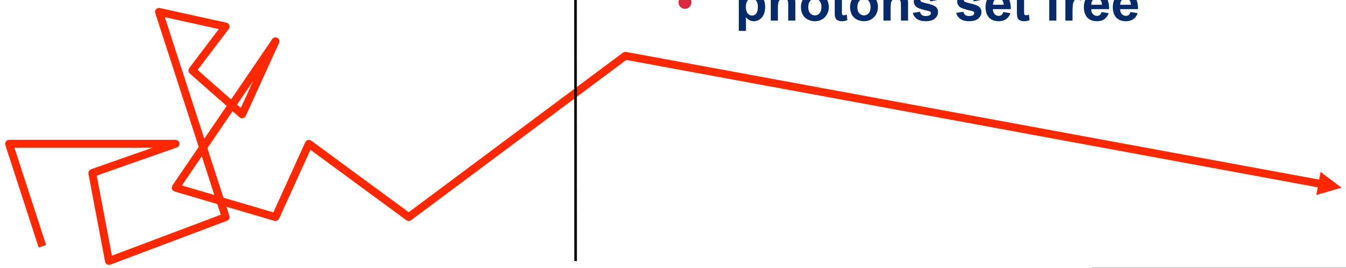
ionised plasma --> neutral gas

- **Redshift z > 1100**
- **Temp T > 3000 K**
- **H ionised**

- **electron -- photon Thompson scattering**

- **z < 1100**
- **T < 3000 K**
- **H recombined**

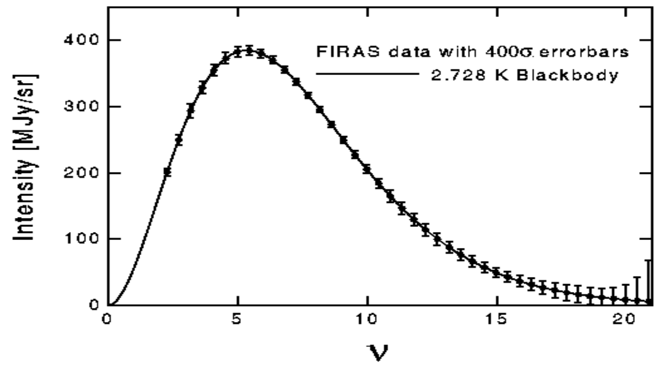
- **almost no electrons**
- **neutral atoms**
- **photons set free**



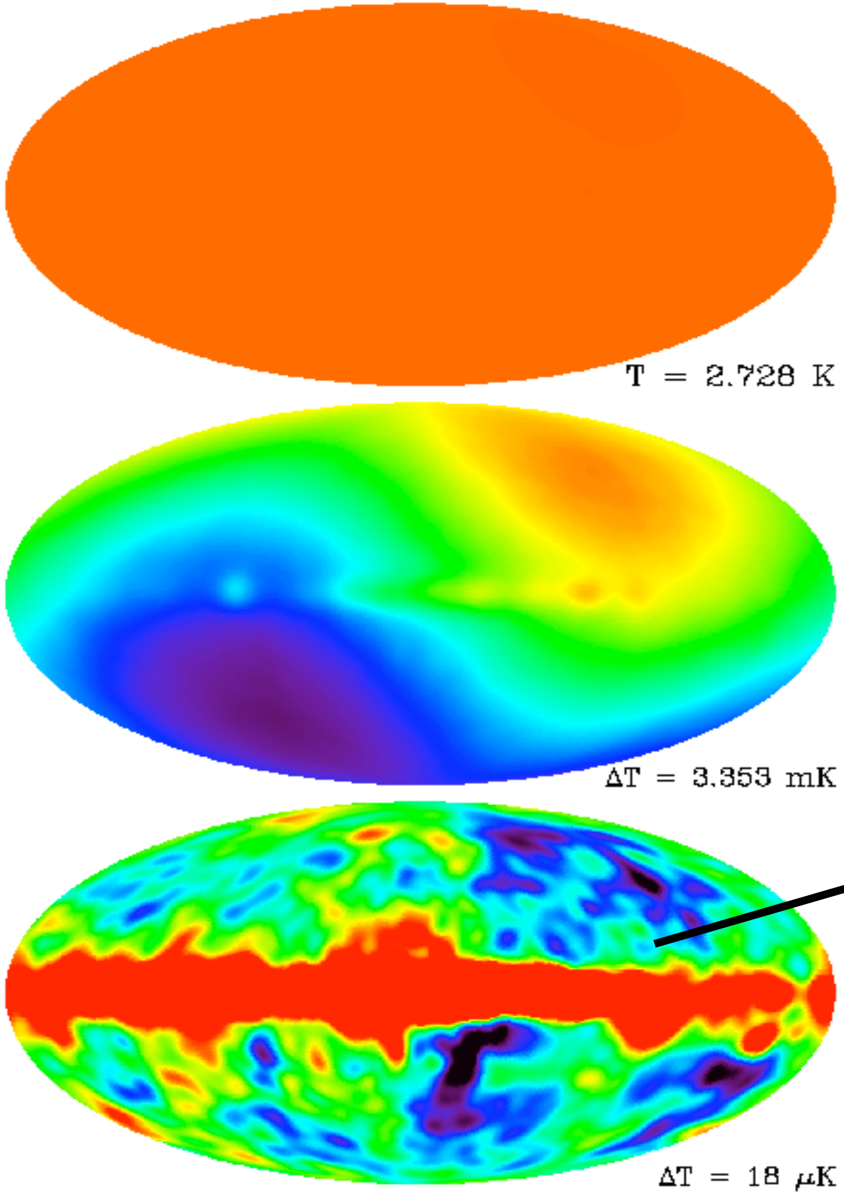
e - scattering optical depth

$$\tau(z) \approx \left(\frac{z}{1080} \right)^{13}$$

thin surface of last scattering



Cosmic Microwave Background



Almost isotropic

$T = 2.728$ K

Dipole anisotropy

$$\frac{V}{c} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta T}{T} \approx 10^{-3}$$

Our velocity:

$$V \approx 600 \text{ km/s}$$

Milky Way sources

$$+ \text{ anisotropies } \frac{\Delta T}{T} \sim 10^{-5}$$

Theory of CMB Fluctuations

Linear theory of structure growth predicts that the perturbations:

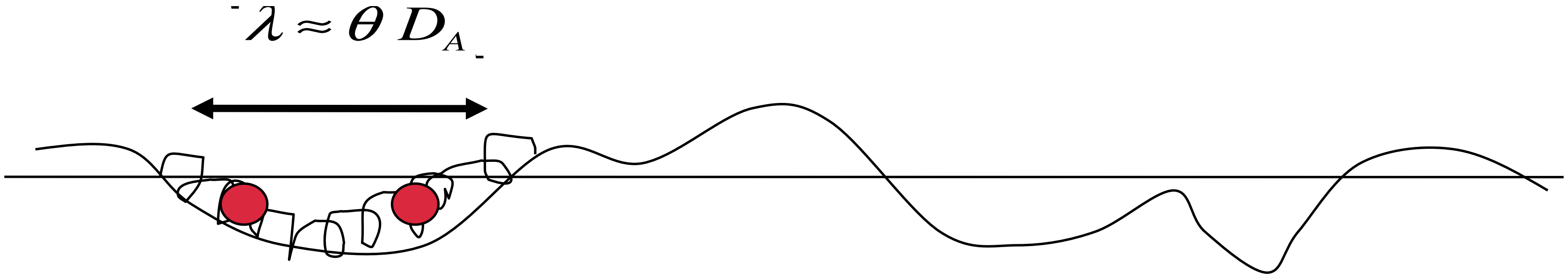
δ_D in dark matter $\dfrac{\delta\rho_{\mathrm{D}}}{\rho_{\mathrm{D}}}$

δ_B in baryons $\dfrac{\delta\rho_{\mathrm{B}}}{\rho_{\mathrm{B}}}$

δ_r in radiation $\dfrac{\delta\rho_{\mathrm{r}}}{\rho_r}$ Or $\tilde{\delta}_r = \dfrac{3}{4}\delta_r = \dfrac{\delta n_\gamma}{n_\gamma}$

will follow a set of coupled Harmonic Oscillator equations.

Acoustic Oscillations



Dark Matter potential wells - many sizes.

photon-electron-baryon fluid

fluid falls into DM wells

photon pressure pushes it out again

oscillations starting at t = 0 (post-inflation)

stopping at z = 1100 (recombination)

The solution of the Harmonic Oscillator [within sound horizon] is:

$$\delta(t) = A_1 \cos kc_s t + A_2 \sin kc_s t + A_3$$

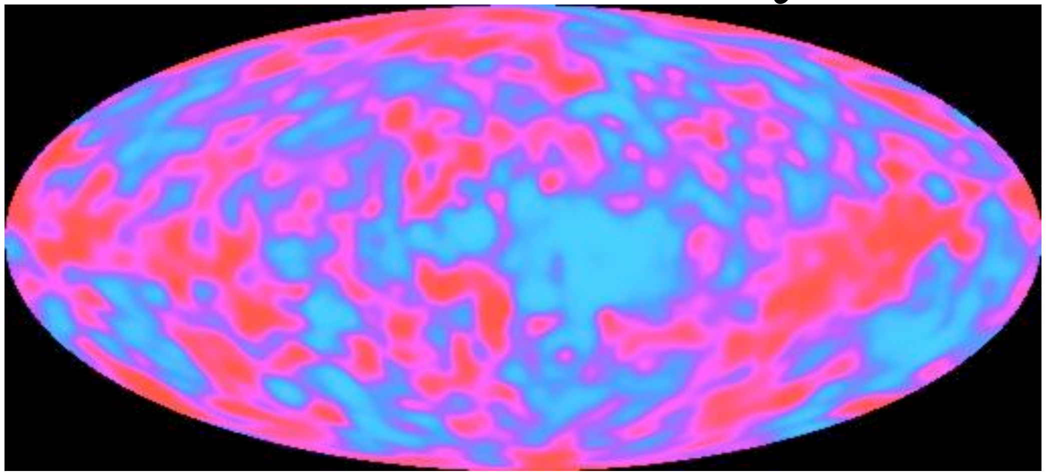
Amplitude is sinusoidal function of $k\ c_s\ t$

if k=constant and oscillate with t [evolution]

or t=0.3Myr and oscillate with k (CMB peaks)

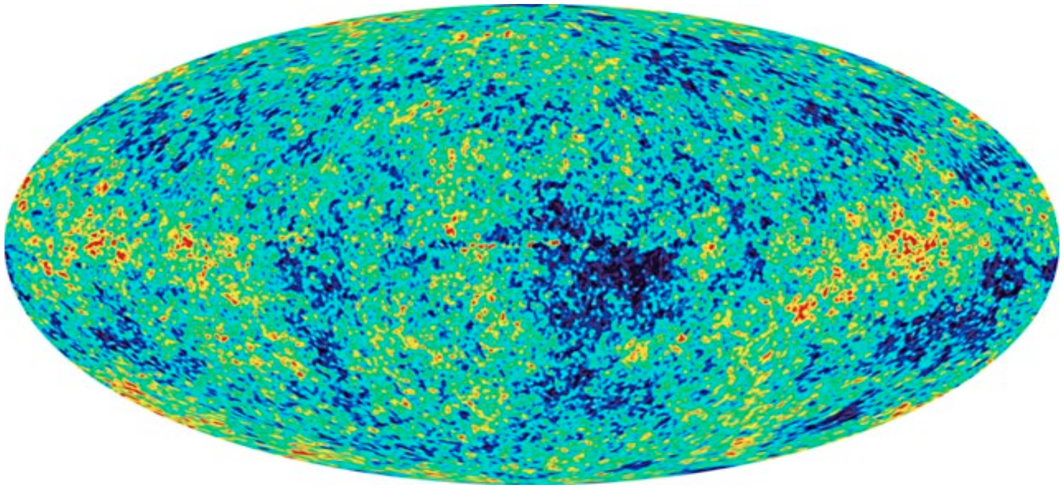
CMB Anisotropies

COBE
1994



$$\frac{\Delta T}{T} \sim 10^{-5}$$

WMAP
2004



$$\Delta\theta \sim 1^{\circ}$$

Snapshot of Universe at z = 1100
Seeds that later form galaxies.

Resonant Oscillations

size of potential well λ

oscillation period $P \approx \dfrac{\lambda}{c_s}$

sound speed $c_s = \dfrac{c}{\sqrt{3}}$

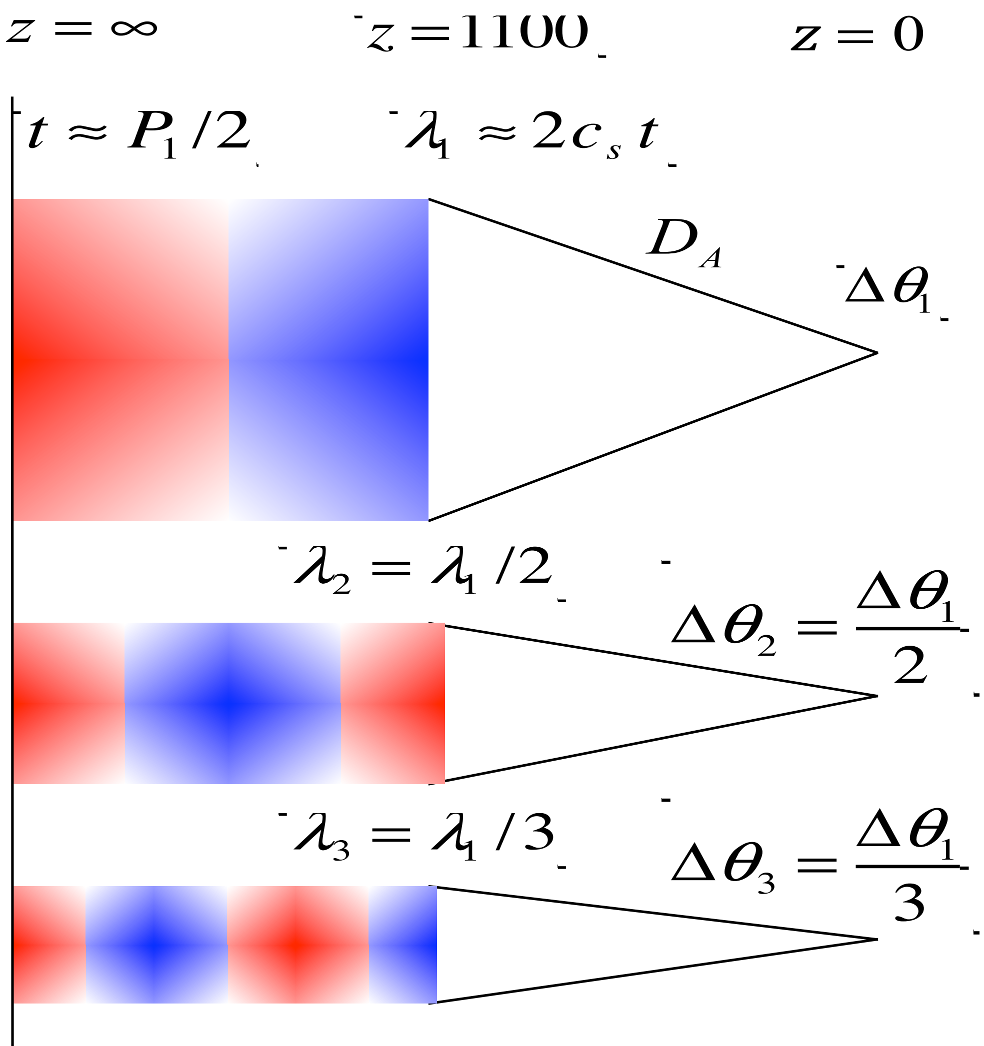
temperature oscillations

$$\Delta T(t) = \Delta T(0)\cos(2\pi\, t / P)$$

$$\max|\Delta T| \text{ at } t = \frac{n\,P}{2} \sim \frac{n\,\lambda}{2c_s}$$

angular size

$$\Delta\theta_n = \frac{\lambda_n}{D_A} = \frac{\Delta\theta_1}{n} \qquad \Delta\theta_1 \approx \frac{2\,c_s\,t}{D_A} \sim 0.8^\circ$$



Smaller wells oscillate faster.

We don't observe the baryon overdensity δ_B directly

-- what we actually observe is temperature fluctuations.

$$\frac{\Delta T}{T} = \frac{\Delta n_\gamma}{3 n_\gamma}$$

$$n_\gamma \sim R^{-3} \propto T^3$$

$$\varepsilon_\gamma \sim n_\gamma k T \propto T^4$$

$$= \frac{\delta_B}{3} = \frac{\tilde{\delta}_R}{3}$$

The driving force is due to dark matter over densities.

The observed temperature is:

$$\left(\frac{\Delta T}{T}\right)_{obs} = \frac{\delta_B}{3} + \frac{\psi}{c^2}$$

Effect due to having to climb out of gravitational well

The observed temperature also depends on how fast the Baryon Fluid is moving.

$$\text{Velocity Field} \quad \nabla v = -\frac{d\delta_B}{dt}$$

$$\left(\frac{\Delta T}{T}\right)_{obs} = \frac{\delta_B}{3} + \frac{\psi}{c^2} \pm \frac{v}{c}$$

Doppler Term

Tutorial: perturbation questions

The edge of the void are lined up by galaxies. What direction is their peculiar gravity and peculiar motion? A patch of void is presently cooler in CMB by 3 micro Kelvin than average. How much was it cooler than average at the last scattering (z=1000)? Argue that a void in universe now originates from an under-dense perturbation at $z=10^{10}$ with δ about 10^{-17}.

List of keys

- Scaling relations among
  - Redshift z, wavelength, temperature, cosmic time, energy density, number density, sound speed
  - Definition formulae for pressure, sound speed, horizon
  - Metrics in simple 2D universe.
- Describe in words the concepts of
  - Fundamental observers
  - thermal decoupling
    - Common temperature before,
    - Fixed number to photon ratio after
    - Hot and Cold DM.
  - gravitational growth.
    - Over-density,
    - direction of peculiar motion driven by over-density, but damped by expansion
    - pressure support vs. grav. collapse

Expectations

Remember basic concepts (or analogies)

See list

Can apply various scaling relations to do some of the short questions at the lectures.

See list

Relax.

thermal history, structure formation, DM counting are advanced subjects, just be able to recite the big picture.

Why Analogies in Cosmology

Help you memorizing

- Cosmology calls for knowledge of many areas of physics. Analogies help to you memorize how things move and change in a mind-boggling expanding 4D metric.

Help you reason, avoid "more equations, more confusions".

- If unsure about equations, e.g. at exams, the analogies *help you recall* the right scaling relations, and get the big picture right.

Years after the lectures,

- Analogies go a long way

Look back, Look forward?

Have done: Malcolm S. Longair's "Galaxy Formation" 2nd edition [Library]

Chpt 1-2,5-8: expanding metrics, energy density, curvature, distances

Chpt 11,15,20: Structure growth, inflation

Heading to:

Chpt 4,9-10,13: Thermal History of Particle Reaction, Neutrinos, WIMPs, DM budget

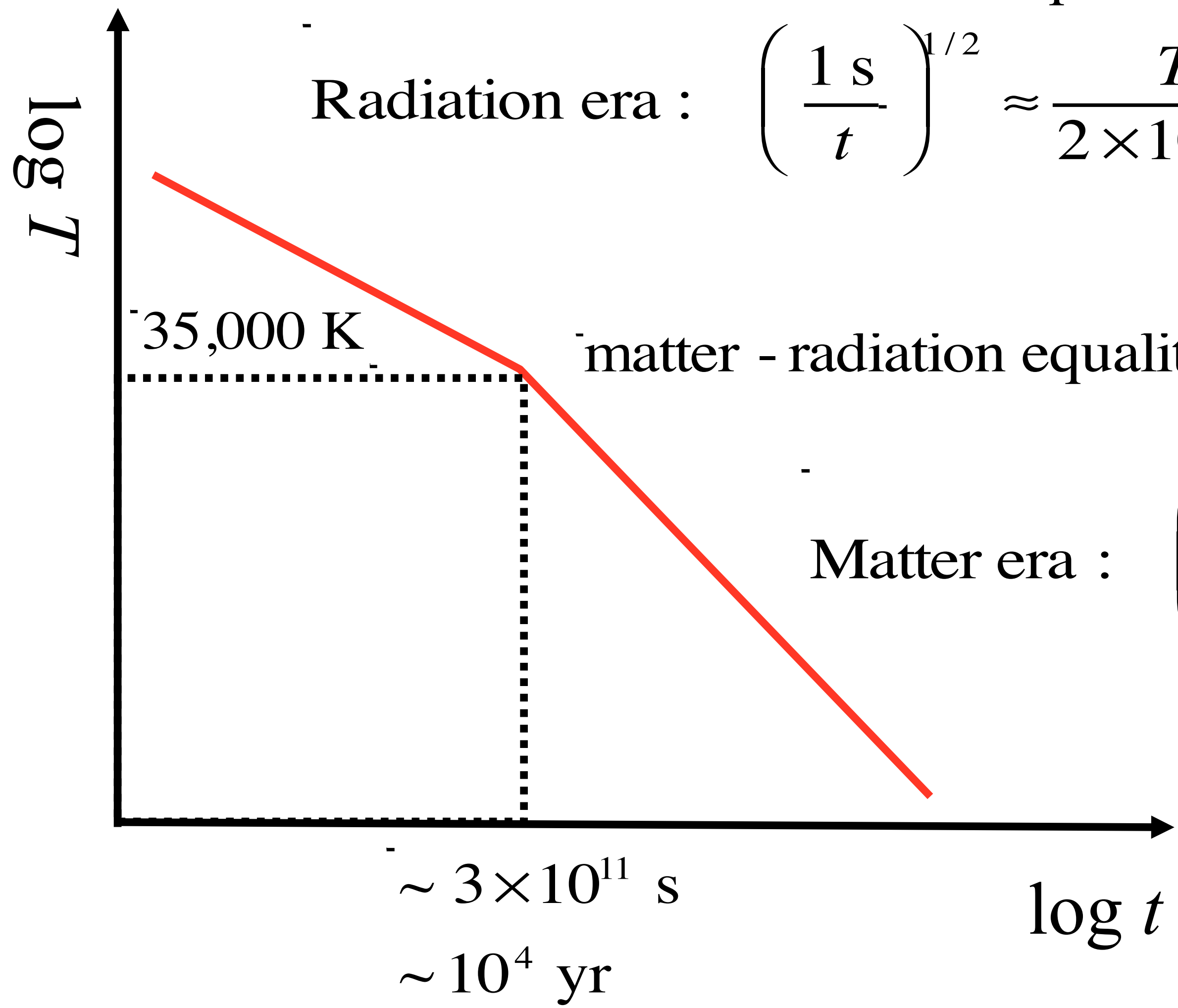
Tutorial: Cooling History T(t)

Derive with concordance parameters

Radiation era : $\left(\frac{1\ \text{s}}{t}\right)^{1/2} \approx \frac{T}{2\times10^{10}\,\text{K}} = \frac{k\,T}{2\ \text{MeV}}$

matter - radiation equality : $\rho_M = \rho_R$

Matter era : $\left(\frac{10^{10}\ \text{yr}}{t}\right)^{2/3} = \frac{T}{2.7\ \text{K}}$



Reading: A busy schedule for the universe

Universe crystalizes with a sophisticated schedule, much more confusing than simple expansion!

- many bosonic/fermionic players changing (numbers conserved except in phase transition!)
- p + p^- <-> g+g (baryongenesis)
- e + e^+ <-> g+g, v + e <-> v + e (neutrino decouple)
- n <→ p + e^- + v, p + n <→ D + g (BBN)
- H^+ + e^- <→ H + g, g + e <-> g + e (recombination)

$$\dot{n} + 3Hn = -\langle \sigma \mathrm{v} \rangle (n^2 - n_T^2)$$

Reading: Significant Events

Event	T	kT	g_{eff}	z	t
Now	2.7 K	0.0002 eV	3.3	0	13 Gyr
First Galaxies	16 K	0.001 eV	3.3	5	1 Gyr
Recombination	3000 K	0.3 eV	3.3	1100	300,000 yr
Equal $r_M = r_R$	9500 K	0.8 eV	3.3	3500	50,000 yr
$e^+ e^-$ pairs	$10^{9.7}$ K	0.5 MeV	11	$10^{9.5}$	3 s
Nucleosynthesis	10^{10} K	1 MeV	11	10^{10}	1 s
Nucleon pairs	10^{13} K	1 GeV	70	10^{13}	$10^{-6.6}$ s
E-W unification	$10^{15.5}$ K	250 GeV	100	10^{15}	10^{-12} s
Quantum gravity	10^{32} K	10^{19} GeV	100(?)	10^{32}	10^{-43} s

Here we will try to single out some rules of thumb.

We will caution where the formulae are not valid, exceptions.

You are not required to reproduce many details, but might be asked for general ideas.

Reading: Big Bang Nuclear Fusion

Big Bang + 3 minutes

T ~ 10^9 K

First atomic nuclei forged.

Calculations predict:

75% H and 25% He

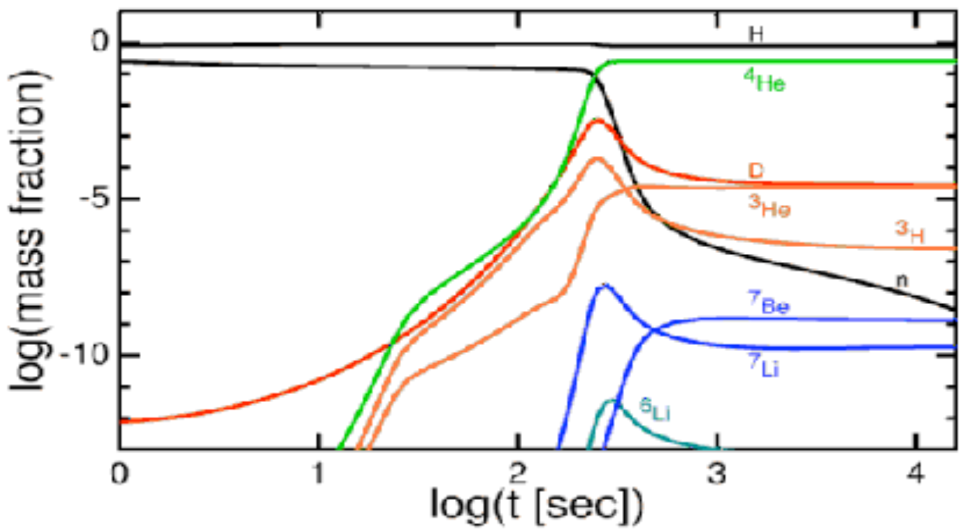
AS OBSERVED !

Helium abundance

Oxygen abundance =>

+ traces of light elements D, ^{3}H, ^{3}He, ^{7}Be, ^{7}Li

=> normal matter only 4% of critical density**.**



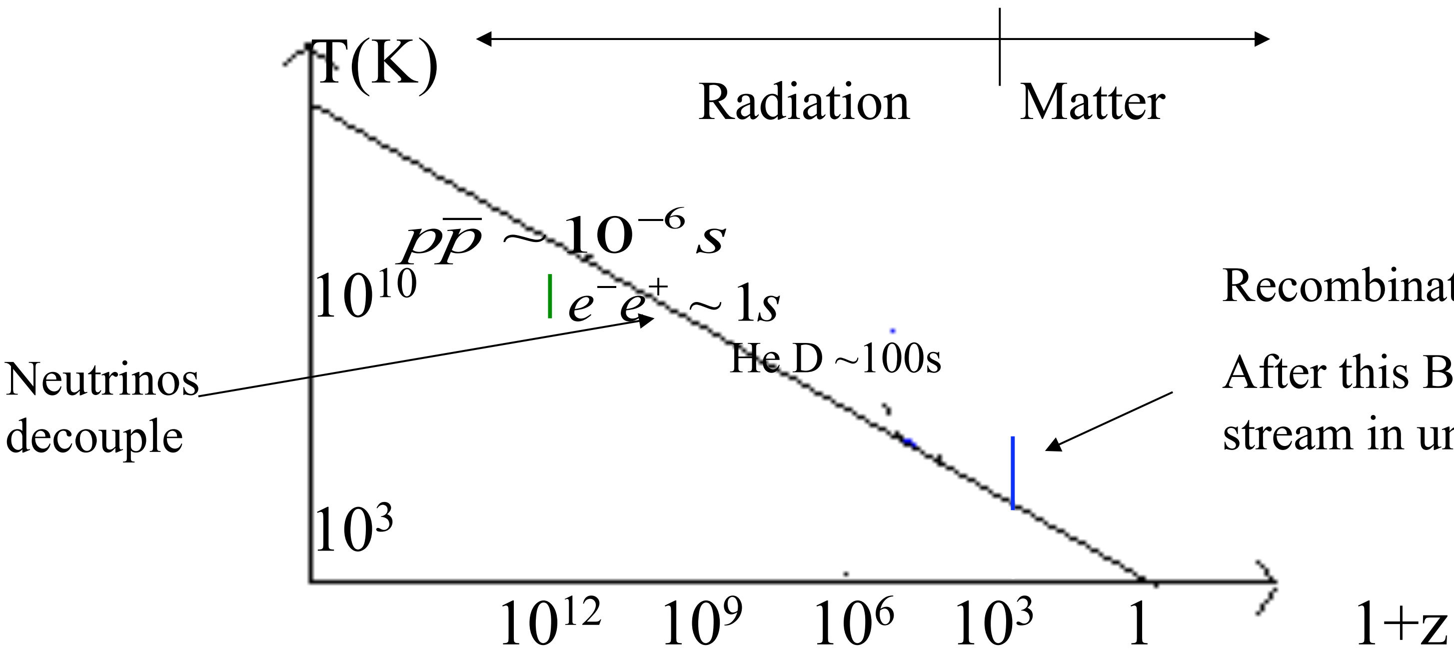
Reading: Thermal Schedule of Universe

At very early times, photons are typically energetic enough that they interact strongly with matter so the whole universe sits at a temperature dictated by the radiation.

The energy state of matter changes as a function of its temperature and so a number of key events in the history of the universe happen according to a schedule dictated by the temperature-time relation.

Crudely

$(1+z) \sim 1/R \sim (T/3) \sim 10^9 (t/100s)^{(-2/n)} \sim 1000 (t/0.3Myr)^{-2/n}$, $H \sim 1/t$, where n~4 during radiation domination



Reading: Competition of two processes

Interactions keeps equilibrium:

E.g., a particle A might undergo the annihilation reaction:

$$A + \overline{A} \rightarrow \gamma + \gamma$$

depends on cross-section σ and speed v. & most importantly

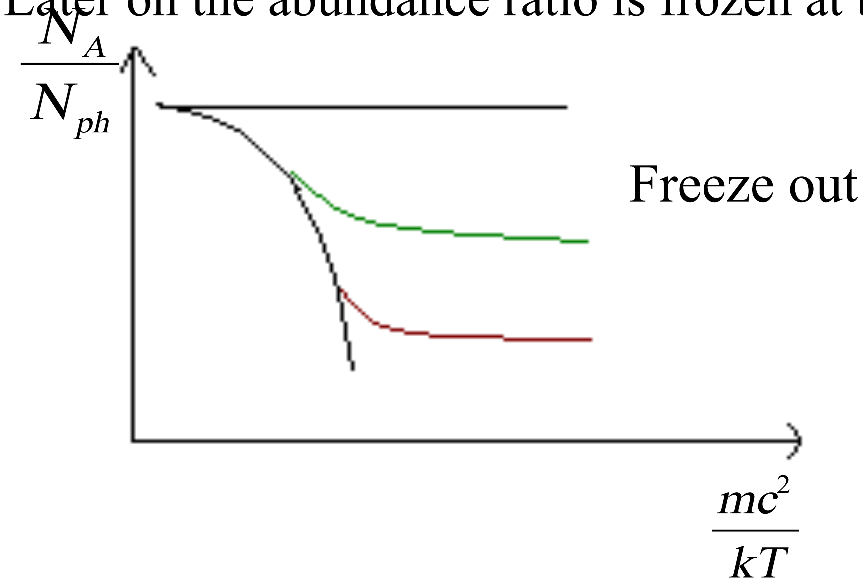
the number density n of photons (falls as $R^{-3} \sim t^{(-6/n)}$)

Decouples because of the increasing gap of space between particles due to Hubble expansion $H \sim t^{-1}$.

e.g., equilibrium process dominates at small time; Hubble expansion process dominates later because it falls slower.

Reading: Rule of thumb
survival of the weakest particle

While in equilibrium, $n_A/n_{ph} \sim \exp(-q)$. $q = mc^2/kT$ →(Heavier is rarer)
At decoupling, $n_A = H_{decouple} / (\langle\sigma_A v\rangle)$, $n_{ph} \sim T^3_{decoupl}$,
Later on the abundance ratio is frozen at this value n_A/n_{ph} ,



$\langle\sigma_A v\rangle$ LOW→ smallest interaction, early freeze-out while relativistic, Hot Matter

$\langle\sigma_A v\rangle$ HIGH→ later freeze-out at lower T, reduced abundance, Cold Matter

Question: why frozen while n_A, n_{ph} both drop as $T^3 \sim R^{-3}$.
Energy density of species A $\sim n_{ph}/(\langle\sigma_A v\rangle)$, if $m \sim kT_{freeze}$

reading: number ratio of non-relativistic particles to photons are const
except for sudden reduction by certain transitions.

Reduction factor ~ exp(-y), $y = mc^2/kT$, which drops sharply for heavier particles.

Non-relativistic particles (relic) become *much rarer* by exp(-y) as universe cools below mc^2/y,
y~10−25.

So rare that infrequent collisions can no longer maintain coupled-equilibrium.

Reading: for example,

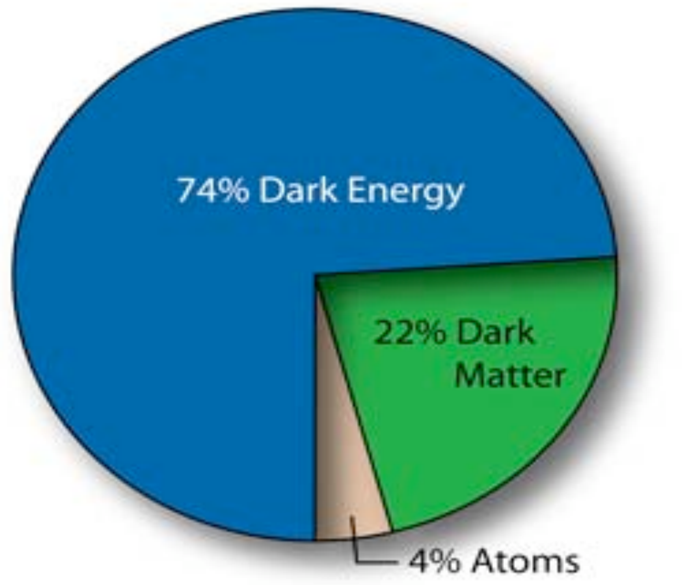
Antiprotons freeze-out $t=10^{-6}$ sec,

earlier than positrons freeze-out t=1sec:

- Because: anti-proton is ~1000 times heavier than positron.
- Hence factor of 1000 hotter in freeze-out temperature
- t goes as T^2 in radiation-dominated regime

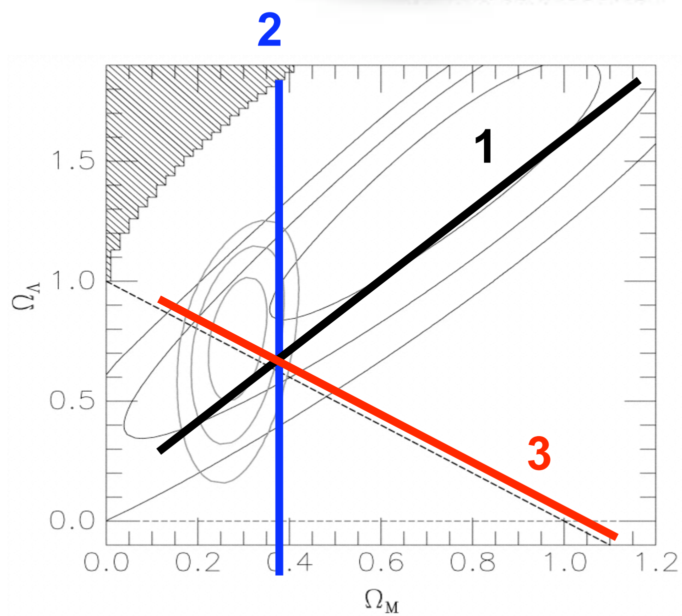
Dark Matter Evidences?

Galaxies: (r ~ 20 Kpc)
Flat Rotation V ~ 200 km/s
Galaxy Clusters: (r ~ 200 Kpc)
Galaxy velocities V ~ 1000 km/s
X-ray Gas T ~ 10^8 K
Giant Arcs



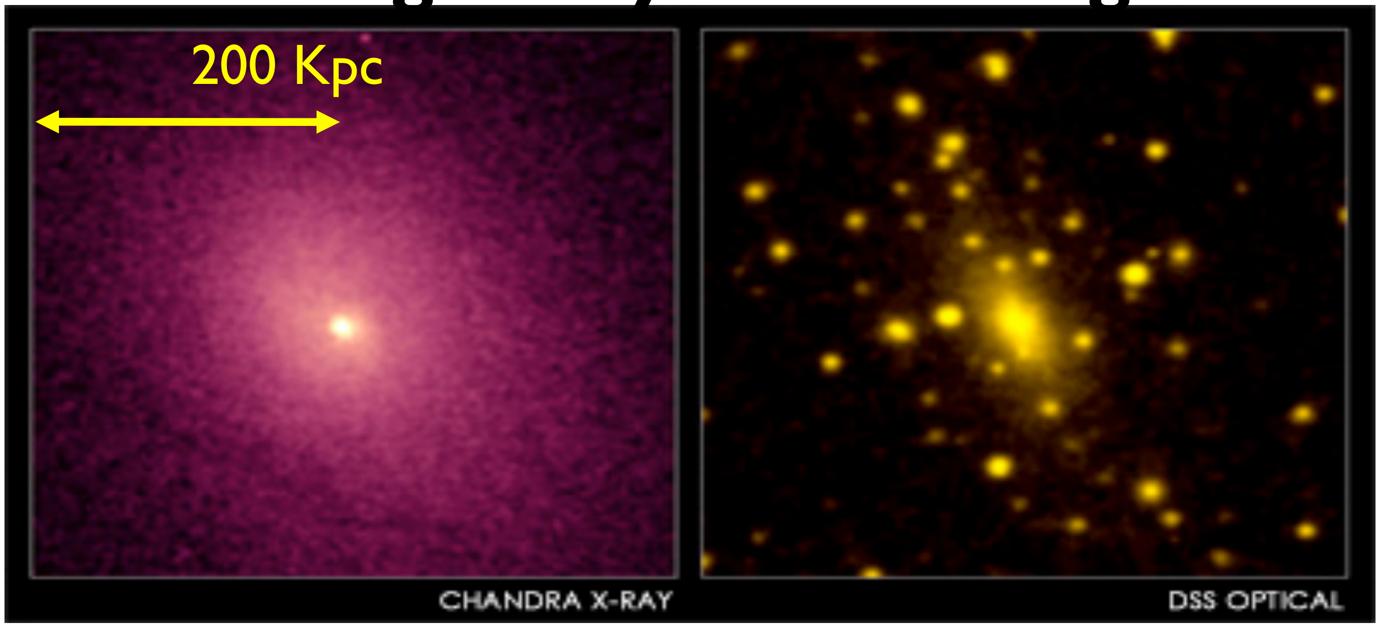
$$\Omega_M \sim 0.3$$

$$\Omega_b \approx 0.04$$



reading: Dark Matter in Galaxy Clusters

Probes gravity on 10x larger scales



$$z = 0.0767$$

$$d \approx \frac{c\,z}{H_0}$$

$$= 320 \text{ Mpc}$$

X-ray **Optical**

Chandra X-ray Image of Abell 2029

The galaxy cluster Abell 2029 is composed of thousands of galaxies enveloped in a gigantic cloud of hot gas, and an amount of **dark matter** equivalent to more than **a hundred trillion Suns**. At the center of this cluster is an enormous, elliptically shaped galaxy that is thought to have been formed from the mergers of many smaller galaxies.

reading: Cluster Masses from X-ray Gas

hydrostatic equilibrium :

$$\frac{dP}{dr} = -\rho\, g = -\rho\, \frac{G\, M(<r)}{r^2}$$

gas law :

$$P = \frac{\rho\, k\, T}{\mu\, m_H}$$

X - ray emission from gas gives : $T(r),\, n_e(r) \rightarrow \rho(r), P(r)$

$$M(<r) = -\frac{r^2}{G\, \rho(r)} \frac{dP}{dr}$$

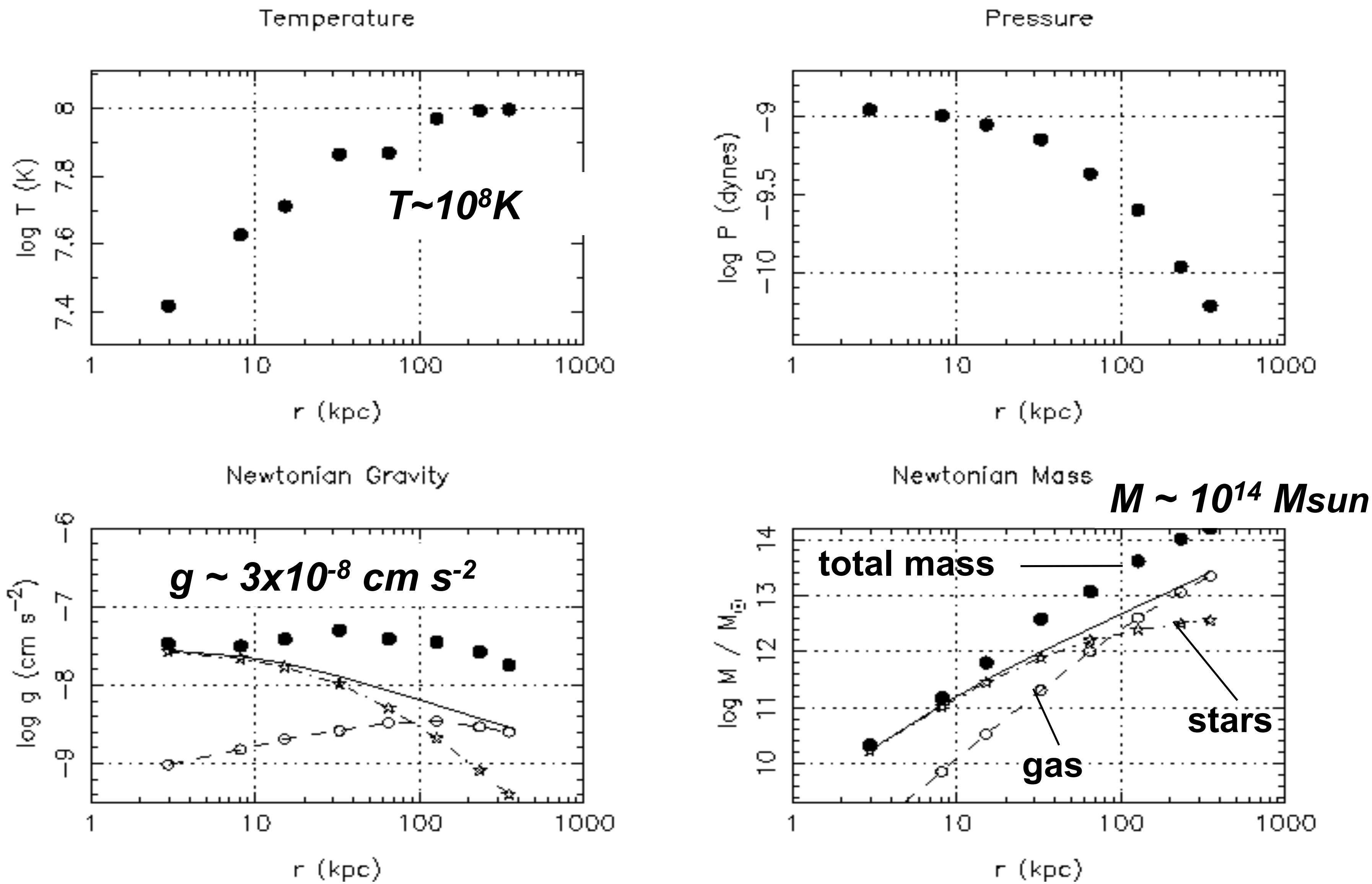
Coma Cluster:

M(gas)~M(stars)~3x10^{13} Msun

often M(gas) > M(stars)

M/L~100-200

Reading: Cluster Mass from fitting X-ray Gas



reading: Galaxy Rotation Curves

HI velocities

Flat rotation curves

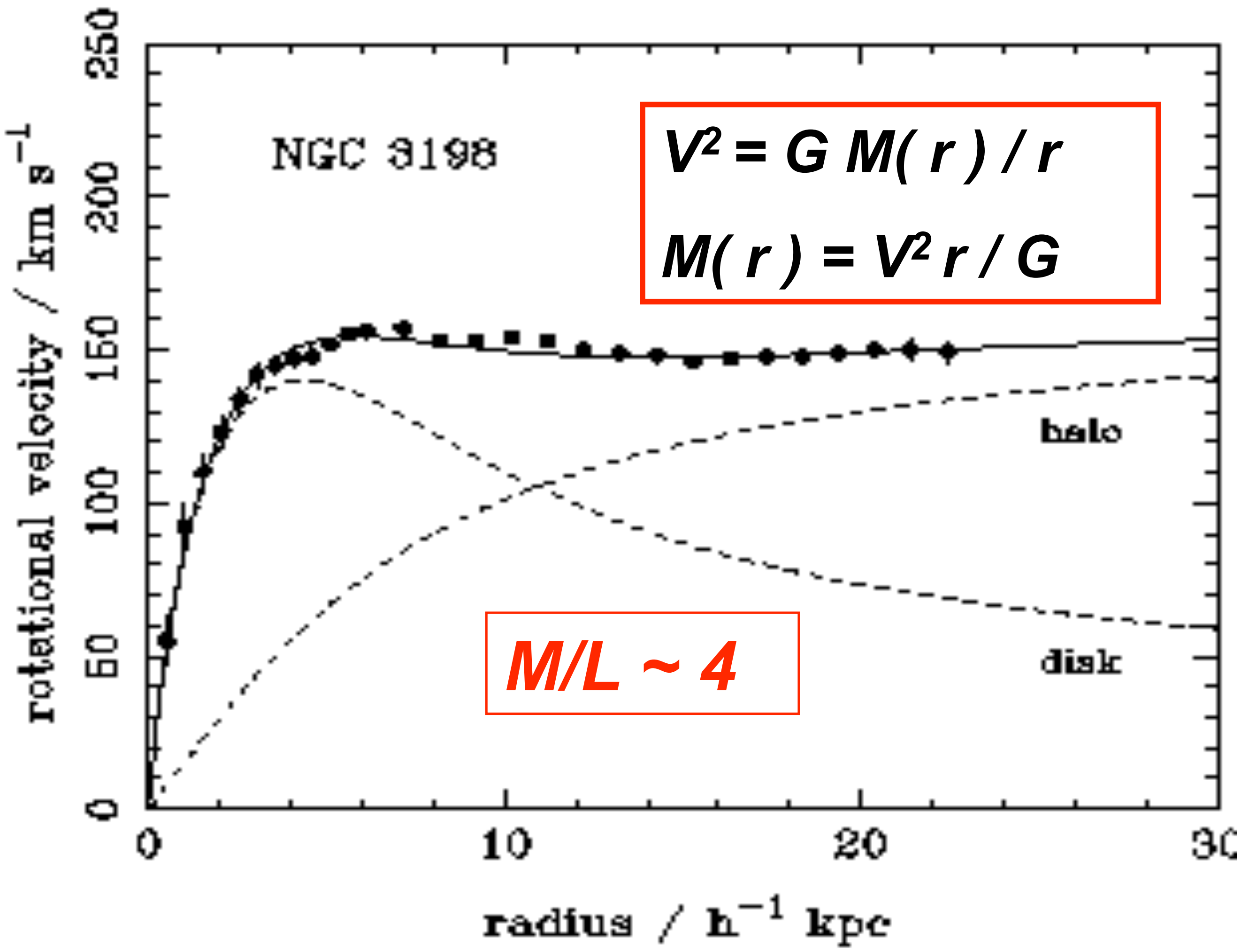
Dark Matter Halos

Spirals, Ellipticals:

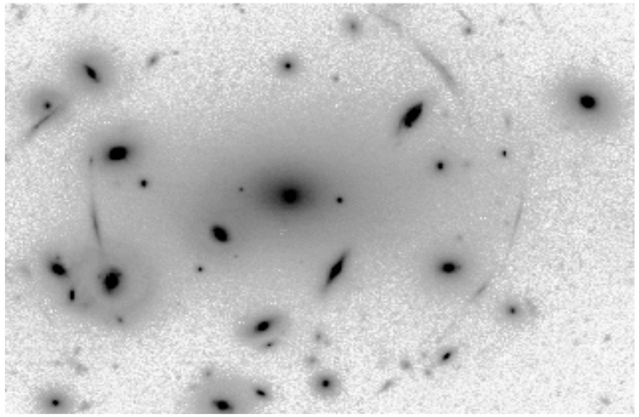
M/L~4-10

Some dwarf galaxies:

M/L ~ 100.



Reading: Masses from Gravitational Lensing



$$\theta_E = \frac{R_E}{D_L} = \left(\frac{4\,G\,M}{c^2} \frac{D_{LS}}{D_L D_S} \right)^{1/2}$$

$$\frac{M}{10^{11}\,M_{sun}} = \frac{D_L D_S / D_{LS}}{\mathrm{Gpc}} \left(\frac{\theta_E}{\mathrm{arcsec}} \right)^2$$

Use redshifts, z_L, z_S,

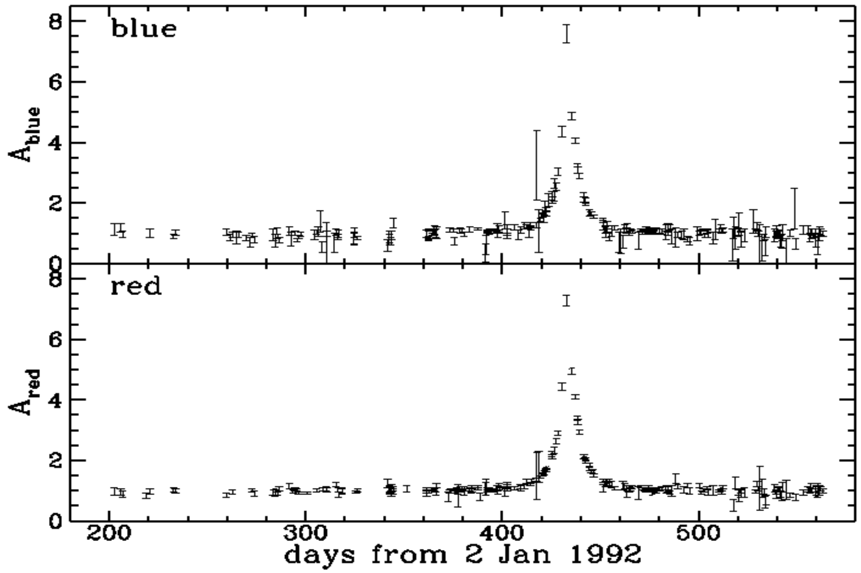
for the angular diameter distances.

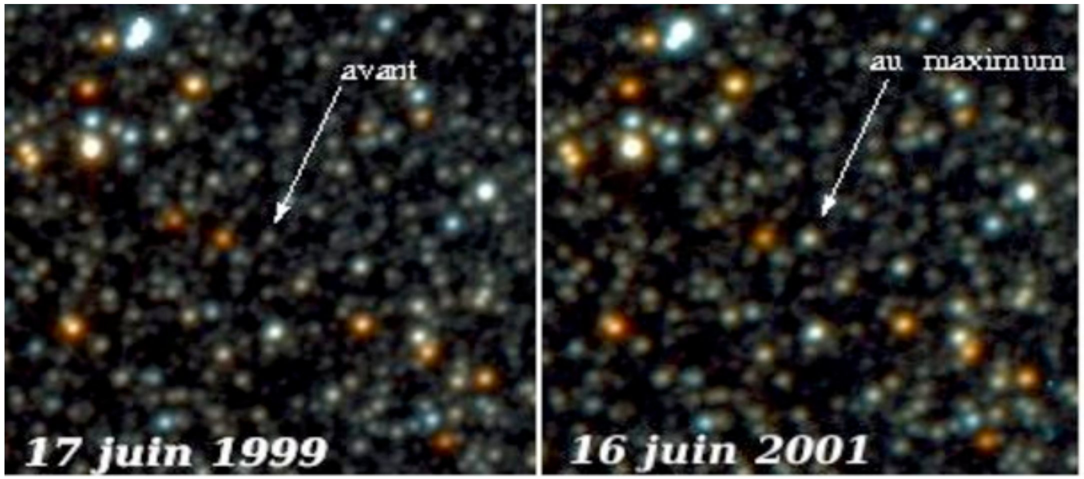
General agreement with Virial Masses.

Dark Matter Candidates

- **MACHOS = Massive Compact Halo Objects**
  - Black holes
  - Brown Dwarfs
  - Loose planets
  - Ruled out by gravitational lensing experiments.
- **WIMPS = Weakly Interacting Massive Particles**
  - Massive neutrinos
  - Supersymmetry partners
  - Might be found soon by Large Hadron Collider in CERN

reading: Microlensing in the LMC



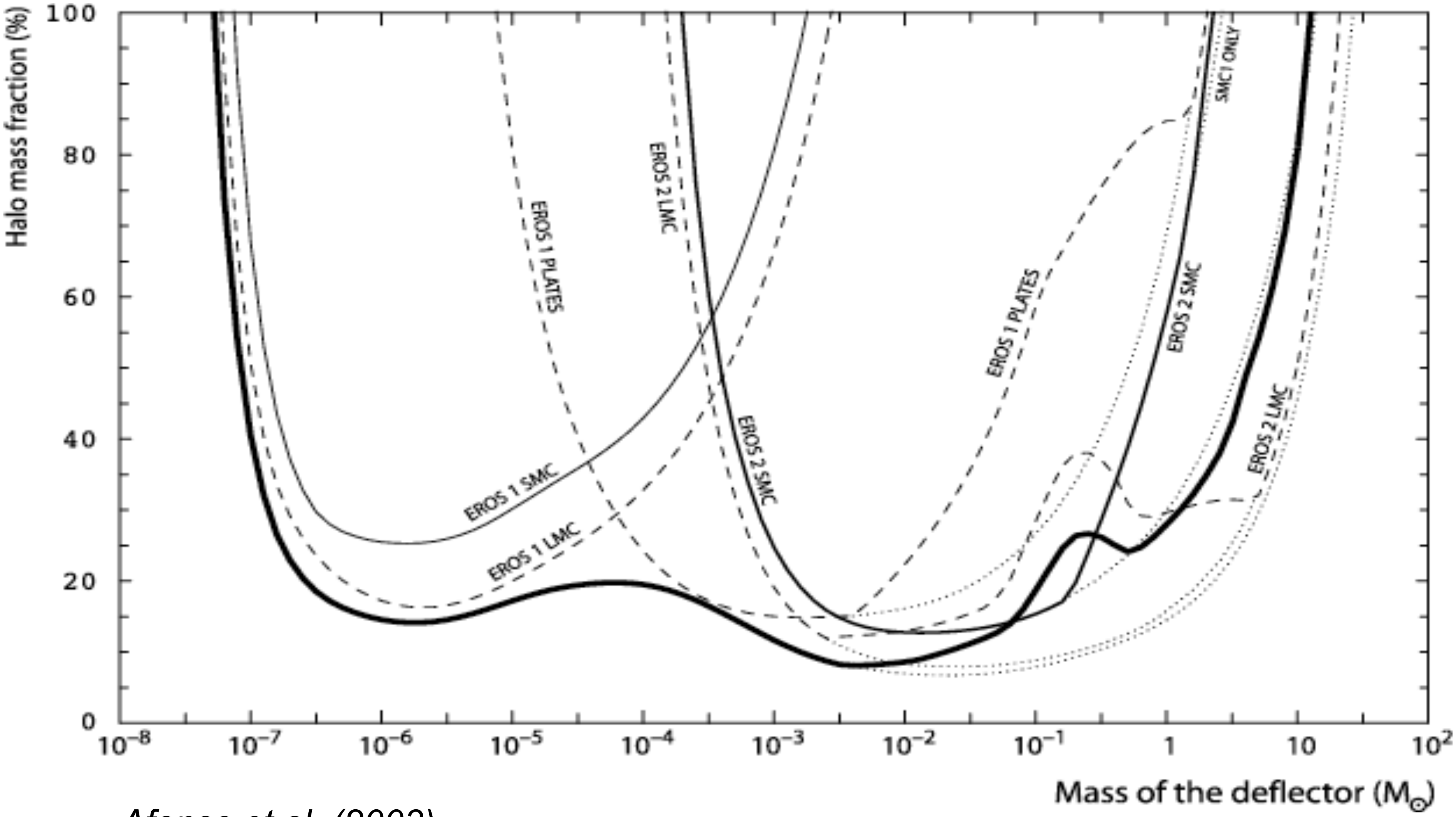


Massive Compace Halo Objects (MACHOS) would magnify LMC stars dozens of times each year. Only a few are seen.

Long events -> high mass

Short events -> low mass

Reading: LMC Microlensing says NO to MACHOs



Afonso et al. (2003)

reading: Mass Density by Direct Counting

- **Add up the mass of all the galaxies per unit volume**
  - **Volume calculation as in Tutorial problem.**
- **Need representative volume > 100 Mpc.**
- **Can't see faintest galaxies at large distance. Use local Luminosity Functions to include fainter ones.**
- **Mass/Light ratio depends on type of galaxy.**
- **Dark Matter needed to bind Galaxies and Galaxy Clusters dominates the normal matter (baryons).**
- **Hot x-ray gas dominates the baryon mass of Galaxy Clusters.**

reading: Schechter Luminosity Function

3 Schechter parameters :

$$\alpha \quad L^* \quad \Phi^*$$

luminosity of a typical big galaxy

$$L^* \approx 10^{11}\ \mathrm{L}_{\mathrm{sun}}$$

luminosity of any galaxy :

$$L = x\,L^* \qquad x \equiv \frac{L}{L^*}$$

number of galaxies per unit luminosity

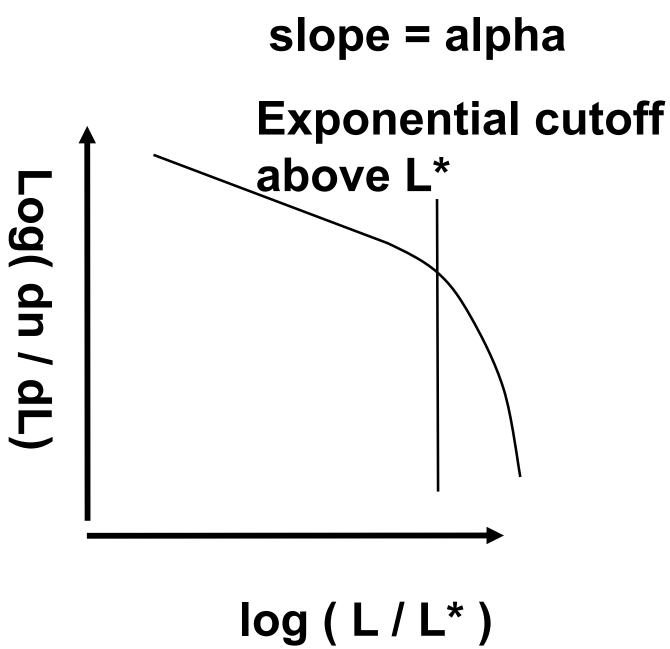
$$\Phi(x) \equiv \frac{dn}{dx} = \Phi^*\ x^{\alpha}\ e^{-x}$$

add up the luminosities

$$\rho_L = \int_0^\infty L\,\frac{dn}{dx}\,dx = L^*\Phi^* \int_0^\infty x^{\alpha+1}\ e^{-x}\ dx$$

add up the mass (need mass/light ratio)

$$\rho_M = \int_0^\infty \frac{M}{L}\,L\,\frac{dn}{dx}\,dx = \left\langle \frac{M}{L} \right\rangle \rho_L$$



Measure Schechter parameters using:

galaxy clusters

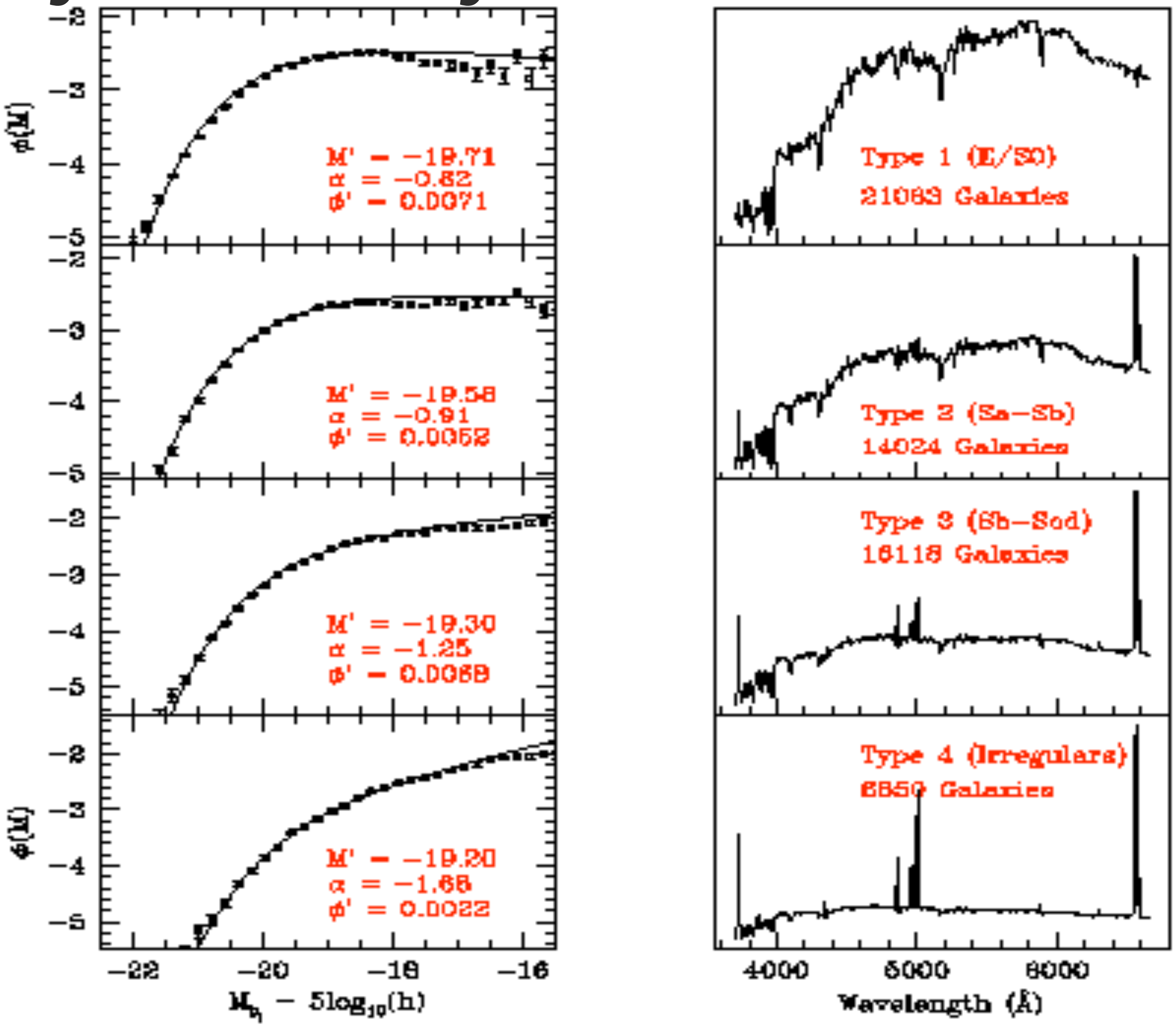
galaxy redshift surveys

Measure M/L for :

Nearby galaxies, galaxy clusters

Reading: Galaxy Luminosity Function

Schechter parameters depend on galaxy type.



reading: 14th Concept: Mass / Light ratios

galaxy luminosity distribution

$$\frac{dn}{dL} = \Phi(L) = \Phi^* \left(\frac{L}{L^*}\right)^\alpha \exp\left(-\frac{L}{L^*}\right)$$

luminosity density $\rho_L = \int L\,\Phi(L)\,dL$

e.g. blue light $\approx 2 \pm 0.7 \times 10^8 h\ L_{sun}\ \mathrm{Mpc}^{-3}$

mass density $\rho_M = \int \left(\frac{M}{L}\right) L\,\Phi(L)\,dL$

$$= \Omega_M\ \rho_{\mathrm{crit}} = 2.8 \times 10^{11}\ \Omega_M\ \ h^2\ \ M_{sun}\ \mathrm{Mpc}^{-3}$$

Universe: $M/L = 1400\,\Omega_M\ h^2 \sim 200\ (\Omega_M/0.3)(h/0.7)^2$

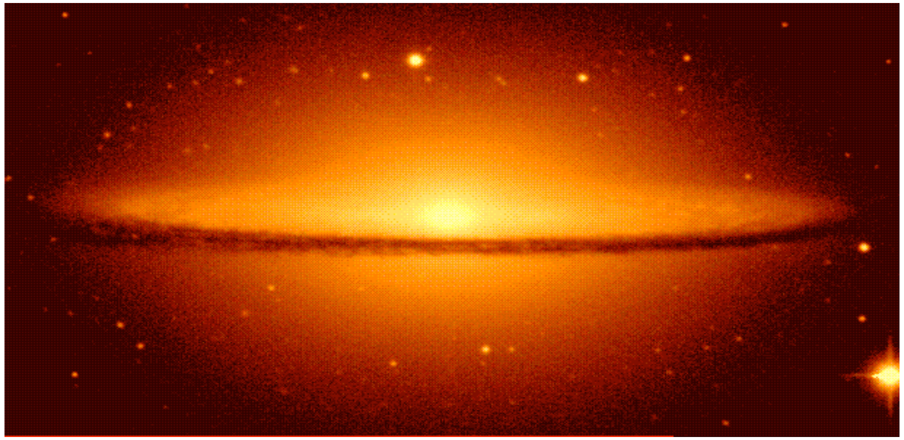
Sun: $M/L = 1$ (by definition)

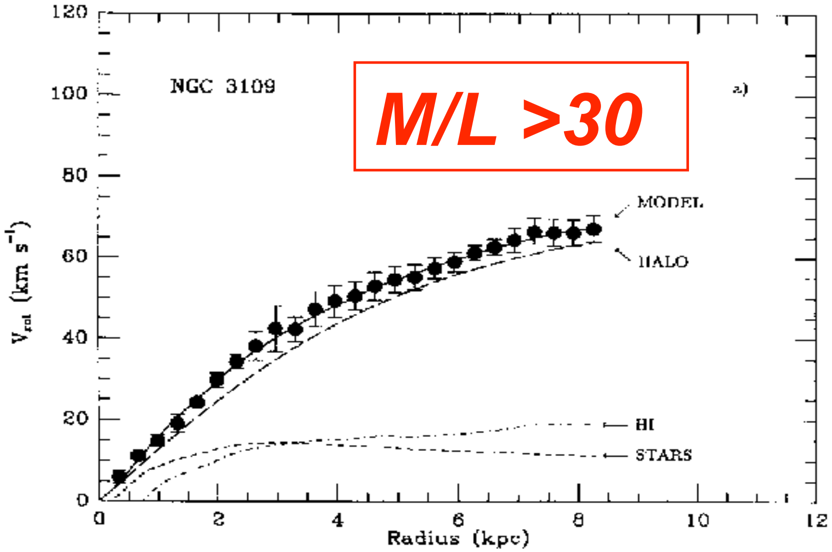
main sequence stars: $M/L \propto M^{-3}$ (since $L \propto M^4$)

comets, planets: $M/L \sim 10^{9-12}$

Is our Dark Matter halo filled with MACHOs ? NO.
Gravitational Lensing results rule them out.

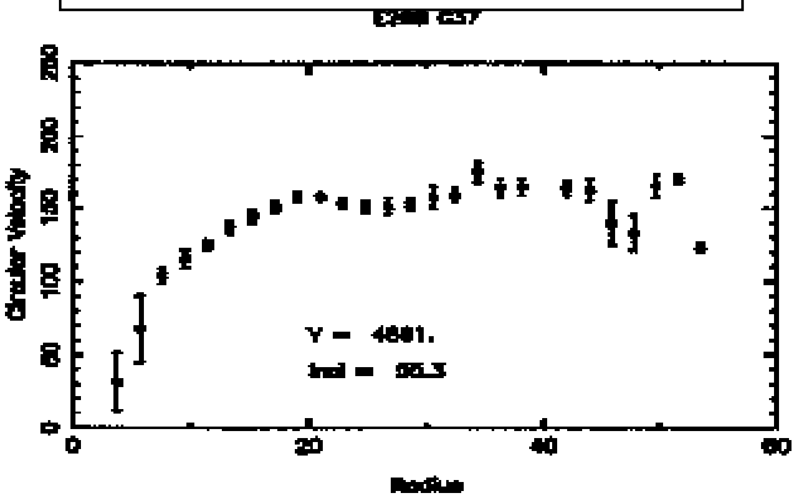
reading: Galaxy Rotation Curves

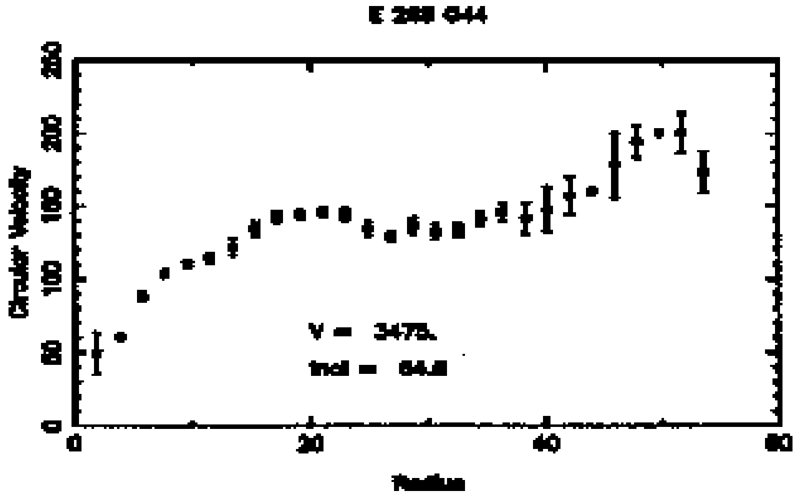




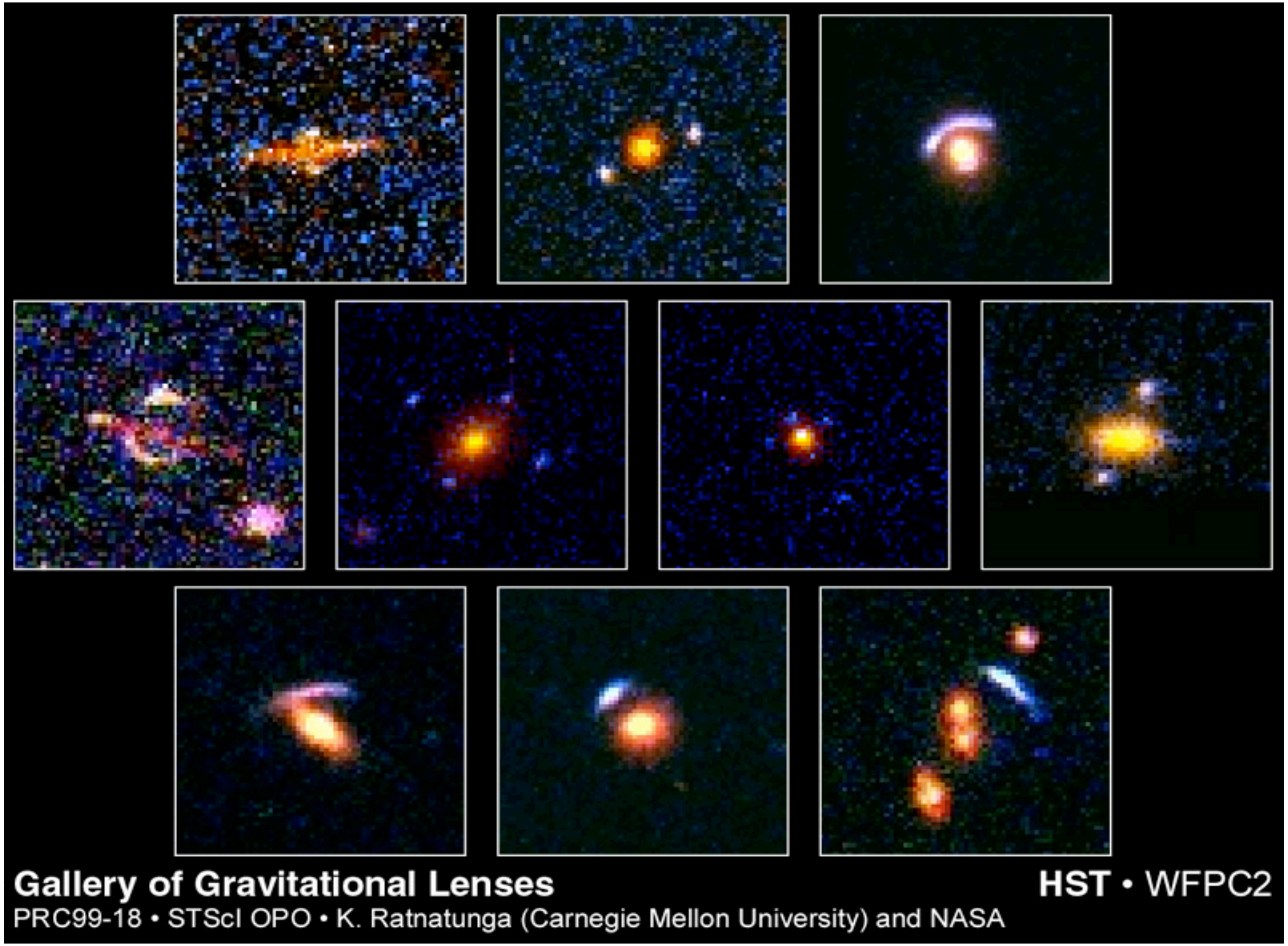
Small galaxies : V(r) rises

Large galaxies : V(r) flat





Quasars Lensed by Galaxies



Some formulae to remember

$$R(t)\, d\chi = c\, dt, \quad c\, dt = \frac{-c\, dz}{(1+z)\, H(z)}, \mathrm{D}_A = \frac{R_0}{(1+z)} \cdot S_k(\chi), \mathrm{D}_L = (1+z)\, R_0 \cdot S_k(\chi)$$

$$\rho_i = \frac{3\, H_0^2}{8\, \pi\, G}\, \Omega_i (1+z)^{n_i}, \; H(z)^2 = \Sigma_{i=m,\, r,\, v,\, c} \frac{8\, \pi\, G \rho_i}{3};$$

$$c_s^2 = \frac{dP}{d\rho}, P = w\, \rho\, c^2, \quad P = -\frac{d\left(\rho\, c^2 R^3\right)}{d\left(R^3\right)};$$