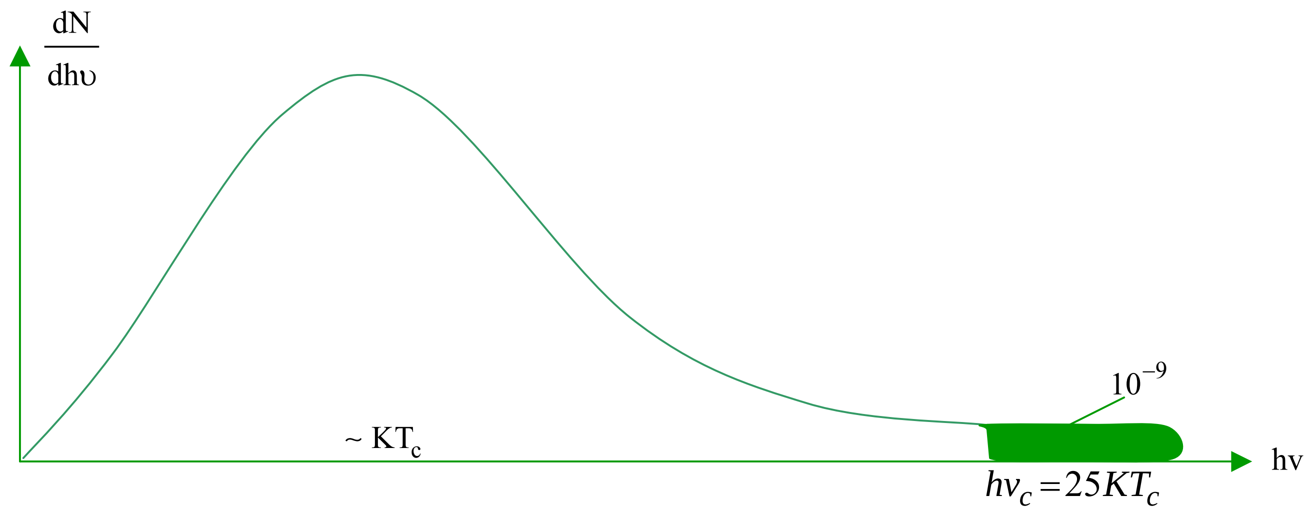
Energetic Tail of Photon Bath



$$\frac{N(>h\nu_c)}{N_{ph}} \approx e^{-\frac{h\nu_c}{KT_{c.}}} \left(\frac{h\nu_c}{KT_c}\right)^2 .O(1)$$

$$\sim e^{-25} \times 25^2$$

$$\sim 10^{-9} \sim \frac{N_B}{N_{ph}}$$

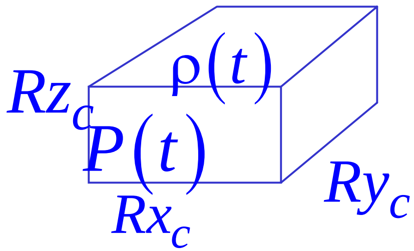
hardest photons

~ # baryons

If run short of hard photon to unbind => "Freeze-out" => $KT_c \sim \frac{h\nu_c}{25}$

Evolution of Sound Speed

Expand a box of fluid



Sound Speed $\quad C_s^2 \equiv \frac{\partial P / \partial (vol)}{\partial \rho / \partial (vol)},$

$$= \frac{\partial P / \partial R}{\partial \rho / \partial R}$$

$$Vol = R^3(t) \cdot x_c y_c z_c$$

$$\propto R^3(t)$$

	Radiation	Matter
Where fluid density $\rho(t) =$	ρ_r	ρ_m
Fluid pressure $P(t) =$	$\frac{c^2}{3}\rho_r$	$\frac{\rho_m}{\mu} \cdot KT_m$
		Matter number density; Random motion energy Non-Relativistic IDEAL GAS

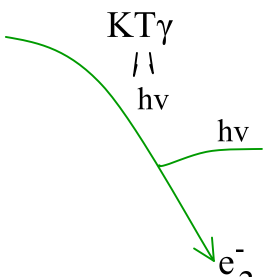
Note $\rho_r \propto R^{-4}$

$\rho_m \propto R^{-3}$

Neglect $\frac{1}{\mu} KT_m << c^2$

Show $C^2_s = c^2/3 /(1+Q)$, $Q = (3\ ?_m) /(4\ ?_r)$, ➔ Cs drops

- from c/sqrt(3) at radiation-dominated era
- to c/sqrt(5.25) at matter-radiation equality



Compton-scatter $\frac{3}{2}KTe$

electrons in γ – bath

Keep electrons hot Te ~ Tr until redshift z

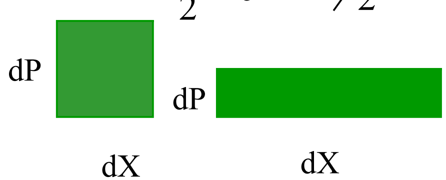
$$Tr \sim 1500 \times \left(\frac{1+z}{500}\right)$$

After decoupling (z<500),
Cs ~ 6 (1+z) m/s because

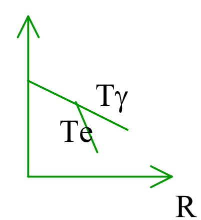
$\underline{d}^{\,3}\underline{P}\ \underline{d}_x^3$ invarient phase space volume

So: $P \propto x^{-1} \propto R^{-1}$

$$\frac{3}{2}\times T_e = m\upsilon^2/2 \propto R^{-2} \qquad Te \sim 1500\,K\times\left(\frac{1+z}{500}\right)^2$$



$C_s \sim 6\ (1+z)$ m/s



Until reionization z ~ 10 by stars quasars

- Growth of Density Perturbations and peculiar velocity

Peculiar Motion

- The motion of a galaxy has two parts:

$$\vec{v} = \frac{d}{dt}\left[R(t)\theta(t)\right]$$

$$= \underline{\underline{\dot{R}(t).\theta}} + \underline{\underline{R(t)\dot{\theta}(t)}}$$

$$\vec{x}_c \leftrightarrow \vec{\theta}$$

Proper length vector

Uniform expansion v_o

Peculiar motion δv

Damping of peculiar motion (in the absence of overdensity)

-

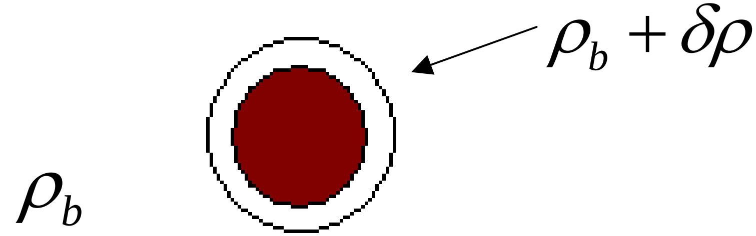
- Generally peculiar velocity drops with expansion.

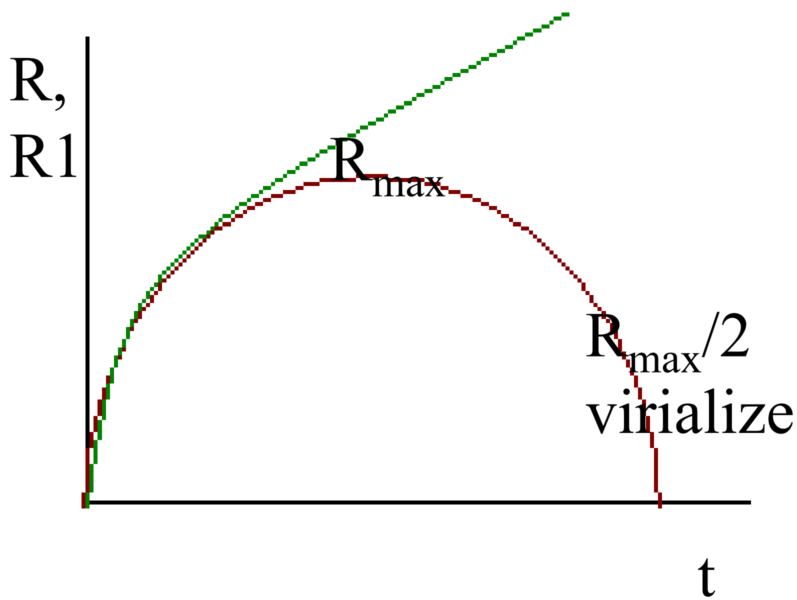
$$R^2\dot{\theta} = R*(R\dot{\theta}) = \text{constant}\sim\text{"Angular Momentum"}$$

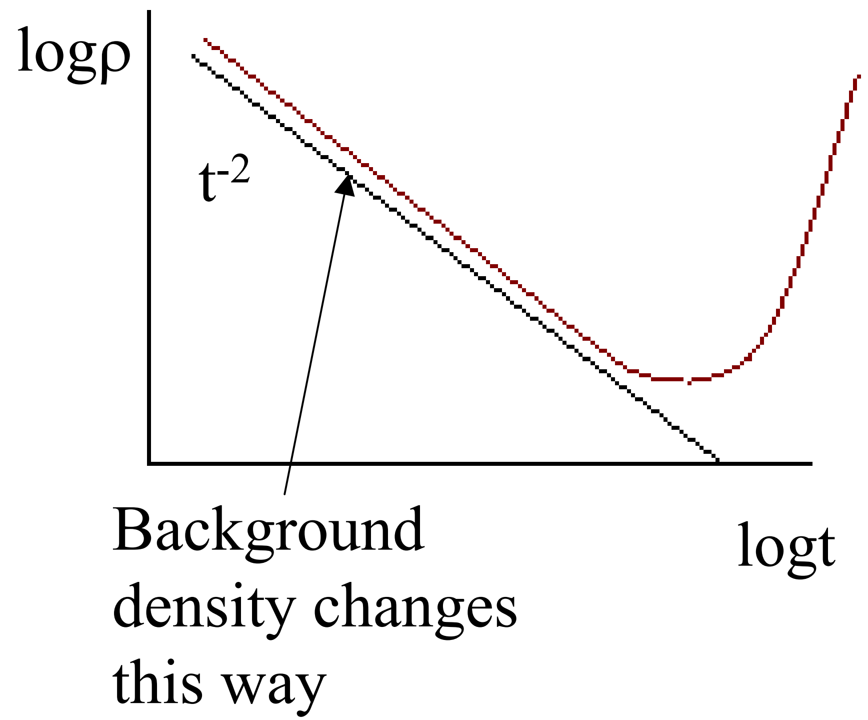
$$\delta v = R(t)\dot{x}_c = \frac{\text{constant}}{\text{R(t)}}$$

Non-linear Collapse of an Overdense Sphere

- An overdense sphere is a very useful non linear model as it behaves in exactly the same way as a closed sub-universe.
- The density perturbations need not be a uniform sphere: any spherically symmetric perturbation will clearly evolve at a given radius in the same way as a uniform sphere containing the same amount of mass.







$$\rho_b = \frac{1}{6\pi G t^2}$$

Gradual Growth of perturbation

$$\rightarrow \frac{\delta\rho}{\rho} = \frac{3c^2}{8\pi G}\frac{1}{\rho R^2} \propto \begin{cases} R^2 \ \text{(mainly radiation } \rho \propto R^{-4}) \\ R \ \text{(mainly matter } \rho \propto R^{-3}) \end{cases}$$

Perturbations Grow!

Equations governing Fluid Motion

$$\nabla^2 \phi = 4\pi G\rho \qquad \text{(Poissons Equation)}$$

$$\frac{1}{\rho}\nabla\rho = \frac{d\ln\rho}{dt} = -\vec{\nabla}.\vec{v} \qquad \text{(Mass Conservation)}$$

$$\frac{\mathrm{dv}}{\mathrm{dt}} = -\vec{\nabla}\phi - \underline{\underline{c_s^2\nabla\ln\rho}} \qquad \text{(Equation of motion)}$$

$$\searrow \frac{\nabla P}{\rho} \quad \text{since } \partial P = c_s^2\partial\rho$$

- Let

$$\rho = \rho_o + \delta\rho$$

$$v = v_o + \delta v = \dot{R}\chi_c + R\dot{\chi}_c$$

$$\phi = \phi_o + \delta\phi$$

$$x(t) = R(t)\chi_c$$

- We define the Fractional Density Perturbation:

$$\delta = \frac{\delta\rho}{\rho_o} = \delta(t)\exp(-i\vec{k} \bullet \vec{x})$$

- Motion driven by gravity: $\vec{g}_o(t) + \vec{g}_1(\theta, t)$

  due to an overdensity: $\rho(t) = \rho_o(1 + \delta(\theta, t))$

- Gravity and overdensity by Poissons equation:

$$-\vec{\nabla}_1 . g_1 = 4\pi G \rho_o \delta$$

- Continuity equation:

$$-\vec{\nabla}.\delta\vec{v} = \frac{d}{dt}\left(\delta(\theta, t)\right)$$

← The over density will rise if there is an inflow of matter

Peculiar motion and peculiar gravity both scale with d and are in the same direction.

the equation for linear growth

- At high z>>1 $\delta \propto R(t)$ $\rho \propto R^{-3}$

& matter domination

$\delta\rho \propto R^{-2}$

$\delta\phi \propto R^{0}$

- In the equation

$-c_s^2 k^2$

$$\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{R}}{R}\frac{\partial \delta}{\partial t} = (4\pi G\rho_o + c_s^2 \nabla^2)\delta$$

Gravity has the tendency to make the density perturbation grow exponentially.

Pressure makes it oscillate

Nearly Empty Pressure-less Universe

$$\Omega_M \sim 0$$

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{2}{t}\frac{\partial \delta}{\partial t} = 0, \quad \dot{H} = \frac{\dot{R}}{R} = \frac{1}{t} \quad (R \propto t)$$

$$\delta \propto t^0 = \text{constant}$$

$\rightarrow$ no growth

The Jeans Instability

- Case 1- no expansion
  - Assume the density contrast δ has a wave-like form

$$\delta = \delta_o \exp(i\vec{k}.\vec{r} - i\omega t)$$

  - Assume no expansion $\dot{R} = 0$

$$\frac{\partial^2 \delta}{\partial t^2} + 2 * 0 * \frac{\partial \delta}{\partial t} = -\omega^2 \delta$$

  - → the dispersion relation

$$\omega^2 \equiv \underline{\underline{c_s^2 k^2}} - \underline{\underline{4\pi G\rho}}$$

Pressure support → $c_s^2 k^2$

gravity → $4\pi G\rho$

- At the (proper) JEANS LENGTH scale we switch from
  - standing sound waves for shorter wavelengths to
  - the exponential growth of perturbations for long wavelength modes

$$\lambda_J = c_s \sqrt{\frac{\pi}{G\rho}}$$

- $\lambda < \lambda_J$, $\omega^2 > 0$ → oscillation of the perturbation.
- $\lambda \geq \lambda_J$, $\omega^2 \leq 0$ → exponential growth/decay

$$\delta \propto \exp(\pm \Gamma t) \text{ where } \Gamma = \sqrt{-\omega^2}$$

- Timescale:

$$\tau = (G\rho)^{-\frac{1}{2}}$$

$=$ dynamical collapse time

for region of density ρ.

- Application: Collapse of clouds, star formation.

Jeans Instability

- Case 2: on very large scale $\lambda >> \lambda_J$ of Expanding universe
  - Neglect Pressure (restoring force) term

$$c_s^2 k^2 << 4\pi G\rho = c_s^2 k_J^2$$

$$\frac{\partial^2 \delta}{\partial t^2} + 2H\frac{\partial \delta}{\partial t} = \frac{3}{2}H^2\Omega_M . \delta$$

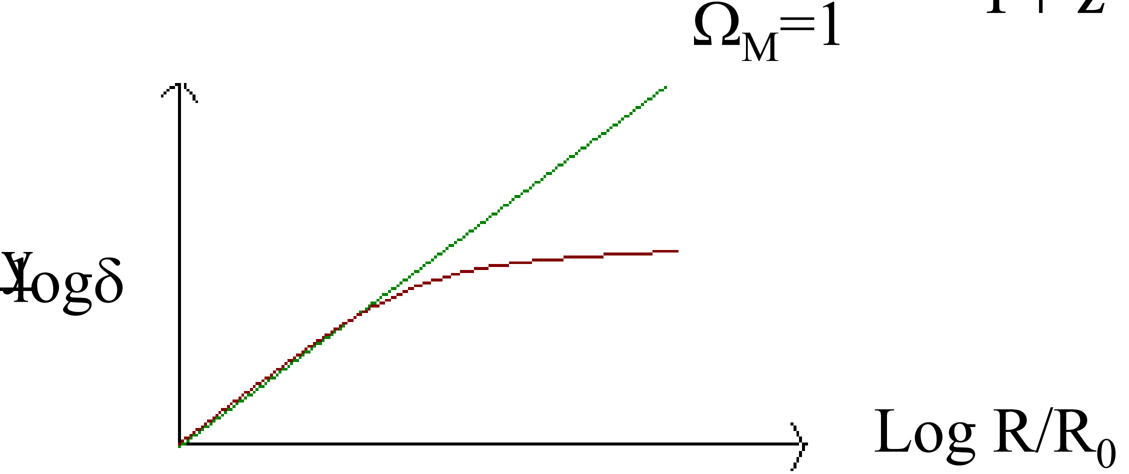
$$4\pi G\rho_m$$

- Einstein de Sitter

$$\Omega_M = 1, H = \frac{\dot{R}}{R} = \frac{2}{3t}$$

$$\text{Verify Growth Solution } \delta \propto R \propto t^{\frac{2}{3}} \propto \frac{1}{1+z}$$

- Generally



Case III: Relativistic Fluid

- equation governing the growth of perturbations being: $$\Rightarrow \frac{d^2\delta}{dt^2} + 2H\frac{d\delta}{dt} = \delta.\left(\frac{32\pi G\rho}{3} - k^2 c_s^2\right)$$

$$\Rightarrow \delta \propto t \propto R^2 \text{ for length scale } \lambda >> \lambda_J \sim ct$$

Jeans Mass Depends on the Species of the Fluid that dominates

- If Photon dominates:

$$M_J^{\gamma} = \rho_{\gamma}(t)\frac{4\pi}{3}\left(\frac{\lambda_J}{2}\right)^3 \propto \frac{1}{6t^2}\left[\underline{\underline{\frac{c}{\sqrt{3}}t}}\right]^3 \propto t^1 \propto \left(1+z\right)^{-2}$$

$c_s t$=distance travelled since big bang

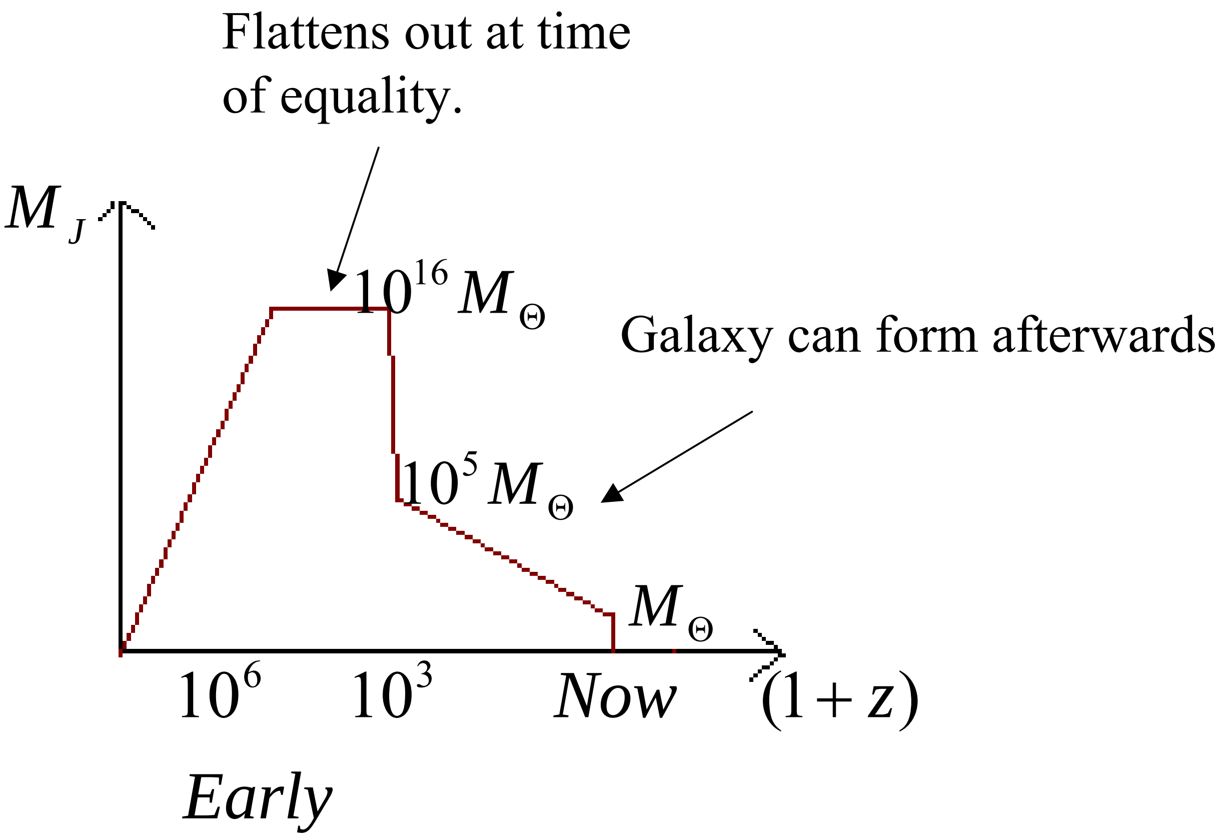
- If DarkMatt dominates & decoupled from photon:

$$M_J^{D} = \rho_{D}(t)\frac{4\pi}{3}\left(\frac{\lambda_J}{2}\right)^3 \propto \left(1+z\right)^3\left[c_s t\right]^3 \propto t^{-1}$$

$t \propto (1+z)^{-3/2} \propto R^{3/2}$,

non-relativistic cooling of random motion $c_s \propto 1/R \propto (1+z)$

- Jeans Mass past and now



Note: $R \propto (1+z)^{-1}$

Dark Matter Overdensity Growth Condition

- GROW Possible only if
  - During matter-domination ($t > t_{eq}$) or
  - during radiation domination, but on proper length scales larger than
    - sound horizon ($\lambda > c_s\, t$) &
    - free-streaming length of relativistic dark matter ($\lambda > c\, t_{fs}$)

Theory of CMB Fluctuations

- Linear theory of structure growth predicts that the perturbations:

δ_D in dark matter $\dfrac{\delta\rho_{\mathrm{D}}}{\rho_{\mathrm{D}}}$

δ_B in baryons $\dfrac{\delta\rho_{\mathrm{B}}}{\rho_{\mathrm{B}}}$

δ_r in radiation $\dfrac{\delta\rho_{\mathrm{r}}}{\rho_r}$ Or $\tilde{\delta}_r = \dfrac{3}{4}\delta_r = \dfrac{\delta n_\gamma}{n_\gamma}$

will follow the following coupled equations.

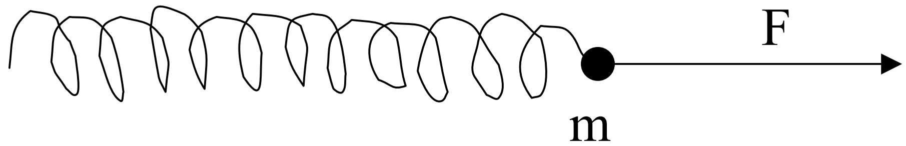
$$\frac{d^2}{dt^2}\begin{pmatrix}\delta_D\\ \delta_B\\ \tilde{\delta}_r\end{pmatrix} + 2H(t)\frac{d}{dt}\begin{pmatrix}\delta_D\\ \delta_B\\ \tilde{\delta}_r\end{pmatrix} + k^2\begin{pmatrix}c_{s,D}^2\delta_D\\ c_{s,B}^2\delta_B\\ c_{s,r}^2\tilde{\delta}_r\end{pmatrix} = \nabla^2\Psi = -k^2\Psi$$

- Where ψ is the perturbation in the gravitational potential, with $\Psi_{x,t} \propto \Psi(t)\exp(i\vec{k}.\vec{x})$

Gravitational Coupling →

$$\Psi = 4\pi G\delta\rho_D + 4\pi G\delta\rho_B + 8\pi G\delta\rho_k$$
$$= 4\pi G\rho_{crit} \times \left[\Omega_D\delta_D + \Omega_B\delta_B + 2\Omega_r\delta_r\right]$$

- This is similar to a spring with a restoring force:



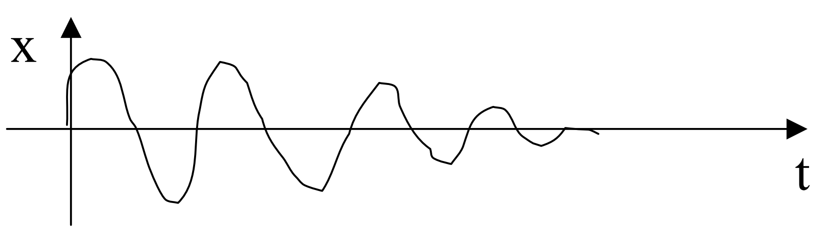
- $F_{restoring}=-m\omega^2 x$

$$\frac{d^2x}{dt^2}=\frac{F}{m}-\omega^2 x-\underline{\underline{\mu\frac{dx}{dt}}}$$

Term due to friction

$$\frac{d^2x}{dt^2}+\mu\frac{dx}{dt}+\omega^2 x=\frac{F(t)}{m}$$

(Displacement for Harmonic Oscillator)



- The solution of the Harmonic Oscillator equation is:

$$\delta(t) = A_1 \cos kc_s t + A_2 \sin kc_s t + A_3$$

For B or R

$$c_s^2 = \frac{c^2}{3(1+Q)} \qquad Q = \frac{3\rho_B}{4\rho_R} \qquad \left(\begin{array}{l}\text{varies with time} \\ Q \propto \Omega_B\end{array}\right)$$

- Amplitude is sinusoidal function of $k\, c_s\, t$
  - if k=constant and oscillate with t
  - or t=constant and oscillate with k.

- We don't observe δ_B directly-what we actually observe is temperature fluctuations.

$$\frac{\Delta T}{T} = \frac{\Delta n_\gamma}{3 n_\gamma}$$

$$= \frac{\delta_B}{3} = \frac{\tilde{\delta}_R}{3}$$

$$n_\gamma \sim R^{-3} \propto T^3$$

$$\varepsilon_\gamma \sim n_\gamma kT \propto T^4$$

- The driving force is due to dark matter over densities.
- The observed temperature is:

$$\left(\frac{\Delta T}{T}\right)_{obs} = \frac{\delta_B}{3} + \frac{\psi}{c^2}$$

Effect due to having to climb out of graviatational well

- The observed temperature also depends on how fast the Baryon Fluid is moving.

Velocity Field $\nabla v = -\frac{d\delta_B}{dt}$

$$\left(\frac{\Delta T}{T}\right)_{obs} = \frac{\delta_B}{3} + \frac{\psi}{c^2} \pm \frac{v}{c}$$

Doppler Term