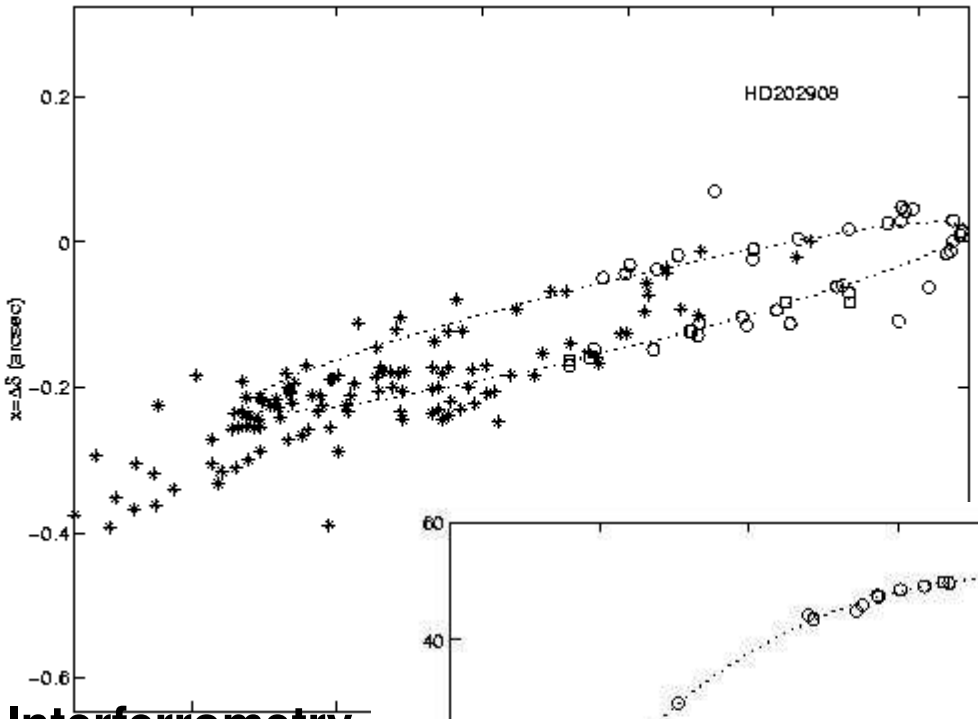
Inclinations

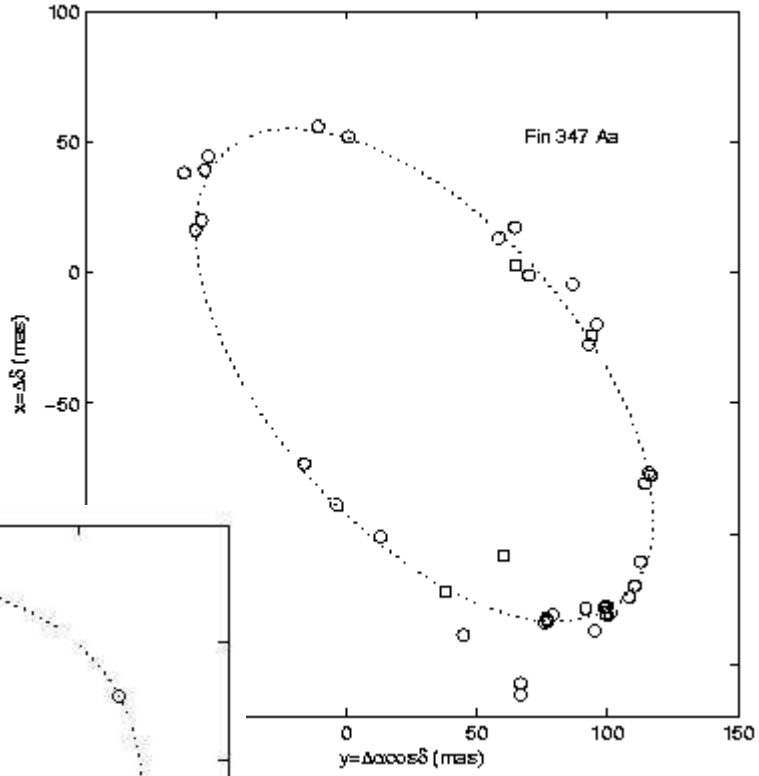
- **Radial velocities give minimum masses**

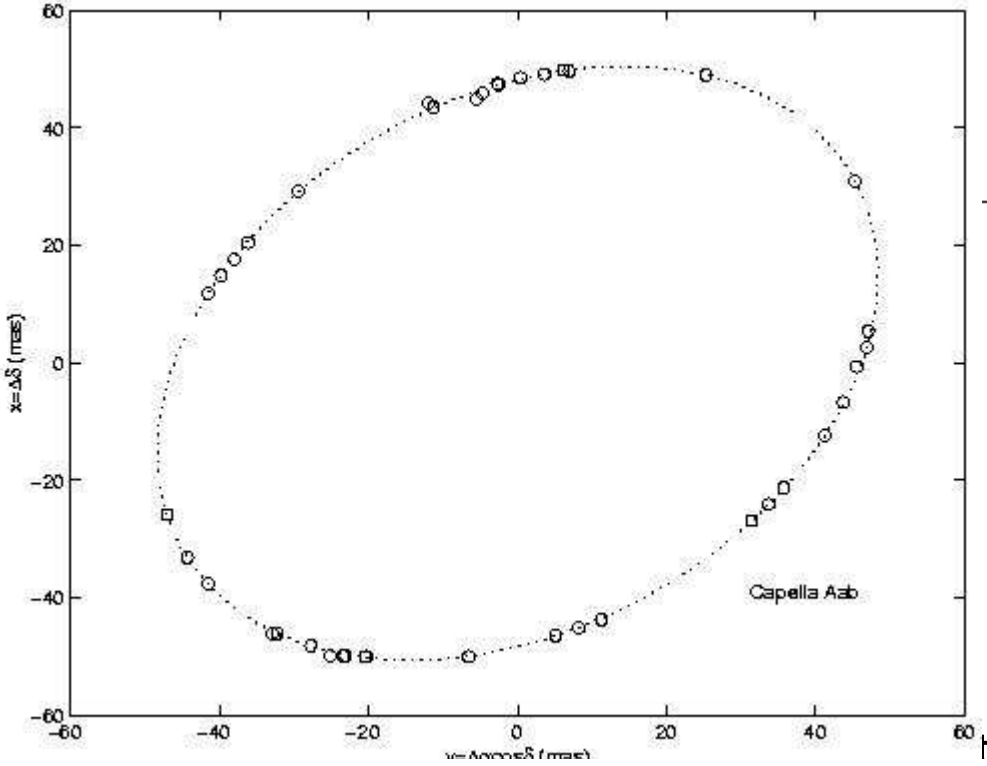
$$\text{SB2:} \quad m \sin^3 i \qquad \text{SB1:} \quad \frac{m^3 \sin^3 i}{M^2}$$

- **need inclinations**
  - **astrometric orbits**
  - **eclipsing binaries**
  - **polarimetry**
  - **apsidal motion**

Astrometric Orbits







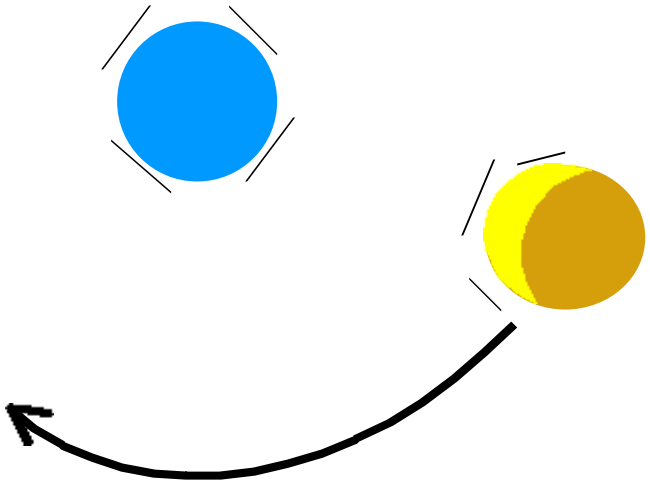
Interferrometry measures the relative orbit (milli-arcsec accuracy).

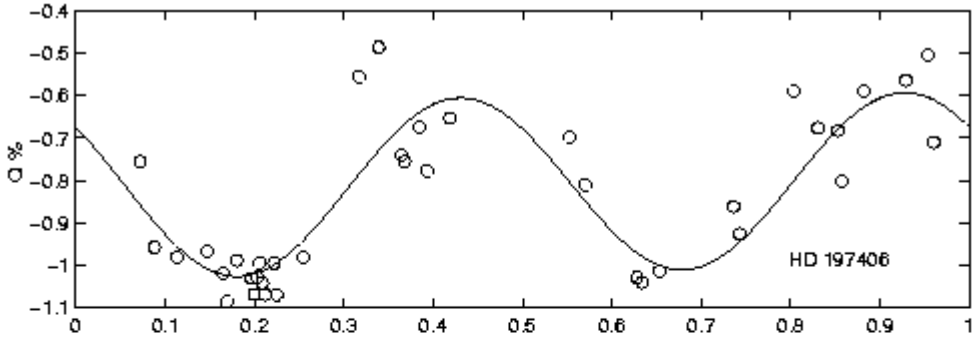
Fit elliptical orbit model to x(t) and y(t).

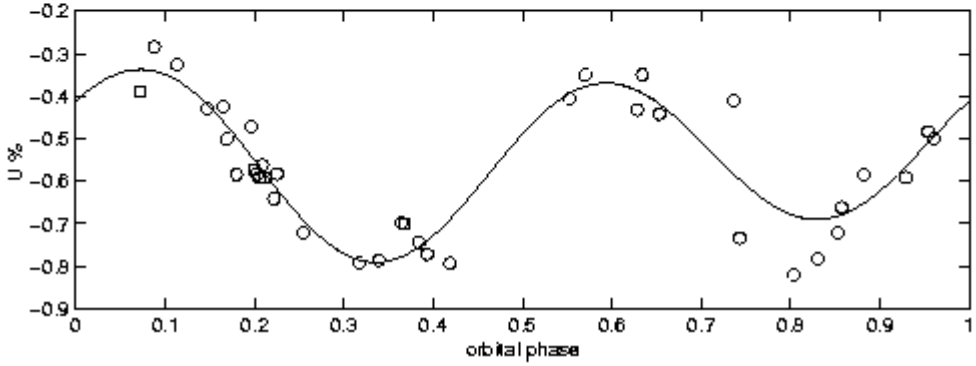
Get inclination, hence masses

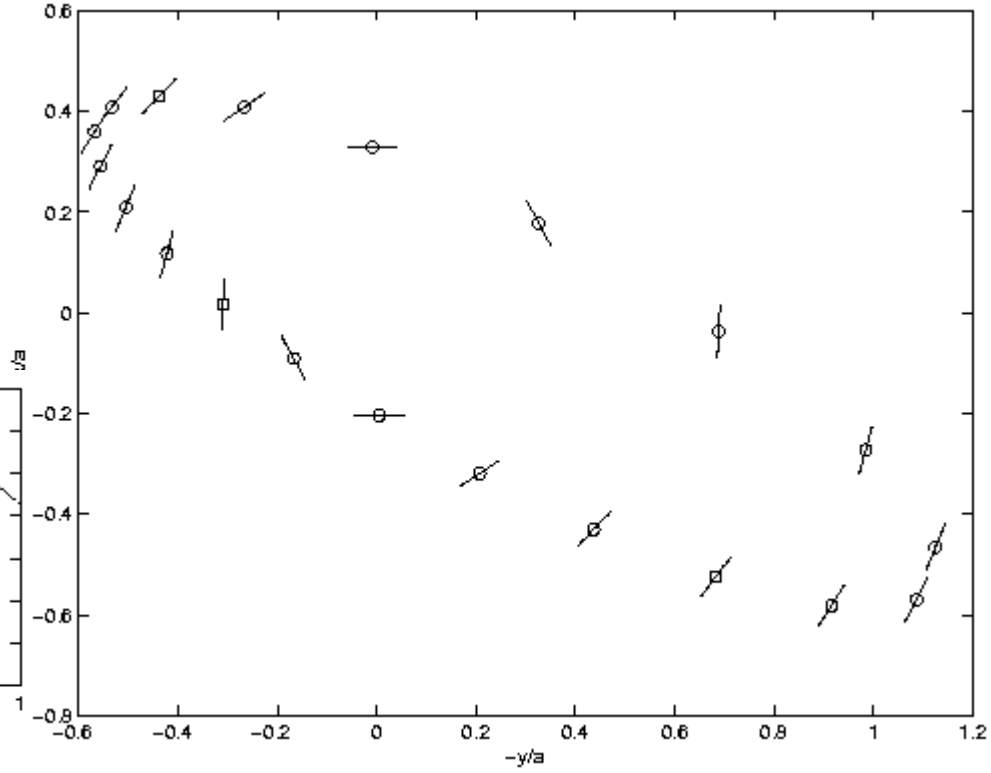
Polarimetry

Scattered light is polarised (e.g. our sky)



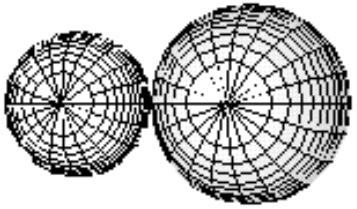
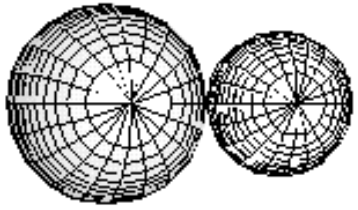


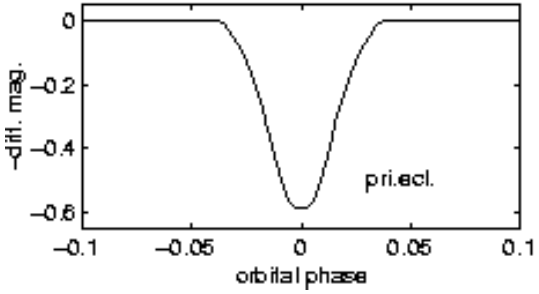


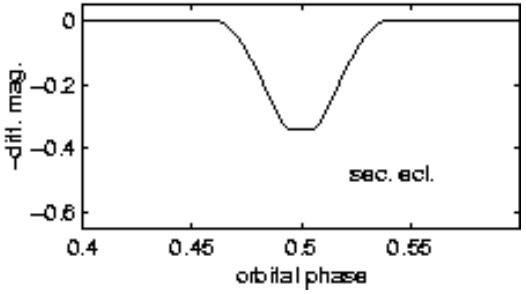


Get inclination, hence masses

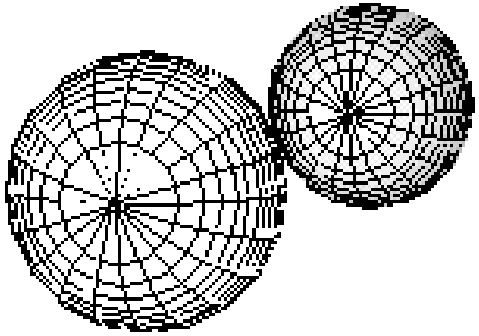
Eclipsing Binaries

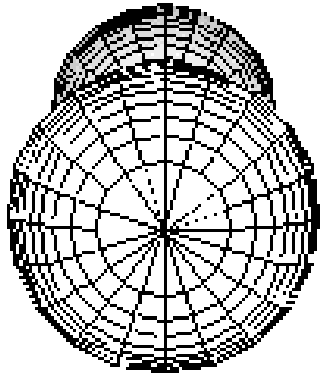
- **Binary properties from eclipses**
  - sizes, shapes of stars, inclination,
  - temperatures, limb darkening, apsidal motion
  - 4 contact phases:
  - 1  2
  - 3 4 
  - primary eclipse secondary eclipse

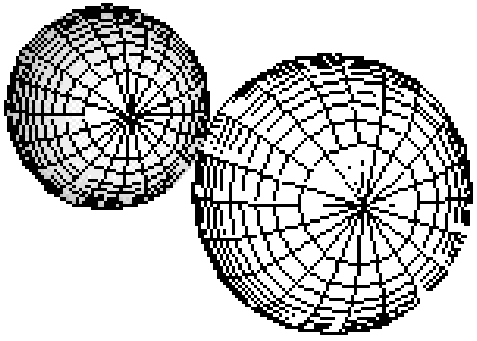


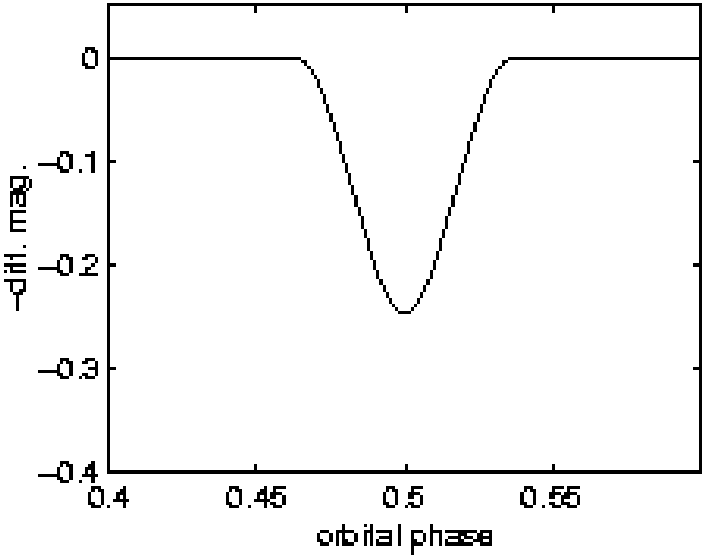


Partial Eclipse









Eclipses

$$R_1 > R_2 \quad \theta \equiv 2\pi\,\phi$$

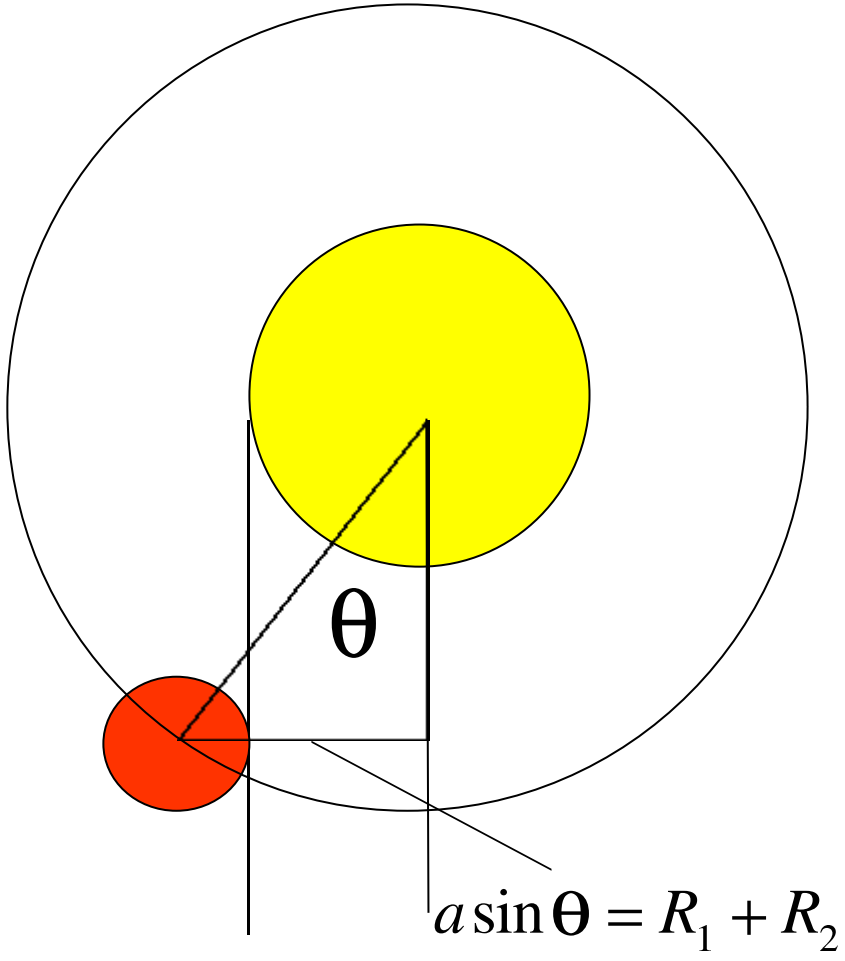
edge-on $(i = 90^\circ)$:

4 contact phases:

$$R_1 \pm R_2 = a\sin\theta$$

$$\frac{R_2}{a} = \frac{\sin\theta_1 - \sin\theta_2}{2}$$

$$\frac{R_1}{a} = \frac{\sin\theta_1 + \sin\theta_2}{2}$$



Eclipses

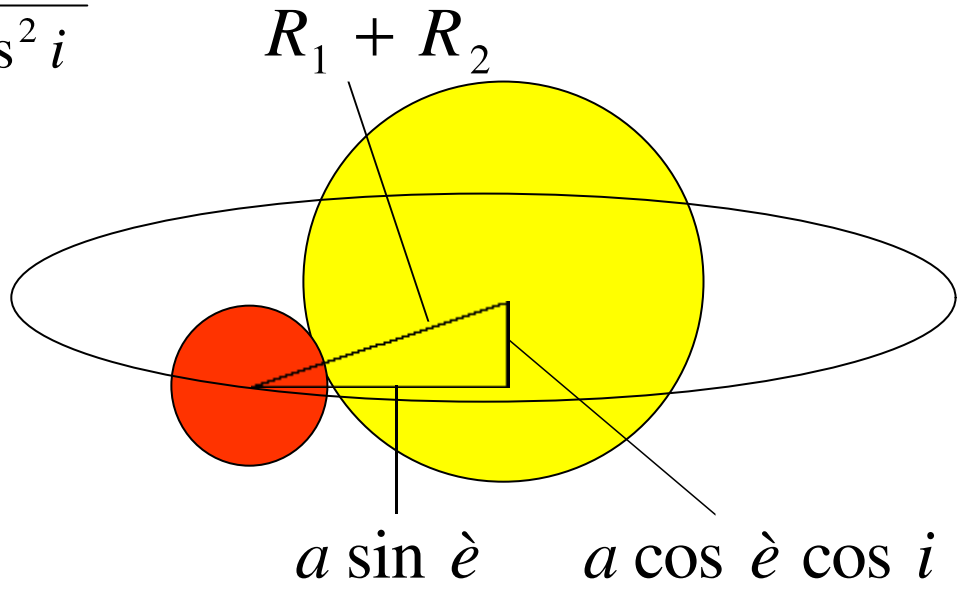
4 contact phases :

$$R_1 \pm R_2 = a\sqrt{\sin^2\theta + \cos^2\theta\ \cos^2 i}$$

$$= a\sqrt{1 + \cos^2\theta\ \sin^2 i}$$

4 measurements : $\phi_1\ \ \phi_2\ \ \phi_3\ \ \phi_4$

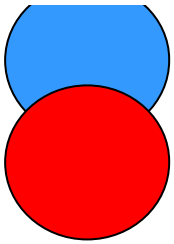
4 parameters : $\frac{R_1}{a}\quad \frac{R_2}{a}\quad i\quad \phi_0$

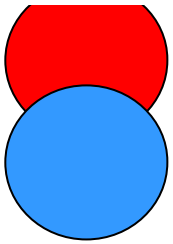


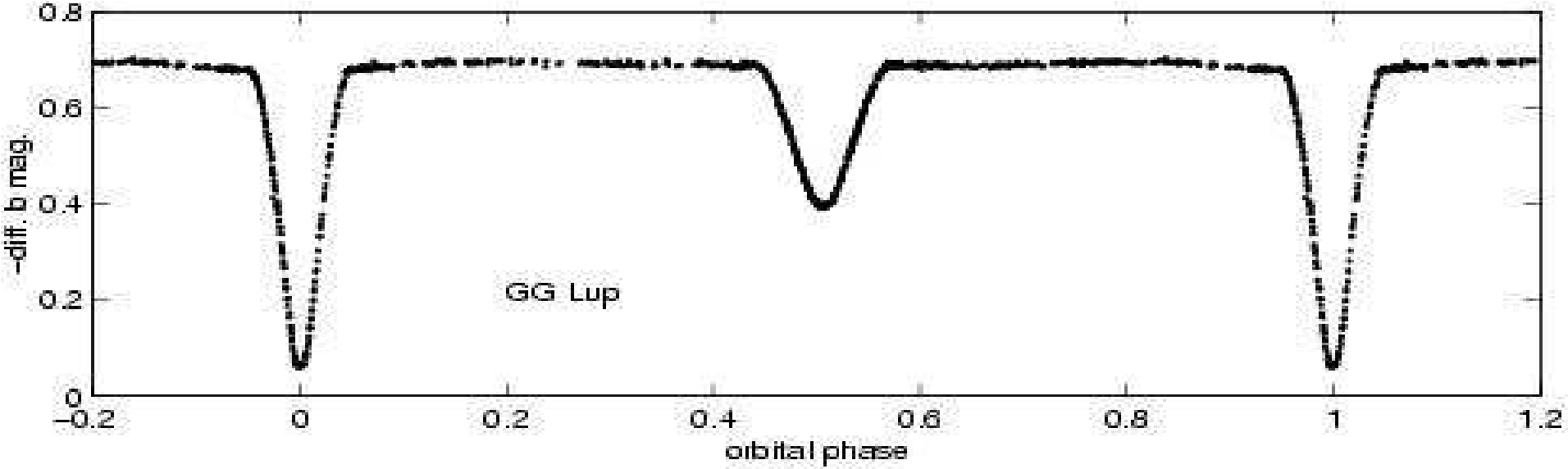
mid eclipse : $\theta = 0$

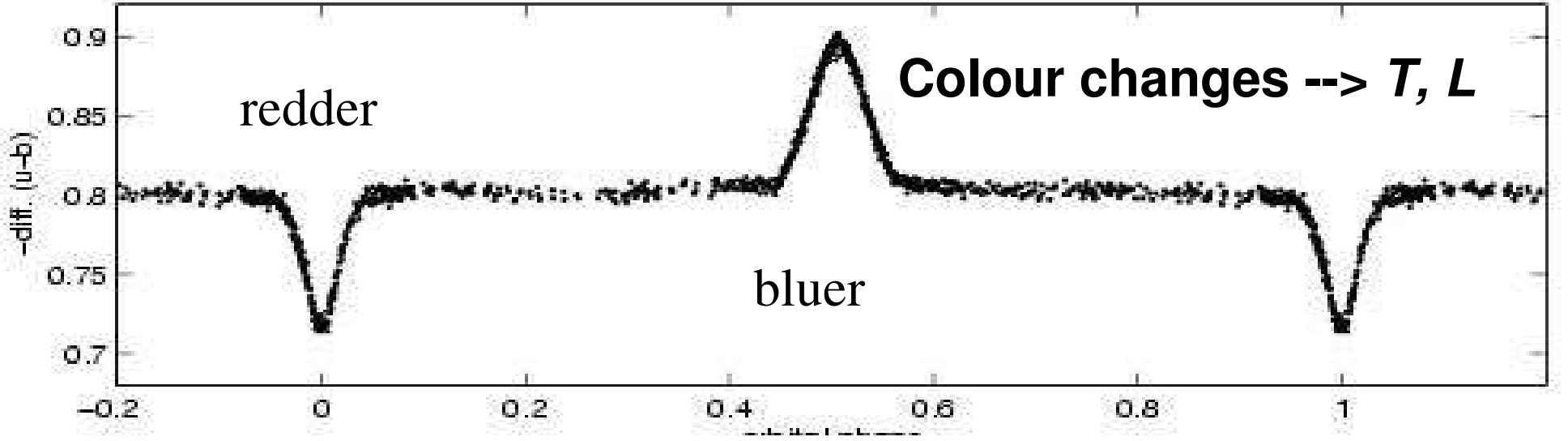
total eclipse : $a\cos i < R_1 - R_2$

partial eclipse : $R_1 - R_2 < a\cos i < R_1 + R_2$






0.8
0.6
0.4
0.2
0
–diff. b mag.
GG Lup
–0.2
0
0.2
0.4
0.6
0.8
1
1.2
orbital phase

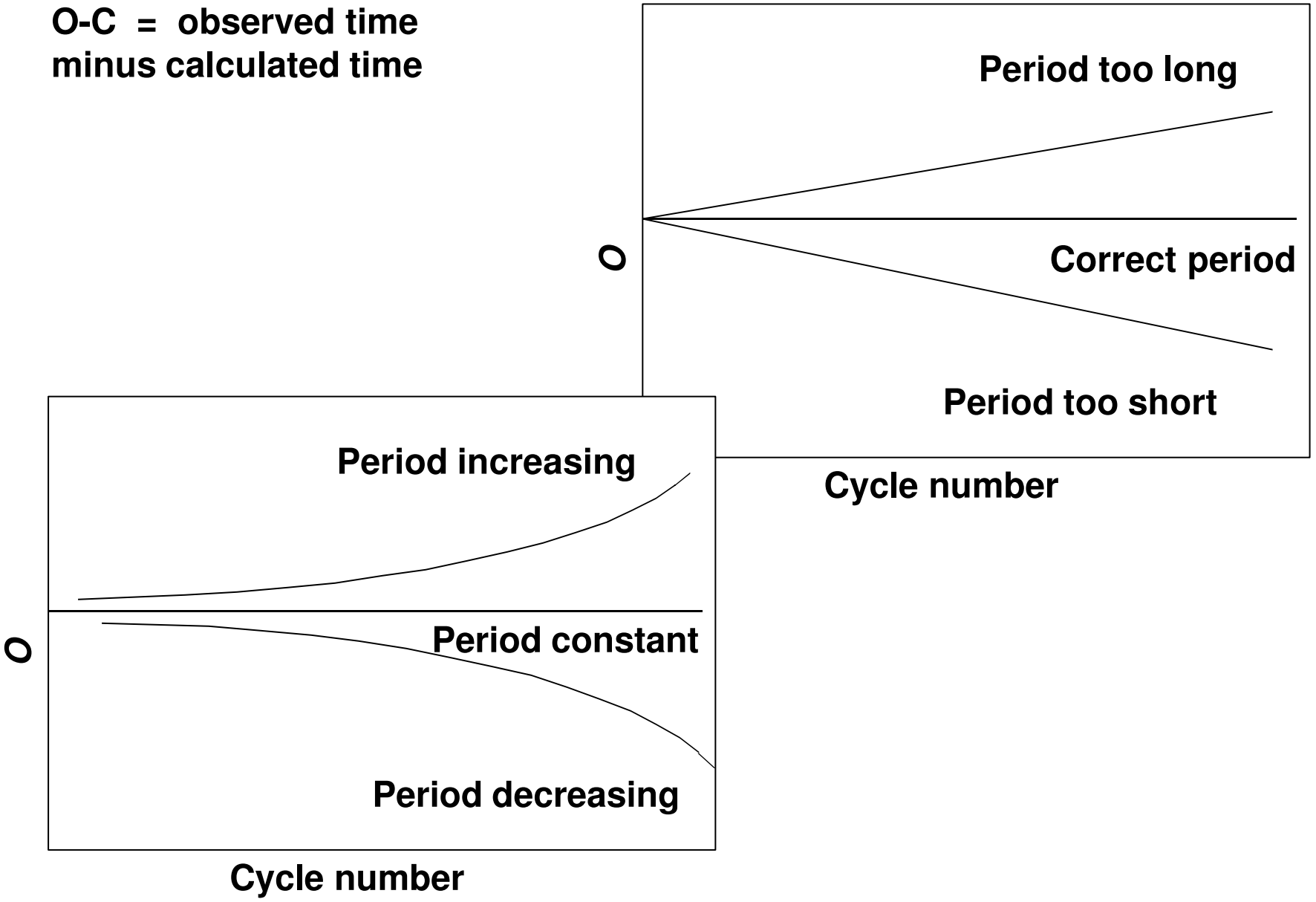

0.9
0.85
0.8
0.75
0.7
–diff. (u–b)
redder
bluer
Colour changes --> T, L
–0.2
0
0.2
0.4
0.6
0.8
1

Application to Binary Pulsars

- **binary system where one star is a pulsar**
  - emits ‘pulses’ of radiation
  - accurate timing possible (accurate clocks)
  - need narrow pulses
    - radio signals from neutron stars
- **solitary pulsar**
  - if at 0 velocity relative to us
    - time between pulses, dt = constant (unless being spun up/down)
  - if at V_{rel} relative velocity
    - dt = constant x pulse number
  - if pulsar spins up, dt decreases with pulse number
    - concave curve
  - if pulsar spins down, dt increases with pulse number
    - convex curve

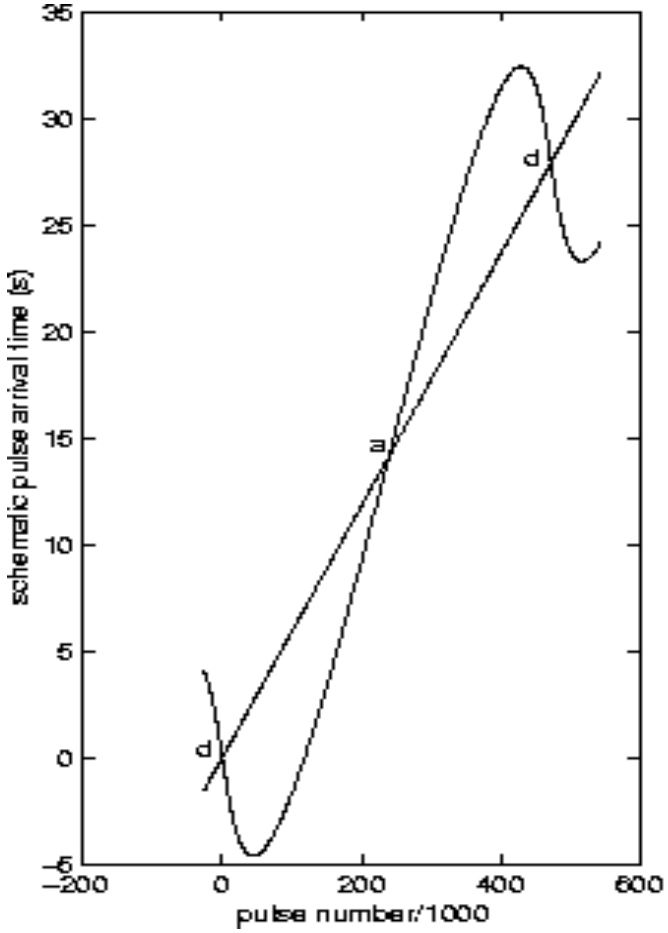
Timing Residuals

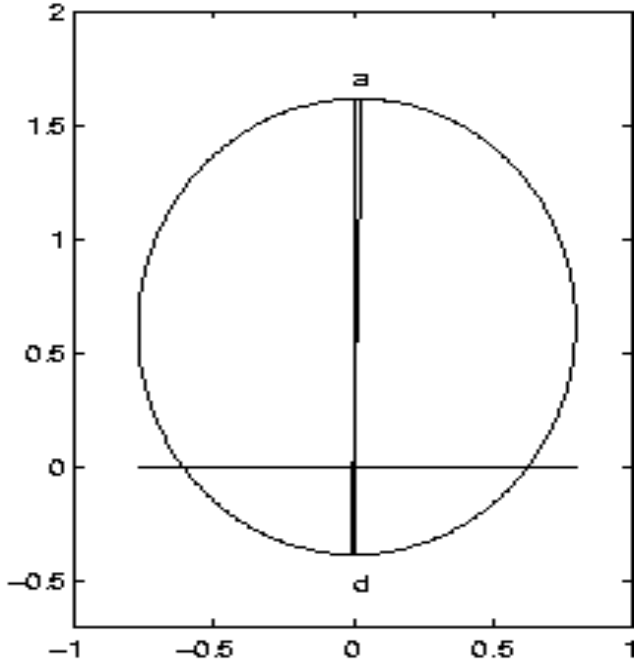
O-C = observed time minus calculated time



Binary Pulsars

- **In binary system, time between pulses affected by orbital motions**
  - **due to light travel time (distance) changing along orbit**





Observer-->

Light travel time

$$\text{pulsar orbit}: \quad r_p = \frac{a_p\,(1+e^2)}{1+e\cos\theta}$$

distance along line of sight:

$$z_p = r_p \sin i \sin(\theta+\omega)$$

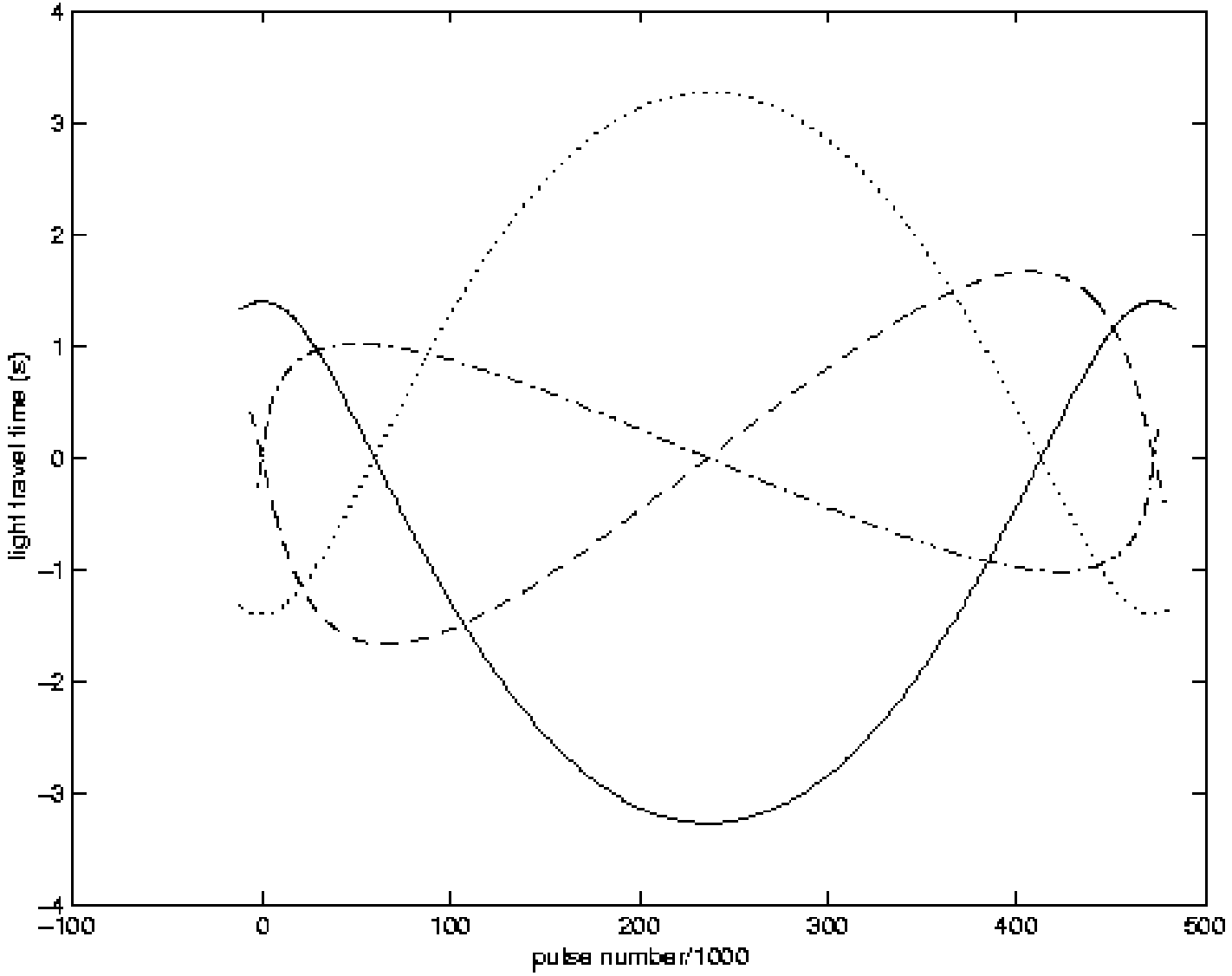
light travel time:

$$\frac{z_p}{c} = \frac{a_p \sin i}{c}\,(1+e^2)\,\frac{\sin(\theta+\omega)}{1+e\cos\theta}$$

circular orbit

$$\frac{z_p}{c} = \frac{a_p \sin i}{c}\sin\left(\frac{2\pi}{P}(t-T_0)\right)$$

light travel time



Binary Pulsar timing residuals

- **Time difference between predicted , $j_n \tau$, and actual (binary) pulse arrival times, t_n is**

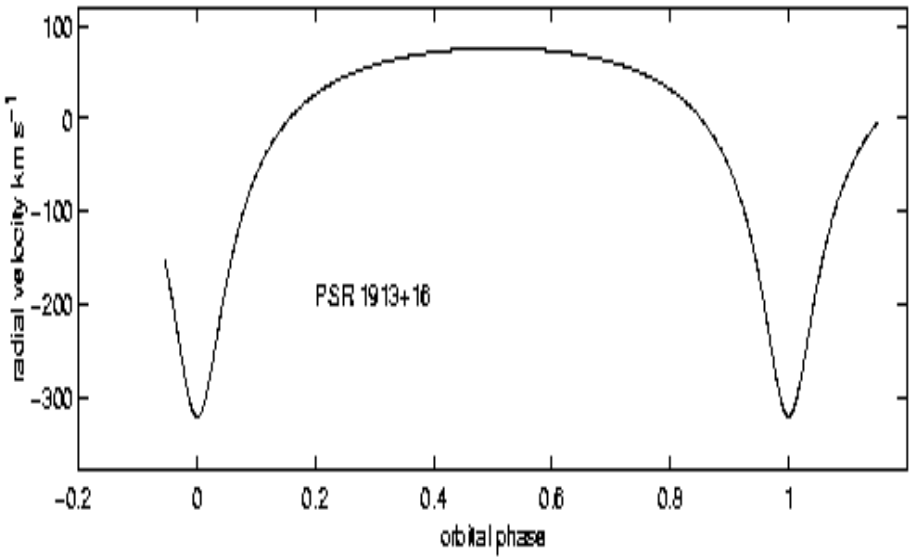
$$\Delta t = t_n - j_n \tau = a\, t + b \sin\left[\frac{2\pi\ (t - T_0)}{P}\right]$$

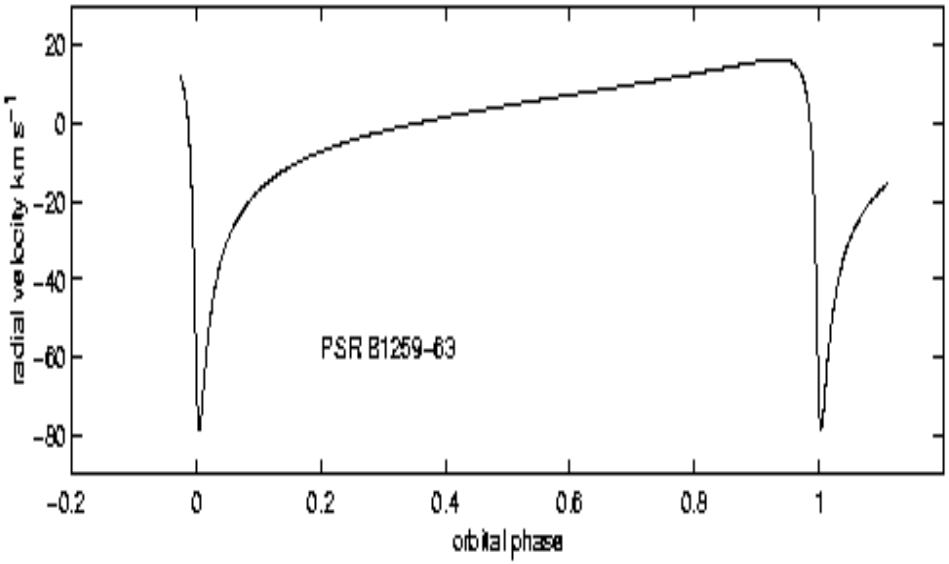
  - **P is the orbital period, T_0 is a reference time**
  - **a,b are determined by the velocity of the pulsar**
    - **a: from systematic velocity**
    - **b: from Keplerian velocity**

  - **for circular orbits: b = (a_p/c) sini**

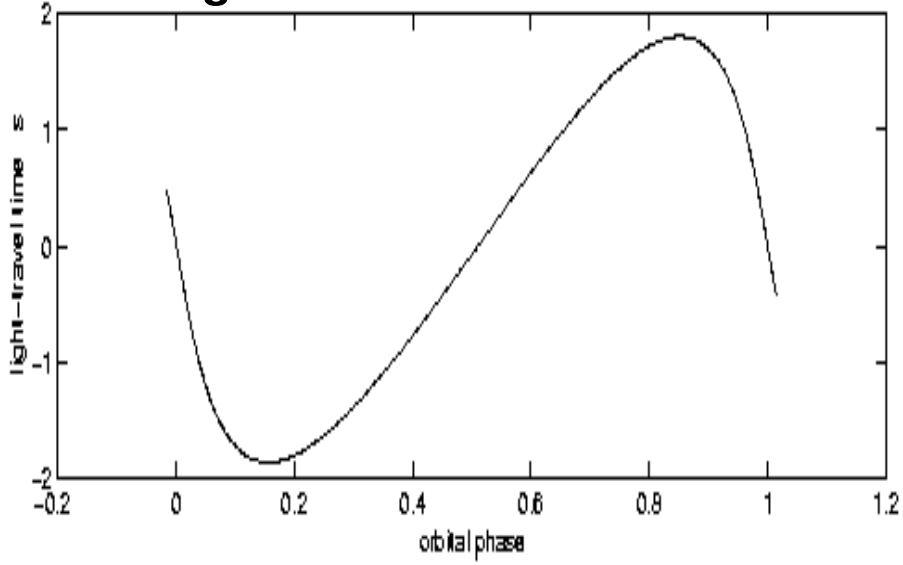
Binary Pulsar Orbits

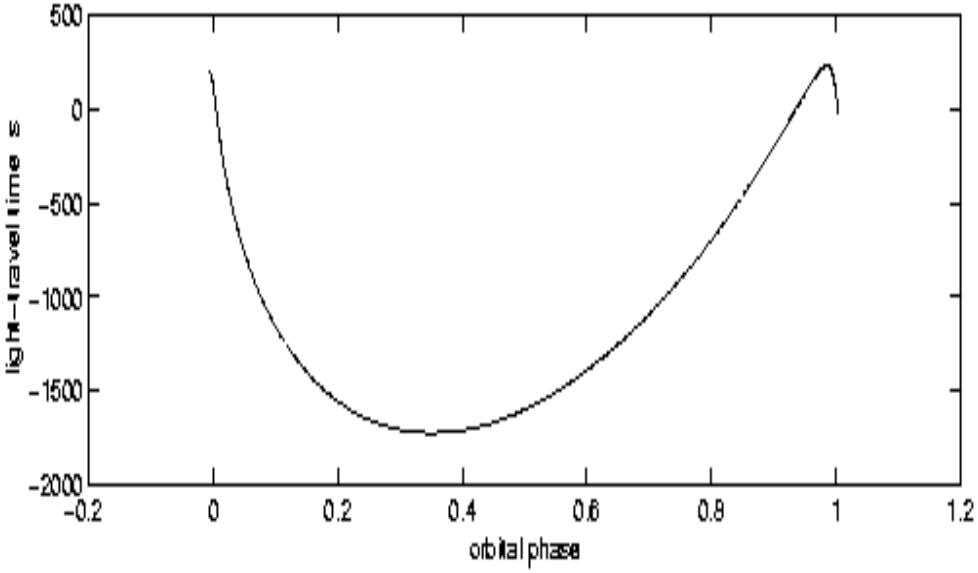
radial velocity





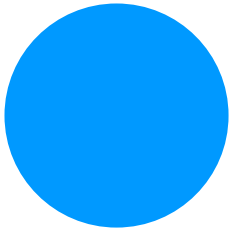
timing





Mass determinations

- **visible companion star**
  - **O-B star in High-Mass X-ray Binaries (HMXB)**
  - **A-K star in Low-Mass X-ray Binaries (LMXB)**



$$a_c \sin i = \frac{(1-e^2)^{1/2} K_c P}{2\pi}$$

mass function

$$f(m_p) = \frac{m_p{}^3 \sin^3 i}{M^2} = \frac{(1-e^2)^{3/2} K_c{}^3 P}{2\pi G}$$

mass ratio, q,

$$q = \frac{m_p}{m_c} = \frac{a_c \sin i}{a_p \sin i}$$

- **If inclination, *i*, can be found, then masses follow**

Frequency shifts

- **Binary orbit also affects pulsar frequency**
  - radio pulsars , very narrow pulse widths
  - pulse frequency affected by orbital velocity
  - Doppler shift:

$$\Delta f = f \frac{V_{rad}}{c} = f \frac{\dot{z}}{c}$$

  - gives a phase lag of:

$$\Delta \phi = \int_{T_0}^{t} \Delta f \, dt \approx f_0 \frac{\dot{z}}{c} (t - T_0)$$

$$= f_0 \frac{\dot{z}}{c} \left[\frac{z}{c} \right]$$

Pulsar Phase lag

- **Combined phase lag is**
  - **from light travel time due to orbit**

$$\Delta\phi_L = -f\,\frac{z}{c}$$

  - **and from Doppler shift**

$$\Delta\phi_D = f_0\,\frac{\dot{z}}{c}\left[\frac{z}{c}\right]$$

  - **hence**

$$\Delta\phi \;= \Delta\phi_D + \Delta\phi_L \approx \left[\frac{z}{c}\right]\left[f_0\,\frac{\dot{z}}{c} - f_0 - \dot{f}_0(t - T_0)\right]$$

  - **generally**

$$\Delta\phi_D \cong 0.001\Delta\phi_L$$

  - **but measurable in radio pulsars**