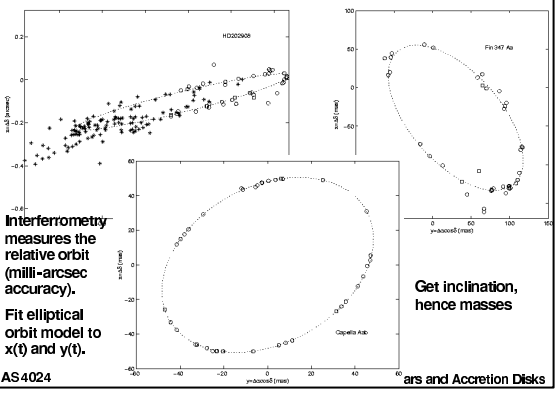
Inclinations

- **Radial velocities give minimum masses**

SB2: $m \sin^3 i$ SB1: $\frac{m^3 \sin^3 i}{M^2}$

- **need inclinations**
  - astrometric orbits
  - eclipsing binaries
  - polarimetry
  - apsidal motion

Astrometric Orbits



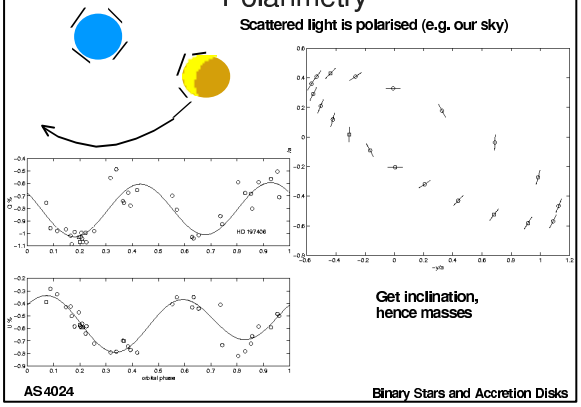
Interferrometry measures the relative orbit (milli-arcsec accuracy).

Fit elliptical orbit model to x(t) and y(t).

Get inclination, hence masses

Polarimetry

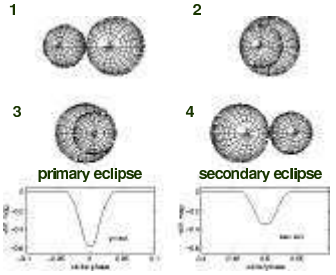
Scattered light is polarised (e.g. our sky)



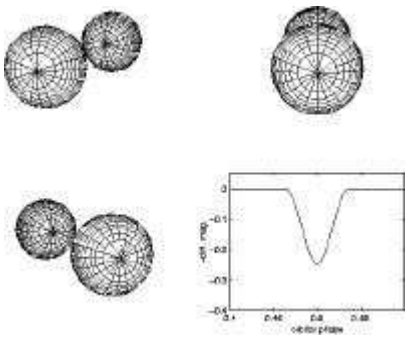
Get inclination, hence masses

Eclipsing Binaries

- **Binary properties from eclipses**
  - sizes, shapes of stars, inclination,
  - temperatures, limb darkening, apsidal motion
  - 4 contact phases:
  - 1 2
  - 3 4
  - primary eclipse secondary eclipse



Partial Eclipse



Eclipses

$R_1 > R_2 \quad \theta \equiv 2\pi\phi$

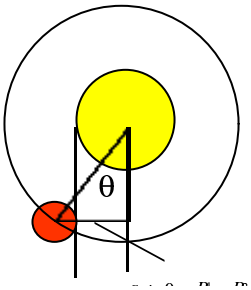
edge-on ($i = 90^\circ$):

4 contactphases:

$R_1 \pm R_2 = a \sin\theta$

$\frac{R_2}{a} = \frac{\sin\theta_1 - \sin\theta_2}{2}$

$\frac{R_1}{a} = \frac{\sin\theta_1 + \sin\theta_2}{2}$



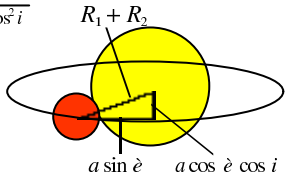
Eclipses

4 contact phases:

$$R_1 \pm R_2 = a\sqrt{\sin^2\theta + \cos^2\theta\,\cos^2 i}$$
$$= a\sqrt{1+\cos^2\theta\,\sin^2 i}$$

4 measurements: $\phi_1\ \phi_2\ \phi_3\ \phi_4$

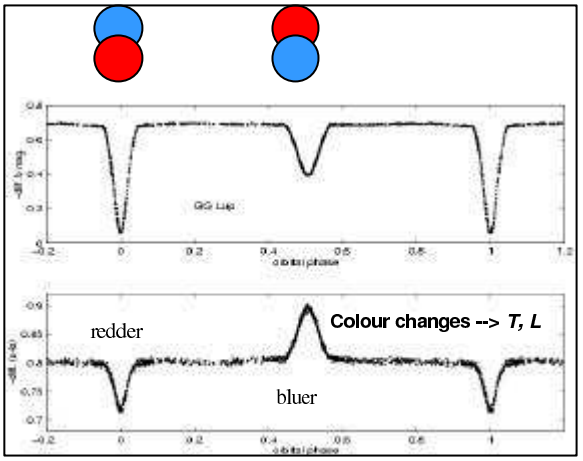
4 parameters: $\frac{R_1}{a}\ \frac{R_2}{a}\ i\ \phi_0$



mid eclipse $\theta = 0$

total eclipse $a\cos i < R_1 - R_2$

partial eclipse $R_1 - R_2 < a\cos i < R_1 + R_2$

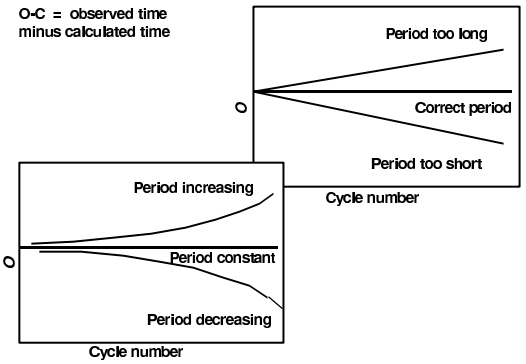


Application to Binary Pulsars

- **binary system where one star is a pulsar**
  - emits 'pulses' of radiation
  - accurate timing possible (accurate clocks)
  - need narrow pulses
    - radio signals from neutron stars
- **solitary pulsar**
  - if at 0 velocity relative to us
    - time between pulses, dt = constant (unless being spun up/down)
  - if at V_{rel} relative velocity
    - dt = constant x pulse number
  - if pulsar spins up, dt decreases with pulse number
    - concave curve
  - if pulsar spins down, dt increases with pulse number
    - convex curve

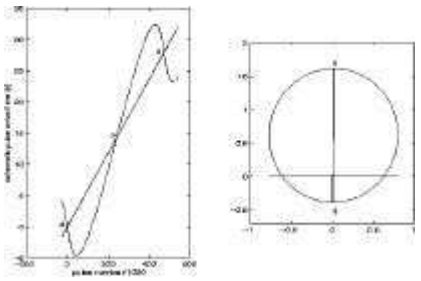
Timing Residuals

O-C = observed time minus calculated time



Binary Pulsars

- **In binary system, time between pulses affected by orbital motions**
  - due to light travel time (distance) changing along orbit



Observer-->

Light travel time

pulsar orbit: $r_p = \frac{a_p\,(1+e^2)}{1+e\cos\theta}$

distance along line of sight:

$z_p = r_p \sin i \sin(\theta + \omega)$

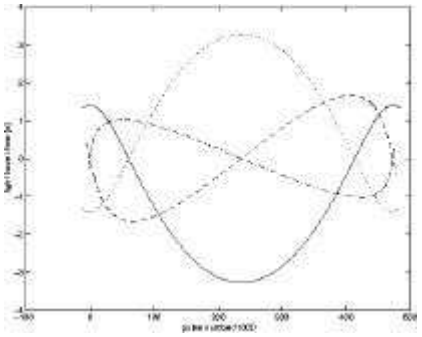
light travel time:

$$\frac{z_p}{c} = \frac{a_p \sin i}{c}(1+e^2)\frac{\sin(\theta+\omega)}{1+e\cos\theta}$$

circular orbit

$$\frac{z_p}{c} = \frac{a_p \sin i}{c}\sin\left(\frac{2\pi}{P}(t-T_0)\right)$$

light travel time



Binary Pulsar timing residuals

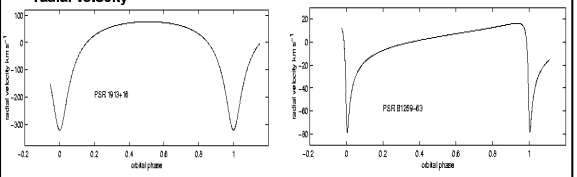
- **Time difference between predicted , $j_n \tau$, and actual (binary) pulse arrival times, t_n is**

$$\Delta t = t_n - j_n\tau = a\,t + b\sin\left[\frac{2\pi\,(t-T_0)}{P}\right]$$

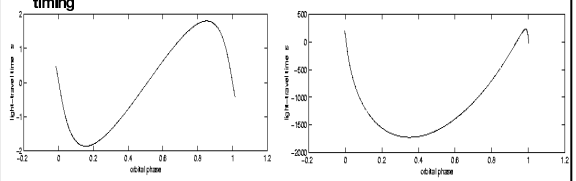
  - P is the orbital period, T_0 is a reference time
  - a,b are determined by the velocity of the pulsar
    - a: from systematic velocity
    - b: from Keplerian velocity
  - for circular orbits: b = (a_p/c) sini

Binary Pulsar Orbits

radial velocity



timing



Mass determinations

- **visible companion star**
  - O-B star in High-Mass X-ray Binaries (HMXB)
  - A-K star in Low-Mass X-ray Binaries (LMXB)

$$a_c \sin i = \frac{(1-e^2)^{1/2} K_c\, P}{2\pi}$$

mass function

$$f(m_p) = \frac{m_p{}^3 \sin^3 i}{M^2} = \frac{(1-e^2)^{3/2} K_c{}^3\, P}{2\pi\, G}$$

mass ratio, q,

$$q = \frac{m_p}{m_c} = \frac{a_c \sin i}{a_p \sin i}$$

  - If inclination, *i*, can be found, then masses follow

Frequency shifts

- **Binary orbit also affects pulsar frequency**
  - radio pulsars , very narrow pulse widths
  - pulse frequency affected by orbital velocity
  - Doppler shift:

$$\Delta f = f\,\frac{V_{rad}}{c} = f\,\frac{\dot{z}}{c}$$

  - gives a phase lag of:

$$\Delta\phi = \int_{T_0}^{t} \Delta f\, dt \approx f_0\,\frac{\dot{z}}{c}(t - T_0)$$

$$= f_0\,\frac{\dot{z}}{c}\left[\frac{z}{c}\right]$$

Pulsar Phase lag

- **Combined phase lag is**
  - from light travel time due to orbit

$$\Delta\phi_L = -f\,\frac{z}{c}$$

  - and from Doppler shift

$$\Delta\phi_D = f_0\,\frac{\dot{z}}{c}\left[\frac{z}{c}\right]$$

  - hence

$$\Delta\phi = \Delta\phi_D + \Delta\phi_L \approx \left[\frac{z}{c}\right]\left[f_0\,\frac{\dot{z}}{c} - f_0 - \dot{f}_0(t-T_0)\right]$$

  - generally $\Delta\phi_D \cong 0.001\,\Delta\phi_L$
  - but measurable in radio pulsars