Q1: What changes to the particle content of the expanding Universe occur at the epochs of:

- Annihilation:
- pair soup -> quark soup ( $10^{9}$ photons/quark)
- Baryogenesis:
- quarks bound (by strong force) into baryons.
- UUD = proton $\mathrm{DDU}=$ neutron
- Nucleosynthesis:
- Atomic nuclei: $75 \% \mathrm{H}, 25 \% \mathrm{He}$, traces of Li , Be
- Recombination:
- Neutral atoms form as free electrons recombine
- photons fly free


## Q2: Given present-day density parameters

$\Omega_{M}=0.3$ for matter and $\Omega_{R}=5 \times 10^{-5}$ for radiation, at what redshift $z$ were the energy densities equal?
volume $R^{3} \quad N$ particles of mass $m$ photon wavelengths stretch:


$$
\begin{aligned}
& \lambda \propto R \propto \frac{1}{1+z} \\
\varepsilon_{M}= & \rho_{M} c^{2}=\Omega_{M}\left(\rho_{c r i t} c^{2}\right)(1+z)^{3} \quad \rho_{M}=\frac{N_{b} m}{R^{3}} \propto(1+z)^{3} \\
\varepsilon_{R}= & \Omega_{R}\left(\rho_{c r i t} c^{2}\right)(1+z)^{4} \quad \varepsilon_{R}=\frac{N_{\gamma} h v}{R^{3}} \propto R^{-4} \propto(1+z)^{4} \\
1= & \frac{\varepsilon_{M}}{\varepsilon_{R}}=\frac{\Omega_{M}}{\Omega_{R}} \frac{1}{1+z} \Rightarrow 1+z=\frac{\Omega_{M}}{\Omega_{R}}=\frac{0.3}{5 \times 10^{-5}}=6000
\end{aligned}
$$

Q3 a) Evaluate the neutron/proton ratio in thermodynamic equilibrium at high and low T .

$$
\begin{aligned}
& m_{n}=m_{p}+\Delta m=1.0014 m_{p} \quad \frac{N_{n}}{N_{p}}=\left(\frac{m_{n}}{m_{p}}\right)^{3 / 2} \exp \left(-\frac{\Delta m c^{2}}{k T}\right) \\
& T \rightarrow \infty \quad \frac{N_{n}}{N_{p}} \rightarrow\left(\frac{m_{n}}{m_{p}}\right)^{3 / 2} \exp (0)=(1.0014)^{3 / 2} \approx 1 \\
& T \rightarrow 0 \quad \frac{N_{n}}{N_{p}} \rightarrow\left(\frac{m_{n}}{m_{p}}\right)^{3 / 2} \exp (-\infty)=0
\end{aligned}
$$

b) Evaluate the $n / p$ ratio and $Y_{p}$ if $m_{n}=m_{p}$.

$$
\begin{aligned}
& m_{n}=m_{p} \Rightarrow \Delta m=0 \quad \frac{m_{n}}{m_{p}}=1 \quad \frac{N_{n}}{N_{p}} \rightarrow(1)^{3 / 2} \exp (0)=1 \\
& N_{n}=N_{p} \Rightarrow 100 \% \mathrm{He} \quad Y_{p}=1
\end{aligned}
$$

## Q4 Alien's CMB-meter reads 5.1 K and 4.9 K in the

 fore and aft directions. Evaluate the velocity.$$
\frac{V}{c}=\frac{\Delta T}{T}=\frac{0.1 K}{5 K} \quad \Rightarrow V=\frac{c}{50}=6000 \mathrm{~km} / \mathrm{s}
$$

Are humans present on Earth at this time?

$$
T=5 K \quad T_{0}=2.7 K \quad \lambda \propto R \quad \Rightarrow \quad T \propto \frac{1}{R}
$$

mater dominated expansion: $R \propto t^{2 / 3}$
time : $\frac{t}{t_{0}}=\left(\frac{R}{R_{0}}\right)^{3 / 2}=\left(\frac{T_{0}}{T}\right)^{3 / 2}=\left(\frac{2.7 K}{5 K}\right)^{3 / 2}=0.40$
now: $t_{0} \sim \frac{1}{H_{0}} \sim 13 \times 10^{9} \mathrm{yr} \quad$ Age of Sun: $\sim 5 \times 10^{9} \mathrm{yr}$
look-back time: $t_{0}-t=0.6 t_{0} \sim 8 \times 10^{9} \mathrm{yr}$ (Before Sun was born!)

