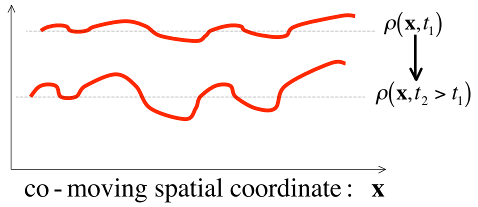
Large Scale Structure

- Galaxies are (biased) tracers of the mass
- "Bubbly" structure observed
  - Voids
  - Walls = edges of voids
  - Filaments = intersections of walls
  - Clusters = intersections of filaments
- Compare with supercomputer simulations.
  - initial density perturbations grow
  - stars / galaxies form when density high enough
- Determine Ω_M
  - High Ω_M => faster growth
  - => clusters form at earlier redshifts
  - => stronger clustering today
  - $\Omega_M \sim 0.3$ matches observed structure.

Density Perturbations Grow

$$\rho(\mathbf{x},t) = \bar{\rho}(t)\,\left(1+\delta(\mathbf{x},t)\right)$$

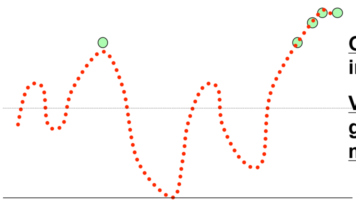


Linear regime: $|\delta|<1$ $\quad \delta \equiv \frac{\rho-\bar{\rho}}{\bar{\rho}}$

$$\delta \propto R(t) \propto \frac{1}{1+z}$$

Biased Galaxy Formation

Non-Linear regime $\delta \geq 1$



Galaxy clusters form in density maxima.

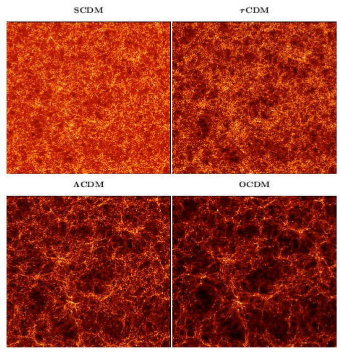
Voids (almost no galaxies) in density minima.

$$\delta_{galaxies} = b\,\delta_M$$

b = "bias parameter"

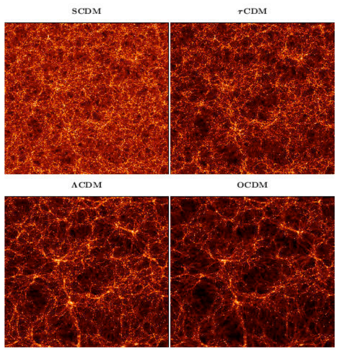
b > 1 --> Galaxies more strongly clustered than matter

Simulations: z = 3



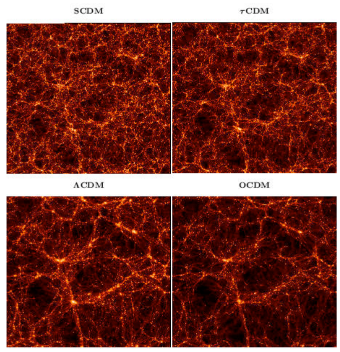
The VIRGO Collaboration 1996

Simulations: z = 1

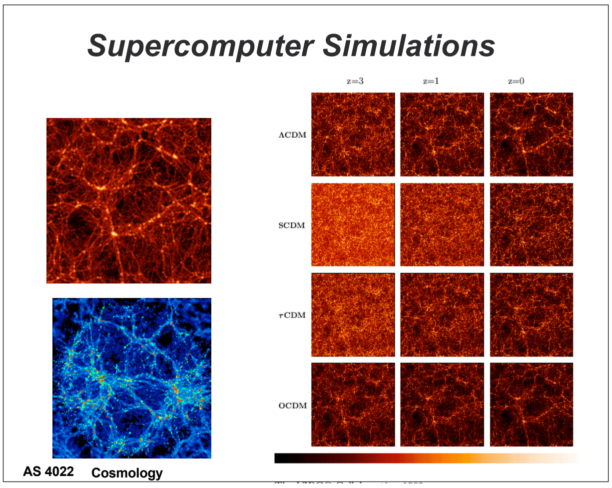


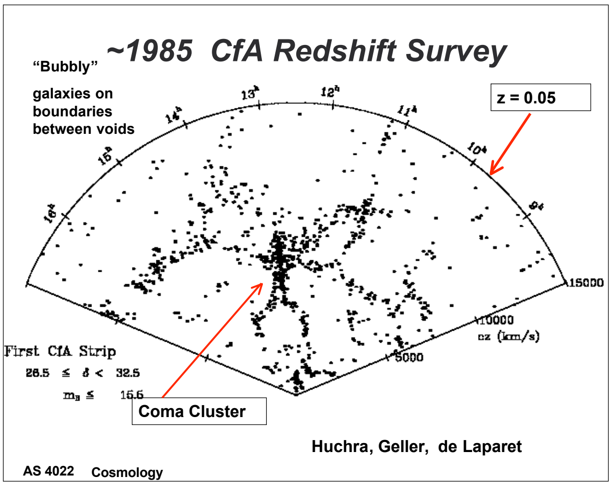
The VIRGO Collaboration 1996

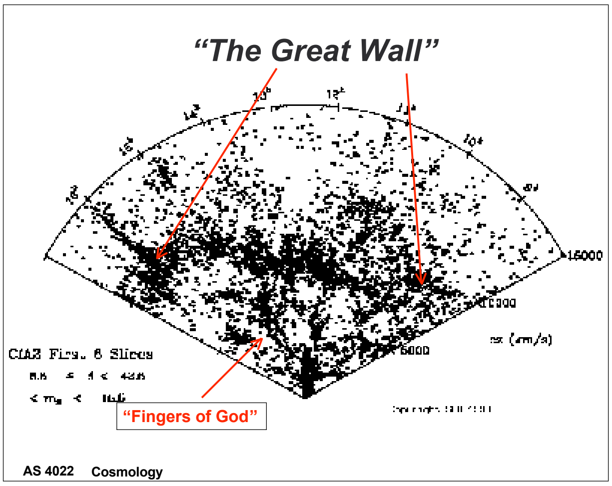
Simulations: z = 0

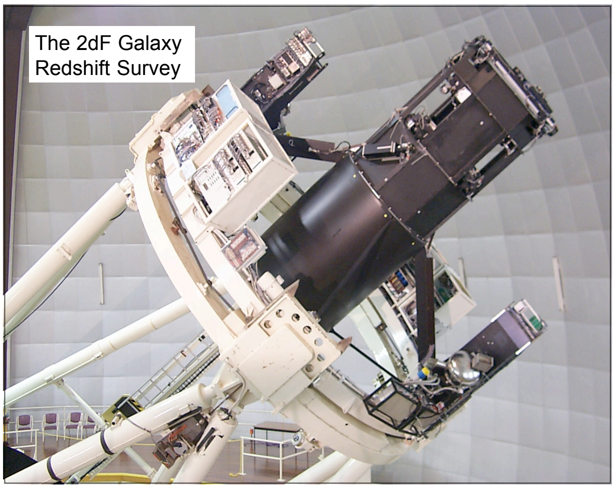


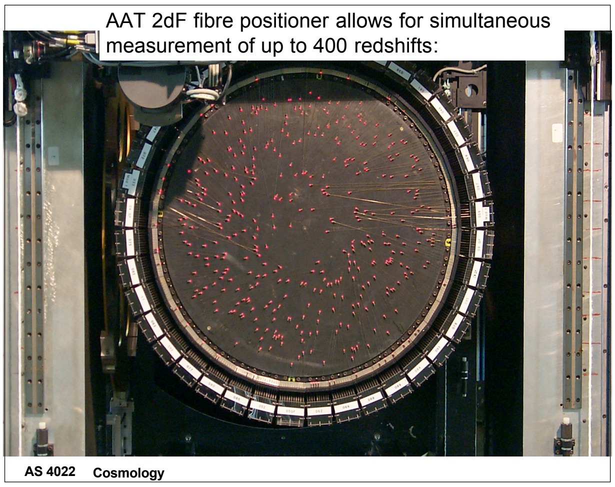
The VIRGO Collaboration 1996

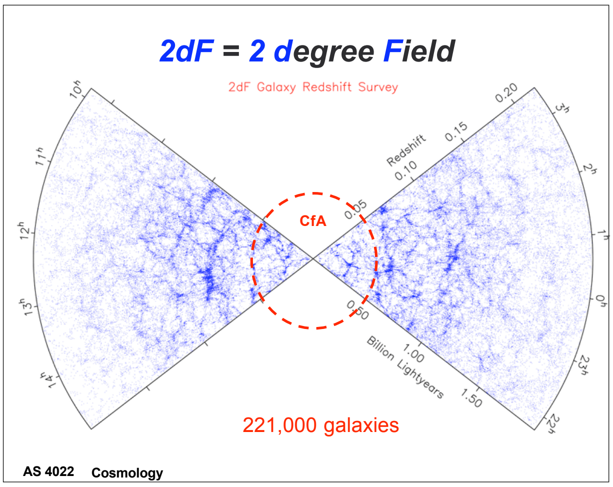

Supercomputer Simulations
z=3
z=1
z=0
ΛCDM
SCDM
τCDM
OCDM
AS 4022 Cosmology


~1985 CfA Redshift Survey
"Bubbly"
galaxies on
boundaries
between voids
z = 0.05
First CfA Strip
26.5 ≤ δ < 32.5
m_B ≤ 15.5
Coma Cluster
15000
10000
cz (km/s)
5000
Huchra, Geller, de Laparet
AS 4022 Cosmology


"The Great Wall"
15000
10000
5000
cz (km/s)
"Fingers of God"
AS 4022 Cosmology


The 2dF Galaxy
Redshift Survey


AAT 2dF fibre positioner allows for simultaneous
measurement of up to 400 redshifts:
AS 4022 Cosmology


2dF = 2 degree Field
2dF Galaxy Redshift Survey
Redshift
0.05
0.10
0.15
0.20
CfA
0.50
1.00
1.50
Billion Lightyears
221,000 galaxies
AS 4022 Cosmology

Galaxy Redshift Surveys

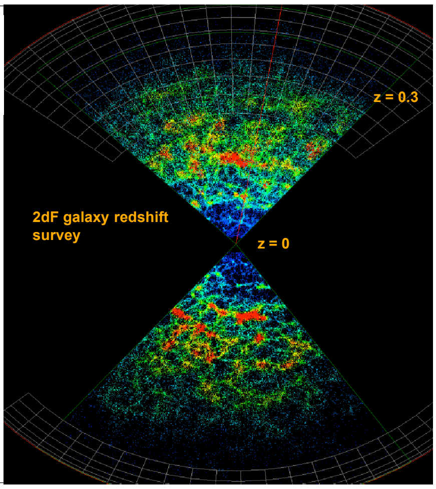
Large Scale Structure:

Empty **voids**

~50Mpc.

Galaxies are in
1. **Walls** between voids.
2. **Filaments** where walls intersect.
3. **Clusters** where filaments intersect.

Like Soap Bubbles !



Theory vs Observations

- **Can't directly compare simulations and observations**
  - details (exactly where density is high/low) don't matter.
- **Amplitude of structure vs size of structure is what matters. Quantify this using:**
  - Power Spectrum: P(k) wavenumber $k = 2\pi / \lambda$
  - 2-point Correlation Function : $\xi(r)$
- **Biased galaxy formation:**
  - bias parameter b.
- **Initial conditions:**
  - Power-law power spectrum for initial amplitude vs scale.
  - Amplitude A, slope n $P_0(k) \sim A^2 k^n$

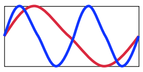
Fourier Analysis (Parceval's Theorem)

density perturbations

$$\delta(\mathbf{x}) \equiv \frac{\rho(\mathbf{x}) - \langle \rho \rangle}{\langle \rho \rangle} = \frac{\delta\rho}{\bar{\rho}}$$

fourier amplitudes $\delta_{\mathbf{k}}$

$$\delta(\mathbf{x}) = \sum_{\mathbf{k}} \delta_{\mathbf{k}} \exp(-i\,\mathbf{k}\cdot\mathbf{x})$$



mean $\langle \delta \rangle = 0$

variance (average over volume V)

$$\langle \delta^2 \rangle = \frac{1}{V}\int \delta^2(\mathbf{x})\, d^3\mathbf{x}$$

$$= \frac{1}{V}\int \left| \sum_{\mathbf{k}} \delta_{\mathbf{k}} \exp(-i\,\mathbf{k}\cdot\mathbf{x}) \right|^2 d^3\mathbf{x}$$

$$= \frac{1}{V}\int \sum_{\mathbf{k}}\sum_{\mathbf{j}} \delta_{\mathbf{k}}\delta_{\mathbf{j}}^* \exp(-i\,(\mathbf{k}-\mathbf{j})\cdot\mathbf{x})\, d^3\mathbf{x}$$

$$= \sum_{\mathbf{k}}\sum_{\mathbf{j}} \delta_{\mathbf{k}}\delta_{\mathbf{j}}^* \underline{\int \exp(-i\,(\mathbf{k}-\mathbf{j})\cdot\mathbf{x})\, \frac{d^3\mathbf{x}}{V}}$$

1 if k = j
0 otherwise

$$\langle \delta^2 \rangle = \sum_{\mathbf{k}} |\delta_{\mathbf{k}}|^2 \approx \frac{V}{(2\pi)^3}\int |\delta_{\mathbf{k}}|^2 d^3\mathbf{k}$$

mode spacing:

$$\lambda_x = L_x / n$$

$$k_x = \frac{2\pi\, n}{L_x} = n\,\Delta k_x$$

$$\Delta k_x = \frac{2\pi}{L_x}$$

$$d^3\mathbf{k} = \frac{(2\pi)^3}{L_x L_y L_z} = \frac{(2\pi)^3}{V}$$

= k - space volume per mode

Power Spectrum

For isotropic structure (consistent with observations) :

power spectrum (average over directions)

$$P(k) \equiv \left\langle |\delta_{\mathbf{k}}|^2 \right\rangle = \int\int \delta^2(k,\theta,\phi)\, \frac{\sin\theta\, d\theta\, d\phi}{4\pi} \qquad k = |\mathbf{k}| = \frac{2\pi}{\lambda}$$

variance of density fluctuations:

$$\langle \delta^2 \rangle = \sum_{\mathbf{k}} |\delta_{\mathbf{k}}|^2 \approx \frac{V}{(2\pi)^3}\int |\delta_{\mathbf{k}}|^2 d^3\mathbf{k}$$

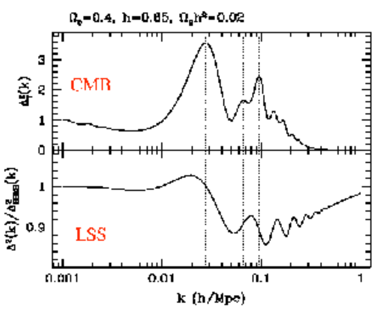
$$= \frac{V}{(2\pi)^3}\int P(k)\, 4\pi k^2 dk = \frac{V}{2\pi^2}\int P(k)\, k^2 dk$$

dimensionless power spectrum :

$$\Delta^2(k) \equiv \frac{d\langle \delta^2 \rangle}{d\ln k} \approx \frac{V}{2\pi^2}\, k \frac{d}{dk}\left(\int P(k)\, k^2 dk \right) = \frac{V}{2\pi^2} k^3\, P(k)$$

Predicted Power Spectra

Independent constraints from CMB and Large-Scale Structure

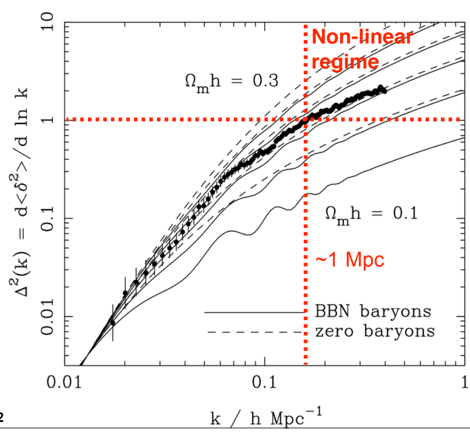


CMB and LSS out of phase:

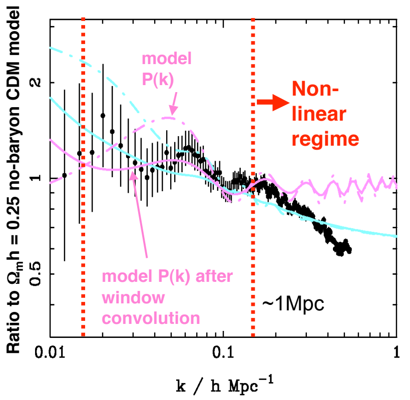
'velocity overshoot'

LSS amplitude smaller than CMB

CDM Model Fits to Galaxy Power Spectrum



CDM Model Fits to Galaxy Power Spectrum



Fit model CDM P(k) (with n=1) after convolution with survey window function.

Parameters:

(1) $\Omega_m h$

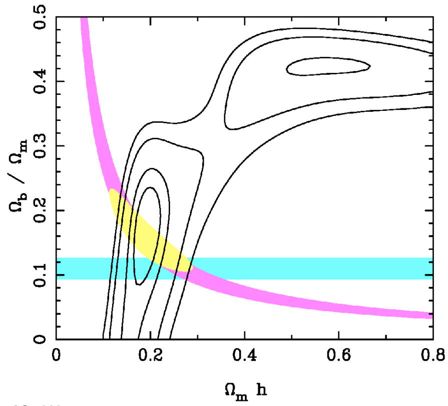
(2) Ω_b / Ω_m

(3) h (marginalise)

Window flattens P(k) and depresses baryon features.

Fits limited to $0.015 < k < 0.15$.

Consistency with Other Constraints



Galaxy-Galaxy Correlation Function

correlation function = fourier transform of power spectrum

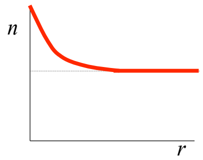
$$\xi(\mathbf{r}) \equiv \langle\, \delta(\mathbf{x})\, \delta(\mathbf{x}+\mathbf{r})\,\rangle = \sum_{\mathbf{k}} |\delta_{\mathbf{k}}|^2 \exp(-i\,\mathbf{k}\cdot\mathbf{r}\,)$$

radial correlation function

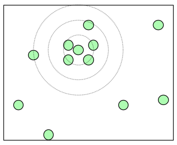
$$\xi(r) = \langle\, \xi(\mathbf{r})\,\rangle \approx \left(\frac{r}{8\ \mathrm{Mpc}}\right)^{-1.8}$$

measures galaxy clustering

$$n(r) = n_0\,(\,1 + \xi(r)\,)$$



Galaxy counts at separation *r* <u>larger than expected</u> for random distribution.



Power-Law Models

power-law power spectrum

$$P(\mathbf{k}) = \langle\, \delta_{\mathbf{k}}^{\,2}\,\rangle = P_0 \left(\frac{k}{k_0}\right)^n$$

$n =$ "tilt"

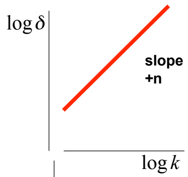
$n = +1 \Rightarrow$ "scale-invariant"

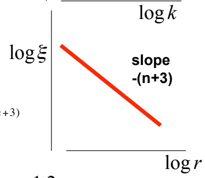
variance after smoothing on scale r

$=$ sum of power for $k = |\mathbf{k}| \le k_{max} = 2\pi / r$

$$\langle\, \delta^2\,\rangle = \sum_{\mathbf{k}} P(\mathbf{k}) \propto \int_0^{k_{max}} k^n \left(4\pi\, k^2 dk\right) \propto k_{max}^{n+3} \propto r^{-(n+3)}$$

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma} \quad r_0 \approx 8\ \mathrm{Mpc} \quad \gamma = n + 3 \approx 1.8 \to n \approx -1.2$$





Easier to derive γ = 1.8 from 2-point correlation function.

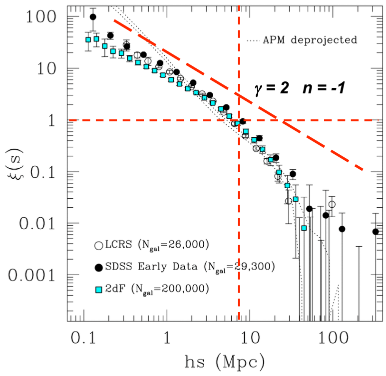
Galaxy-Galaxy Correlation Function

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma}$$

$r_0 \approx 8$ Mpc

$\gamma = n + 3 \approx 1.8$

$\to n \approx -1.2$



2-D Correlation Function

angular separation

$$\sigma \equiv \frac{\Delta\theta}{D_A}$$

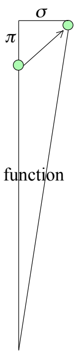
radial separation

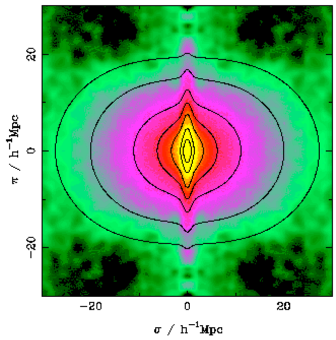
$$\pi \equiv \frac{c\,\Delta z}{H_0}$$

2 – point correlation function

$\xi(\,\sigma, \pi\,)$

Why flattened ?

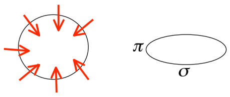




Hawkins et al. 2002

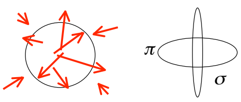
Redshift Distortions

1. Kaiser effect:



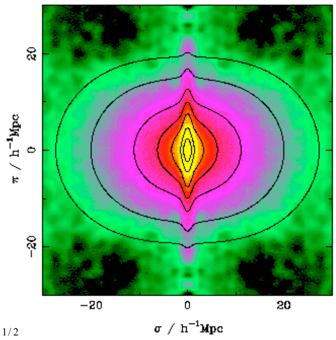
Infall velocities reduce π

2. "Fingers of God"



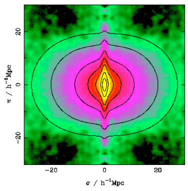
Virialised cluster cores

$$V \sim \left(\frac{G\,M_c}{r_c} \right)^{1/2}$$

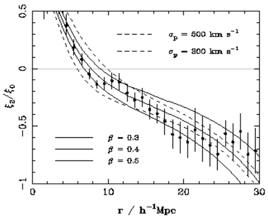


Hawkins et al. 2002

Kaiser Effect

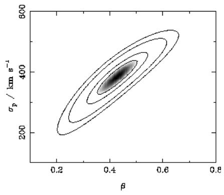


Flattening vs size of $\sigma - \pi$ contours



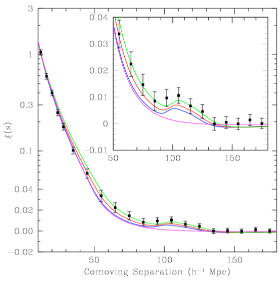
$$\beta \equiv \frac{\Omega_M^{\,0.6}}{b} = 0.43 \pm 0.07$$

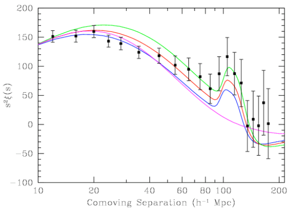
b = bias parameter > 1



σ_p = dispersion of galaxy peculiar velocities.

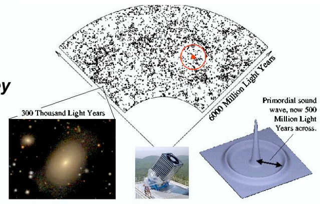
Baryon Acoustic Oscillations

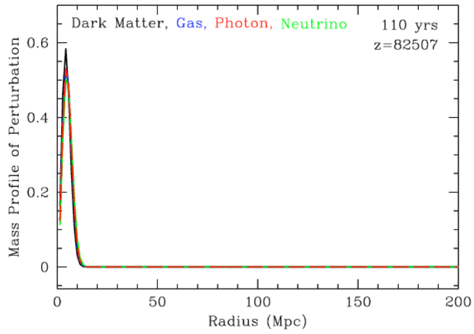




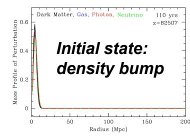
SDSS = Sloan Digital Sky Survey

Eisenstein et al. 2001.

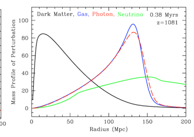




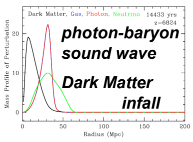
Baryon Acoustic Oscillations



Initial state: density bump

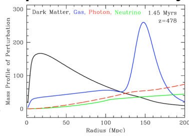


baryons fall into dark matter well

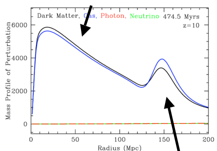


photon-baryon sound wave

Dark Matter infall

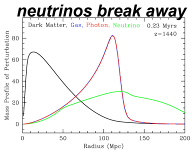


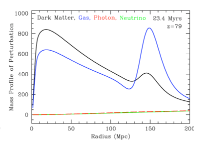
photons break away



Dark matter falls into baryon "shell"

neutrinos break away





observed structure: