Exoplanet Discovery Methods

(1) Direct imaging

Today: Star Wobbles

(2) Astrometry → position

(3) Radial velocity → velocity

Later:

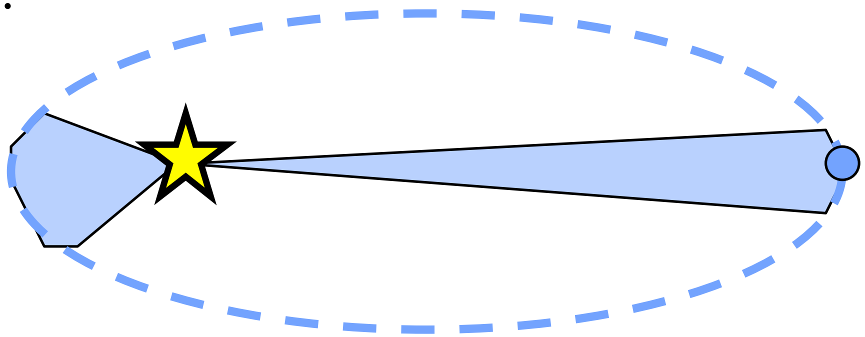
(4) Transits

(5) Gravitational microlensing

(6) Pulsar timing

Kepler Orbits

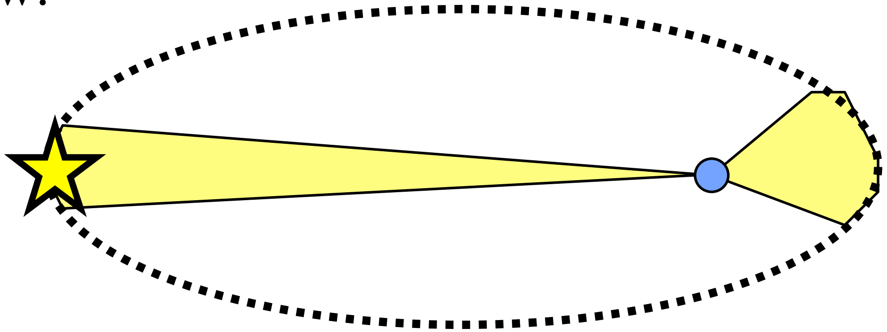
Star’s view:



Kepler 1: Planet orbit is an ***ellipse with star at one focus*** (Newton showed this is due to gravity’s inverse-square law).

Kepler 2: Planet seeps out ***equal area in equal time*** (angular momentum conservation).

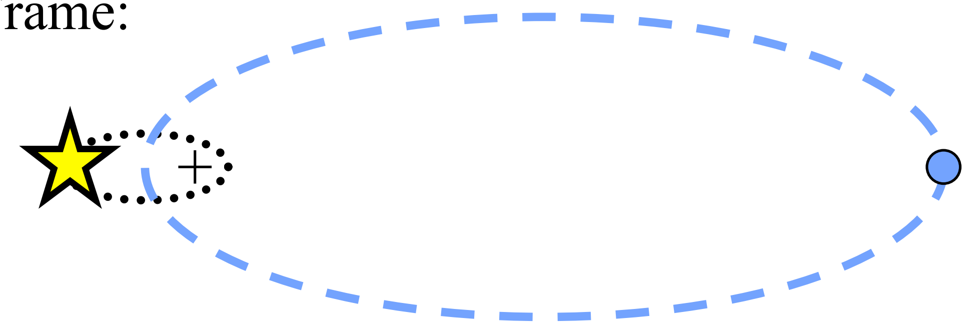
Planet’s view:



Planet at the focus.

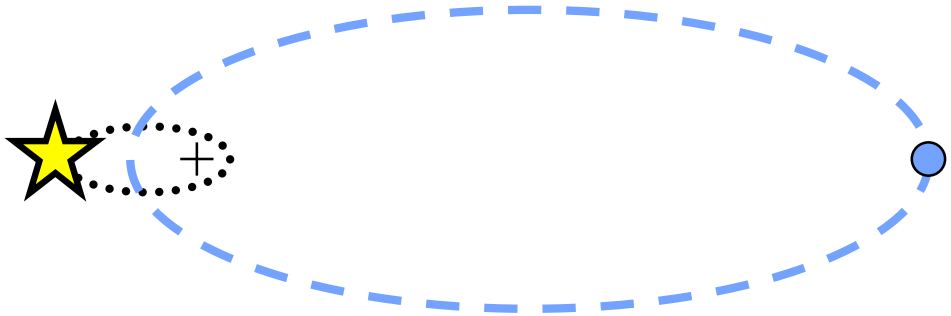
Star sweeps equal area in equal time

Inertial Frame:



Star and planet both orbit around the ***centre of mass***.

Kepler Orbits



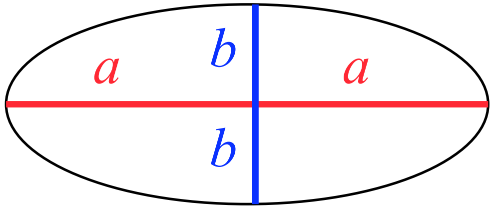
$$M = M_* + m_p = \text{total mass}$$

$$a = a_p + a_* = \text{semi-major axis}$$

$$a_p\, m_p = a_*\, M_* = a\, M \quad \text{Centre of Mass}$$

$$P = \text{orbit period}$$

$$a^3 = G\, M \left(\frac{P}{2\pi}\right)^2 \quad \text{Kepler's 3rd Law}$$



$$e \equiv \frac{a - b}{a} = \text{eccentricity} \quad 0 = \text{circular} \ 1 = \text{parabolic}$$

Astrometry

- Look for a periodic "wobble" in the ***angular position*** of host star
- Light from the star+planet is dominated by star
- Measure star's motion in the plane of the sky due to the orbiting planet
- Must correct measurements for ***parallax*** and ***proper motion*** of star
- ***Doppler*** (radial velocity) more sensitive to planets ***close to the star***
- ***Astrometry*** more sensitive to planets ***far from the star***

Stellar wobble: Star and planet orbit around centre of mass.
Radius of star's orbit scales with planet's mass:

$$\frac{a_*}{a} = \frac{m_p}{M_* + m_p} \qquad \frac{a_p}{a} = \frac{M_*}{M_* + m_p}$$

Angular displacement for a star at distance d:

$$\Delta\theta = \frac{a_*}{d} \approx \left(\frac{m_p}{M_*}\right)\left(\frac{a}{d}\right)$$

(Assumes small angles and $m_p << M_*$)

Scaling to Jupiter and the Sun, this gives:

$$\Delta\theta \approx 0.5 \left(\frac{m_p}{\mathrm{m_J}}\right)\left(\frac{M_*}{\mathrm{M_{sun}}}\right)^{-1}\left(\frac{a}{5\mathrm{AU}}\right)\left(\frac{d}{10\mathrm{pc}}\right)^{-1} \ \mathrm{mas}$$

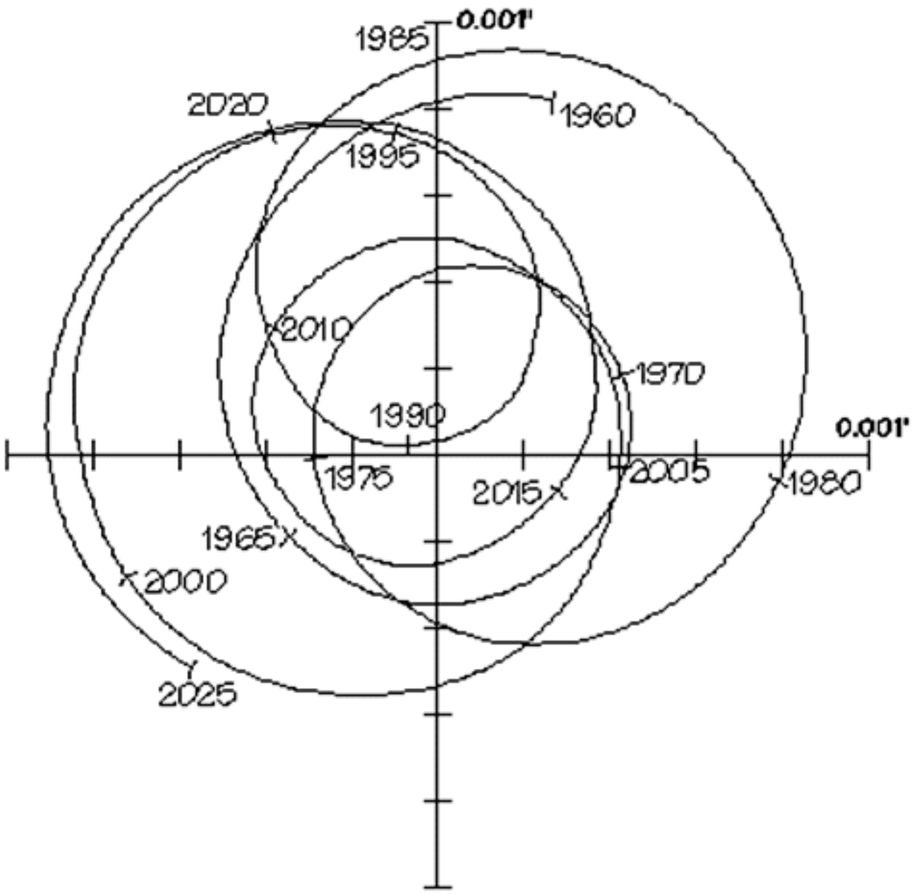
Note:

- Units are milliarcseconds -> very small effect
- Amplitude increases at **large orbital separation, *a***
- Amplitude decreases with distance to star *d*.

- Detecting planets at large orbital radii requires a **long search time**, comparable to the orbital period.

$$\frac{P}{\mathrm{yr}} = \left(\frac{M_*}{M_{sun}}\right)^{-1/2}\left(\frac{a}{\mathrm{AU}}\right)^{2/3}$$

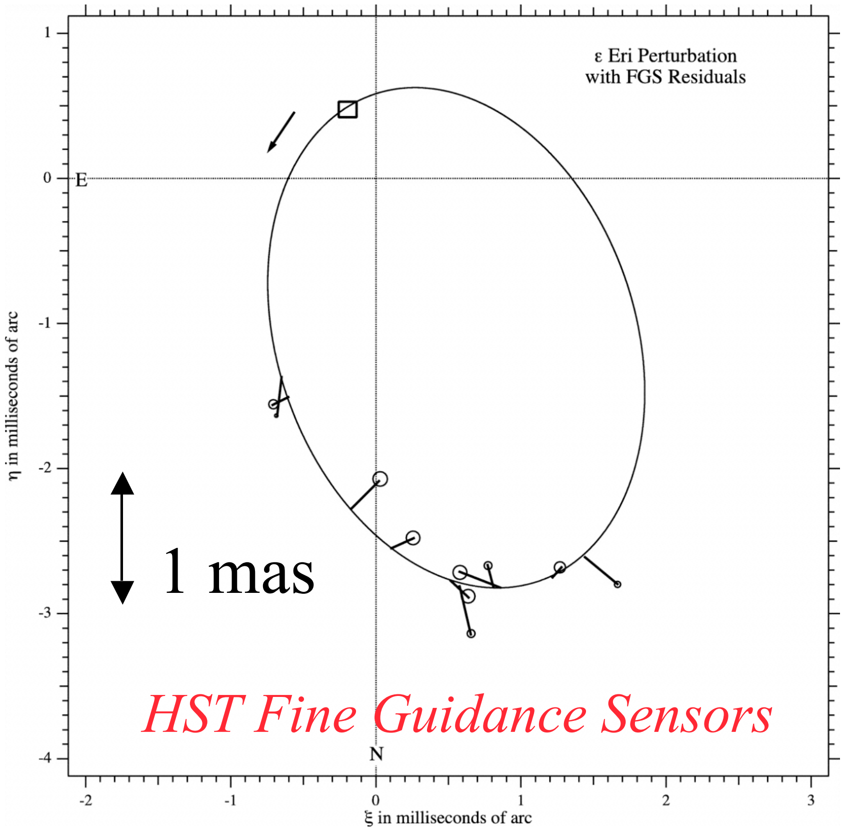
Epsilon Eridani



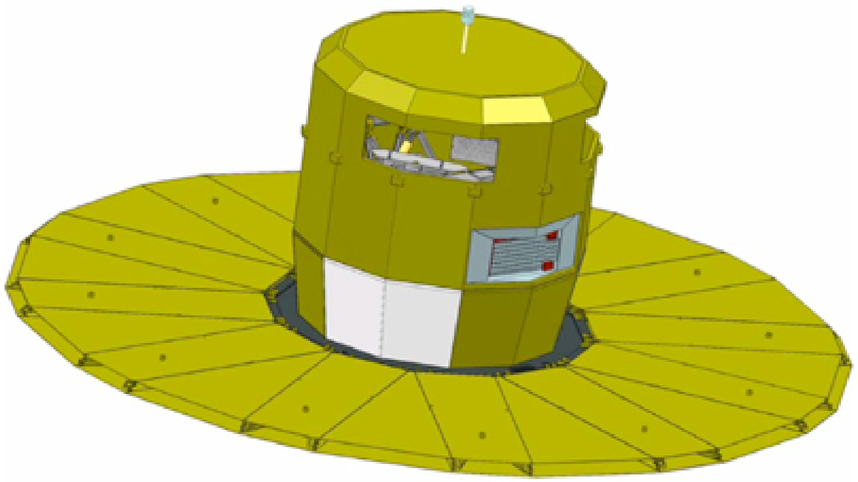
The wobble of the Sun’s projected position due to the influence of all the planets as seen from 10 pc

Data obtained 1980-2006 to track the orbit

$P = 6.9$ yr, $m_p = 1.55\ M_J$



Future Astrometric Experiments



- PRIMA on VLT Interferometer (Paranal, Chile)
- ESA's **GAIA** (2011 launch) and NASA's SIM (not yet funded)
- Planned astrometric errors ***~10 micro-arcsecond***
- May detect planets of a few Earth masses at 1 AU around nearby stars

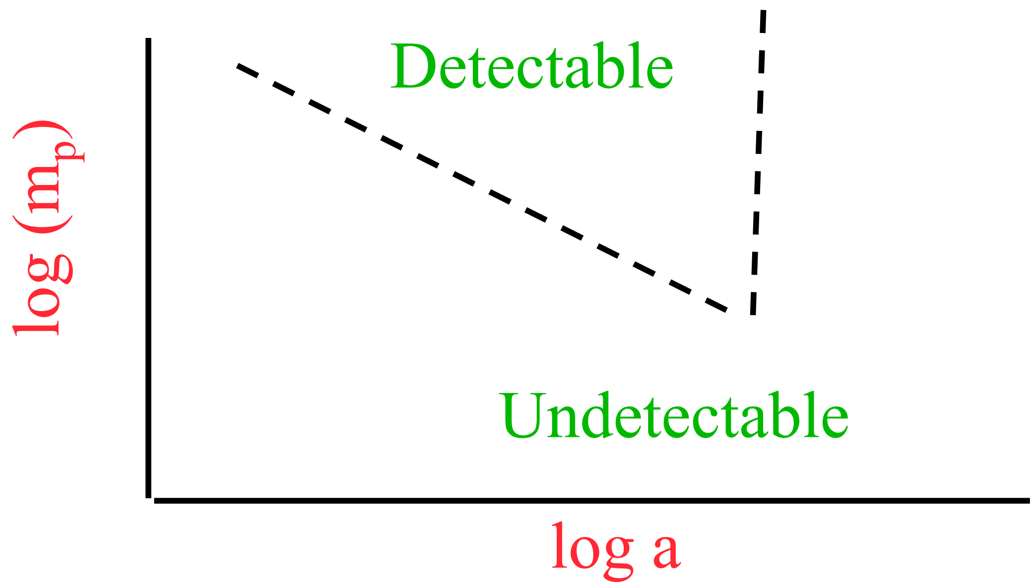
Astrometry Selection Function

Need to observe (most of) a full orbit of the planet:

No discovery for planets with $P > P_{survey}$

For $P < P_{survey}$, planet detection requires a star wobble several times larger than the accuracy of the measurements. ==> minimum detectable planet mass.

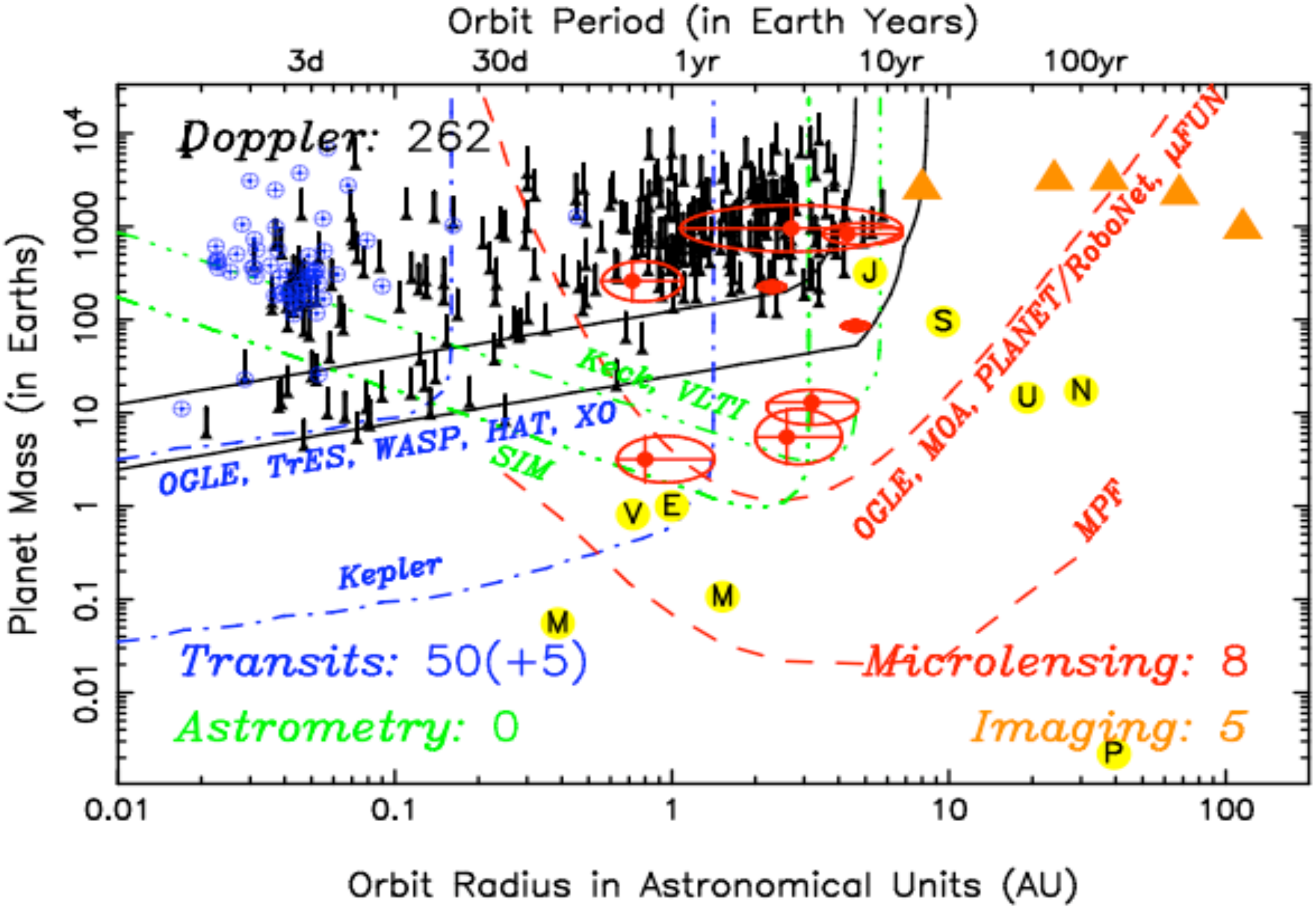
Planet mass sensitivity as a function of orbital separation



$$\Delta\theta = \frac{a_*}{d} \approx \left(\frac{m_p}{M_*}\right)\left(\frac{a}{d}\right)$$

$$m_p \propto a^{-1}$$

Exoplanets: 50+262+8+5=325 (Mar 2009)



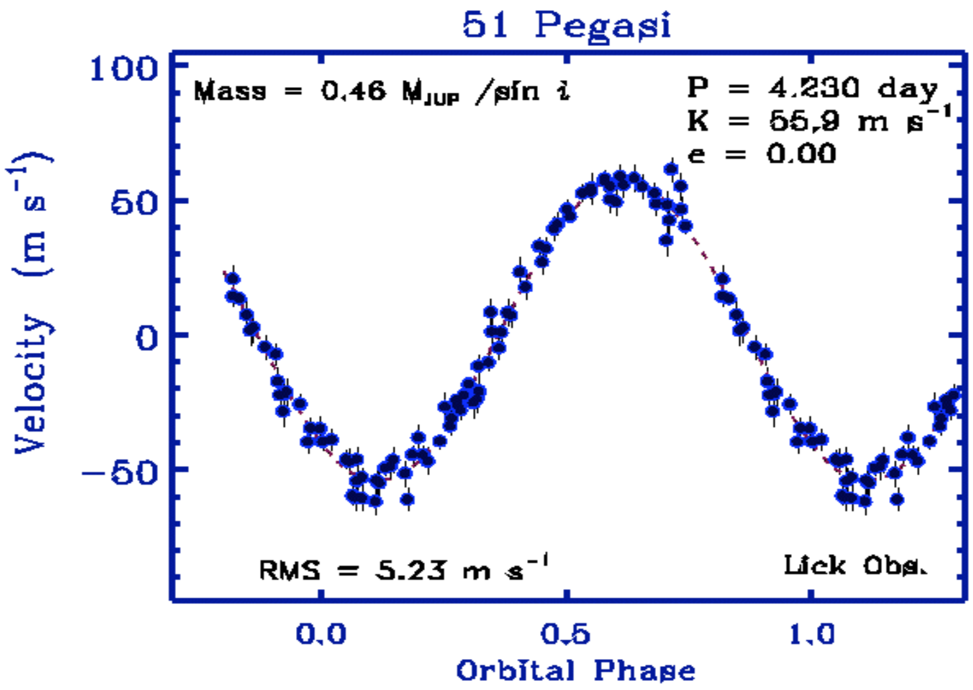
Doppler Wobbles: Radial Velocity

Periodic variations in the Radial Velocity of the Host Star

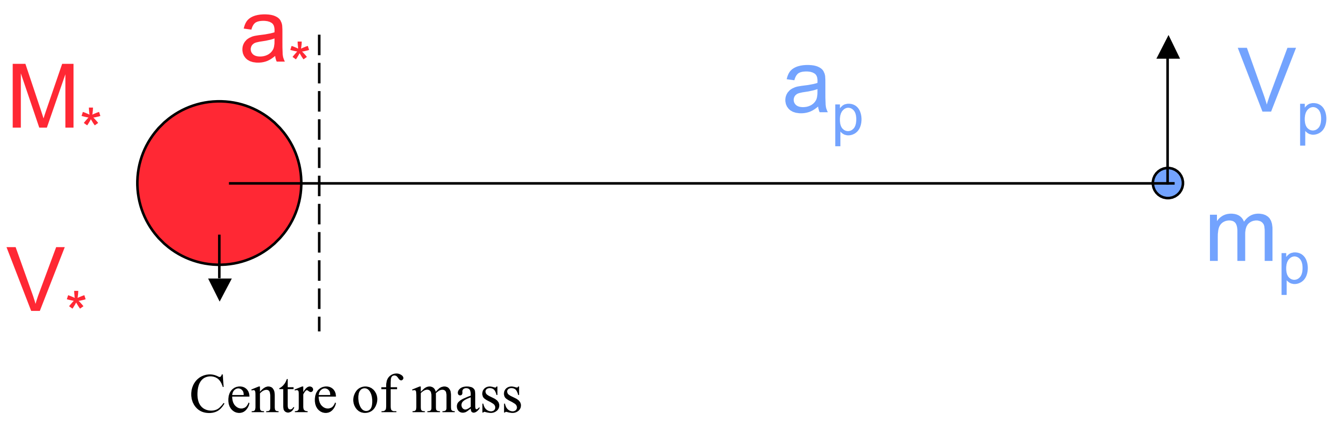
- Most successful method:
  >300 planets detected

- The first planet around a normal star, 51 Peg, was detected by doppler wobbles in 1995.

- Doppler shift of starlight caused by the star orbiting the center of mass with 1 or more orbiting planets



Star's Orbit Velocity



Consider first a circular orbit.

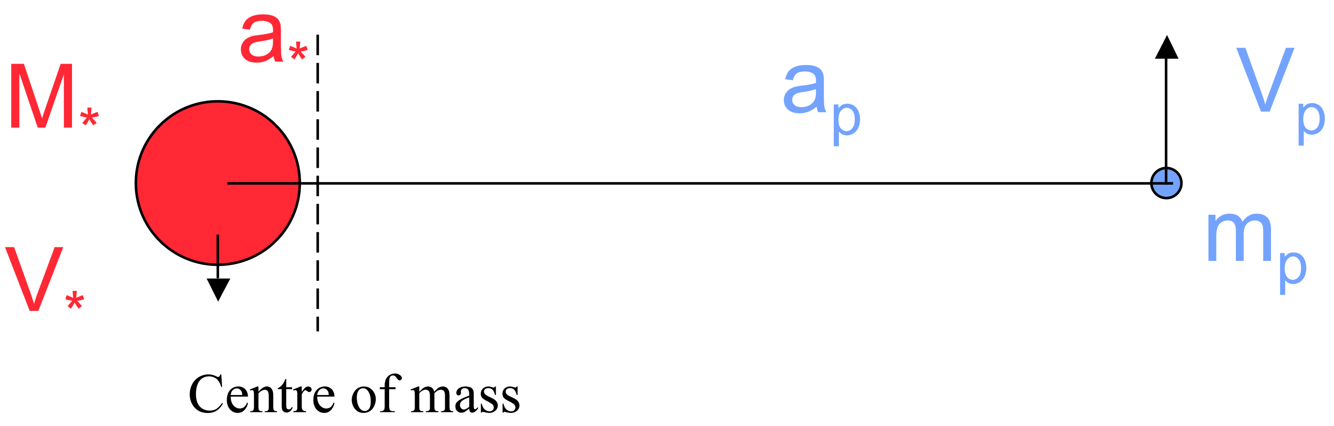
Velocities: $V_* = (2\pi a_*) / P$ $V_p = (2\pi a_p) / P$

Conservation of momentum: $M_* V_* = m_p V_p$ thus $M_* a_* = m_p a_p$

Kepler's 3rd Law: $a^3 = G M (P / 2\pi)^2$ $M = M_* + m_p$

$$V_* = \frac{2\pi a_*}{P} = \frac{2\pi}{P}\frac{m_p}{M} a = \frac{2\pi}{P}\frac{m_p}{M}\left(G M\left(\frac{P}{2\pi}\right)^2\right)^{1/3} = m_p\left(\frac{2\pi G}{P M^2}\right)^{1/3}$$

Star’s Orbit Velocity



Kepler’s law applies for V = relative velocity, M = total mass

$$\frac{V^2}{a} = \frac{G\,M}{a^2} \quad \Rightarrow \quad V = \left(\frac{G\,M}{a}\right)^{1/2} = \frac{2\pi\,a}{P} \qquad M \equiv M_* + m_p$$

$$V_* = \frac{a_*}{a} V = \frac{m_p}{M}\left(\frac{G\,M}{a}\right)^{1/2} = m_p\left(\frac{G}{a\,M}\right)^{1/2}$$

Star’s centrifugal acceleration due to planet’s gravity:

$$\frac{V_*^{\,2}}{a_*} = \frac{G\,m_p}{a^2} \quad \Rightarrow \quad V_* = \left(\frac{G\,m_p}{a^2}\,a_*\right)^{1/2} = \left(\frac{G\,m_p}{a^2}\left(\frac{a\,m_p}{M}\right)\right)^{1/2} = m_p\left(\frac{G}{a\,M}\right)^{1/2}$$

Star's Orbit Velocity

From Kepler's Law and $a_* M_* = a_p m_p$ (center of mass),
The star's velocity is:

$$V_* \approx \left(\frac{m_p}{M}\right)\sqrt{\frac{GM}{a}} \qquad M \equiv M_* + m_p \approx M_*$$

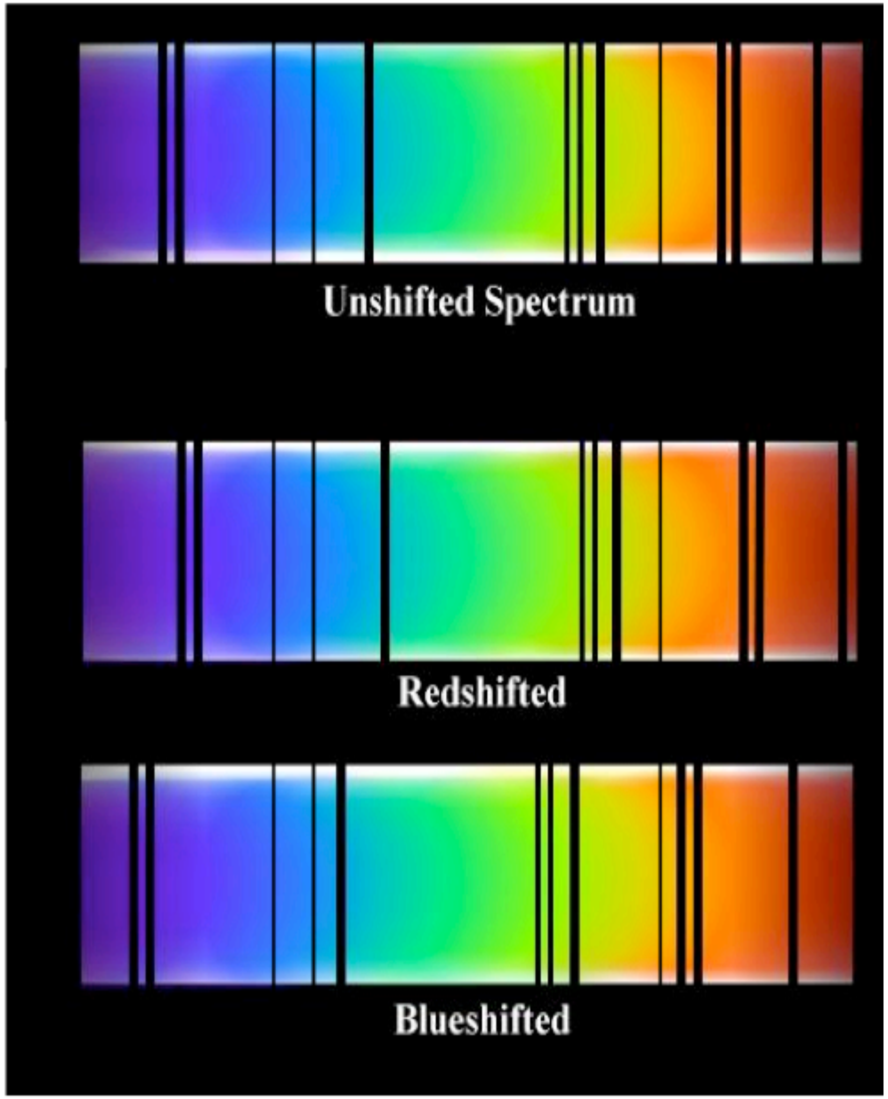
Star's velocity scales with planet's mass.
Hot Jupiter (P = 5 days) orbiting a 1 M_{sun} star: 125 m/s
Jupiter orbiting the Sun: 12.5 m/s
Sun due to Earth: 0.1 m/s

Thermal velocity width of spectral lines ~ 10 km/s $(T/10^4\mathrm{K})^{1/2}$

Special techniques and spectrographs needed to measure such tiny radial velocity shifts stably over many years.

Key Technology:

Spectra of Stars



Iodine Gas Cell

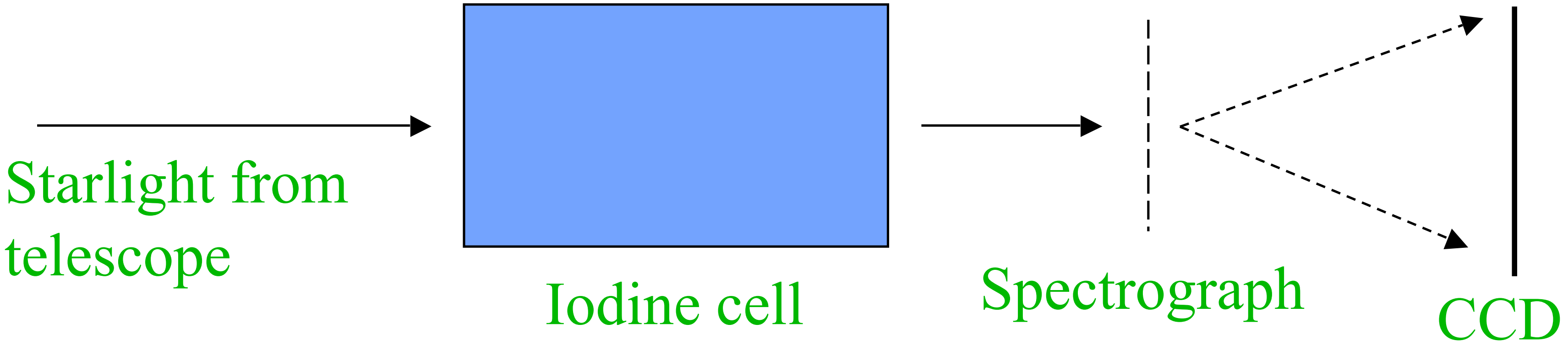
Pass the starlight through an Iodine Cell
and then into a Spectrograph

High sensitivity to small radial velocity shifts:

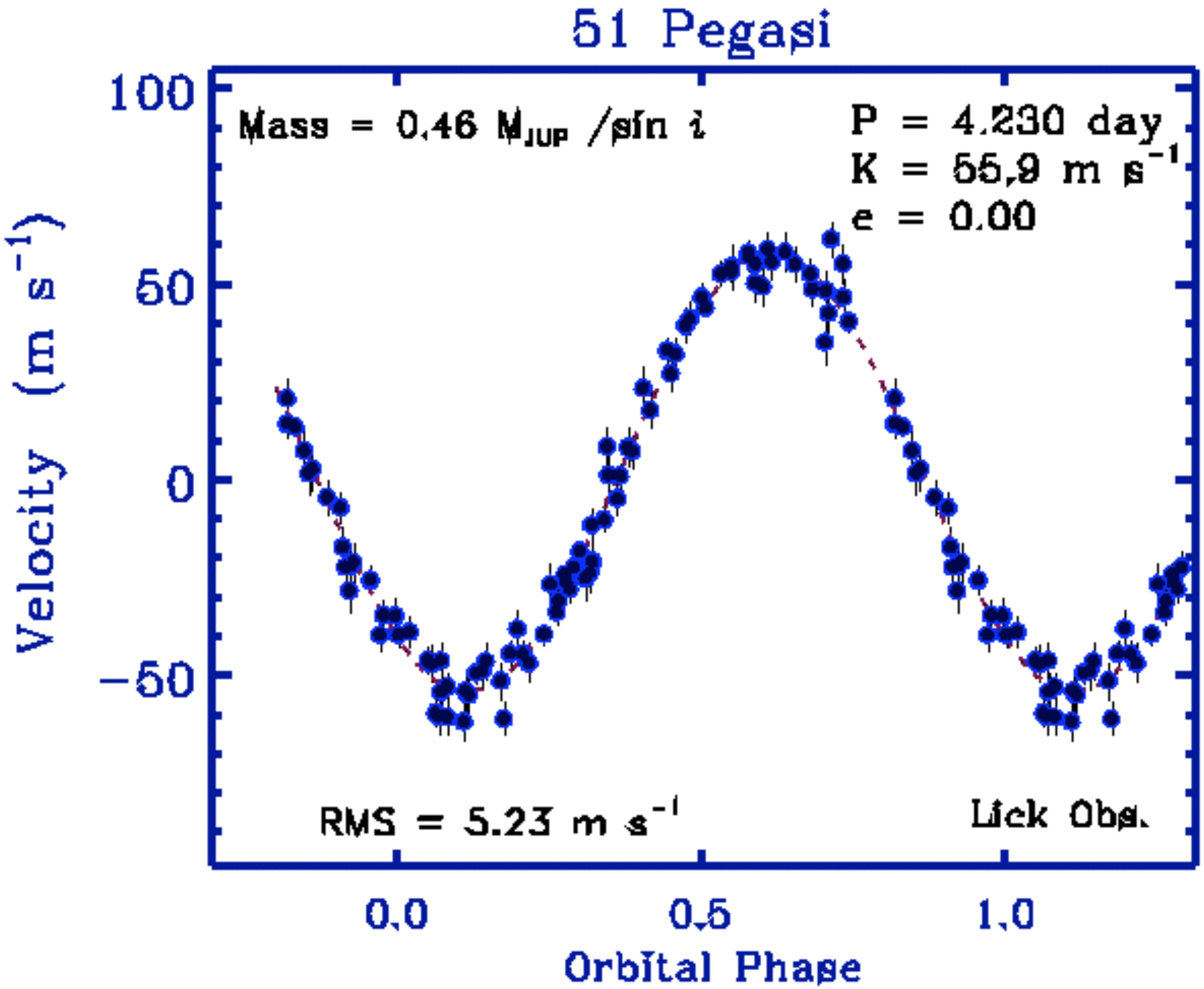
- Achieved by comparing high S/N ~ 200 spectra with template stellar spectra
- Large number of lines in spectrum allows shifts of much less than one pixel to be determined

Absolute wavelength calibration and stability over long timescales:

- Achieved by passing stellar light through a cell containing iodine, imprinting large number of additional lines of known wavelength into the spectrum.
- Calibration suffers identical instrumental distortions as the data



Examples of radial velocity data



51 Peg b, the first known exoplanet, with a 4.2 day circular orbit.

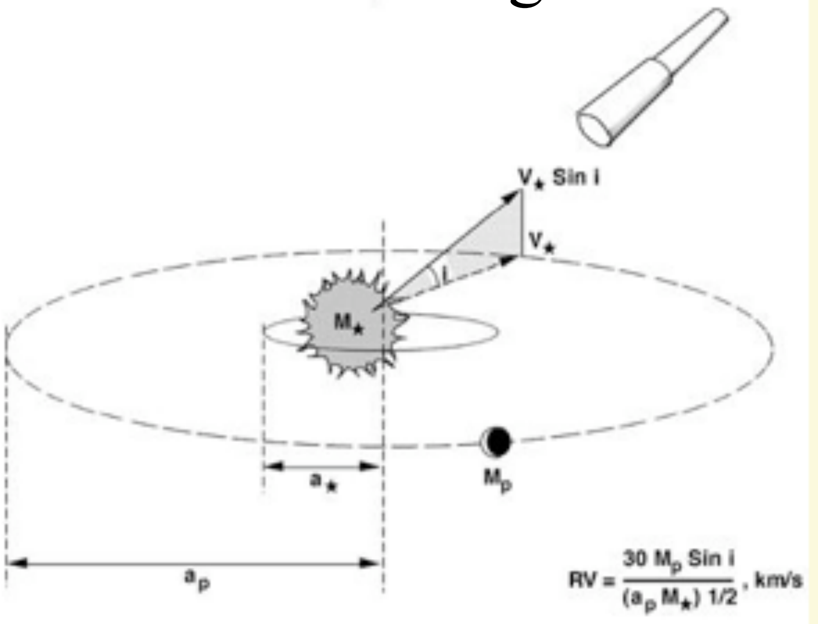
Orbital inclination => lower limits

The *observed* velocity is component along the line of sight, thus reduced by the sine of the orbit's inclination angle :

$$V_{obs} = V_* \sin(i)$$

With $V_* \approx \left(\dfrac{m_p}{M_*}\right)\sqrt{\dfrac{G\,M_*}{a}}$

The measured quantity is: $m_p \sin(i)$



(assuming M* is well determined e.g. from spectral type)

V_{obs} gives us $m_p\, sin(i)$, a **lower limit** on the planetary mass, if there are no other constraints on the inclination angle.

Error sources

(1) Theoretical: photon noise limit

- flux in a pixel that receives N photons uncertain by ~ $N^{1/2}$
- implies absolute limit to measurement of radial velocity
- depends on spectral type - more lines improve signal
- < 1 m/s for a G-type main sequence star with spectrum recorded at S/N=200
- practically, S/N=200 can be achieved for V=8 stars on a 3m class telescope in survey mode

(2) Practical:

- stellar activity - young or otherwise active stars are not stable at the m/s level
- remaining systematic errors in the observations

Currently, best observations achieve:

Best RV precision ~ 1 m/s

...in a single measurement. Allowing for the detection of low mass planets with peak Vobs amplitudes of ~ 3 m/s

HD 40307, with a radial velocity amplitude of ~ 2 m/s, has the smallest amplitude wobble so far attributed to a planet.

Radial velocity monitoring detects massive planets (gas giants, especially those at small a. It is now also detecting super-Earth mass planets (< 10 M_E)

Selection Function

Need to observe (most of) a full orbit of the planet:

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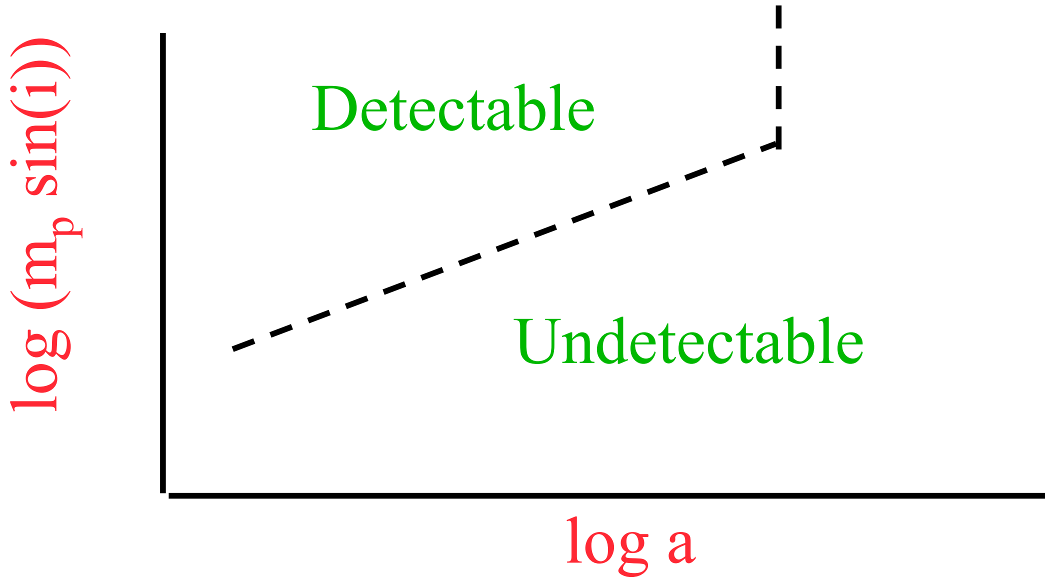
For $P < P_{survey}$, planet detection requires a star wobble Vobs several times larger than the accuracy of the measurements. ==> minimum mass of detectable planet.

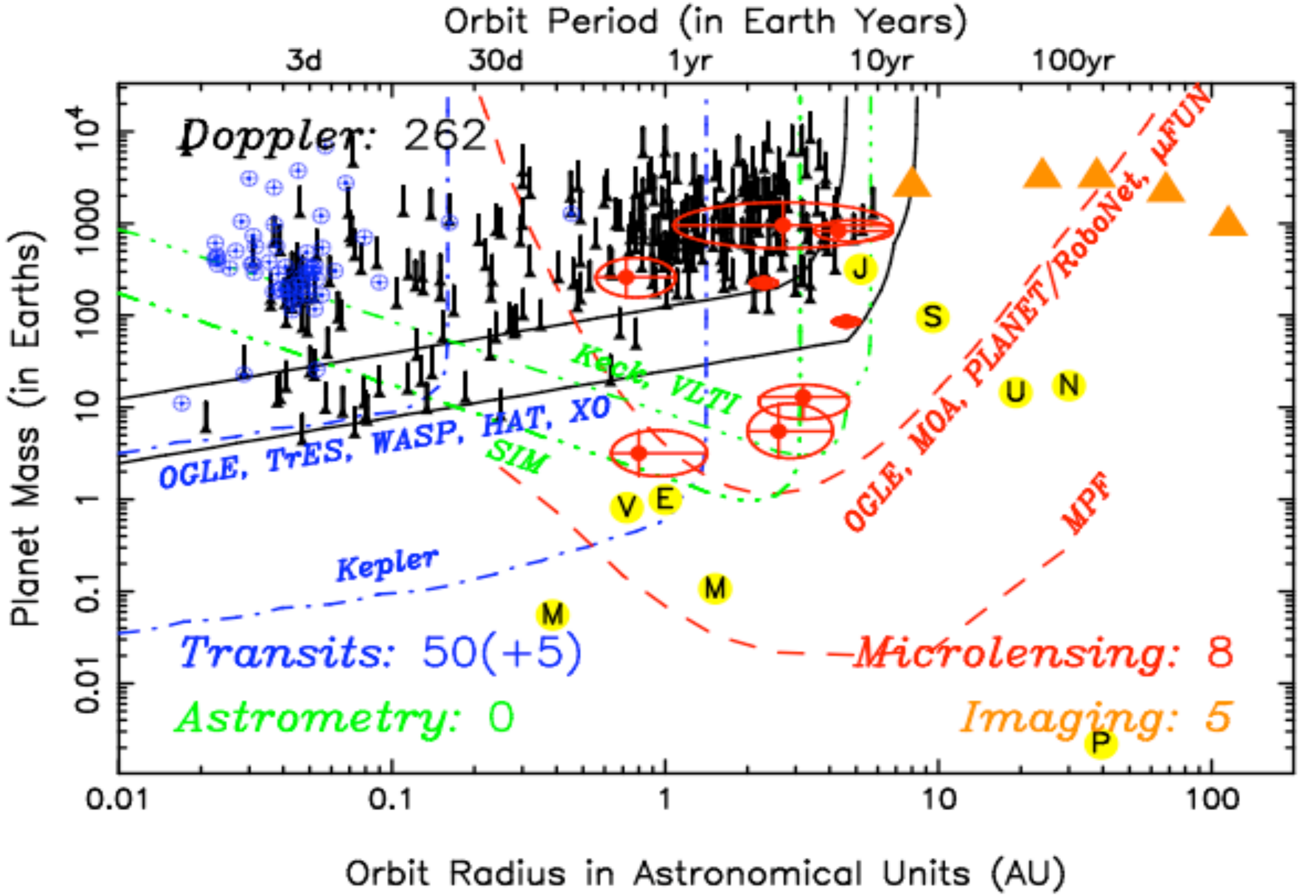
Planet mass sensitivity as a function of orbital separation



$$V_* \approx \left(\frac{m_p}{M_*}\right)\sqrt{\frac{GM_*}{a}}$$

$$m_p \sin(i) \propto a^{1/2}$$

Exoplanets: 50+262+8+5=325 (Mar 2009)



Eccentric Orbits

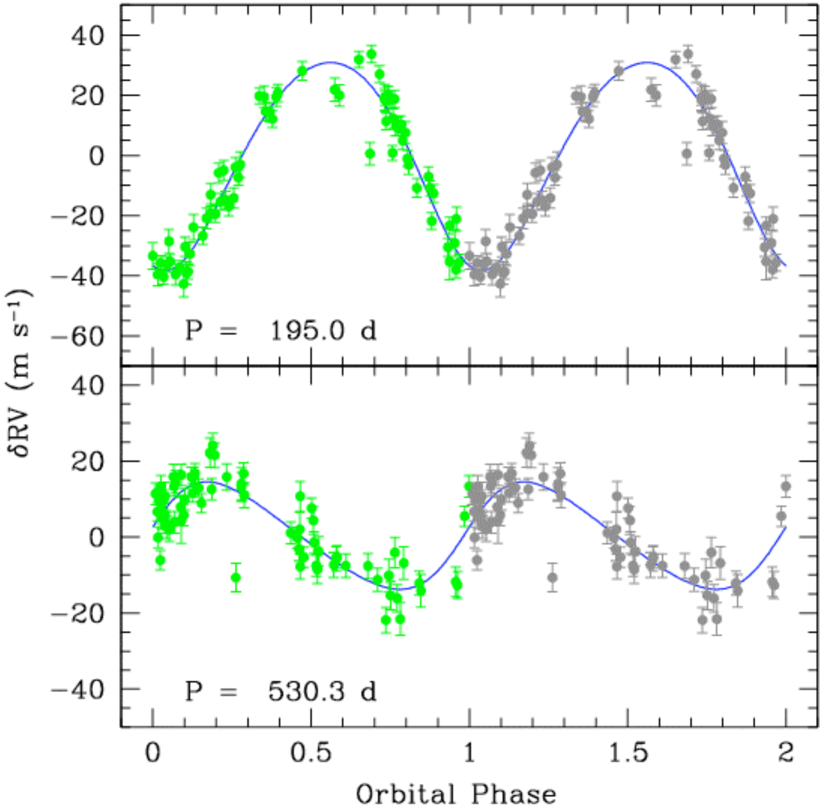
Circular orbit: velocity curve is a sine wave.

Elliptical orbit: velocity curve more complicated, but still varies periodically.

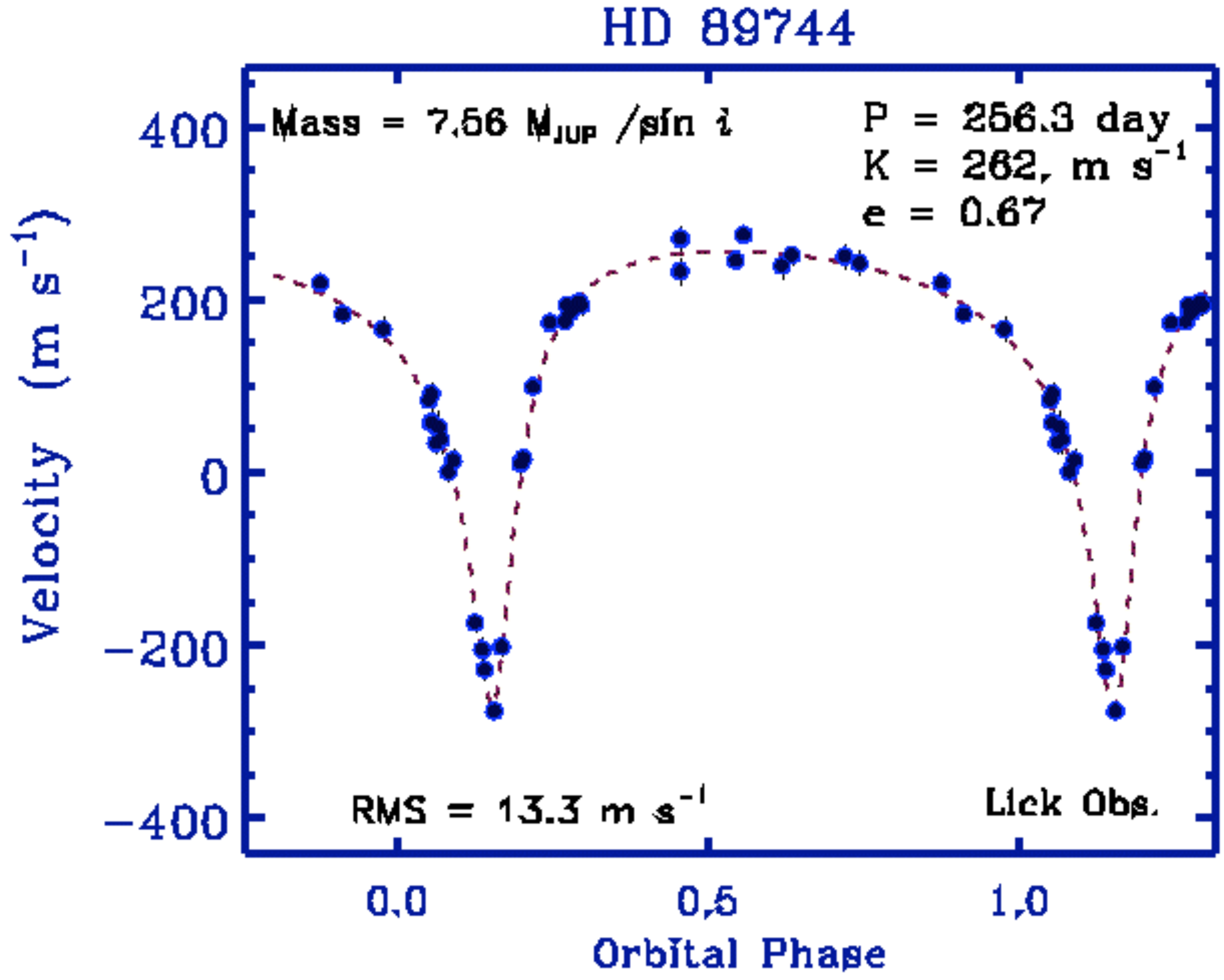
Eccentric orbit:

$$V_{rad} = \frac{2\pi a sin(i)}{P(1-e^2)^{1/2}}[cos(\theta - \omega) + e cos(\omega)]$$

Circular orbit: e --> 0, ω --> 0



Example of a planet with an eccentric orbit: e=0.67



Eccentric (non-circular) Orbits

Not yet well understood.

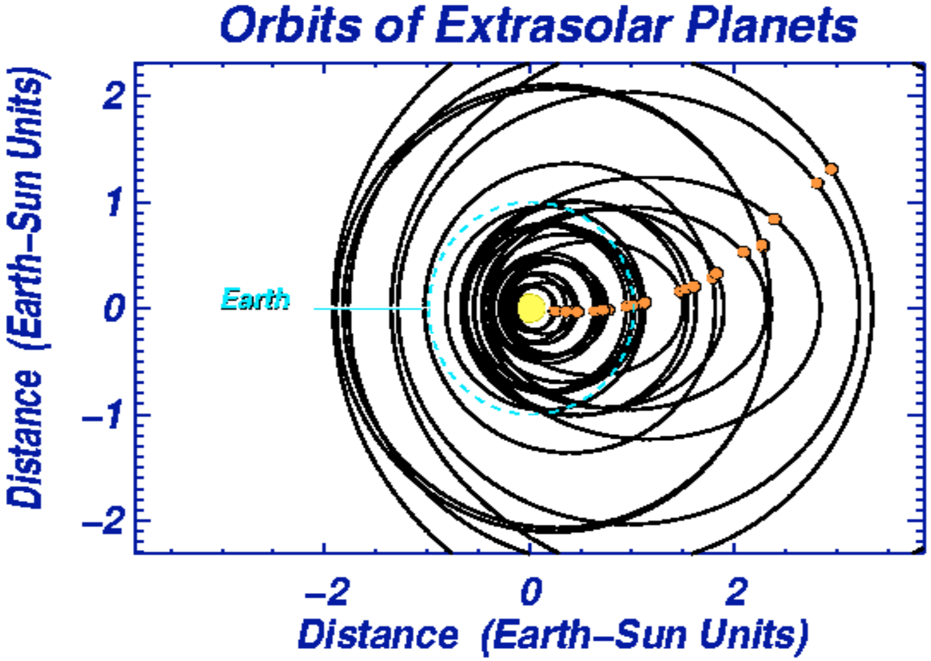
Early star-star encounters?

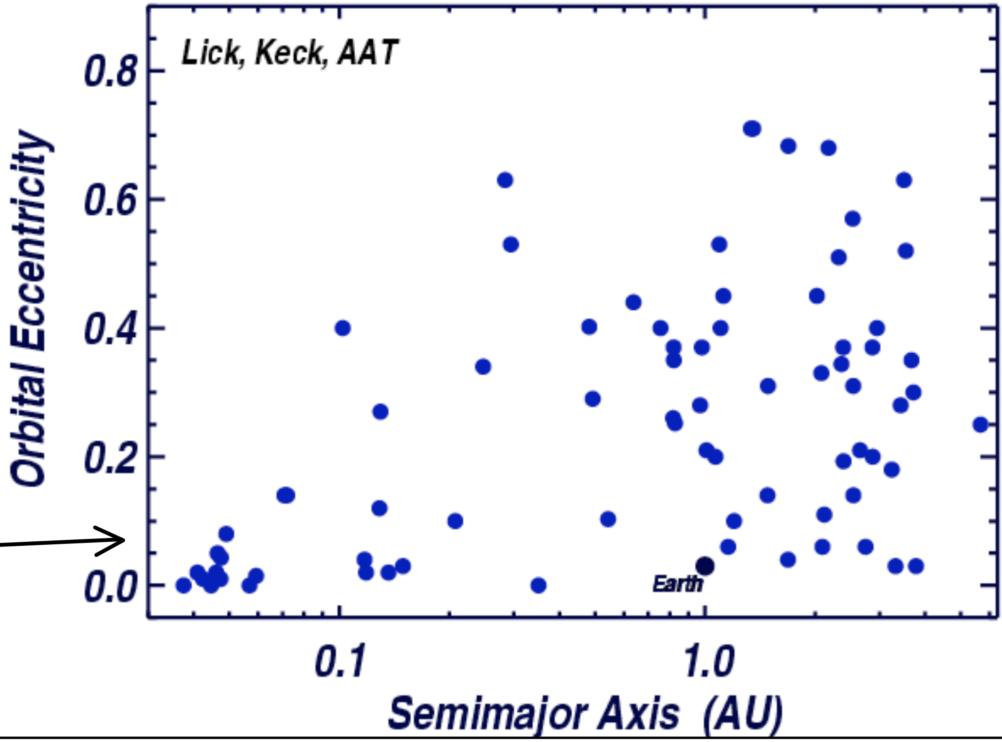
Planet-planet interactions?

Eccentricity pumping.

Small planets ejected?

Tidal circularisation. →

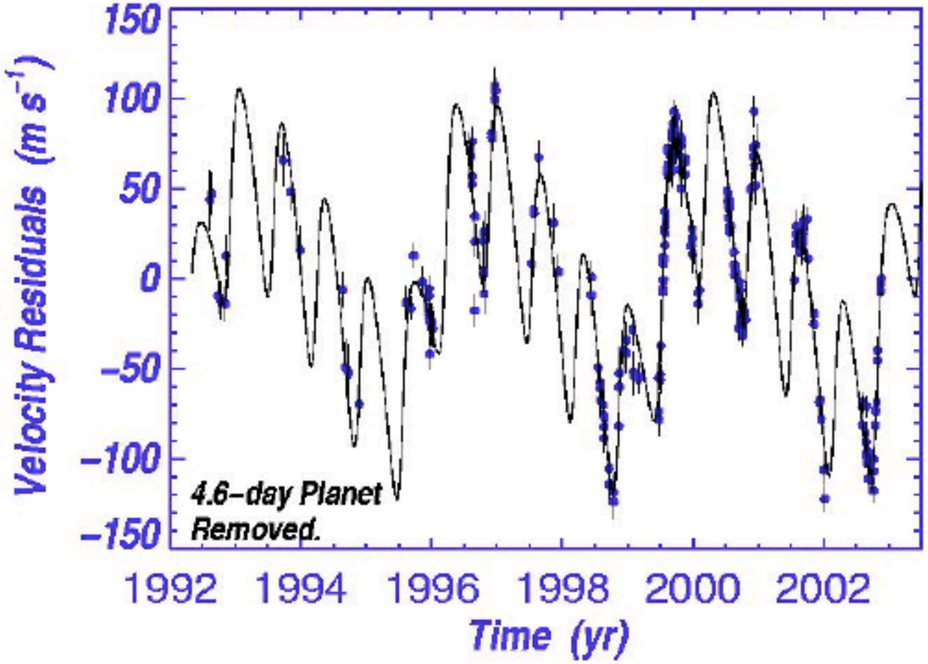




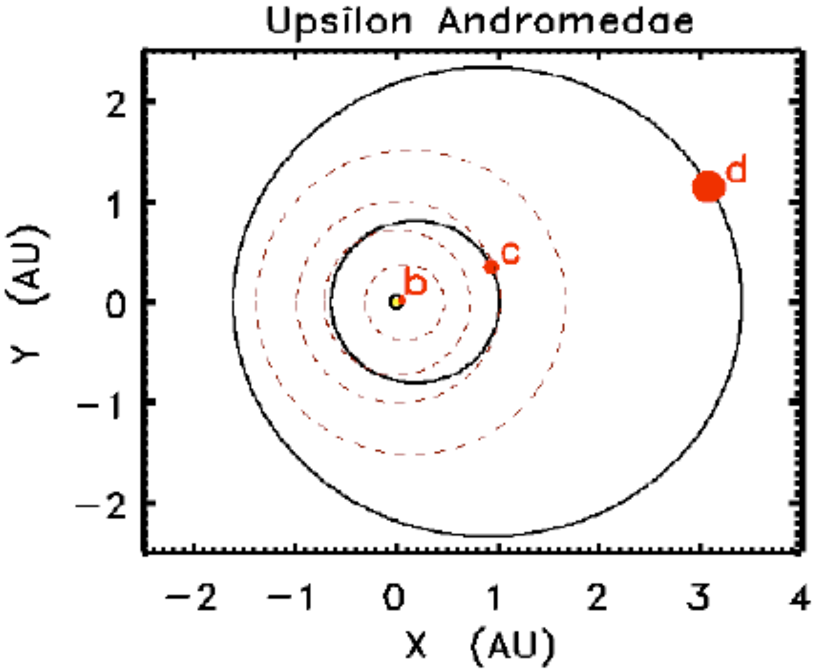
A planetary system with 3 gas giants

Planet b: 0.059 AU, 0.72 M_J e=0.04
Planet c: 0.83 AU, 1.98 M_J, e=0.23
Planet d: 2.5 AU, 4.1 M_J, e=0.36

RV Curve



Upsilon And: F8V star
M_*=1.28 Msun
Teff=6100 K

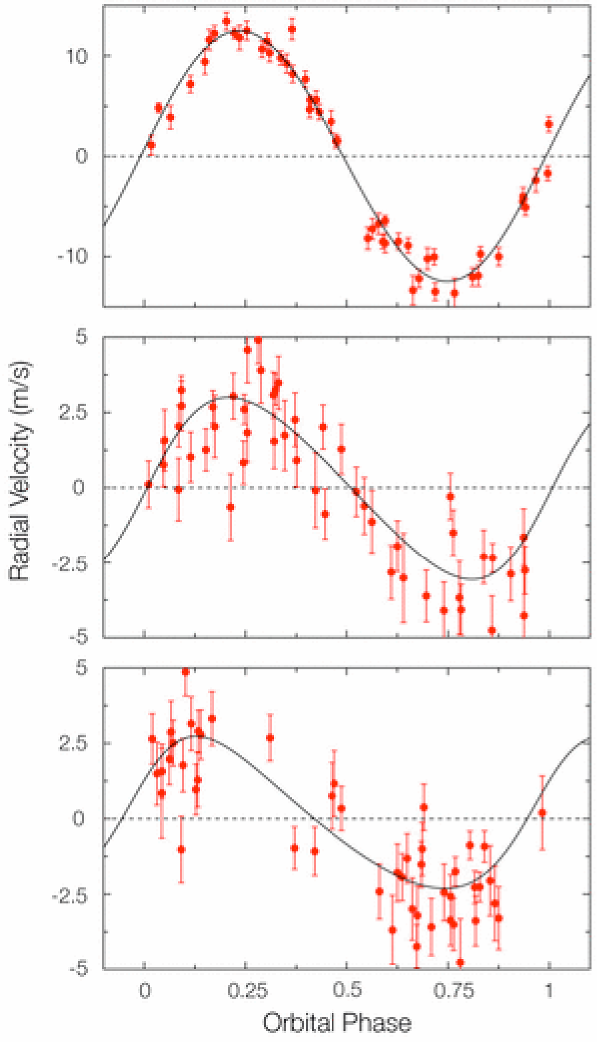


A system of “super Earths”

- 22 planets discovered with $m_p < 30\ M_E$
- 9 super-Earths ($2\ M_E < m_p < 10\ M_E$)
- Found at a range of orbital separations:
  - Microlensing detection of $5.5\ M_E$ at 2.9 AU
  - RV detections at P~ few days to few hundred days
- 80% are found in multi-planet systems

GJ 581	M3V	$0.31\ M_{sun}$
581b	5.3d	$15.7\ M_E$
581c	12.9d	$5.0\ M_E$
581d	83.6d	$7.7\ M_E$

Bonfils, et al. 2005
Udry, et al. 2007



Observed Velocity Variation of Gliese 581

ESO Press Photo 22d/07 (25 April 2007)

Summary

Observables:

(1) Planet mass, up to an uncertainty from the normally unknown inclination of the orbit. Measure $m_p \sin(i)$

(2) Orbital period -> radius of the orbit given the stellar mass

(3) Eccentricity of the orbit

Current limits:

- Maximum ~ 6 AU (ie orbital period ~ 15 years)
- Minimum mass set by activity level of the star:
    - ~ 0.5 M_J at 1 AU for a typical star
    - 4 M_E for short period planet around low-activity star
- No strong selection bias in favour / against detecting planets with different eccentricities