AS2001:
Stellar Structure and Evolution

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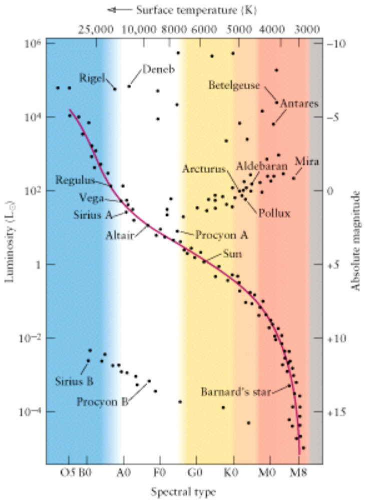
THE HR DIAGRAM

- **Notice specific regions:**
  - **main sequence**
  - **red giants**
  - **white dwarfs**
- **Also - links between regions**
- **and several empty regions**
- **luminosity range**
  - 10^5 L_{Sun} - 10^{-4} L_{Sun}
- **surface temp range**
  - 10^5 K - 2×10^3 K
- **radius range**
  - 10^3 R_{Sun} - 10^{-2} R_{Sun}



Quick reminder

- The luminosity of a star is $L = 4 \pi R^2 \sigma T^4$
- hence dividing by the luminosity of the Sun

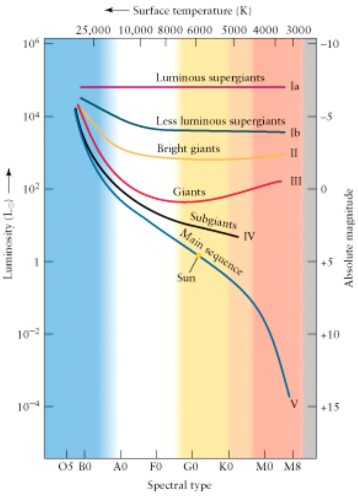
$$\frac{L}{L_{Sun}} = \left(\frac{R}{R_{Sun}}\right)^2 \left(\frac{T}{T_{Sun}}\right)^4$$

$$\text{ie} \quad \log\left(\frac{L}{L_{Sun}}\right) = 4\log\left(\frac{T}{T_{Sun}}\right) + 2\log\left(\frac{R}{R_{Sun}}\right)$$

So, on the HR diagram which shows log(L) versus log(T), we can draw a series of straight lines for different stellar radii.

- For stars of same spectral type (ie surface temp) , the width of the spectral lines varies with the luminosity (eg H lines narrow for supergiants, broader for m-s stars).
- This effect is due to variations in the pressure and density of the stellar atmospheres.
- High pressure and density → many collisions → shift in energy levels of H atoms → broad lines

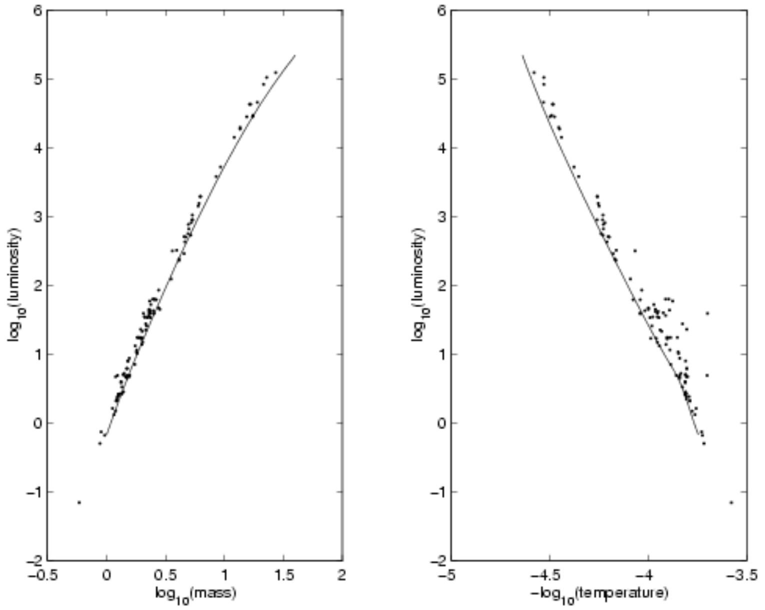
- Hence narrow lines in supergiants, since their pressure and density is low (their mass is spread over a large volume) → few collisions → energy levels undisturbed.
- Hence luminosity classes I - V
- Spectral type + luminosity class tells you a great deal about a star!



Mass - luminosity relation

- Use binary systems to measure stellar masses
- from brightness variations (light curves) of eclipsing binaries, determine orbital inclination i, and sizes of stars relative to their orbit size
- from Doppler effect on positions of spectral lines, measure orbital motion of both stars, and hence absolute size of orbits (e.g. in km).
- combined data give masses, radii, temperatures and luminosities of both stars, usually quoted in solar units.

Mass-luminosity and HR diagram (T-L) for detached eclipsing binaries



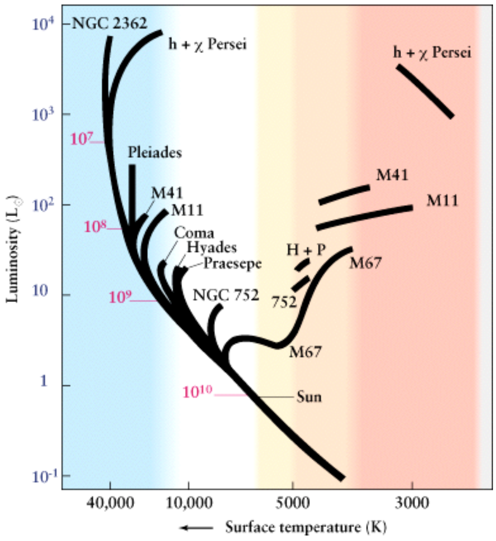
- **From masses of many stars, find for m-s stars**

$$\frac{L}{L_{Sun}} = \left(\frac{M}{M_{Sun}}\right)^{4.0\pm0.02} \quad \text{for} \quad 0.4 < M < 10 M_{Sun}$$

$$\frac{L}{L_{Sun}} = \left(\frac{M}{M_{Sun}}\right)^{3.6\pm0.1} \quad \text{for} \quad 5 \le M \le 40 M_{Sun}$$

But note exponent in this power law decreases rapidly for stars of higher mass

- Composite HR diagrams of different clusters → stellar evolution; also use clusters to get distances



Open clusters:

- distances: main sequence (m-s) fitting
- distribution: Milky Way (MW) plane
- population: I
- Chemical comp. :
  X ~ 0.73; Y ~ 0.25; Z ~ 0.01 - 0.04
- Ages: young - old; 10^6 - n x 10^9 yrs

Globular clusters:

- distances: m-s fitting; RR Lyrae (all have $M_V \sim 0.6$)
- distribution: galactic halo + nucleus
- population: II
- Chemical comp. : $Z \sim 0.0001 - 0.001$
- Ages: very old; n x 10^{10} yrs

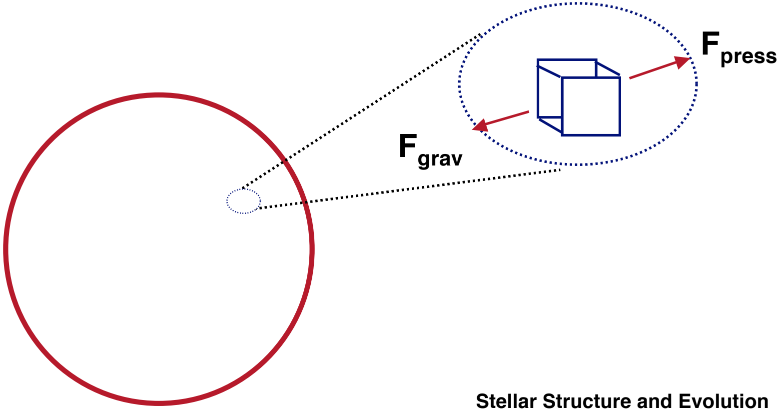
STELLAR STRUCTURE

- Governed by mass, age, initial chemical composition
- The equations of stellar structure describe the complete internal structure of a star
- this in turn determines the observable parameters of the star: its radius, surface temperature and luminosity
  - NB: asteroseismology provides further tests of these equations and our knowledge of stellar structure

End 1

Stellar equilibrium

- Most stars are in a stable state (i.e. there is a balance between all their internal forces)
- the inward force of gravity is balanced by an outward pressure force (hydrostatic equilibrium)
- pressure is higher in centre of star

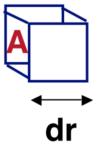


Gravitational force

Newton's Laws:

- the gravitational force F_{grav} is the same as if all the mass closer to the star than the element were concentrated at the centre.
- The outer layers have no effect.

- If each side of the cube is of area A, it is a distance r from the centre and has density ρ then the mass of the element is m= ρ Adr and the mass interior to r is M(r), giving



$$F_{\text{grav}} = \frac{GM(r)\rho A dr}{r^2}$$

Pressure force

- Pressure forces all balance except on the lower and upper faces

- lower face: pressure force outward $\mathrm{P}(r)A$
- upper face: pressure force inward $\mathrm{P}(r+dr)A$

- condition for no net force is:

$$\mathrm{P}(r+dr)A - P(r)A + \frac{GM(r)\rho A dr}{r^2} = 0$$

- But, for an infinitesimal element of side dr,

$$P(r+dr)-P(r)=\left(\frac{dP}{dr}\right)dr$$

- and so force balance becomes

$$\frac{dP(r)}{dr}=-\frac{GM(r)\rho(r)}{r^2} \qquad \textbf{(1)}$$

The equation of hydrostatic equilibrium

- The mass interior to r can be found from

$$\frac{dM(r)}{dr}=4\pi r^2\rho(r) \qquad \textbf{(2)}$$

i.e.

$$M(r)=4\pi\int r^2\rho(r)dr$$