Black Holes

- For a neutron star, the "escape velocity"

$$(2GM/R)^{1/2} \approx 1.6 \times 10^5 \, \mathrm{kms}^{-1} \approx 0.5\mathrm{c}$$

- i.e. ordinary Newtonian gravitation is no longer sufficient. For such a deep gravitational potential, we have to use General Relativity instead.
- If the mass of the imploded core > 3 M_{Sun}, then current theory proposes the complete collapse of the mass to a singularity - a BLACK HOLE.

- e.g. put

$$v_{esc} = \mathrm{c} \quad \text{and} \quad v_{esc} = (2GM/R)^{1/2}$$
$$\text{then} \quad \mathrm{R} = 2\mathrm{GM}/\mathrm{c}^2 = \mathrm{R}_{Sch}$$

- called THE SCWARZCHILD RADIUS or EVENT HORIZON.
- Correct result for two counteracting wrong reasons!

- $(1/2)\ mc^2$ is not the kinetic energy of a photon; it is hc / λ
- Newton's laws of motion and law of gravitation do not apply at relativistic speeds

Aside on gravitational redshift (a simple argument)

Photon has total energy at surface

$$E_{tot} = h\nu - GMm/R$$

As it climbs out of the gravitational potential well, in order to conserve energy, its frequency must decrease as its gravitational p.e. increases.

Hence

$$\Delta\nu = -GMm/Rh$$

but the photon mass is $m = h\nu/c^2$

i.e. $$\Delta\nu = -GMh\nu/Rhc^2$$

and so the shift in wavelength is

$$\frac{\Delta\nu}{\nu} = \frac{\Delta\lambda}{\lambda} = \frac{GM}{Rc^2}$$

- The depth of the gravitational potential well causes gravitational redshift of photons to lower energies. Doing it all properly...
- A photon which starts with a wavelength λ_0 at distance r outside a spherical mass M will have a longer wavelength λ when it is seen at ∞ .

$$\frac{\lambda}{\lambda_0} = \left(1 - \frac{2GM}{rc^2}\right)^{-1/2}$$

- Then for

$$r \rightarrow 2GM/c^2, \quad \lambda \rightarrow \infty$$

and so the photon energy hc / $\lambda \rightarrow 0$

- Hence the photon is redshifted out of existence!
- The event horizon has a surface area of

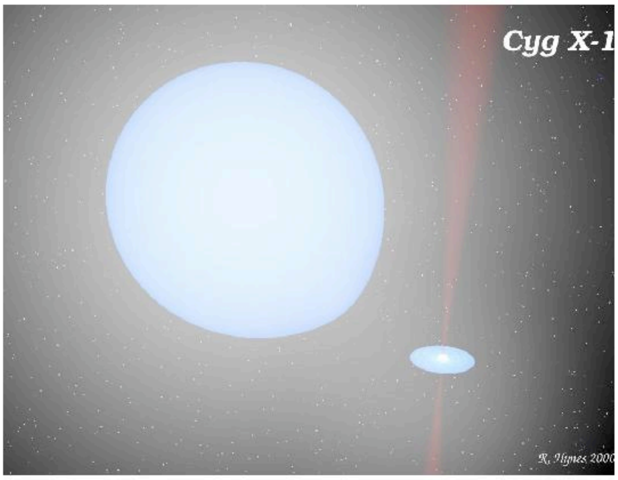
$$4\pi\left(\frac{2GM}{c^2}\right)^2$$

but we cannot measure such a radius since spacetime is very greatly distorted near a deep gravitational potential well.

- Observational evidence for the existence of black holes of stellar mass comes from X-ray binary systems
- There are ~10 binary star systems (e.g. Cyg X-1; V404 Cyg) which have :

- one component which is not visible which has a well-established mass ~ several M_{Sun};
- X-rays from accretion disk around proposed black hole.

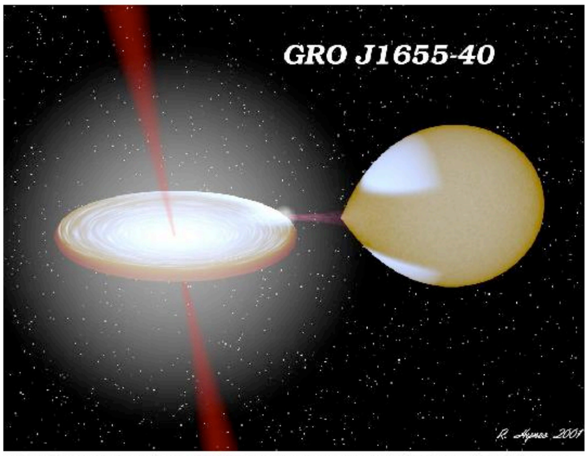
Cyg X-1 http://obelix.as.utexas.edu/~tih/binsite/cygx1.html



rih@astro.soton.ac.uk

The first high-mass x-ray binary - a OB supergiant star filling its Roche lobe transferring matter to an unseen compact object of several solar masses via a stellar wind producing an accretion disc with collimated outflows

GRO J1655-40 http://obelix.as.utexas.edu/~tih/binsite/j1655.html



This unusual X-ray transient was discovered in 1994 and showed a remarkable series of outbursts finally ceasing in 1997. It gained considerable notoriety in its first outburst by ejecting collimated relativistic jets at an estimated speed of 92 per cent of the speed of light. It is believed to contain an F5 subgiant star orbiting a 6-7 Solar mass black hole. The orbital inclination is quite high (around 70 degrees) and partial eclipses probably occur, at least during outburst.

rih@astro.soton.ac.uk

An eclipsing system with well determined properties - Roche-lobe filling F subgiant star of 2.3 Msun, and compact object of 7.0Msun surrounded by an accretion disc and variable collimated outflows

Evolution of Binary Stars

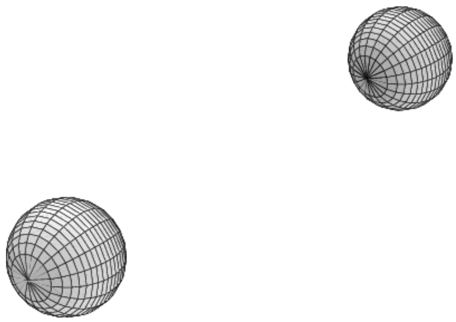
Form of evolution depends on

- **initial masses and the mass ratio** $q = m_1/m_2$
- **chemical composition**
- **initial orbital period (or separation a) and hence total orbital angular momentum J**

$$J = \left[\frac{Ga\left(1-e^2\right)}{\left(m_1+m_2\right)}\right]^{1/2} m_1 m_2$$

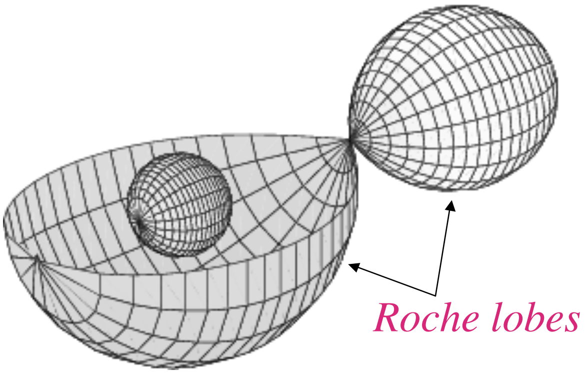
- **In contrast to the evolution of single stars, these factors - the separation and the mass ratio - present UPPER LIMITS to the sizes of stars in a given binary - hence alterations from normal stellar evolution.**
- **ROCHE LOBE size = f (a, m_1/m_2) provides upper limit to possible radius of star.**
- **Size - mainly f(separation a)**
- **shape - mainly f(mass ratio)**

Detached Binaries



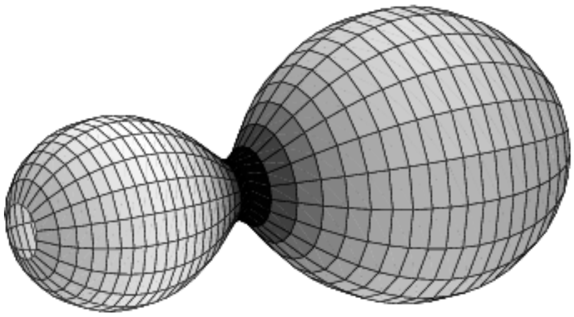
excellent tests of theoretical models of stars because we can determine M,R,T,L directly.

Semi-detached binaries



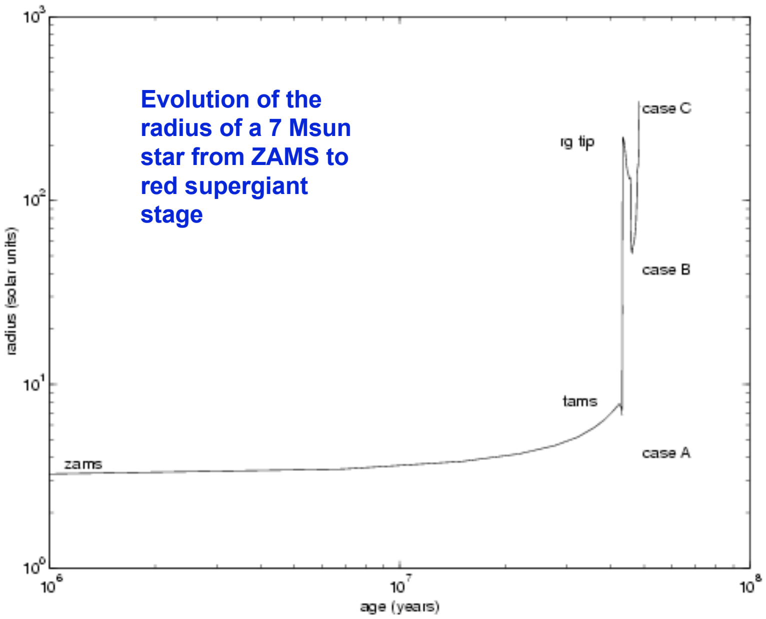
Determine M,R,T,L, and test models of mass exchange and stellar evolution

Contact binaries



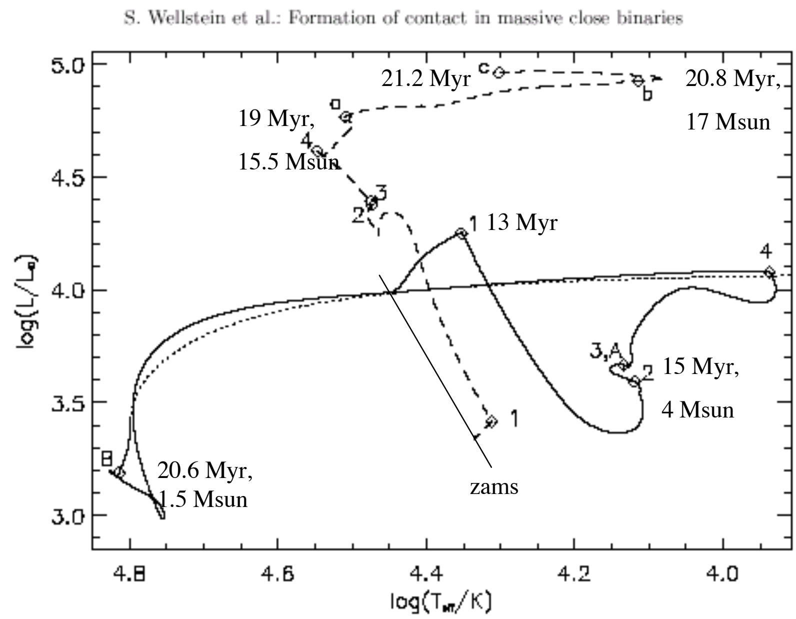
Determine M,R,T,L; but still no adequate theory to explain energy transfer between two stars

- **Hence MASS TRANSFER from one component star to its companion is possible at various stages of evolution, dependent upon (a, m_1/m_2) .**
- **Also MASS LOSS (and ANGULAR MOMENTUM LOSS).**
- **Three broad categories - cases A,B,C - of mass transfer / loss.**

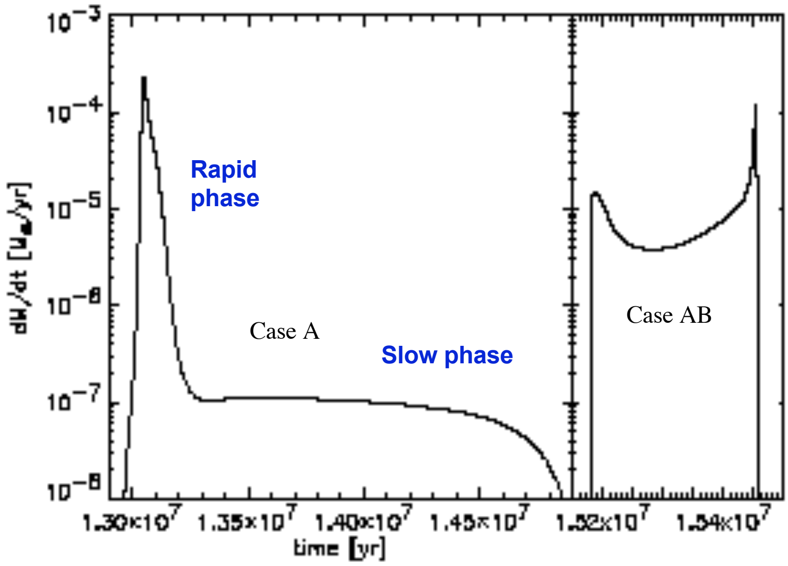


- **Case A: Roche Lobe overflow (RLOF) occurs during main-sequence phase; mass-ratio reversal except for short-period (P < 1 day) systems where transfer of few % of mass leads to a contact system, and subsequent merger.**
- **Hence Algol systems (semi-detached): m-s star (B,A) + evolved low-mass F,G subgiant filling Roche lobe**
  - **subgiant is original more massive star, the loser**
  - **m-s star is original less massive star, the gainer**

Subsequent evolution to wide binaries with white dwarfs, although some shorter period systems (P<1.5 days) will merge into single stars



12+7.5 M(sun); P(init) 2.5 days; case A +AB



Case A + AB mass transfer

- **Case B: RLOF occurs after m-s phase and through red giant phase; dramatic mass-ratio reversal:**

$$\left(\frac{m_1}{m_2}\right)_{\text{initial}} \approx 3-5 \quad \Rightarrow \quad \left(\frac{m_1}{m_2}\right)_{\text{post mass-transfer}} \approx 0.1-0.15$$

- **Hence massive X-ray binaries: original primary evolves through W-R stage to a helium star - then SN event to neutron star or black hole - original secondary is now OB supergiant powering x-ray source by stellar wind**

Case C: RLOF occurs after red giant stage for cataclysmic variables and low-mass X-ray binaries

initial orbital period > 100 days allowing original primary to become red giant before RLOF. When RLOF occurs, deep convective envelope of the giant ensures dynamical timescale mass loss - much of the r-g envelope engulfs entire binary in a common-envelope phase, resulting in a binary with a very short orbital period (hours) containing the stripped core of the r-g - a white dwarf - and the original secondary m-s companion - cataclysmic variables with their dominant accretion discs.

Most massive red supergiants reach SN event and hence neutron stars or black holes; common-envelope event and hence low-mass x-ray binaries.