Transport of energy through interior

Energy transport by radiation:

– (NB convection takes over close to Solar surface)

- In core, $T_c \sim 1.6 \times 10^7$ K
- Wien's Law: $\lambda_{max} = 3 \times 10^7 / T \sim 2$ Angstroms →
- expect γ-ray, X-ray, EUV radiation.
- But surface radiates mostly visible radiation →
- core radiation is degraded to longer wavelengths due to opacity processes

- absorption due to transitions:
  bound-bound,
  bound-free (photoionisation),
  free-free (electron scattering)
- emission:
  inverse of above, but usually via series of smaller transitions (directions need not be outwards)

Some numerical values:

- Remember: the intensity $I_\nu(s)$ of a beam travelling a distance s through an absorbing medium of density ρ and opacity κ_ν is

$$I_\nu(s) = I_\nu(0)e^{-\kappa_\nu \rho s}$$

- Hence the intensity falls by 1 / e in a distance $s = 1 / \kappa_\nu \rho$.
- Call this the photon mean free path λ .

For the Sun:

core density $\rho_c \sim 10^5$ kg m^{-3}

opacity $\kappa_c \sim 10^{-1}$ m^2 kg^{-1}

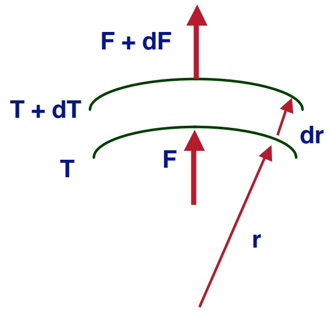
so $\lambda = 1 / \kappa \rho = 10^{-4}$ m (!!!!!)

Temperature gradient over λ :

- mean temp gradient ΔT in Sun is ~ 2×10^{-2} K m^{-1}
- hence over a distance λ, $\Delta T = 2 \times 10^{-6}$ K
- Hence: only a very slight departure from true thermodynamic equilibrium

Equation of radiative transport

- Consider a thin shell from (r) to (r+dr) in the stellar interior



Inner surface:

- black-body temperature T(r)
- so outward radiative flux per unit area $F(r) = \sigma T^4 (r)$

Outer surface:

- Temperature $T(r+dr) = T + dT$
- outward flux $(F + dF) = \sigma (T + dT)^4$
- binomial theorem: $(T + dT)^4 \sim T^4 + 4T^3 dT + \dots$
- hence outward flux $(F + dF) = \sigma T^4 + \sigma 4T^3 d T$

- dT is negative (outer part cooler) so dF is negative, so flux absorbed by shell is

$$dF = 4\sigma T^3 dT$$

- This absorption is due to the opacity κ(r) of the shell material, defined by

$$dF = -\kappa(r)\rho(r)F(r)dr = 4\sigma T^3(r)dT$$

i.e

$$F(r) = \frac{-4\sigma T^3(r)}{\kappa(r)\rho(r)}\frac{dT}{dr} \qquad (11)$$

- We define the luminosity as the energy flux per second through the thin shell:

$$L(r) = 4\pi r^2 F(r)$$

i.e

$$L(r) = \frac{-16\pi\sigma r^2 T^3(r)}{\kappa(r)\rho(r)}\frac{dT}{dr} \qquad (12)$$

- Hence the opacity determines the temperature gradient.
- More detailed studies of absorption / emission processes => L(r) → 4/3 L(r)

Energy transport by convection

- When the temperature gradient is high, the fluid becomes unstable and boils.
- Energy is transported by convection.
- This is modelled adequately in many stars by the “mixing length” theory.