STELLAR ENERGY SOURCES

- Stars must evolve because they lose energy into space (the observed luminosity).
  - Evidence → timescales ~ 10^9 years
- This energy is generated (by gravitational contraction or thermonuclear fusion) in the stellar interior at a rate $\varepsilon = \varepsilon(T, \rho, r)$ W kg $^{-1}$
- NB $\varepsilon = 0$ for most parts of the star, except the core and certain localised shell regions, which become important at different stages of evolution.

- Energy generation within a shell adds to the luminosity entering the lower face of the shell, so (L) enters, but (L + dL) leaves
- Shell mass is $4\pi r^2 \rho(r) dr$, hence

$$dL = 4\pi r^2 \rho(r) \varepsilon(r) dr \quad (13)$$

Energy lost

Energy generated

The Equations of Stellar Structure

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$ **hydrostatic equilibrium**

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$ **mass equation**

$$P(r) = \frac{k\rho(r)T(r)}{m_0 \bar{\mu}(r)}$$ **equation of state**

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho(r)\varepsilon(r)$$ **energy equation**

$$L(r) = \frac{-64\pi\sigma r^2 T^3(r)}{3\kappa(r)\rho(r)} \frac{dT(r)}{dr}$$ **energy transport (radiative)**

- **There are no completely satisfactory, realistic, analytic solutions to these equations**
- **To solve them numerically, we need to specify values of parameters at known positions in the star (ie boundary conditions)**

- **At the centre (r=0):** **M(r) = 0, L(r) = 0**
- **At the surface (r = R):** **M(R) = M*, L(R) = L*, T(R) = T_{eff}, P(r) → 0, ρ (r) → 0**

- Such numerical solutions provide stellar models, which give values of P,T, ρ etc throughout the star, and also the total luminosity L and radius R for a given mass M and chemical composition $\bar{\mu}$.
- $\bar{\mu}$ is important because it influences P(r), κ(r) and ρ(r)
- (note the possible effects of mixing by convection)

Gravitational contraction -- how long can it last?

- From (5) $$\frac{dP}{dM} = -\frac{GM(r)}{4\pi r^4}$$

i.e. $$4\pi r^3 dP = -\frac{GM(r)}{r} dM \quad (14)$$

Integrate over the whole star with volume V = (4 / 3) πr ^{3}

$$3\int_{P_c}^{P_r} V dP = -\int_0^{M_s} \frac{GM}{r} dM = \Omega \quad (15)$$

where Ω is the gravitational potential energy of the whole star (NB Ω < 0 !)

- We can integrate the LHS by parts to get

$$3\int_{P_c}^{P_r} V\,dP = 3[PV]_c^s - 3\int_0^{V_s} P\,dV$$

Since V = 0 at r = 0 and dV = dM / ρ equation (15) becomes

$$4\pi r_s^3 P_s = 3\int\left(\frac{P}{\rho}\right)dM + \Omega$$

But $P_s \ll P_c$ so that

$$3\int\left(\frac{P}{\rho}\right)dM + \Omega = 0 \qquad \text{Virial Theorem} \qquad (16)$$

Making sense of the Virial Theorem

- In a perfect gas, total thermal energy / unit vol is

$$u^* = n n_f kT/2$$

where n = total no. of particles

n_f = no. of degrees of freedom for each particle

n_f is related to the ratio of specific heats:

$$\gamma = \frac{n_f + 2}{n_f} \Rightarrow n_f = \frac{2}{\gamma - 1}$$

For a perfect monatomic gas, $n_f = 3$, $\gamma = 5/3$

- With P = nkT per unit volume, we may write u* as

$$u^* = \frac{nkT}{2}\frac{2}{\gamma - 1} = \frac{P}{\gamma - 1}$$

and the thermal energy per unit mass as

$$u = \frac{P}{\rho(\gamma - 1)}$$

- Hence rewrite the Virial Theorem as

$$3(\gamma - 1)U + \Omega = 0 \qquad (17)$$

where U is the total thermal energy of the star (for a perfect gas with no radiation pressure).

For $\gamma = 5/3$ $2U + \Omega = 0 \Rightarrow -\Omega = 2U$ (18)

(-ve) gravitational potential energy = 2 x thermal energy

NB: Total energy $E < 0$ since

$$E = U + \Omega = -U = \Omega / 2.$$

As the star radiates energy into space,

E decreases (becomes more negative)

→ Ω decreases
→ U increases

and the so the star contracts and heats up as it radiates energy.

Thermal time scale τ_{th}

- This is the time over which the thermal energy of a star can supply its luminosity.
- As star contracts, release of gravitational energy → increase of thermal energy.
- Since the thermal energy U and the gravitational potential energy Ω differ only by a factor of 2, we just look at Ω to find τ_{th} .
- The luminosity L is the rate of loss of energy, i.e. $L \sim \Omega / \tau_{th}$.

$$3\int_{P_c}^{P_r} V dP = -\int_0^{M_s} \frac{GM}{r} dM = \Omega$$

Now, from (15), noting that r < r_s everywhere

i.e

$$-\Omega = \int_0^{M_s} \frac{GM}{r} dM > \int_0^{M_s} \frac{GM}{r_s} dM = \frac{GM_s^2}{2r_s}$$

$$\tau_{th} \approx GM_s^2 / RL$$

For the Sun, τ_{th} ~ 3 x 10^7 years. Slow gravitational contraction is not sufficient to explain timescales > 10^9 years , but it is important at some stages of stellar evolution.