THERMONUCLEAR ENERGY

- Only energy source capable of producing observed stellar luminosities for known time scales is thermonuclear fusion -

  the coalescence of light elements to form heavier nuclei, releasing energy in the process.

- How likely is it for two protons to coalesce?

- Average thermal energy of particles

  $E_{th} = 3 kT / 2$

  $= 4 \times 10^{-16}$ J for T ~ 2×10^{7} K (core temp)

- But Coulomb potential between 2 protons separated by $r = 10^{-15}$ m is much bigger...

$$E_{\mathrm{pot}} = \frac{q_1 q_2}{4\pi\varepsilon_0 r} = 2.3 \times 10^{-13}\,\mathrm{J}$$

with

$$q_1 = q_2 = 1.6 \times 10^{-19}\,\mathrm{C}$$

$$\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{J^{-1} m^{-1} C^{2}}$$

There is, however, a finite probability of overcoming this huge potential barrier.

At T ~ 2 x 10^7 K, about 10^{-8} of all protons have sufficient energy. The strong nuclear force takes over to build 2 protons into a nucleus of deuterium.

The term thermonuclear arises because the necessary velocities are of thermal origin.

Basic particle interactions in nuclear reactions

Conservation laws

- mass - energy $E = mc^2 = m_0c^2 / \sqrt{1 - v^2/c^2}$
- total electric charge
- number of particles and antiparticles
- difference between number of leptons and antileptons (eg electrons, neutrinos)
- difference between number of baryons and antibaryons (eg protons, neutrons)

Most common nuclear reactions in stars involve:

(1) Beta decay: exergonic, spontaneous

$$n \rightarrow p + e^{-} + \overline{\upsilon}$$

(weak interaction)

neutron proton electron antineutrino

also inverse beta decay

$$p + e^{-} \rightarrow n + \upsilon$$

neutrino

---

(2) Positron decay: endergonic, requires threshold input energy
(since mass of neutron and positron > mass of proton)

$$p \rightarrow n + e^{+} + \upsilon$$

(weak interaction)

(3) (p, γ) process: proton reacts with nucleus of charge Z and mass A to produce a more massive particle (A+1) with charge (Z+1):

$$^{A}_{Z}E + p \rightarrow {}^{A+1}_{Z+1}E + \gamma$$

$$e.g. \quad {}^{12}_{6}C + {}^{1}_{1}H \rightarrow {}^{13}_{7}N + \gamma$$

(4) (α, γ) process: similar to (p, γ) process but with α - particle (^{4}He nucleus) added to nucleus

also: (γ, α) process where photon liberates an α - particle

Important in nuclei with even numbers of p and n - very stable nuclei

(5) (n, γ) and (γ, n) processes: as above with neutrons

The first nuclear reaction in stars

- These occur when core temp T ~ 10^6 K (this is not a critical temperature, rather the temp at which reactions proceed at a certain rate).
- The first reactions involve elements like ^{2}D, ^{6,7}Li at T ~ n x 10^6 K with $1 \leq n < 10$
- These elements are readily destroyed, even from the surfaces of stars (since stars are fully convective at the early stages).

HYDROGEN NUCLEAR REACTIONS

1. PROTON - PROTON CHAIN

(a) ${}^1_1\mathrm{H} + {}^1_1\mathrm{H} \rightarrow {}^2_1\mathrm{D} + e^+ + \upsilon$ $\quad 14 \times 10^9\,\mathrm{yrs}$ $\quad 1.44\ \mathrm{MeV}$

(positron decay)

(b) ${}^2_1\mathrm{D} + {}^1_1\mathrm{H} \rightarrow {}^3_2\mathrm{He} + \gamma$ $\quad 6\ \mathrm{sec}$ $\quad 5.49\ \mathrm{MeV}$

(p, γ)

(c) ${}^3_2\mathrm{He} + {}^3_2\mathrm{He} \rightarrow {}^4_2\mathrm{He} + 2\,{}^1_1\mathrm{H}$ $\quad 10^6\,\mathrm{yrs}$ $\quad 12.85\ \mathrm{MeV}$

Steps (a) and (b) repeated before (c) can occur. Reaction rate $\varepsilon \sim T^4$. Dominant in Sun, G, K stars. ε ~26MeV

---

2. CARBON - NITROGEN - OXYGEN (CNO) CYCLE

(a) ${}^{12}_6\mathrm{C} + {}^1_1\mathrm{H} \rightarrow {}^{13}_7\mathrm{N} + \gamma$ $\quad 5 \times 10^3\,\mathrm{yrs}$ $\quad 1.94\ \mathrm{MeV}$

(p, γ)

(b) ${}^{13}_7\mathrm{N} \rightarrow {}^{13}_6\mathrm{C} + e^+ + \upsilon$ $\quad 15\ \mathrm{min}$ $\quad 2.22\ \mathrm{MeV}$

$(e^+\ \mathrm{decay})$

(c) ${}^{13}_6\mathrm{C} + {}^1_1\mathrm{H} \rightarrow {}^{14}_7\mathrm{N} + \gamma$ $\quad 2 \times 10^3\,\mathrm{yrs}$ $\quad 7.55\ \mathrm{MeV}$

(p, γ)

(d) ${}^{14}_7\mathrm{N} + {}^1_1\mathrm{H} \rightarrow {}^{15}_8\mathrm{O} + \gamma$ $\quad 10^6\,\mathrm{yrs}$ $\quad 7.29\ \mathrm{MeV}$

(p, γ)

(e) ${}^{15}_8\mathrm{O} \rightarrow {}^{15}_7\mathrm{N} + e^+ + \upsilon$ $\quad 3\ \mathrm{min}$ $\quad 2.76\ \mathrm{MeV}$

(f) ${}^{15}_7\mathrm{N} + {}^1_1\mathrm{H} \rightarrow {}^{12}_6\mathrm{C} + {}^4_2\mathrm{He}$ $\quad 30\mathrm{yrs}$ $\quad 4.97\ \mathrm{MeV}$

Reaction rate $\varepsilon \sim T^{15}$; dominant in O, B, A stars, total ε ~ 25 MeV

In both cases, net result is the same

$$4\mathrm{p} + 2e^{-} \rightarrow 1\ {}^{4}_{2}\mathrm{He}\ \text{nucleus},\ 5\gamma + 2\upsilon$$

Since CNO cycle requires ^{12}C as a catalyst, it can only occur in stars with sufficient heavy element abundance

=> ineffective in pop II stars .

Rough calculation of energy conversion:

$4p \rightarrow {}^{4}He$ + energy

i.e. 4 x (1.008 amu) -> 1 x (4.004 amu) + energy

Hence 0.028 amu = 4.6 x 10^{-29} kg

converted (by $E = mc^2$) to an energy of

E ~ 4 x 10^{-12} J ~ 25 MeV (since 1eV = 1.6 x 10^{-19} J)

This result compares well with more detailed summations of the individual reactions, namely 26.3 MeV (pp chain) and 25.0 MeV (CNO cycle). If you sum up the energy released in each part of these reaction chains, the values are ~1MeV greater, but the neutrinos escape with that extra energy.

NB: for each H nucleus, 0.007 amu ~ 0.007 of H is lost as energy.