HYDROGEN - BURNING MAIN SEQUENCE

Russell - Vogt Theorem

- **If .... a star is in hydrostatic and thermal equilibrium, and**

  .... it derives all its energy from nuclear reactions, i.e. if ε = ε (T, P, μ),
- **then ... its structure is completely and uniquely determined by its total mass, and by the distribution of chemical elements throughout its interior.**

We have seen that, from (7) and (10)

$$T_c \propto \frac{\mu M}{R}; \quad P_c \propto \frac{\rho M}{R}; \quad \varepsilon = \varepsilon(T, P, \mu)$$

hence

$$\varepsilon = \varepsilon(M, R, \mu)$$

$$\text{But} \quad \mathrm{R} \propto \mathrm{M}^{\alpha} \quad \text{with} \;\; 0 < \alpha < 1 \quad \Rightarrow$$

$$\varepsilon = \varepsilon(M, \mu)$$

$$\text{and since } \mathrm{L} \propto \mathrm{R}^2 \mathrm{T}_{\mathrm{eff}}^4, \quad \Rightarrow$$

$$\mathrm{L} \propto \mathrm{M}^{\beta} \quad \beta > 0$$

Hence we expect for stars of same initial chemical composition m_i, and same form of energy production, that there will be a unique (M - L) relationship + unique (L - T_{eff}) relationship.

i.e. the MAIN SEQUENCE is the locus of all H - burning stars of similar m_i

(changing m_i changes the position of the m-s)

Observationally, for main-sequence stars:

$$\frac{L}{L_{Sun}} = \left(\frac{M}{M_{Sun}}\right)^{4.0-3.6} \quad \text{and} \quad R \propto M^{0.9-0.6}$$

High -mass stars $M \geq 4M_{Sun}$

High core temperature ∴ CNO cycle dominates - provides highly-concentrated source of energy since the energy generation rate $\varepsilon \sim T^{15}$

$$\frac{d\varepsilon}{\varepsilon} = 15\frac{dT}{T}$$

Hence transport of energy through the CORE region is by CONVECTION - means that core material is well-mixed. Outer envelope is radiative.

Low -mass stars

Lower core temperature ∴ pp-chain dominates with $\varepsilon \sim T^4$. CORE is RADIATIVE; but outer envelope is convective due to high opacity induced by H and He ionization zones.

Limits on mass:

Lower limit ~ 0.07 M_{Sun}: BROWN DWARFS below this value (P_c , T_c) too low for nuclear reactions.

Upper limit ~ 60 - 100 M_{Sun} : EDDINGTON LIMIT due to effects of radiative pressure (also vibrational stability)

Main sequence lifetime

- **Longest interval of star's evolution since energy production by fusion of the most abundant element (H) to He**
- **Remember: each proton converted to He loses 0.007 amu =>**

total mass converted = 0.007 of H available in the core.

- **So - what is the mass of the core?**

- **When all H → He in the core, the core temp is the same in all parts, i.e. it is isothermal.**
- **SCHONBERG & CHANDRASEKHAR showed that the mass of an isothermal region cannot exceed q_{sc} = 10 - 15 % total stellar mass**

Hence total energy available is

$$E = mc^2 = 0.007 X q_{\mathrm{sc}} M c^2$$

(X: fraction of H in total mass M of star)

Energy E is radiated at a rate L (luminosity) for a time t

$$t = E/L = 0.007 X q_{\mathrm{sc}} c^2 \, M/L$$

For the Sun,

$$q_{sc} = 0.1, \;\; X = 0.73, \;\; M_{\mathrm{Sun}} = 2 \times 10^{30}\,\mathrm{kg}, \;\; L_{\mathrm{Sun}} = 4 \times 10^{26}\,\mathrm{W}$$

$$\Rightarrow \mathrm{t}_{\mathrm{MS}} \approx 7 \times 10^9 \,\mathrm{years}$$

In general, we obtain the approximate result that the main-sequence lifetime is the nuclear time scale

$$t_{MS} \approx 7 \times 10^9 \frac{(M / M_{\mathrm{Sun}})}{(L / L_{\mathrm{Sun}})} \quad \mathrm{years}$$

eg for an O5 star:

$$M = 40 M_{\mathrm{Sun}}, \;\; L = 4 \times 10^5 L_{\mathrm{Sun}}, \quad t_{MS} \approx 10^6 \quad \mathrm{years}$$

eg for an M5 star:

$$M = 0.1 M_{\mathrm{Sun}}, \;\; L = 5 \times 10^{-4} L_{\mathrm{Sun}}, \quad t_{\mathrm{MS}} \approx 1.4 \times 10^{12} \quad \mathrm{years}$$