#### 3. Radiation Field Basics II

- Rutten: 2.1, 2.2
- Intensity moments
- Emission, extinction, source function
- Equation of radiation transfer
- Optical depth

## Moments of the Radiation Field

We have defined the moment operator **M** operating on *f*:

$$\mathbf{M}^{(n)}[f] = \frac{1}{2} \int_{-1}^{1} f \,\mu^{n} \,\mathrm{d}\mu$$

The first three moments of the specific intensity are historically named J (zeroth moment), H (first), and K (second):

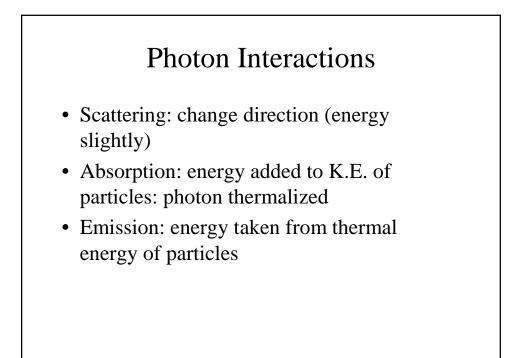
$$J_{\nu} = \frac{1}{4\pi} \int I_{\nu} d\Omega$$
$$H_{\nu} = \frac{1}{4\pi} \int I_{\nu} \cos \theta d\Omega$$
$$K_{\nu} = \frac{1}{4\pi} \int I_{\nu} \cos^{2} \theta d\Omega$$

For a plane parallel atmosphere (no  $\phi$  dependence), we get

$$J_{\nu}(z) = \frac{1}{2} \int_{-1}^{1} I_{\nu}(z,\mu) \,\mathrm{d}\mu$$
$$H_{\nu}(z) = \frac{1}{2} \int_{-1}^{1} I_{\nu}(z,\mu) \,\mu \,\mathrm{d}\mu$$
$$K_{\nu}(z) = \frac{1}{2} \int_{-1}^{1} I_{\nu}(z,\mu) \,\mu^{2} \,\mathrm{d}\mu$$

These moments are used in solving the equation of radiation transfer. As shown previously, physically J and H are related to the mean intensity of radiation and the flux respectively. K is related to the radiation pressure by

$$p_{v} = \frac{4\pi}{c} K_{v}$$



## **Emission Coefficient**

 $j_{v}$  defined by:

 $\mathrm{d}E_{\nu} \equiv j_{\nu}\,\mathrm{d}V\,\mathrm{d}t\,\mathrm{d}\nu\,\mathrm{d}\Omega$ 

Energy  $dE_v$  added to radiation in the volume dV in the frequency range dv in time dt in direction about solid angle  $d\Omega$ . Energy added can be due to stimulated emission, spontaneous emission, thermal emission, or energy scattered into the beam. The intensity contribution to a beam from local emission along a path length ds is:

 $\mathrm{d}I_{\nu}(s) = j_{\nu}(s)\,\mathrm{d}s$ 

#### **Extinction Coefficient**

The energy removed from a beam along a path length ds defines the extinction coefficient. It may be defined per particle, per mass, or per volume

dI (s

$$\mathrm{d}I_{v}(s) = -I_{v}\,\sigma_{v}\,n\,\mathrm{d}s$$

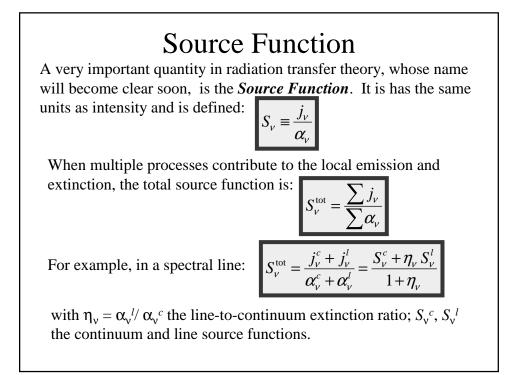
 $\sigma_v$  is the monochromatic extinction coefficient or cross section per particle (m<sup>2</sup>) and *n* the particle density (m<sup>-3</sup>). The definition per path length is

$$\mathrm{d}I_{v}(s) = -I_{v}\,\alpha_{v}\,\mathrm{d}s$$

where  $\alpha_v$  is in units of m<sup>-1</sup>. The definition per mass is

$$\mathrm{d}I_{\nu}(s) = -I_{\nu} \,\kappa_{\nu} \,\rho \,\mathrm{d}s$$

where  $\kappa_{\!\nu}$  has units  $m^2\,kg^{\text{-1}}$  and  $\rho$  is the density (kg m^{\text{-3}}).



# Equation of Radiation Transfer The equation of radiation transfer (ERT) *along a ray* is: $dI_{\nu}(s) = I_{\nu}(s+ds) - I_{\nu}(s) = j_{\nu}(s) ds - \alpha_{\nu}I_{\nu}(s) ds$ Where *s* is measured along the path in the propagation direction. It can be re-written as follows: $dI_{\nu} = j_{\nu} - \alpha_{\nu}I_{\nu}$ $dI_{\nu} = S_{\nu} - I_{\nu}$ The RHS is just the radiation created (or scattered into the beam), minus radiation destroyed (absorbed or scattered out of the beam). Simply put RHS = sources - sinks. Hence, S is the source function

minus radiation destroyed (absorbed or scattered nus the scaling), minus radiation destroyed (absorbed or scattered out of the beam). Simply put, RHS = *sources* – *sinks*. Hence,  $S_v$  is the source function. ERT says photons do not decay spontaneously so that  $I_v$  along a ray doesn't change unless photons are added or taken from the beam.

