9. Model Atmospheres

- Rutten: 7.1, 7.2
- Assumptions
- Gas law, equation of state
- Hydrostatic equilibrium
- Temperature stratification
- Radiative equilibrium

Model Atmospheres

Problem:

Construct a numerical model of the atmosphere to estimate

(a) Variation of physical variables (T, P) with depth

(b) Emergent spectrum in continuum and lines

Compare calculated spectrum with the real star and revise model until agreement is satisfactory.

Calibrations:

A grid of models varying T_{eff}, g, and chemical composition can predict classification parameters such as colour indices and metal abundance indicators, and a main sequence of g vs T_{eff} to compare with interior models.

Model Atmosphere Assumptions

- Homogeneous plane parallel layers
- Hydrostatic equilibrium
- Time independent
- Radiative Equilibrium
- Local Thermodynamic Equilibrium

Plane Parallel:

“Thickness” of photosphere << R_*
g constant throughout photosphere
Adequate for main sequence stars, not for extended envelopes, e.g., supergiants

Homogeneous:

Physical quantities vary with depth only
Reduces problem to one dimension
Ignores sunspots, starspots, granulation,…

Hydrostatic Equilibrium:

Assume pressure stratification is such that balances g
Ignores all movement of matter

Steady State:

Properties do not change with time
ERT assumed independent of time
Energy level populations constant: detailed balance or statistical equilibrium
Neglects: rotation, pulsation, expanding envelopes, winds, shocks, variable magnetic fields, etc

Radiative Equilibrium:

All energy transport by radiative processes
Neglects transport by convection
Neglects hydrodynamic effects
There is much evidence for the existence of velocity fields in the solar atmosphere and mass motions, including convection, are significant in stellar atmospheres, particularly supergiants.
A complete theory must include hydrodynamical effects and show how energy is exchanged between radiative/non-radiative modes of energy transport

Local Thermodynamic Equilibrium (LTE):

All properties of a small volume of material gas are the same as their thermodynamic equilibrium values at the local values of T and P.
A reasonably good approximation for photospheres of stars that are not too hot, or too large, as most of the spectrum is formed at depths where LTE holds.

Gas Law, Equation of State

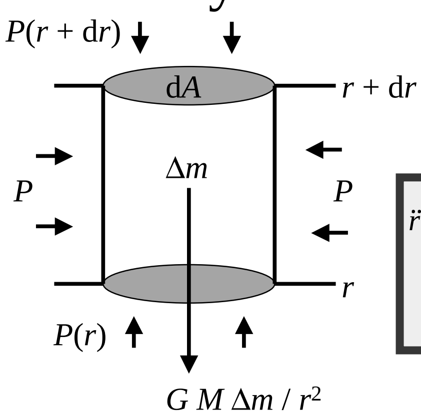
The ideal gas law generally holds in stellar photospheres. The classical version is: $\boxed{P_g V = n_{mole} \mathcal{R} T}$

With n_{mole} the number of moles, the gas constant $\mathcal{R} = kN_A = k/m_H$.
Other versions use total number density $N_g = n_{mole} N_A / V$,
mean "molecular" weight $\mu = m/m_H$, density $\rho = N_g \mu m_H$:

$$\boxed{P_g = \frac{n_{mole} N_A}{V} \frac{\mathcal{R}}{N_A} T = N_g k T = \frac{\rho k T}{\mu m_H} = \frac{\rho \mathcal{R} T}{\mu}}$$

Total pressure is sum over all partial pressures $P_g = \Sigma N_i k T$. Partial electron pressure: $\boxed{P_e = N_e k T}$

Hydrostatic Equilibrium



$$\boxed{\Delta m = \rho \, dr \, dA}$$

$$\boxed{\begin{aligned} \ddot{r} \, \Delta m &= \frac{G M \Delta m}{r^2} + P(r) \, dA - P(r + dr) \, dA \\ &= \frac{G M \Delta m}{r^2} - \frac{\partial P}{\partial r} \frac{\Delta m}{\rho} \end{aligned}}$$

Hydrostatic equilibrium: acceleration negligible: $\boxed{\frac{dP}{dr} = -\rho \frac{G M}{r^2}}$

Plane parallel: $\boxed{\frac{dP}{dz} = -\rho g}$ Optical depth $d\tau = -\kappa \rho \, dz$: $\boxed{\frac{dP}{d\tau} = \frac{g}{\kappa}}$

Simplest case of isothermal atmosphere with constant *m* and only gas pressure gives:

$$\boxed{\frac{\mathrm{d}P}{\mathrm{d}z} = -\frac{\mu g}{\mathcal{R}T} P_{\mathrm{g}} = -\frac{P_{\mathrm{g}}}{H_{\mathrm{P}}}}$$

Where the pressure scaleheight is $H_{\mathrm{P}} = \mathcal{R}T/\mu g$. The solution is the standard barometric exponential decay law:

$$\boxed{P_{\mathrm{g}}(z) = P_{\mathrm{g}}(0)\exp(-z/H_{\mathrm{P}})}$$

Scaleheight indicates extent of atmosphere: spectra come from layers spanning a few H_{P}. For sun $H_{\mathrm{P}} \sim 150$ km, all photosphere ~ 500 km. Plane-parallel holds if $H_{\mathrm{P}} / R_* << 1$. Holds for all except largest supergiants. Note this test does not say anything about horizontal inhomogeneities, and other effects that can mess up plane-parallel approximation.

Radiative Equilibrium

From the Gray Atmosphere, the radiative equilibrium condition was that the flux was constant throughout the atmosphere: $\mathrm{d}F / \mathrm{d}\tau = 0$. Including frequency dependence we get a more rigorous condition:

$$\boxed{\frac{\mathrm{d}\mathcal{F}_{rad}}{\mathrm{d}z} = 4\pi \int_0^\infty \alpha_\nu (S_\nu - J_\nu)\,\mathrm{d}\nu = 0}$$

This says that all emitted energy must equal all extincted energy. It can also be written:

$$\boxed{\int_0^\infty \alpha_\nu S_\nu\,\mathrm{d}\nu = \int_0^\infty \alpha_\nu J_\nu\,\mathrm{d}\nu}$$

In LTE, $S_\nu = B_\nu(T[\mathrm{r}])$, so the above is solved to determine the temperature structure of an atmosphere.

Convection?

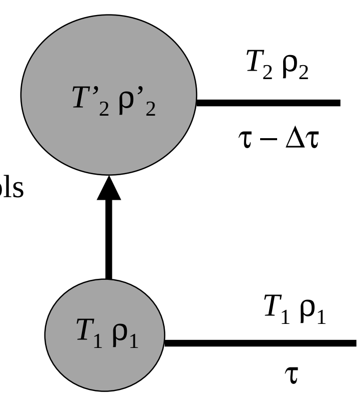
- Classical model assumes radiative equilibrium
- If temperature gradient too large, convective currents can set in e.g., granulation on solar surface, material rising and falling due to convection
- Test model for stability against convection…

Condition established by K. Schwarzschild (1906)

Displace upward gas bubble with T_1, ρ_1
Gas stays in hydrostatic equilibrium
Bubble stays at same pressure as surroundings
Pressure is less, bubble expands and cools

No energy exchange: adiabatic process
Get T_2', ρ_2' from adiabatic gas law:

$$P\,V^{\gamma} = \text{constant}$$



If $\rho_2' > \rho_2$ because gas has cooled below surroundings during expansion, bubble sinks back and atmosphere is stable against convection

If bubble remains warmer despite adiabatic cooling, $\rho_2' < \rho_2$ and it continues to rise, so convection sets in

No convection if $T_2 < T_2$; $T_1 - T_2' < T_1 - T_2$

Adiabatic gradient > radiative gradient: $\Delta T_{\mathrm{adiabatic}} > \Delta T_{\mathrm{radiative}}$

Compute model assuming radiative flux and then check for consistency…

$V = R\,T\,/\,P$ + adiabatic gas law => $\boxed{T^{\gamma}\,P^{\gamma} = \mathrm{constant}}$

$\gamma = C_p\,/\,C_v$ = ratio of specific heats = 5/3 for monatomic gas

$$\boxed{\left[\frac{\mathrm{d}T}{\mathrm{d}\tau}\right]_{ad} = \frac{\gamma-1}{\gamma}\frac{T}{P}\frac{\mathrm{d}P}{\mathrm{d}\tau}}$$

From Eddington solution: $\boxed{\frac{1}{T}\left[\frac{\mathrm{d}T}{\mathrm{d}\tau}\right]_{rad} = \frac{3}{8+12\tau}}$

No convection if: $\boxed{\frac{3}{8+12\tau} < \frac{\gamma-1}{\gamma}\frac{g}{P\,\kappa}}$

P_g and κ are functions of τ…

Increase of κ with depth causes convection

Also γ can change with dissociation

As H begins to ionize, γ drops below 5/3 and doesn’t reach this value again until ionization is complete

Important in A stars where H ionization zone not far beneath surface