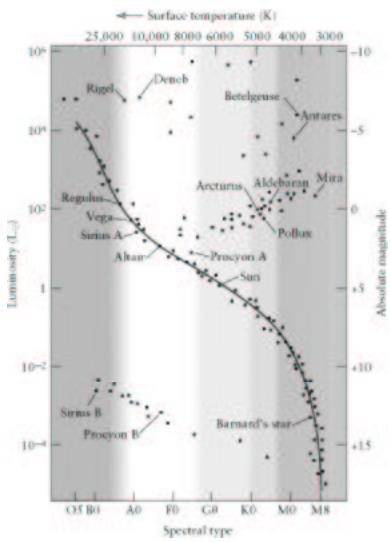
1.1 Motivation

- **Observational: How do we explain stellar properties as demonstrated, e.g. by the H-R diagram?**
- **Theoretical: How does an isolated, self-gravitating body of gas behave?**
- **Aims:**
  - **Identify and understand physical processes**
  - **Make simple models**
  - **Understand fundamental properties of stars**
  - **Learn how stars form and evolve**

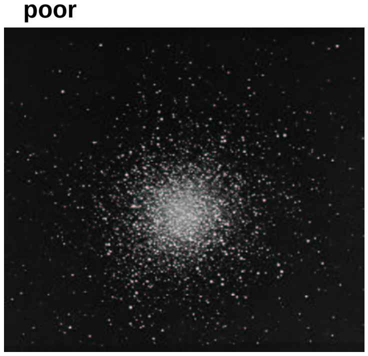
1.2 The H-R diagram

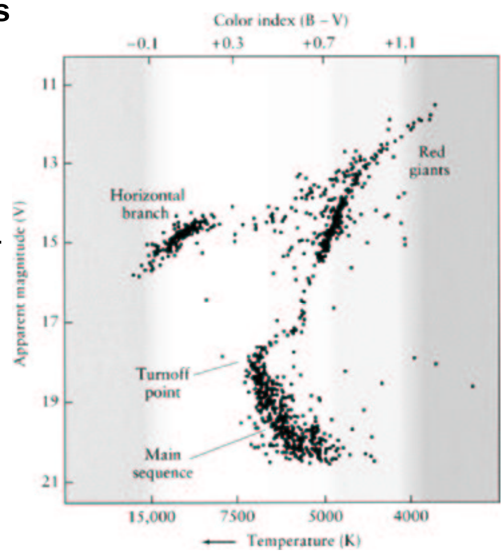
- **Solar neighbourhood**
  - **stars of known distance.**
  - **Main sequence, giants, supergiants, white dwarfs**
  - **All masses and evolutionary stages present**



Cluster H-R diagrams

- **Open and globular clusters**
  - **“Snapshots” at a single age**
  - **Assume all stars formed at same time**
  - **All stars at same distance**
  - **Open clusters: disc population, young, solar-like metallicity**
  - **Globulars: Halo pop, old, metal-poor**





Typical globular-cluster HR diagram.

1.3 Temperature-luminosity relation

- **Most important empirical relation between stellar properties.**
- **Theoretical H-R diagram: plot**

$$\log\left(\frac{L}{L_{Sun}}\right) \text{ against } \log\left(\frac{T_{eff}}{T_{eff,Sun}}\right)$$

- **Main sequence shows luminosity correlated with effective temperature:**

$$\frac{L}{L_{Sun}} = \left(\frac{T_{eff}}{T_{eff,Sun}}\right)^{\alpha} \text{ where on average } \alpha = 6.67.$$

1. 4 Mass - luminosity relation

- **From masses of many binary stars, find for m-s stars:**

$$\frac{L}{L_{Sun}} = \left(\frac{M}{M_{Sun}}\right)^{4.0\pm0.02} \quad \text{for} \quad 0.4 < M < 10M_{Sun}$$

$$\frac{L}{L_{Sun}} = \left(\frac{M}{M_{Sun}}\right)^{3.6\pm0.1} \quad \text{for} \quad 5 \le M \le 40M_{Sun}$$

- **Theory of stellar structure must reproduce this!**

1.6 Definitions

- **Symbols & units commonly used for stellar parameters :**
- **Stellar mass (M) -- solar units M/M_{Sun}**
- **Stellar radius (R) -- solar units R/R_{Sun}**
- **Surface gravity ($g=GM/R^2$) -- ms^{-2}**
- **Effective temperature (T_{eff}) -- K**
- **Stellar luminosity (L) -- solar units L/L_{Sun}**
- **Composition (X, Y, Z) -- mass fractions of H (X), He (Y) and other elements (Z).**
- **Stellar age (t) -- years**

Example: The Sun

$$M = 1\mathrm{M_{Sun}} = 1.99 \times 10^{30}\ \mathrm{kg}$$
$$R = 1\mathrm{R_{Sun}} = 6.96 \times 10^{8}\ \mathrm{m}$$
$$g = 2.74 \times 10^{2}\ \mathrm{m\ s^{-2}}$$
$$T_{\mathrm{eff}} = 5780\ \mathrm{K}$$
$$L = 1\mathrm{L_{Sun}} = 3.86 \times 10^{26}\ \mathrm{W}$$
$$\mathrm{X = 0.71,\ Y = 0.265, Z = 0.025}$$
$$\mathrm{t} \cong 4.6 \times 10^{9}\ \mathrm{y}$$

1.7a Dynamical (free-fall) timescale

- **Ball of gas supported by pressure.**
- **How long to collapse if support removed?**
- **Spherical shell, initially at rest with radius R_0 encloses mass M.**
- **After collapse to radius r, inward velocity conserves PE+KE:**

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 = \frac{GM}{r} - \frac{GM}{R_0}$$

- **Hence** $$t_{\mathrm{ff}} = \int_{R_0}^{0} \frac{dt}{dr} dr = -\int_{R_0}^{0} \left(\frac{2GM}{r} - \frac{2GM}{R_0}\right)^{-1/2} dr$$

Choose negative root to ensure positive answer!

Example: Solar free-fall time

Define $x = r / R_0$:

$$t_{\text{ff}} = \left(\frac{R_0^3}{2GM}\right)^{1/2} \int_0^1 \left(\frac{x}{1-x}\right)^{1/2} dx = \frac{\pi}{2}\left(\frac{R_0^3}{2GM}\right)^{1/2},$$

(use substitution $x = \sin^2\theta$ to evaluate integral).

- **cf. period of particle orbiting at R_0.**
- **Numerical value:**

$$t_{\text{ff}} = 1770\left(\frac{R_0}{\mathrm{R}_{\text{Sun}}}\right)^{3/2}\left(\frac{\mathrm{M}_{\text{Sun}}}{M}\right)^{1/2} \quad \text{s.}$$

1.7b Thermal (Kelvin) timescale

- **Treat star as a reservoir of heat energy U.**
- **Length of time to radiate this energy away:**

$$\tau_{th} \approx U / L$$

- **Virial theorem relates U to star's gravitational binding energy Ω :**

$$2U + \Omega = 0 \Rightarrow -\Omega = 2U$$

- **We'll see later that for a star:**

$$\Omega = -q\frac{GM^2}{R}$$

- **where q is a dimensionless constant of order 1, so:**

$$\tau_{th} = \frac{q}{2}\frac{GM^2}{LR} \approx 3\times10^7\left(\frac{M}{M_{\text{Sun}}}\right)^2\left(\frac{L_{\text{Sun}}R_{\text{Sun}}}{LR}\right) \quad \text{y.}$$

1.7c Nuclear timescale

- **Radioisotope dating => Earth rocks much older than K-H "age" for Sun (oops...!)**
- **Energy available from thermonuclear fusion:**

$$E = mc^2 = 0.007 q_{\mathrm{sc}} X M c^2$$

Mass fraction converted **Schönberg-Chandrasekhar**

- **Nuclear timescale = time to radiate this amount of energy at main-sequence luminosity L:**

$$t_{\mathrm{MS}} = E/L = 0.007 q_{sc} X c^2\, M/L$$
$$\approx 7\times 10^9 \frac{q_{\mathrm{sc}} M}{(q_{\mathrm{sc}} M)_{\mathrm{Sun}}} \Big/ \frac{\mathrm{L}}{L_{\mathrm{Sun}}} \quad \mathrm{y.}$$

Diffusion time

- **If Sun were transparent, γ-rays generated in core would emerge in 2.3s with no energy loss.**
- **Actual escape time is much longer due to multiple absorptions & re-emissions.**
- **Total radiant energy content:**

$$U_{\mathrm{r}} = u_{\mathrm{r}} V = a\overline{T}^4 . \frac{4}{3}\pi R^3$$

- **Crude approximation:** $\overline{T}_{\mathrm{Sun}} \approx T_{\mathrm{c,Sun}}/2 = 7\times 10^6$ K.
- **So time for a photon to diffuse out is roughly:**

$$t_{\mathrm{diff}} = U_{\mathrm{r}}/L \approx 2\times 10^5 \left(\frac{R}{\mathrm{R}_{\mathrm{Sun}}}\right)^3 \left(\frac{T_{\mathrm{c}}}{T_{\mathrm{c,Sun}}}\right)^4 \left(\frac{\mathrm{L}_{\mathrm{Sun}}}{L}\right) \quad \mathrm{y.}$$

2. Equations of stellar structure

- **Object: find a set of n equations in n variables that describe the structure of a star.**
- **Augment them with n boundary conditions.**
- **We'll start with the basics like making sure that mass, momentum and energy are conserved.**

2.1 Mass continuity

- **Gotta conserve mass!**
- **By considering a shell of radius r in a region of local density ρ(r), we found in AS2001 that:**

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

- **This introduces 2 unknowns:**
  - **M (r) = mass enclosed within sphere radius r**
  - **ρ (r) = local gas density at radius r.**
- **So we need at least 1 more equation!**

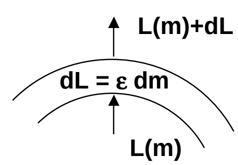
2.2 Hydrostatic equilibrium

- Otherwise known as force balance or conservation of momentum!
- Main forces operating are self-gravity and internal pressure.
- In AS 2001, considered force balance across a thin shell to get:

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

- So now we have a third unknown:
  - P(r) = local pressure at radius r.
- But still only 2 equations so we need another!

2.3 Energy Conservation I: sources & sinks



- Energy moves through the star.
- Consider a shell of mass:

$$dm = \rho dV = 4\pi r^2 \rho dr$$

- Energy flows into bottom of shell at rate L(m)
- Energy leaves top of shell at rate L(m)+dL
- Denote rate at which energy is produced or absorbed within shell (per unit mass) by:

$$\frac{dL}{dm} = \varepsilon, \text{ and so } \frac{dL}{dr} = 4\pi r^2 \rho \varepsilon$$

- That's one more equation but 2 extra unknowns:
  - L(r) = local luminosity at radius r.
  - ε (r) = energy generation per unit mass at radius r.

Energy sources and sinks

- **We'll consider these later. They include:**
- **Nuclear, e.g.** $4p \rightarrow \alpha + 26.72\ \mathrm{MeV}$
- **Neutrino, e.g.** $\gamma + e^- \rightarrow e^- + \gamma$

  or $\gamma + e^- \rightarrow e^- + \nu + \bar{\nu}$
- **Nonadiabatic expansion, i.e.**

$$dQ = TdS = dE + PdV$$

- **Note that these imply a relationship between T, P and energy generation rate, i.e. another equation.**