4. Star formation

• Galactic ISM: roughly uniform gas, $n \sim 5 \times 10^6 \ m^{-3}$.
• Stars form in molecular clouds.
• Dimensions $\sim 10 \text{pc}$, density $\sim 5 \times 10^8 \ m^{-3}$, temperature $\sim 10 \text{K}$.
• Galactic magnetic field strongly tied to ionized plasma in ISM.
• Field lines run parallel to galactic plane.
• Local perturbations $\rightarrow$ potential wells $\rightarrow$ condensations.

4.1 Jeans’ criterion

• Contracting cloud must be compact enough to ensure that dispersive effects of internal pressure don’t overwhelm gravity.
• Energetically, cloud becomes bound if:
  \[ E_{\text{grav}} + E_{\text{kin}} < 0. \]
• Spherical cloud, mass $M$, radius $R$ has gravitational binding energy:
  \[ E_{\text{grav}} = \frac{GM}{r} \ dm = -A \frac{GM^2}{R}, \]
  where $A$ depends on (and increases with) degree of central condensation of internal density distribution.
• $A=3/5$ for uniform density; we’ll use $A=1$. 

Conditions for collapse

• Total thermal (kinetic) energy of cloud is:

\[ E_{\text{kin}} = \frac{3}{2} \frac{M}{\mu m_{\text{H}}} kT \]

• Critical condition: for collapse of a cloud with radius \( R \) to occur, need either:

\[ M_{\text{cloud}} > M_{J} = \frac{3kT}{2G\mu m_{\text{H}}} R \]

• or

\[ \rho_{\text{cloud}} > \rho_{J} = \frac{3}{4\pi M^2} \left[ \frac{3kT}{2G\mu m_{\text{H}}} \right]^3 \]

4.2 Onset of contraction

• Contraction of a massive gas-dust cloud will proceed if not opposed by increasing internal pressure.

• Release of \( E_{\text{grav}} \) tends to increase internal temperature but also excites \( H_2 \) and other molecules into excited rotational levels.

• De-excitation emits photons mainly at IR and mm-wave frequencies where cloud is transparent.

• Hence photons escape, cooling the cloud and allowing contraction to proceed.
4.3 Fragmentation and formation of protostars

- As main cloud contracts, smaller subregions will also reach the Jeans density
- Fragments will contract independently provided gravitational PE is not converted to internal KE.
- Energy released can be absorbed by:
  - Dissociation of H₂ (ε_D = 4.5 eV)
  - Ionisation of atomic H (ε_I = 13.6 eV)
- Amount of energy absorbed by this process is:
  \[ \frac{M}{2m_H} \varepsilon_D + \frac{M}{m_H} \varepsilon_I. \]

Radius of a protostar

- If we know the initial radius of the protostar we can calculate the final radius from:
  \[ GM \left( \frac{1}{R_2} - \frac{1}{R_1} \right) \approx \frac{1}{m_H} \left( \frac{\varepsilon_D}{2} + \varepsilon_I \right). \]
- Example: Radius of a protostar of 1 solar mass is \( \sim 10^{15} \) m with a Jeans density \( \sim 10^{-16} \) kg m\(^{-3}\).
- The dissociation + ionisation energy is \( 3 \times 10^{39} \) J.
- The radius after gravitational contraction is \( R_2 \sim 10^{11} \) m.
- The timescale for this contraction is \( t_{\text{dyn}} \sim 20,000 \) y.
4.4 Approach to hydrostatic equilibrium

- After H is all ionised:
  - the internal pressure rises
  - contraction slows down
  - hydrostatic equilibrium is approached.
- Can use the Virial theorem to estimate the average internal temperature at this point.
- Total thermal KE of protons and electrons is:
  \[ E_{\text{kin}} \sim \frac{3kT}{2} \frac{M}{\mu m_H} = \frac{3kTM}{m_H}. \]
- Gravitational energy at end of collapse is:
  \[ E_{\text{grav}} \sim -\frac{GM^2}{R_2} \sim -\frac{M}{m_H} \left( \frac{\epsilon_D}{2} + \epsilon_i \right). \]

4.5 Thermal contraction

- Virial theorem: \( 2E_{\text{kin}} + E_{\text{grav}} = 0 \), so protostar approaches equilibrium at an average temperature
  \[ kT \sim \frac{\left( \epsilon_D + 2\epsilon_i \right)}{12} \sim 2.6 \text{ eV}, \]
  - Corresponds to \( T \sim 30,000 \text{ K} \).
  - Independent of mass of protostar.
  - Subsequent contraction governed by opacity, which controls loss of radiation from surface.
  - Hence gravitational energy is radiated away on a thermal (Kelvin) timescale, \( t_k \sim 10^7 \text{ } - \text{ } 10^8 \text{ y} \).
  - Star remains close to hydrostatic equilibrium so we can continue to use Virial theorem.
4.6 How far could contraction proceed without nuclear reactions?

- Classical mechanics breaks down when wavefunctions of neighbouring electrons begin to overlap significantly, i.e. at separation
  \[ r = \lambda_B = \frac{h}{m_e v} \]
  de Broglie wavelength

- Since
  \[ \frac{1}{2} m_e v^2 = \frac{3}{2} kT, \quad r = \frac{h}{(3m_e kT)^{1/2}} \]

  \[ \Rightarrow \rho = \frac{\mu m_H}{(4/3)\pi r^3} \]

  \[ \Rightarrow \rho \approx \frac{\mu m_H (m_e kT)^{3/2}}{h^3}. \]

Minimum mass of a star

- At this point a further increase in density does not affect the temperature.

- Virial theorem gives average internal temperature:
  \[ kT \approx \frac{GM \mu m_H}{3R} = G \mu m_H M^{2/3} \rho^{1/3} \]

- Substitute to get maximum temperature:
  \[ kT_{\text{max}} \approx \left[ \frac{G^2 \mu m_H^{8/3} m_e}{h^2} \right] M^{4/3} \Rightarrow T_{\text{max}} \approx 10^7 \left( \frac{M}{M_{\text{Sun}}} \right)^{4/3} \]

- For \( M < 0.08 \, M_{\text{Sun}} \), \( T \) will not be high enough to trigger nuclear reactions.