Thermonuclear Physics

- **Four fundamental forces:**

Force	*Source*	*Range*	*Nuclear reactions?*
gravitational	**mass**	**$1/r^2$**	**No**
electrostatic	**charge**	**$1/r^2$**	**Yes**
weak nuclear	**baryon-lepton**	**$1/r^w$: w>>2**	**Some**
strong nuclear	**baryon-baryon**	**$1/r^s$: s>>2**	**Yes**

- **Electrostatic repulsion between $^+$vely charged nuclei gives potential:**

$$E_{\text{pot}} = \frac{q_1 q_2}{4\pi\varepsilon_0 r} \approx 2.3\times10^{-13}\ \text{J for protons.}$$

- **Average KE of proton at 2×10^7 K is:**

$$E_{\text{kin}} = \frac{3}{2}kT \approx 4\times10^{-16}\ \text{J}\ .$$

Given a Maxwellian velocity distribution, only ~ 1 proton in 10^8 has enough KE to get over this Coulomb barrier.

Quantum mechanical description

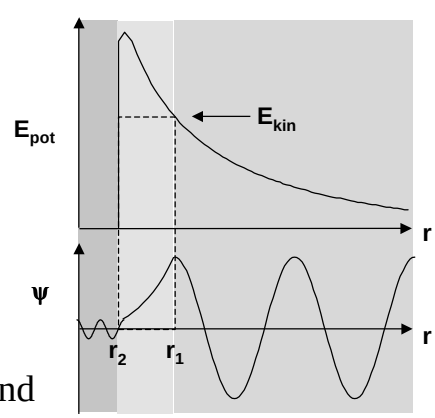
- **Schrödinger equation for proton wavefunction:**

$$\frac{d^2\psi}{dr^2} + \frac{2m}{\hbar^2}(E_{\text{kin}} - E_{\text{pot}})\psi = 0.$$

For $r > r_1$ and $r < r_2$,

$(E_{\text{kin}} - E_{\text{pot}}) > 0 \Rightarrow \psi$ is real.

$r > r_1$:	$\psi \sim \sin kr$
$r_2 < r < r_1$:	$\psi \sim e^{-kr}$
$r < r_2$:	$\psi \sim \sigma \sin kr$



where $k = \frac{2m}{h^2}\left(E_{\text{kin}} - E_{\text{pot}}\right)$ and

$\sigma =$ probability of barrier penetration.

Two things we need to know

- **Reaction rate:**
  - **Related to tunnelling probability and energy distribution of particles.**
  - **Can calculate penetration probability and hence reaction cross-section from Schrödinger eq (see Phillips pp 102-104).**
- **Energy released:**
  - **approximately, the binding energy per nucleon.**
  - **All nuclei consist of Z protons and N neutrons.**
  - **Total rest mass energy of individual particles > rest mass energy of nucleus.**
  - **Deficit represents binding energy of nucleus:**

$$Q(Z,N) = [Zm_{\mathrm{p}} + Nm_{\mathrm{n}} - m(Z,N)]c^2$$

  - **(=net energy released during construction of nuclide.)**
- **Also need to know how much of this energy is "lost" in the form of neutrinos.**

Thermonuclear cross-sections

- **Calculated from**

$$\sigma(E) = \frac{S(E)}{E} e^{-(E_{\mathrm{G}}/E)^{1/2}}$$

where E_{G} is the Gamow energy :

$$E_{\mathrm{G}} = (\pi \alpha Z_i Z_j)^2 \, 2\mu c^2$$

Fine structure constant

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 hc} \approx \frac{1}{137}$$

Reduced mass

$$\mu = \frac{m_i m_j}{m_i + m_j}$$

- **S(E) depends on nuclear physics of the reaction and is in practice very difficult to measure.**

Thermonuclear rate coefficients

- **To convert tunnelling probability to reaction rate r_{ij} (reactions $s^{-1}\,kg^{-1}$):**
  - **average over particle velocities (see, e.g. Phillips p.106) to get temperature dependence:**

$$\langle \sigma v \rangle = \int_0^\infty \sigma v N(v) dv \Big/ \int_0^\infty N(v) dv,$$

e.g. Maxwellian distribution at temperature T

  - **then multiply by number densities:**

$$r_{ij} = n_i n_j \langle \sigma v \rangle / \rho .$$

- **If Q_{ij} is the energy released per reaction ij, total energy released is:**

$$\varepsilon_{ij}(\rho, T, n_i, n_j) = r_{ij} Q_{ij} .$$

Thermonuclear networks

- **Common networks: pp chain, CN(O) cycle.**
- **Reaction rate of a thermonuclear network depends on slowest reaction in network.**
- **Energy released per product nucleon is that for entire cycle:**

$$\varepsilon_{\text{cycle}} = r_{\text{slowest}} Q_{\text{cycle}} .$$

Reactions of the pp chains

$$p + p \rightarrow d + e^{+} + \nu_e$$

1.4x10^{10} y

$$p + d \rightarrow {}^{3}\mathrm{He} + \gamma$$

$${}^{3}\mathrm{He} + {}^{3}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + 2p$$

$${}^{3}\mathrm{He} + {}^{4}\mathrm{He} \rightarrow {}^{7}\mathrm{Be} + \gamma$$

$$e^{-} + {}^{7}\mathrm{Be} \rightarrow {}^{7}\mathrm{Li} + \nu_e$$

$$p + {}^{7}\mathrm{Li} \rightarrow {}^{4}\mathrm{He} + {}^{4}\mathrm{He}$$

$$p + {}^{7}\mathrm{Be} \rightarrow {}^{8}\mathrm{B} + \gamma$$

$${}^{8}\mathrm{B} \rightarrow {}^{8}\mathrm{Be}^{*} + e^{+} + \nu_e$$

$${}^{8}\mathrm{Be}^{*} \rightarrow {}^{4}\mathrm{He} + {}^{4}\mathrm{He}$$

Branch I	Branch II	Branch III
$Q_{\mathrm{eff}} = 26.2$ MeV	$Q_{\mathrm{eff}} = 25.7$ MeV	$Q_{\mathrm{eff}} = 19.1$ MeV
[85%]	[15%]	[0.02%]

Reaction rates for pp chains

- **e.g. slowest reaction in any of the pp chains is the pp reaction itself:**

$$r_{\mathrm{pp}} \sim \frac{\rho}{T_6^{2/3}} X_{\mathrm{H}}^{2} e^{25.44 - 33.81/T_6^{1/3}}. \qquad T_6 \equiv T/10^{6}\ \mathrm{K}$$

- **Energy released depends how far chain goes.**
- **Choice of pp-chain governed mainly by relative abundances of individual species:**
  - **pp II and III dominate when ^{4}He is abundant; deplete ^{7}Li.**
  - **PPIII is rare but produces the most energetic neutrinos.**
- **Approximate reaction rate for pp chain:**

$$\varepsilon_{\mathrm{pp}} \approx 120 \left(\frac{X_{\mathrm{H}}}{0.5}\right)^{2} \left(\frac{\rho}{10^{5}\ \mathrm{m}^{-3}}\right) \left(\frac{T}{15 \times 10^{6}\ \mathrm{K}}\right)^{4}\ \mathrm{Wm}^{-3}.$$

Reactions of the CN cycles

$$p + {}^{12}\mathrm{C} \rightarrow {}^{13}\mathrm{N} + \gamma$$
$${}^{13}\mathrm{N} \rightarrow {}^{13}\mathrm{C} + e^{+} + \nu_e$$
$$p + {}^{13}\mathrm{C} \rightarrow {}^{14}\mathrm{N} + \gamma$$
$$p + {}^{14}\mathrm{N} \rightarrow {}^{15}\mathrm{O} + \gamma$$
$${}^{15}\mathrm{O} \rightarrow {}^{15}\mathrm{N} + e^{+} + \nu_e$$
$$p + {}^{15}\mathrm{N} \rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}$$

CN cycle (most important)

CNO cycle (less important)

3×10^{8} y

$$p + {}^{14}\mathrm{N} \rightarrow {}^{15}\mathrm{O} + \gamma$$
$${}^{15}\mathrm{O} \rightarrow {}^{15}\mathrm{N} + e^{+} + \nu_e$$
$$p + {}^{15}\mathrm{N} \rightarrow {}^{16}\mathrm{O} + \gamma$$
$$p + {}^{16}\mathrm{O} \rightarrow {}^{17}\mathrm{F} + \gamma$$
$${}^{17}\mathrm{F} \rightarrow {}^{17}\mathrm{O} + e^{+} + \nu_e$$
$$p + {}^{17}\mathrm{O} \rightarrow {}^{14}\mathrm{N} + {}^{4}\mathrm{He}$$

Reaction rates for CN cycles

- **Reaction rate is controlled by p+^{14}N $\rightarrow$ ^{15}O +γ:**

$$r_{\mathrm{CN}} \sim \frac{\rho}{T_6^{2/3}} X_{\mathrm{H}} X_{\mathrm{N14}} e^{74.4-152.3/T_6^{1/3}}.$$

- **Rate is cycle dependent since conversion of ^{12}C and ^{16}O to ^{14}N modifies ^{14}N abundance.**
- **High Coulomb barriers: strong T dependence.**
- **Energy produced ~25.02 MeV, independent of cycle.**
- **Abundance of ^{14}N in Sun is 0.6%**
- **Approximate reaction rate for CN cycle:**

$$\varepsilon_{\mathrm{CN}} \approx 2.0\left(\frac{X_{\mathrm{H}}}{0.5}\right)\left(\frac{X_{\mathrm{N14}}}{0.006}\right)\left(\frac{\rho}{10^{5}\ \mathrm{m}^{-3}}\right)\left(\frac{T}{15\times10^{6}\ \mathrm{K}}\right)^{16}\ \mathrm{Wm}^{-3}.$$

anywhere from 13 to 18

Triple-α reactions

- **Once H has been completely converted to He, next most favourable reaction is:**

$$^{4}\mathrm{He} + {}^{4}\mathrm{He} \rightarrow {}^{8}\mathrm{Be} + \gamma \qquad -91.8\ \mathrm{keV\ (endothermic!)}$$

$$^{4}\mathrm{He} + {}^{8}\mathrm{Be} \rightarrow {}^{12}\mathrm{C}^{*} + \gamma \qquad -287.7\ \mathrm{keV\ (\quad "\quad)}$$

$$^{12}\mathrm{C}^{*} \rightarrow {}^{12}\mathrm{C} + \{2\gamma \text{ or } \mathrm{e}^{+} + \mathrm{e}^{-}) \quad +7.65\ \mathrm{MeV}$$

- **High threshold: ^{4}He+^{8}Be fusion needs T>10^{8} K.**
- **Need high energy ^{4}He to collide with ^{8}Be within 10^{-16} s to prevent ^{8}Be decaying again to 2α.**
- **Once formed, most ^{12}C* decay back to ^{8}Be+ ^{4}He, but a few survive to decay via photon cascade or positron-electron pair emission.**
- **To estimate ε, see Phillips pp125-126.**

Neutrinos

- **Produced as electron (or positron) decay capture products.**
- **Capture cross section:** $\sigma_{\nu} \approx 10^{-48} (\varepsilon_{\nu} / \mathrm{MeV})^{2}\ \mathrm{m}^{2}$
- **Hence mean free path:** $\lambda_{\nu} \approx 10^{21} (\varepsilon_{\nu} / \mathrm{MeV})^{-2} / \rho\ \mathrm{m}$
- **i.e. neutrinos escape more or less unimpeded.**
- **Neutrino losses account for only a small fraction of energy produced in pp, CN and 3α.**
- **i.e. little effect on main-sequence or red-giant structure & evolution.**
- **Important in stellar collapse: neutrino flux ~ photon flux during final stages of collapse in SN explosion.**
- **Can get λ_{ν} ~ 25m in supernova core!**