SH ASTRONOMY AND ASTROPHYSICS: AS 3003

Stellar Physics 1: Stellar atmospheres

1. Compute the ratio of ionized to neutral hydrogen in a container of pure hydrogen gas with temperature $T = 10^4$ K and electron pressure $P_e = 30$ Pa. The ionization potential of neutral hydrogen is 13.6 eV, and the partition functions of neutral and ionized hydrogen have values 2 and 1 respectively.

   (note: in exam conditions you will not be expected to derive, but may be expected to know, the Saha equation)

2. An F star has a temperature of 7000 K. Microturbulence in the atmosphere has RMS velocity $\xi_t = 3$ kms^{-1}. Determine the full width at half maximum of an optically thin line of iron with wavelength 400 nm.

3. The transfer equation for radiation emerging from a stellar atmosphere at angle θ to the outward normal is,

   $$\mu \frac{\mathrm{d}I}{\mathrm{d}\tau} = I - S$$

   where $\mu = \cos\theta$.

   If the source function in the Solar photosphere has a linear dependence upon the optical depth τ,

   $$S \simeq a_0 + a_1\tau$$

   show that the specific intensity of light emerging from the photosphere has a linear dependence upon $\cos\theta$.

   If, more generally,

   $$S = \Sigma_{i=0}^{i=n} a_i \tau^i,$$

   then show that,

   $$I(\theta) = \Sigma A_i \cos^i \theta$$

   and derive an expression for the A_i. It may be helpful to know that,

   $$\int_0^\infty x^i e^{-x} dx = i!.$$

   (questions of this general form also occur frequently in exams)

4. Derive the equation of hydrostatic equilibrium in the form used to model a stellar atmosphere.

   If κ is directly proportional to the gas pressure P_g, show how P_g and κ would vary with optical depth τ. For a given optical depth show how the gas pressure varies with gravity. If the temperature distribution is,

   $$T^4 = \frac{3}{4} T_e^4 (\tau + 2/3)$$

then show that the temperature will be constant at small optical depths, but that in deeper layers $T \propto \sqrt{P_g}$.

5. At fixed pressure $P_e = 10$ Pa, calculate the fraction of hydrogen that is H^- as a function of temperature T. Likewise, calculate the fraction of hydrogen in the $n = 3$ level that contributes to absorption in the visible. Sketch these functions.

   If the absorbtion cross-sections for H^- and in the Paschen continuum are similar, estimate the temperature below which H^- is the most important source of continuous absorption.

   (relevant ionization and excitation potentials are in the notes)