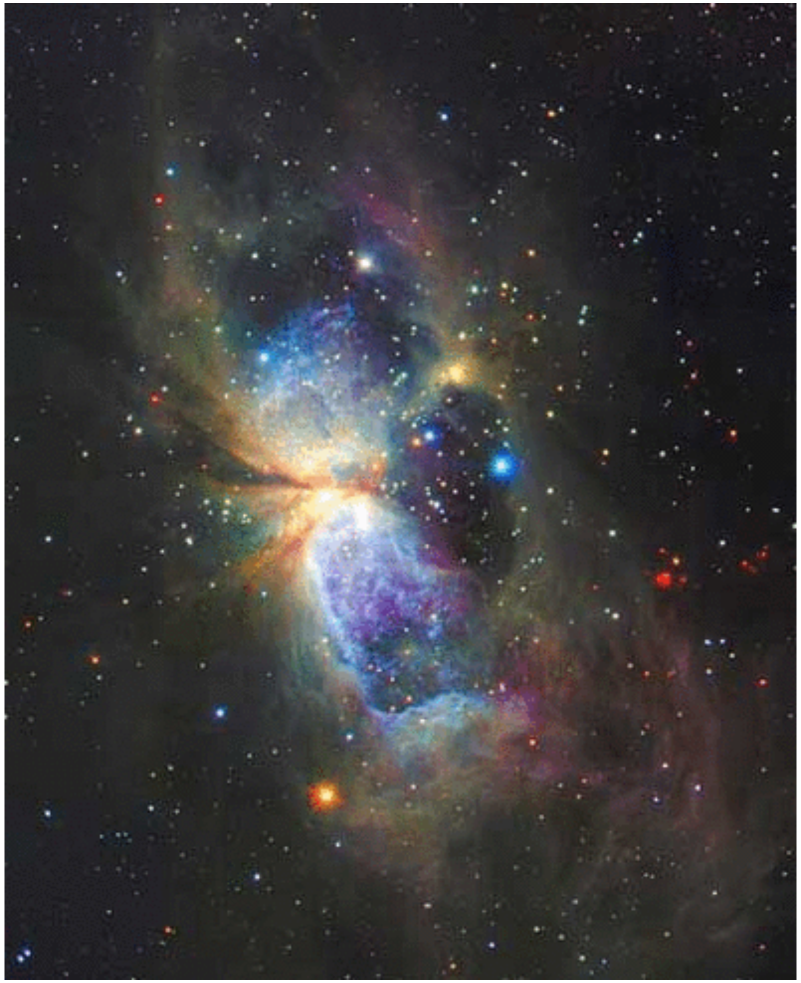
MCRT L12: Photoionization

- Regions of ionized hydrogen in star forming regions and the interstellar medium
- Photoionization and recombination
- Stromgren spheres
- Monte Carlo photoionization

HII Regions

- Massive (hot) stars produce large numbers of ionizing photons (energy above 13.6eV) which ionize hydrogen
- Detailed structure of a nebula depends on density distribution of surrounding gas
- Consider an idealized picture: a star in a uniform medium of pure hydrogen
- Real situation can be much more messy: blue in image is ionized hydrogen



Orion Nebula

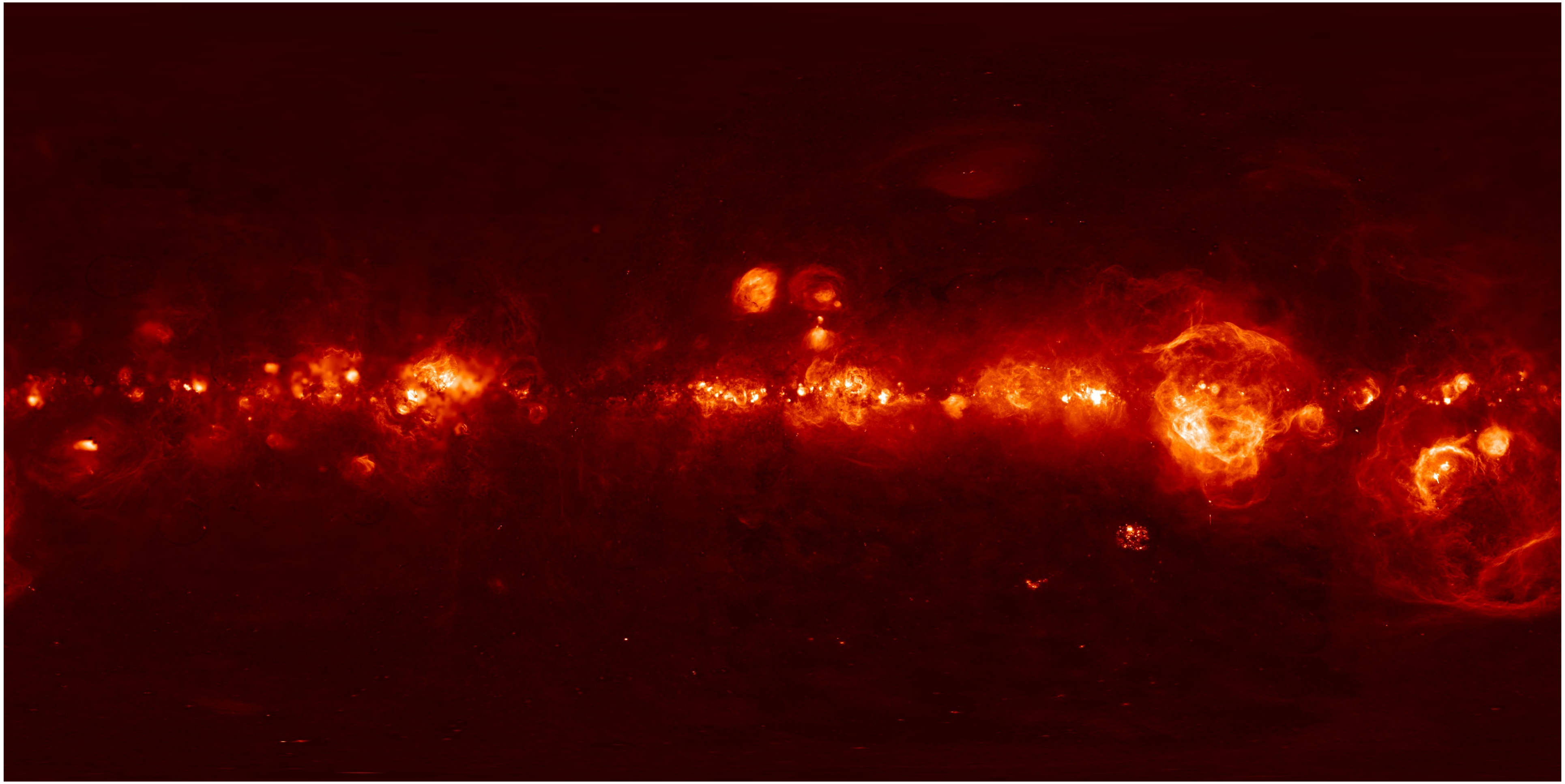
Glowing ionized hydrogen

Hydrogen ionized by photons with
$E > 13.6eV$ or $\lambda < 912A$
$1eV = 1.602E\text{-}19$ J

Four bright O stars emit most of the ionizing photons that produce the Orion Nebula

Running Man and Orion Nebulae, December 2011, Ruben Kier

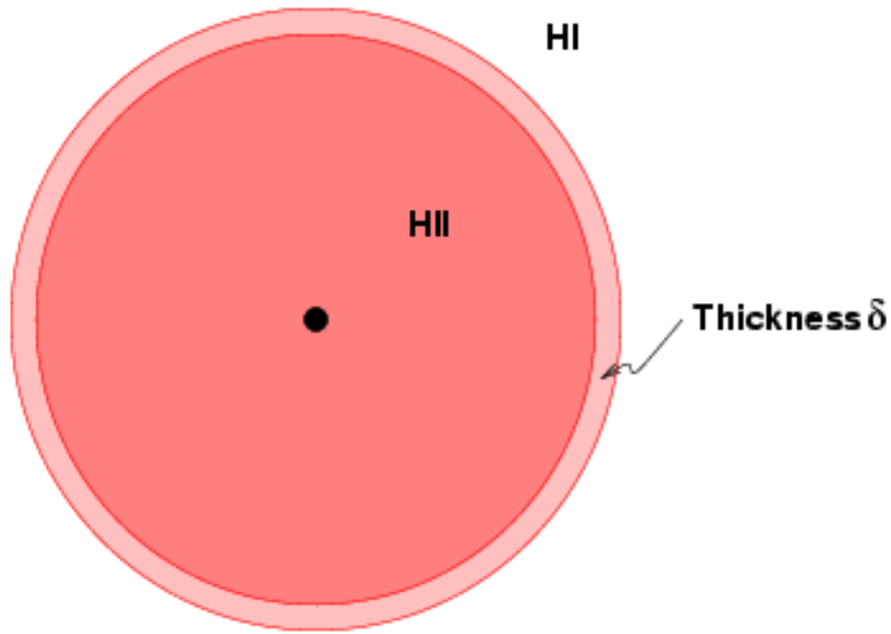
HII Regions & Diffuse Ionized Gas



WHAM: Wisconsin Hα Mapper

Strömgren spheres

- Consider a uniform region of pure hydrogen
- Massive star turns on instantly
- Ionization cross section $\sigma \sim 10^{-21}$ m^2 for neutral H
- If gas density $n_H \sim 10^9$ m^{-3}, mean free path $\sim 10^{12}$ m before ionizing an atom, so ionizing photons cannot escape
- For ionized gas, the much smaller Thomson cross section $\sim 6.7 \times 10^{-29}$ m^2 applies, so stellar photons travel freely through ionized gas to the edge of the neutral material (long mean free path)
- The ionized HII region around the star is called a *Strömgren sphere*
- HII region is separated from the surrounding neutral hydrogen by a thin layer of thickness, $\delta \sim (n_H \sigma)^{-1}$, the mean free path of an ionizing photon



Development of an HII region

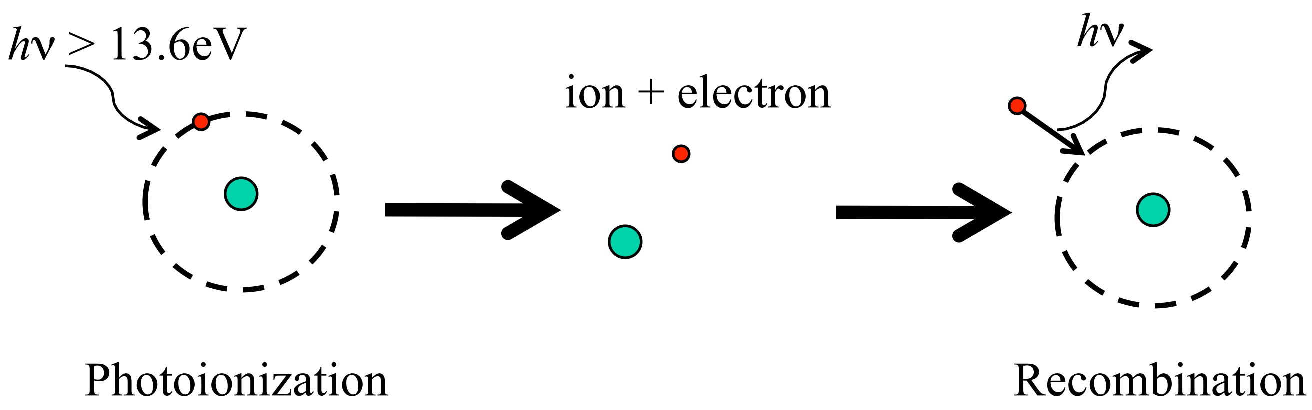
In time dt, the star emits dN_i photons energetic enough to photoionize the gas

atom + photon → ion + electron

These photons either expand the HII region, or compensate for radiative recombinations in the already ionized gas close to the star

ion + electron → atom + photon

i.e. to reionize atoms that have recombined to the ground state



Ionizing photon budget

- To expand the HII region by dR, the star must ionize $4\pi R^2 n_H dR$ atoms.
- In the interior of the region,

$$\frac{4}{3}\pi R^3 n_e n_i \alpha(H^0,T)$$

recombinations occur per unit time. $\alpha(H^0,T)$ is the recombination rate coefficient, n_i and n_e are the number densities of ions and electrons,

- $\alpha(H^0,T)$ is weakly dependent on temperature $\propto T^{-0.5}$
- $\alpha(H^0,8000K) \sim 4 \times 10^{13}$ cm^3 s^{-1}

Expansion rate

The equation for the time development of the HII region is then,

$$\frac{dN_i}{dt} = 4\pi R^2 n_H \frac{dR}{dt} + \frac{4}{3}\pi R^3 n_i n_e \alpha(T)$$

Rearranging,

$$\frac{dR}{dt} = \frac{Q}{4\pi R^2 n_H} - \frac{n_i n_e \alpha(T) R}{3 n_H}$$

where $Q = dN_i/dt$ is the number of ionizing photons emitted per second

Properties of this model

- Volume of HII region is small at first
- Most photons ionize gas at boundary, which expands rapidly
- Expansion speed can be >> sound speed in gas, which has no time to react to the passing ionization front
- As region grows, need more photons to balance recombinations. When all photons are needed, get size of region by setting $dR/dt = 0$:

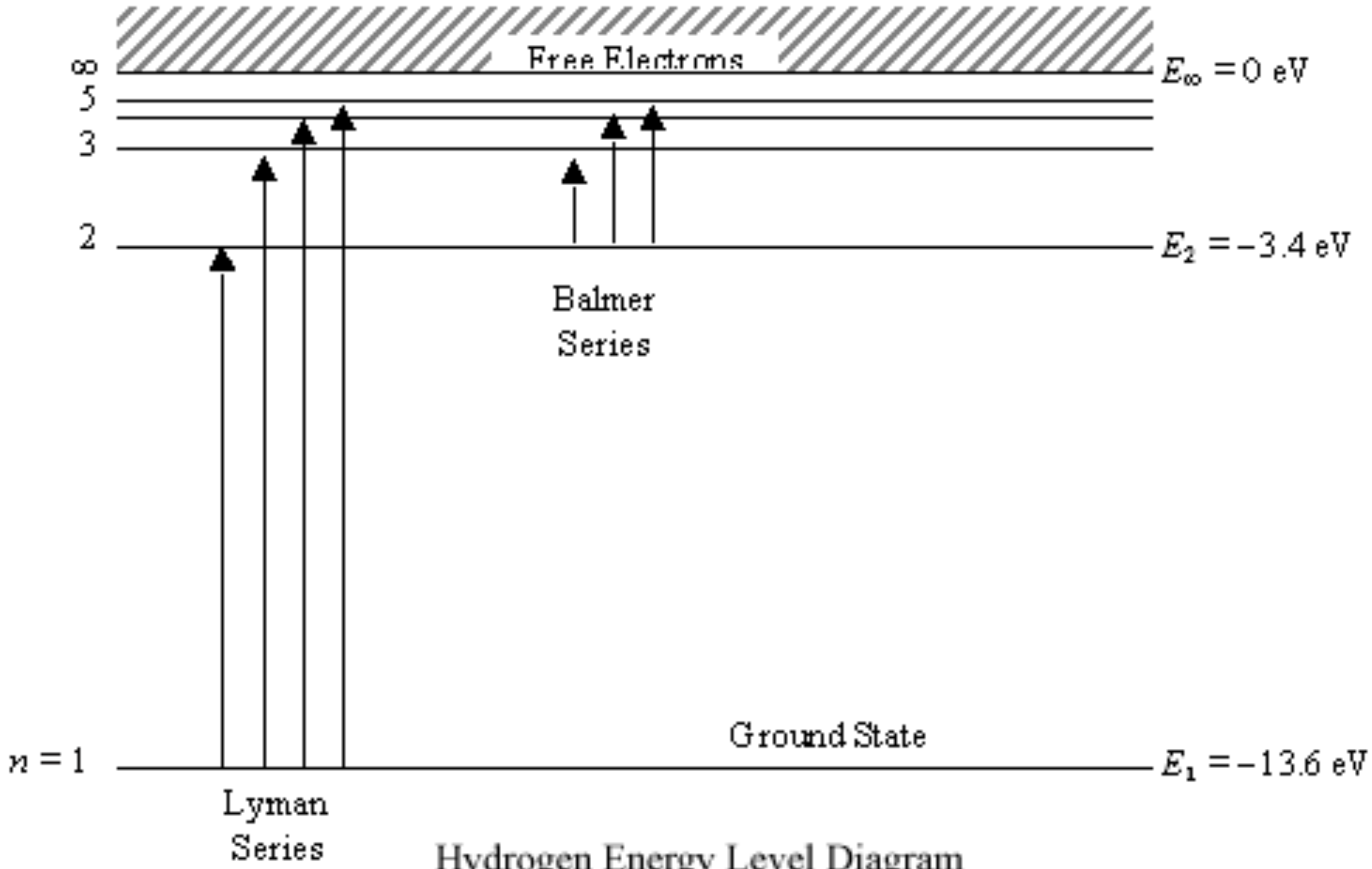
$$R_s^3 = \frac{3Q}{4\pi n_i n_e \alpha(T)}$$

- An O star with $T_{\mathrm{eff}} = 35{,}000$K has $Q \cong 10^{49}\ \mathrm{s}^{-1}$
- Use $\alpha \sim 4 \times 10^{-19}\ \mathrm{m}^3\ \mathrm{s}^{-1}$, and $n_i = n_e = 10^9\ \mathrm{m}^{-3}$, we find $R_s \sim 0.6$ pc
- 1pc = parsec = 3.086×10^{18}cm

Hydrogen in the interstellar medium

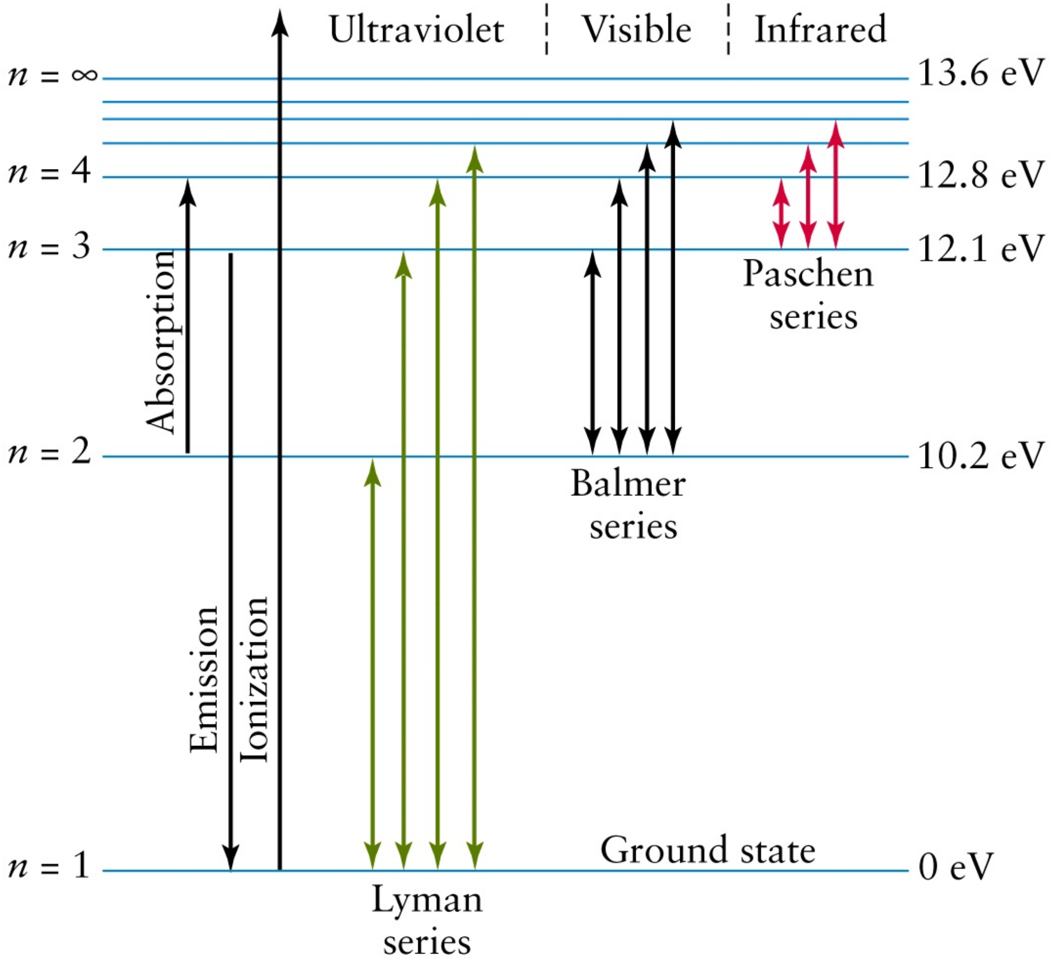
- Photons with $h\nu > 13.6$eV can ionize hydrogen from ground state
- Recombinations of electrons to upper levels, quickly followed by cascade to low levels: H is either ionized or in ground state
- Recombination: as electrons cascade down they emit line radiation which can escape from the nebula
- Recombinations direct to $n = 1$ produce photons with $h\nu > 13.6$eV and can ionize H
- Heating from ionization, cooling from escaping photons

Photoionization of Hydrogen



Hydrogen Energy Level Diagram

Line & continuum emission



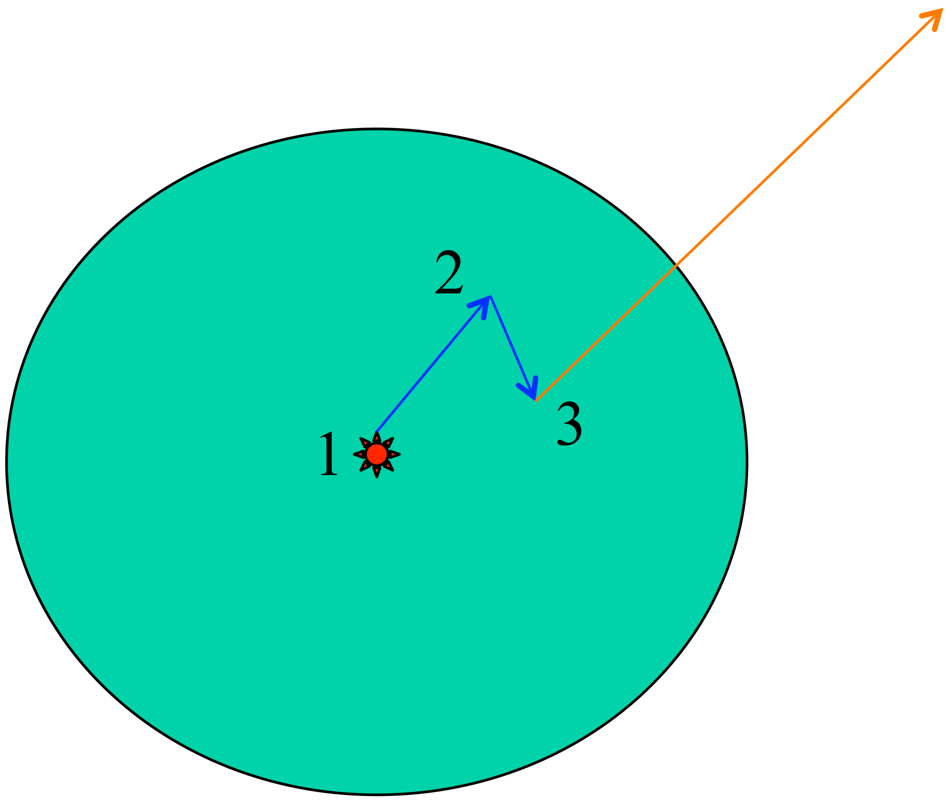
Recombinations of electrons direct to ground, $n = 1$, produce a continuum with $h\nu > 13.6$eV. This diffuse recombination emission can ionize hydrogen.

Assumptions for a simple Monte Carlo code for 3D HII regions

- All stellar photons are emitted at a single frequency
- Spectrum of diffuse ionizing photons is strongly peaked to just above 13.6eV, so assume all diffuse photons are emitted with $h\nu = 13.6$eV
- Temperature of ionized gas is $T \sim 8000$K

Monte Carlo Radiation Transfer

1. Choose frequency of stellar packet and set σ
2. Emit packets isotropically from point source(s)
3. Generate random optical depth
4. Find location where packet interacts with H^0: photoionization event occurs
5. Random test if recombination photon can ionize
6. Repeat 3 – 5 until packet exits simulation
7. Emit new source packet: do all source packets
8. Update ionization structure: new opacity grid
9. Iterate until ionization structure converges



1. Star emits ionizing photon
2. Photoionization, recombination, emission of diffuse ionizing photon
3. Photoionization, recombination, emission of non-ionizing photon

Photon Packets

- Until now have described Monte Carlo codes using photon ENERGY or LUMINOSITY packets:

$$\varepsilon = L\,\Delta t / N$$

- Now use PHOTON packets:

$$\varepsilon = Q\,h\nu\,\Delta t / N$$

- Can easily use atomic probabilities for photon interactions

Isotropic emission and optical depth

Initial direction for random walk:

$$\theta = \cos^{-1}(2\xi - 1)$$

$$\phi = 2\pi\,\xi$$

$P(\tau) =$ exp(-τ): packet travels τ before interaction:

$$\tau = -\log\xi$$

Physical distance, S, that the photon has traveled:

$$\tau_\nu = \int_0^S n\,\sigma_\nu\,\mathrm{d}s \qquad n\sigma_\nu = n(\mathrm{H}^0)\,\sigma_\nu(\mathrm{H}^0)$$

- Packet will be absorbed and photoionize H, immediately followed by recombination and emission of a packet

- Total recombination rate to all levels is α_A, recombination rate to ground level is α_1, recombination rate to excited levels ($n > 1$) is α_B

- Probability of emission as ionizing packet is α_1/α_A, or as non-ionizing is $\alpha_B / \alpha_A = 1-\alpha_1 / \alpha_A$

- At T = 8000K, $\alpha_1/\alpha_A \sim 0.4$

- If recombination packet has $h\nu > 13.6$eV, it can ionize H and random walk continues

- If recombination packet has $h\nu < 13.6$eV, it cannot ionize H, random walk terminated

Ionizing & non ionizing photons

- Lyman continuum, $h\nu > 13.6$ eV

- Probability of ionizing emission: $P_{\text{Ly-c}} = \alpha_1 / \alpha_A$

- Recombination coefficients at 8000K, $P_{\text{Ly-c}} \sim 0.4$

- If $\xi < P_{\text{Ly-c}}$: set cross section for $h\nu = 13.6$eV
  re-emit isotropically
  continue random walk

- If $\xi > P_{\text{Ly-c}}$: $h\nu < 13.6$ eV, non-ionizing line + continuum spectrum, terminate packet as it cannot ionize, nebula optically thin so packet escapes

Photoionization

$$n(\mathrm{H}^0)\int_{\nu_H}^{\infty}\frac{4\pi J_\nu}{h\nu}\sigma_\nu(\mathrm{H}^0)\,\mathrm{d}\nu = n(\mathrm{H}^+)\,n_\mathrm{e}\,\alpha(\mathrm{H}^0,T)$$

\# photoionizations/sec \# recombinations/sec

Need to know mean intensity throughout grid, use pathlengths:

$$4\pi J_\nu\,\mathrm{d}\nu = \sum \varepsilon_\nu\, l/(\Delta t\,\Delta V) \qquad \varepsilon_\nu = Q\,h\nu\,\Delta t/N$$

Sum ($l\,\sigma_\nu$) in each cell, so at end of iteration have integral *I*:

$$I = \int_{\nu_H}^{\infty}\frac{4\pi J_\nu}{h\nu}\sigma_\nu(\mathrm{H}^0)\,\mathrm{d}\nu = Q/(N\,\Delta V)\sum l\,\sigma_\nu(\mathrm{H}^0)$$

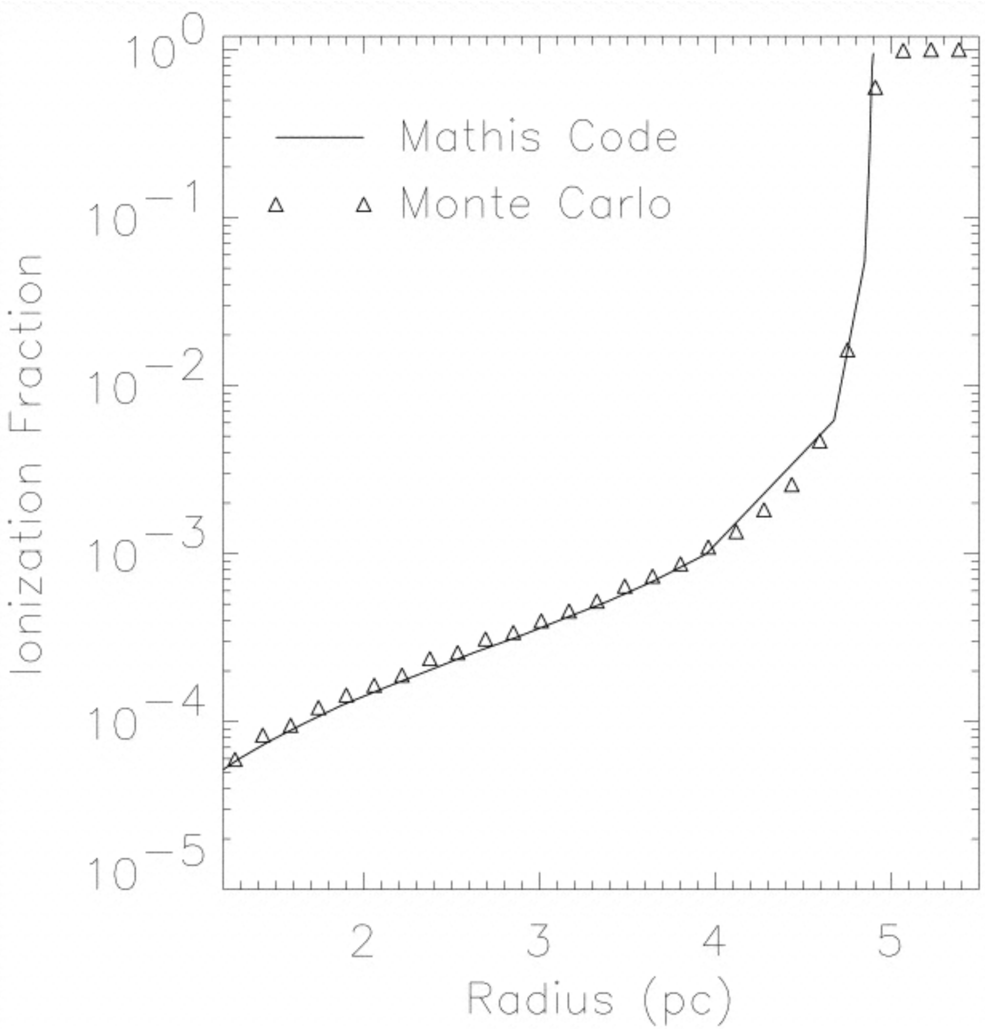
Solve the photoionization equation

- Total number density of hydrogen is $n(\mathrm{H}) = n(\mathrm{H}^+) + n(\mathrm{H}^0)$
- For pure hydrogen gas in ionization equilibrium, $n_\mathrm{e} = n(\mathrm{H}^+)$
- Solve quadratic equation to determine ionization fractions in each cell

$$n(\mathrm{H}^0)\int_{\nu_H}^{\infty}\frac{4\pi J_\nu}{h\nu}\sigma_\nu(\mathrm{H}^0)\,\mathrm{d}\nu = n(\mathrm{H}^+)n_\mathrm{e}\,\alpha(\mathrm{H}^0,T)$$

$$\alpha\, n(\mathrm{H}^+)^2 + I\,n(\mathrm{H}^+) - I\,n(\mathrm{H}) = 0$$

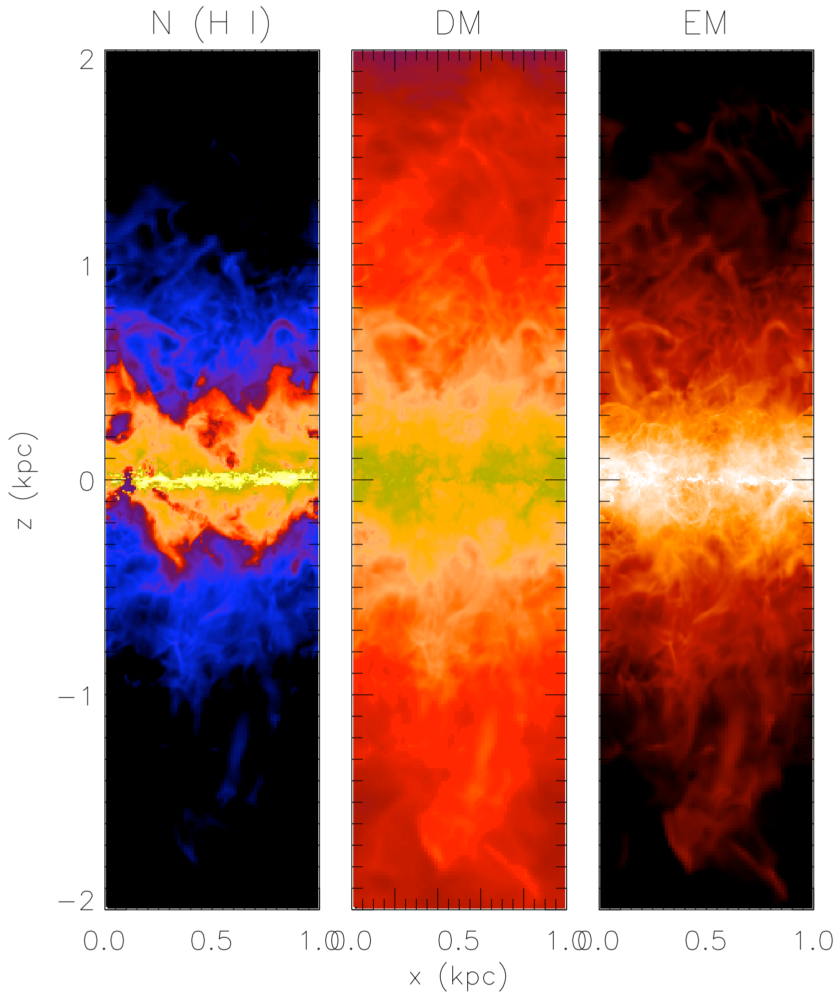
- Update the opacity for next iteration based on ionization fraction
- Regions close to source are ionized, so have low opacity (Thomson scattering), therefore packets for subsequent iterations can travel further before interacting with a neutral hydrogen atom



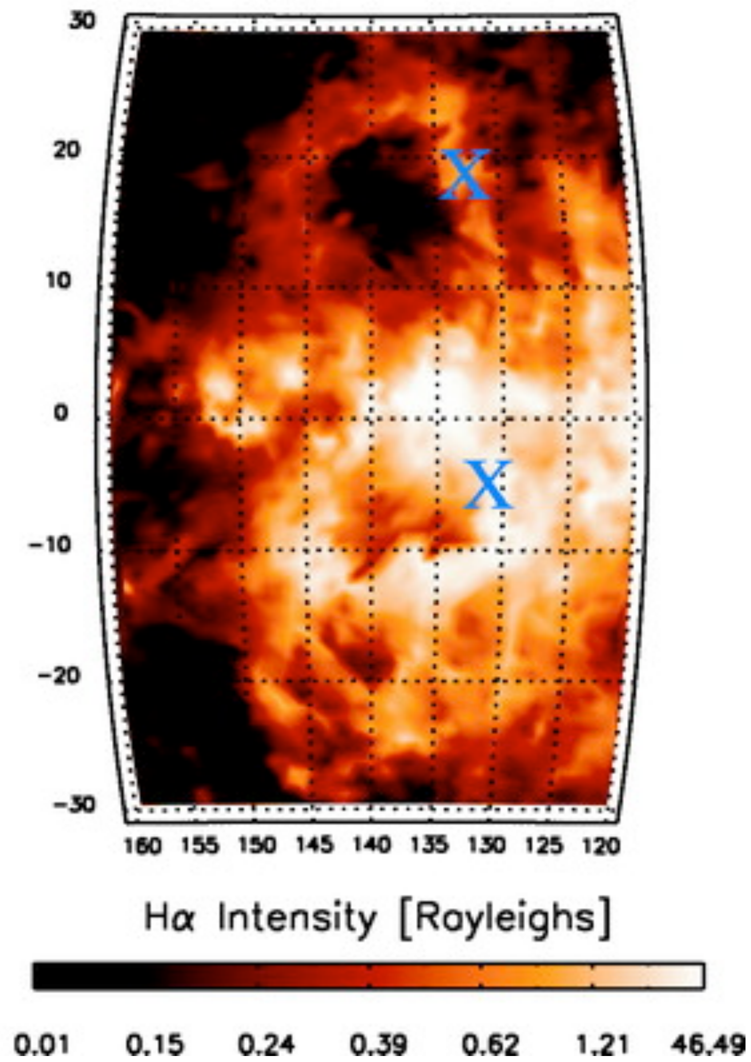
Ionization of uniform medium
$n(H)=100\ cm^{-3}$
$T_*=40000K$, $Q(H)=4.26E49\ s^{-1}$,

- Mathis Code – detailed 1D model of ionization, heating & cooling
- Monte Carlo – 2 frequency approximation good for ionization structure

Photoionize MHD snapshots



Simulation gives ionized and neutral gas



Hill et al. (2012), Barnes et al. (2014)