MCRT L12: Photoionization

- Regions of ionized hydrogen in star forming regions and the interstellar medium
- Photoionization and recombination
- Stromgren spheres
- Monte Carlo photoionization
**HII Regions**

- Massive (hot) stars produce large numbers of ionizing photons (energy above 13.6eV) which ionize hydrogen.
- Detailed structure of a nebula depends on density distribution of surrounding gas.
- Consider an idealized picture: a star in a uniform medium of pure hydrogen.
- Real situation can be much more messy: blue in image is ionized hydrogen.
Orion Nebula

Glowing ionized hydrogen

Hydrogen ionized by photons with $E > 13.6$eV or $\lambda < 912$A
$1$eV = $1.602E-19$ J

Four bright O stars emit most of the ionizing photons that produce the Orion Nebula

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HII Regions & Diffuse Ionized Gas

WHAM: Wisconsin Hα Mapper
• Consider a uniform region of pure hydrogen
• Massive star turns on instantly
• Ionization cross section \( \sigma \sim 10^{-21} \text{ m}^2 \) for neutral H
• If gas density \( n_H \sim 10^9 \text{ m}^{-3} \), mean free path \( \sim 10^{12} \text{ m} \) before ionizing an atom, so ionizing photons cannot escape
• For ionized gas, the much smaller Thomson cross section \( \sim 6.7 \times 10^{-29} \text{ m}^2 \) applies, so stellar photons travel freely through ionized gas to the edge of the neutral material (long mean free path)
• The ionized HII region around the star is called a Strömgren sphere
• HII region is separated from the surrounding neutral hydrogen by a thin layer of thickness, \( \delta \sim (n_H \sigma)^{-1} \), the mean free path of an ionizing photon
Development of an HII region

In time $dt$, the star emits $dN_i$ photons energetic enough to photoionize the gas

$$\text{atom} + \text{photon} \rightarrow \text{ion} + \text{electron}$$

These photons either expand the HII region, or compensate for radiative recombinations in the already ionized gas close to the star

$$\text{ion} + \text{electron} \rightarrow \text{atom} + \text{photon}$$

i.e. to reionize atoms that have recombined to the ground state
Ionizing photon budget

- To expand the HII region by $dR$, the star must ionize $4\pi R^2 n_H dR$ atoms.
- In the interior of the region,

$$\frac{4}{3} \pi R^3 n_e n_i \alpha(H^0,T)$$

recombinations occur per unit time. $\alpha(H^0,T)$ is the recombination rate coefficient, $n_i$ and $n_e$ are the number densities of ions and electrons,
- $\alpha(H^0,T)$ is weakly dependent on temperature $\propto T^{-0.5}$
- $\alpha(H^0,8000K) \sim 4 \times 10^{13} \text{ cm}^3 \text{ s}^{-1}$
Expansion rate

The equation for the time development of the HII region is then,

\[
\frac{dN_i}{dt} = 4\pi R^2 n_H \frac{dR}{dt} + \frac{4}{3} \pi R^3 n_i n_e \alpha(T)
\]

Rearranging,

\[
\frac{dR}{dt} = \frac{Q}{4\pi R^2 n_H} - \frac{n_i n_e \alpha(T) R}{3n_H}
\]

where \(Q = dN_i/dt\) is the number of ionizing photons emitted per second
Properties of this model

- Volume of HII region is small at first
- Most photons ionize gas at boundary, which expands rapidly
- Expansion speed can be $>>$ sound speed in gas, which has no time to react to the passing ionization front
- As region grows, need more photons to balance recombinations. When all photons are needed, get size of region by setting $dR/dt = 0$:

$$R_s^3 = \frac{3Q}{4\pi n_i n_e \alpha(T)}$$

- An O star with $T_{eff} = 35,000K$ has $Q \approx 10^{49} \text{ s}^{-1}$
- Use $\alpha \sim 4 \times 10^{-19} \text{ m}^3 \text{ s}^{-1}$, and $n_i = n_e = 10^9 \text{ m}^{-3}$, we find $R_s \sim 0.6 \text{ pc}$
- $1 \text{ pc} = \text{parsec} = 3.086 \times 10^{18} \text{ cm}$
Hydrogen in the interstellar medium

- Photons with $h\nu > 13.6\text{eV}$ can ionize hydrogen from ground state
- Recombinations of electrons to upper levels, quickly followed by cascade to low levels: H is either ionized or in ground state
- Recombination: as electrons cascade down they emit line radiation which can escape from the nebula
- Recombinations direct to $n = 1$ produce photons with $h\nu > 13.6\text{eV}$ and can ionize H
- Heating from ionization, cooling from escaping photons
Photoionization of Hydrogen

Hydrogen Energy Level Diagram

Lyman Series

Ground State

Balmer Series

Free Electrons

$E_\infty = 0$ eV

$E_2 = -3.4$ eV

$E_1 = -13.6$ eV
Recombinations of electrons direct to ground, $n = 1$, produce a continuum with $h\nu > 13.6\text{eV}$. This diffuse recombination emission can ionize hydrogen.
Assumptions for a simple Monte Carlo code for 3D HII regions

- All stellar photons are emitted at a single frequency
- Spectrum of diffuse ionizing photons is strongly peaked to just above 13.6eV, so assume all diffuse photons are emitted with $h\nu = 13.6\text{eV}$
- Temperature of ionized gas is $T \sim 8000\text{K}$
Monte Carlo Radiation Transfer

1. Choose frequency of stellar packet and set $\sigma$
2. Emit packets isotropically from point source(s)
3. Generate random optical depth
4. Find location where packet interacts with $H^0$: photoionization event occurs
5. Random test if recombination photon can ionize
6. Repeat 3 – 5 until packet exits simulation
7. Emit new source packet: do all source packets
8. Update ionization structure: new opacity grid
9. Iterate until ionization structure converges
1. Star emits ionizing photon
2. Photoionization, recombination, emission of diffuse ionizing photon
3. Photoionization, recombination, emission of non-ionizing photon
Photon Packets

• Until now have described Monte Carlo codes using photon ENERGY or LUMINOSITY packets:

\[ \varepsilon = \frac{L \Delta t}{N} \]

• Now use PHOTON packets:

\[ \varepsilon = \frac{Q h \nu \Delta t}{N} \]

• Can easily use atomic probabilities for photon interactions
Isotropic emission and optical depth

Initial direction for random walk:

\[ \begin{align*}
\theta &= \cos^{-1}(2\xi - 1) \\
\phi &= 2\pi \xi
\end{align*} \]

\[ P(\tau) = \exp(-\tau): \text{packet travels } \tau \text{ before interaction:} \]

\[ \tau = -\log \xi \]

Physical distance, \( S \), that the photon has traveled:

\[ \tau_v = \int_{0}^{S} n \sigma_v \, ds \quad n \sigma_v = n(H^0) \sigma_v(H^0) \]
• Packet will be absorbed and photoionize H, immediately followed by recombination and emission of a packet

• Total recombination rate to all levels is $\alpha_A$, recombination rate to ground level is $\alpha_1$, recombination rate to excited levels ($n > 1$) is $\alpha_B$

• Probability of emission as ionizing packet is $\alpha_1/\alpha_A$, or as non-ionizing is $\alpha_B/\alpha_A = 1 - \alpha_1/\alpha_A$

• At $T = 8000K$, $\alpha_1/\alpha_A \sim 0.4$

• If recombination packet has $h\nu > 13.6eV$, it can ionize H and random walk continues

• If recombination packet has $h\nu < 13.6eV$, it cannot ionize H, random walk terminated
Ionizing & non ionizing photons

- Lyman continuum, $h\nu > 13.6$ eV

- Probability of ionizing emission: $P_{\text{Ly-c}} = \alpha_1 / \alpha_A$

- Recombination coefficients at 8000K, $P_{\text{Ly-c}} \sim 0.4$

- If $\xi < P_{\text{Ly-c}}$: set cross section for $h\nu = 13.6$eV re-emit isotropically continue random walk

- If $\xi > P_{\text{Ly-c}}$: $h\nu < 13.6$ eV, non-ionizing line + continuum spectrum, terminate packet as it cannot ionize, nebula optically thin so packet escapes
Photoionization

\[ n(H^0) \int_{\nu_H}^{\infty} \frac{4\pi J_\nu}{h\nu} \sigma_\nu(H^0) d\nu = n(H^+) n_e \alpha(H^0, T) \]

\[ \# \text{ photoionizations/sec} \quad \# \text{ recombinations/sec} \]

Need to know mean intensity throughout grid, use pathlengths:

\[ 4\pi J_\nu \, d\nu = \sum \varepsilon_\nu \frac{l}{(\Delta t \Delta V)} \quad \varepsilon_\nu = \frac{Q h\nu \Delta t}{N} \]

Sum \((l \sigma_\nu)\) in each cell, so at end of iteration have integral \(I\):

\[ I = \int_{\nu_H}^{\infty} \frac{4\pi J_\nu}{h\nu} \sigma_\nu(H^0) d\nu = \frac{Q}{(N \Delta V) \sum l \sigma_\nu(H^0)} \]
Solve the photoionization equation

- Total number density of hydrogen is \( n(H) = n(H^+) + n(H^0) \)
- For pure hydrogen gas in ionization equilibrium, \( n_e = n(H^+) \)
- Solve quadratic equation to determine ionization fractions in each cell

\[
n(H^0) \int_{\nu_H}^{\infty} \frac{4\pi J}{h\nu} \sigma_\nu(H^0) d\nu = n(H^+) n_e \alpha(H^0, T) \\
\alpha n(H^+)^2 + I n(H^+) - I n(H) = 0
\]

- Update the opacity for next iteration based on ionization fraction
- Regions close to source are ionized, so have low opacity (Thomson scattering), therefore packets for subsequent iterations can travel further before interacting with a neutral hydrogen atom
Mathis Code – detailed 1D model of ionization, heating & cooling
Monte Carlo – 2 frequency approximation good for ionization structure

Ionization of uniform medium
\( n(H) = 100 \text{ cm}^{-3} \)
\( T_\ast = 40000 \text{K}, \quad Q(H) = 4.26 \times 10^{49} \text{ s}^{-1} \)
Photoionize MHD snapshots

Simulation gives ionized and neutral gas

Hill et al. (2012), Barnes et al. (2014)