1. Given a probability distribution, \( P(x) \), show in integral form how to randomly sample values of \( x \) so that after many random samplings a histogram of the \( x \)s will reproduce \( P(x) \).

2. With the aid of a sketch, describe the rejection technique for randomly sampling from a probability distribution function.

3. Using equations and logic statements, describe the method of “Russian roulette” as used in Monte Carlo radiation transfer codes and in what situations it would be useful.

4. With the aid of sketches and appropriate equations, describe the next event estimator as used in Monte Carlo radiation transfer codes.

5. In a Monte Carlo code for neutron transport in U235 the cross sections for neutron scattering, neutron absorption, and fission are \( \sigma_s \), \( \sigma_a \), and \( \sigma_f \). When a neutron reaches a randomly chosen interaction location in the simulation, write down an if-then-else statement that determines whether the neutron is scattered, absorbed, or creates a fission event.

6. In the 1940s John von Neumann used the middle-square algorithm to produce random numbers for Monte Carlo simulations of neutron transport. For a simulation of neutron transport in \( ^{235}\text{U} \), with a middle-square algorithm using the squares of 4-digit numbers, what are the largest and smallest distances that Monte Carlo neutrons of 1MeV can travel? Use the following parameters for \( ^{238}\text{U} \): mass density of 19 g cm\(^{-3}\) and the total cross section of a \( ^{238}\text{U} \) atom for 1MeV neutrons is 5.4 barns.

7. Derive the pathlength formulae for radiation pressure and absorbed energy. Explain why such pathlength summing techniques for determining energy deposition give better statistics than sampling photon absorption events in Monte Carlo radiation transfer codes.

8. A Monte Carlo simulation is designed to study the transport of photons in a spherically symmetric medium where the interaction cross section per particle is \( \sigma \) and the density structure (particles per unit volume) as a function of radius is given by

\[
n(r) = n_0 \left( \frac{r}{R_0} \right)^2
\]

where \( R_0 \) is a constant. A photon packet is launched from location \((x_0, y_0, z_0)\) in a direction given by the unit vector \((n_x, n_y, n_z)\). Derive an expression for the distance the photon packet will travel from its launch location to reach a randomly sampled optical depth \( \tau \). The expression should include \( \tau \), \( \sigma \), \( n_0 \), \( R_0 \), \( x_0 \), \( y_0 \), \( z_0 \), \( n_x \), \( n_y \), \( n_z \).

9. Derive the formula for choosing an optical depth for forced first scattering

\[
\tau = -\ln\left(1 - \xi \left[1 - \exp(-\tau_e)\right]\right)
\]

where \( \tau_e \) is the optical depth along the direction of travel from the photon’s location to the edge of the medium and \( \xi \) is a random number in the range (0,1).