Atomic processes : Bound-bound transitions (Einstein coefficients)

Radiative processes from electron transitions:

- **Bound-bound**: electron moves between two bound states in an atom or ion. Photon emitted or absorbed.

$$h\nu = \chi_{\mathrm{u}} - \chi_{\mathrm{l}}$$

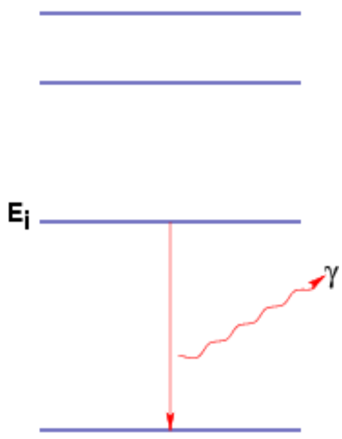
- **Bound-free**: electron moves between bound and unbound states. Bound-unbound: ionization. Unbound-bound: recombination

$$h\nu = \chi_{\mathrm{ion}} - \chi_n + \frac{1}{2} m u^2$$

- **Free-free**: Free electron gains energy by absorbing a photon as it passes an ion, or loses energy by emitting a photon. This emission process is called Bremsstrahlung (braking).

$$h\nu = \frac{1}{2} m u_2^2 - \frac{1}{2} m u_1^2$$

Transition between two atomic energy levels:



Photon frequency, $h\nu_{ij} = | E_i - E_j |$

Hydrogen-like atoms (nucleus + one electron):

$$E_n = -Z^2 \frac{m_e e^4}{2n^2\hbar^2} \equiv -\frac{Z^2 R}{n^2}$$

where

n is an integer (the principal quantum number),
Z is nuclear charge in units of e, and
$R \simeq 13.6$ eV is a constant.

Spectrum consists of a series of lines, labelled by the final n of downward transition, e.g., the Lyman series are transitions to n=1.

Lyman α is the transition n=2 to n=1, with wavelength $\lambda(\mathrm{Ly}\alpha)$ = 121.6 nm.

Boltzmann’s Law

- In thermodynamic equilibrium at temperature T, the populations n_1 and n_2 of any two energy levels are given by Boltzmann's law,

$$\frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-(E_2 - E_1)/kT}$$

- E_1 and E_2 are the energies of the levels relative to the ground state.
- Some energy levels are degenerate (i.e., can hold >1 electron). Statistical weights g_1, g_2 give the number of sublevels.
- In terms of photon frequency: $\frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-h\nu/kT}$

HYDROGEN ATOM

Excitation energy

$$\chi_n = \chi_{ion}(1 - \frac{1}{n^2})$$

Statistical weight of level n is $2n^2$

n=1, Lyman series 1216- 912 Å

n=2, Balmer series 6563-3647 Å

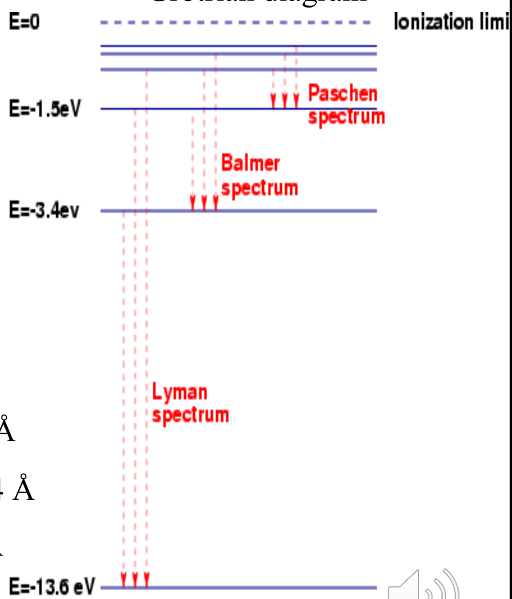
n=3, Paschen series 18751-8204 Å

n=4, Brackett series 40512-14584 Å

n=5, Pfund series 74578-22788 Å

Astrophysical Formulae, Lang

Grotrian diagram



Bound-bound transitions: Einstein coefficients

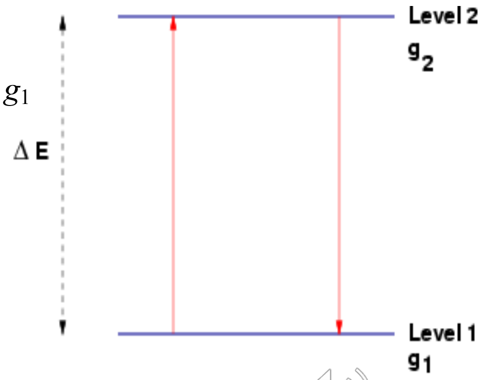
- Kirchhoff's Law was introduced in Lecture 4 and relates the absorption and emission coefficients for black body radiation,

$$B_\nu = \frac{j_\nu}{\alpha_\nu}$$

- This law
  - was derived without using any knowledge of microscopic processes
  - Must imply some relation between emission and absorption processes at an atomic level

2-level atom

- Einstein considered the case of a two level atom:
  - Two energy levels
  - Energy E_1, statistical weight g_1
  - Energy $E_1 + \Delta E = E_1 + h\nu_0$, statistical weight g_2
  - 3 important radiative processes follow



1. Spontaneous emission

- Atom decays spontaneously from level 2 to level 1
- Photon emitted
- Occurs independently of the radiation field
- **Define**: The Einstein A-coefficient, A_{21}, is the transition rate per unit time for spontaneous emission, typically $\sim 10^8\ s^{-1}$

2. Absorption

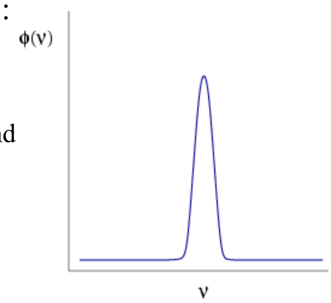
- Photons with energies close to $h\nu_0$ cause transitions from level 1 to level 2
- The probability per unit time for this process will evidently be proportional to the mean intensity at the frequency ν_0

Line profile $\phi(\nu)$

Need to define a *line profile function* $\phi(\nu)$:

- describes the probability that a photon of frequency ν will cause a transition
- $\phi(\nu)$ is sharply peaked at ν_0, with width $\Delta\nu$ and normalization,

$$\int_0^\infty \phi(\nu)d\nu = 1$$



Define: The transition rate per unit time for absorption is $B_{12}\bar{J}$

where, $$\bar{J} \equiv \int_0^\infty J_\nu \phi(\nu)d\nu$$

with J_ν being the mean intensity and $\phi(\nu)$ the line profile function

B_{12} is one of the Einstein B-coefficients

Note: we have been careful to distinguish between J_ν and $\bar{J}$, but this is a technicality. If J_ν changes slowly over the line width $\Delta\nu$ of the line, then $\phi(\nu)$ is almost $\delta(\nu - \nu_0)$ and $\bar{J} \cong J_{\nu_0}$

3. Stimulated emission

Planck's law does not follow from considering only spontaneous emission and absorption. Must also include *stimulated emission*, which like absorption is proportional to $\bar{J}$

Define: $B_{21}\bar{J}$ is the transition rate per unit time for stimulated emission.

B_{21} is a second Einstein B-coefficient. Stimulated emission occurs into the same state (frequency, direction, polarization) as the photon that stimulated the emission.

Lecture 6 revision quiz

- Make sketches to illustrate the physical processes involved in b-b, b-f, and f-f transitions.
- Calculate the wavelengths of the first 3 lines of the hydrogen Balmer series: Hα, Hβ, Hγ.
- Define the statistical weight *g* of an atomic energy level.
- Write down Boltzmann's Law and define all symbols used and their units.