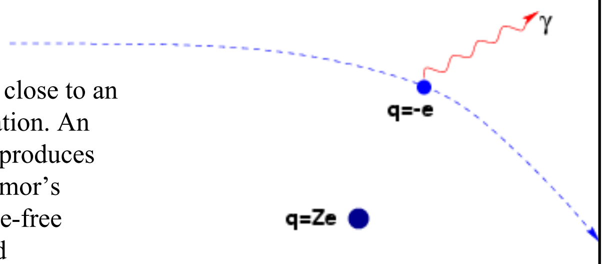
Free-free transitions and Scattering



- An electron passing close to an ion feels an acceleration. An accelerating charge produces radiation (recall Larmor's formula). This is free-free emission, also called *Bremsstrahlung*:
  - Important at high temperatures, where the plasma is highly ionized.
  - Depends upon temperature T, ion charge Z, and electron, ion densities n_e and n_i. For *Thermal Bremsstrahlung*, the power per unit volume is proportional to (see Rybicki & Lightman, 5.2):

$$T^{1/2} n_e n_i Z^2$$

Free-Free absorption–I

- A free electron can also *gain* energy during a collision with an ion by absorbing a photon. This is *free-free absorption*.
- A free electron passing an ion can emit or absorb radiation while it is close enough. At temperature T, thermal velocity is

$$(1/2)m_e\upsilon^2 \approx kT$$

- and the time they are close enough will be proportional to

$$\upsilon^{-1} \propto T^{-1/2}$$

Free-Free absorption–II

- If the density is ρ, the number of systems able to participate is

$$\propto \rho T^{-1/2}$$

- A single system has an absorption coefficient proportional to $Z^2 \nu^{-3}$, where Z is the charge number of the ion. Then,

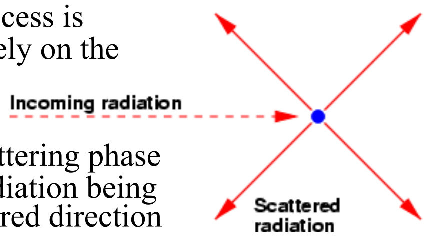
$$\kappa_\nu \propto Z^2 \rho T^{-1/2} \nu^{-3}$$

- Most absorption happens near the peak of $B_\nu(T)$, so since $\nu_{\max} \sim T$, an intensity-weighted integral over frequency gives *Kramers' Law*:

$$\kappa_{ff} \propto \rho T^{-7/2}$$

Scattering

- For thermal emission the source function $S_\nu = B_\nu(T)$ is:
  - Independent of the incident radiation field
  - A function only of the local temperature
- Another important emission process is *scattering*, which depends entirely on the incident radiation field



- Differential cross section or scattering phase function gives probability of radiation being scattered from incident to scattered direction
- Example:
  - *Electron scattering*: photons scatter off free electrons. This is called Thomson scattering for low energies, and Compton scattering at high energies.

Electron scattering

- Free electrons scatter radiation with the same efficiency at all wavelengths. Absorption cross-section (m^2 per electron) is:

$$\sigma_{es} = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\varepsilon_0 m_e c^2}\right)^2$$

- which numerically is 6.7×10^{-29} m^2. The opacity is

$$\kappa_{es} = \frac{N}{\rho}\sigma_{es}$$

- which for pure hydrogen is,

$$\kappa_{es} = 0.04 m^2 kg^{-1}$$

- This process is called *Thomson scattering* or *electron scattering*. It becomes the most important opacity source at high temperatures.
- At high energies, when $h\nu \sim m_e c^2$, this description breaks down. Relativistic effects need to be taken into account – this is the regime of *Compton scattering* or *inverse Compton scattering*.

Isotropic scattering

Consider the simplest case:

- Isotropic scattering: Scattered radiation is emitted uniformly across all solid angles $\rightarrow$ emission coefficient is independent of solid angle
- Coherent scattering: Scattered radiation has the same frequency distribution as the incoming radiation (also called *elastic scattering*) i.e., no redistribution of energy across frequencies
- The astrophysically important case of electron scattering by non-relativistic electrons approximately meets these restrictions

Transfer equation for pure scattering

- For coherent isotropic scattering, power j_ν emitted per unit volume per unit frequency range = power absorbed:

$$j_\nu = \sigma_\nu J_\nu$$

- Here, σ_ν is the absorption coefficient for the scattering process (the scattering coefficient), and J_ν is the mean intensity as before.
- Source function:

$$S_\nu \equiv \frac{j_\nu}{\sigma_\nu} = J_\nu \equiv \frac{1}{4\pi}\int I_\nu \, d\Omega$$

- For pure scattering, the transfer equation becomes,

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu = -\sigma_\nu (I_\nu - J_\nu)$$

- Notes:
  - J_ν is the integral of I_ν over all directions. The transfer equation for scattering no longer depends simply on local T.
  - scattering coefficient is usually called σ_ν. Don't confuse with the cross section per atom. σ_ν has the same units as the absorption coefficient α_ν, i.e., [length^{-1}].

Scattering and absorption

- Suppose we have a material in which there is *both* thermal emission with an absorption coefficient α_ν, and isotropic coherent scattering with scattering coefficient σ_ν. The transfer equation then has two terms,

$$\frac{dI_\nu}{ds} = -\alpha_\nu (I_\nu - B_\nu) - \sigma_\nu (I_\nu - J_\nu)$$
$$= -(\alpha_\nu + \sigma_\nu)(I_\nu - S_\nu)$$

- The source function is still the ratio of the emission to the absorption coefficients,

$$S_\nu = \frac{\alpha_\nu B_\nu + \sigma_\nu J_\nu}{\alpha_\nu + \sigma_\nu}$$

- The net absorption coefficient $(\alpha_\nu + \sigma_\nu)$ can be used to define the optical depth $d\tau_\nu = (\alpha_\nu + \sigma_\nu)\, ds$ exactly as before.

Lecture 10 revision quiz

- The differential cross section (or scattering phase function) for electron scattering is $\pi r_e^2 (1 + \cos^2 \theta)$ where r_e is the classical electron radius and θ is the angle between the incident and scattered radiation. Show that the total cross section for electron scattering is given by the formula in the notes.
- Evaluate the absorption cross-section (in m^2 per electron) for electron scattering.
- Work through the steps to derive the equation of radiation transfer for a purely scattering medium and for a medium that has scattering and absorption.

Lecture 10 revision quiz

- Evaluate the absorption cross-section (in m^2 per electron) for electron scattering.
- What type of source function (thermal or scattering) is most appropriate to compute the spectrum emanating from:
  - The Orion Nebula?
  - The solar white-light corona?
  - The solar photosphere?
  - A neon discharge tube?
  - A candle flame?
  - White clouds?
  - Blue sky?