Monte Carlo II
Scattering Codes

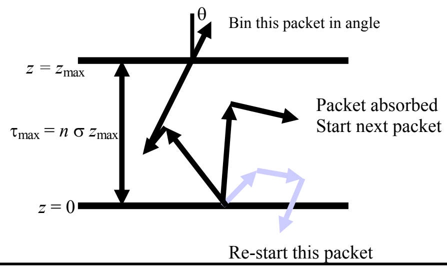
- Plane parallel scattering atmosphere
- Optical depths & physical distances
- Emergent flux & intensity
- Internal intensity moments

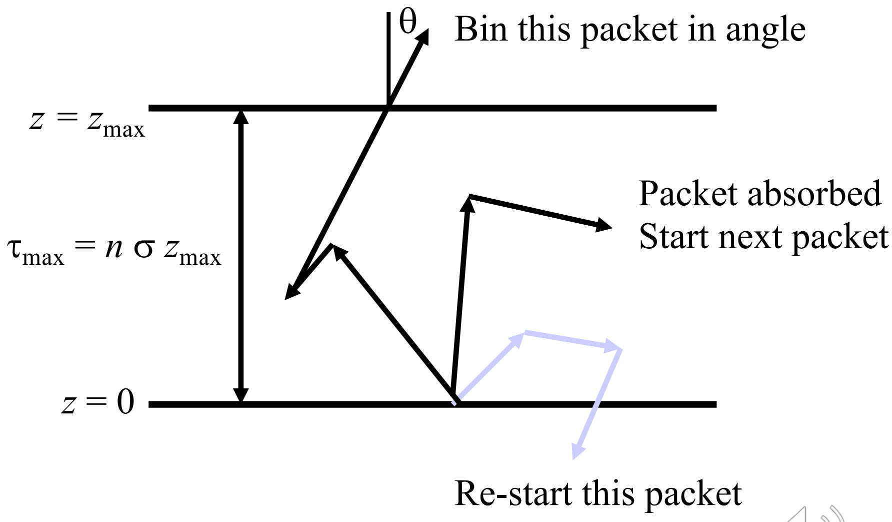
Constant density slab, vertical optical depth $\tau_{max} = n \sigma z_{max}$
Normalized length units $z = z / z_{max}$.

Emit packet
Packet scatters in slab until:

absorbed: terminate, start new packet
$z < 0$: re-emit packet
$z > 1$: escapes, "bin" packet

Loop over packets
Pick optical depths, test for absorption, test if still in slab





Emitting Packets: Packets need an initial starting location and direction. Uniform specific intensity from a surface.

Start packet at $(x, y, z) = (0, 0, 0)$

$$I_\nu(\mu) = \frac{dE}{\mu\, dA\, dt\, d\nu\, d\Omega} \Rightarrow \frac{dE}{dA\, dt\, d\nu\, d\Omega} \propto \frac{dN}{d\Omega} \propto \mu I_\nu(\mu)$$

Sample μ from $P(\mu) = \mu\, I\,(\mu)$ using cumulative distribution.
Normalization: emitting outward from lower boundary, so $0 < \mu < 1$

$$\xi = \frac{\int_0^\mu P(\mu)\,\mathrm{d}\mu}{\int_0^1 P(\mu)\,\mathrm{d}\mu} = \mu^2 \quad \Rightarrow \quad \mu = \sqrt{\xi}$$

Distance Traveled: Random optical depth $\tau = -\log \xi$, and $\tau = n\,\sigma\,L$, so distance traveled is:

$$L = \frac{\tau}{\tau_{max}} z_{max}$$

Scattering: Assume isotropic scattering, so new packet direction is:

$$\theta = \cos^{-1}(2\xi - 1)$$
$$\phi = 2\pi\,\xi$$

Two different ξ values!!

Absorb or Scatter: Scatter if $\xi < a$, otherwise packet absorbed, exit "do while in slab" loop and start a new packet.

Structure of FORTRAN 77 program:

```
do i = 1, npackets
1       call emit_packet
        do while ( (z .ge. 0.) .and. (z .le. zmax) )    ! packet is in slab
                L = -log(ran) * zmax / taumax
                z = z + L * nz              ! update packet position, x,y,z
                if ((z.lt.0.).or.(z.gt.zmax)) goto 2    ! packet exits
                if  (ran .lt. albedo) then
                        call scatter
                else
                        goto 3              ! terminate packet
                end if
        end do
2       if (z .le. 0.) goto 1       ! re-start packet
        bin packet according to direction
3 continue      ! exit for absorbed packets, start a new packet
end do
```

Intensity Moments

The moments of the radiation field are:

$$J_\nu = \frac{1}{4\pi}\int I_\nu \, d\Omega \quad H_\nu = \frac{1}{4\pi}\int I_\nu \, \mu \, d\Omega \quad K_\nu = \frac{1}{4\pi}\int I_\nu \, \mu^2 \, d\Omega$$

Compute these moments throughout the slab. First split the slab into layers, then tally number of packets, weighted by powers of their direction cosines to obtain *J*, *H*, *K*. Contribution to specific intensity from a single packet is:

$$\Delta I_\nu = \frac{\Delta E}{|\mu| \Delta A \Delta t \Delta \nu \, \Delta\Omega} = \frac{F_\nu}{|\mu| N_0 \, \Delta\Omega} = \frac{\pi B_\nu}{|\mu| N_0 \, \Delta\Omega}$$

Substitute into intensity moment equations and convert the integral to a summation to get:

$$J_\nu = \frac{B_\nu}{4N_0}\sum_i \frac{1}{|\mu_i|} \quad H_\nu = \frac{B_\nu}{4N_0}\sum_i \frac{\mu_i}{|\mu_i|} \quad K_\nu = \frac{B_\nu}{4N_0}\sum_i \frac{\mu_i^2}{|\mu_i|}$$

Note the mean flux, *H*, is just the net energy passing each level: number of packets traveling up minus number traveling down.

Lecture 13 revision quiz

- Check the normalization and formula for choosing angles from the distribution $\mu I(\mu)$.
- For a packet traveling in a direction given by polar and azimuthal angles (θ, ϕ), in Cartesian coordinates what are the x, y, and z components of the unit vector for the direction of travel?
- Explain why the Monte Carlo estimator of mean intensity given in the notes will be susceptible to noise.