Nebular emission spectra

- Four types of emission from a Stromgren sphere:
  - Recombination lines of H, He
  - Collisionally excited lines of metals. Optical: mostly forbidden lines of O, N, S, Ar, Ne. UV: C, Si, Mg.
  - Some continuum (f-f, f-b, 2-photon) emission. f-f and f-b significant only at radio wavelengths.
  - Other emission lines: resonance-fluorescence, di-electronic recombination, etc

Hydrogen recombination spectrum

- Einstein A coefficients typically $A \sim 10^6$ s^{-1}
- Collision rates typically $q \sim 10^{-4}$ cm^3 s^{-1}
- Hence collisions for H become important only when

$$N_e \geq \frac{A}{q} \sim 10^{10} \text{ cm}^{-3}.$$

- So collisions are unimportant at nebular densities
- Detailed balance for H gives level pops N_{nL} :

$$N_p N_e \alpha_{nL}(T) + \sum_{n'>n}^{\infty} \sum_{L'} N_{n'L'} A_{n'L',nL} = N_{nL} \sum_{n''=1}^{n-1} \sum_{L''} A_{nL,n''L''}$$

Recombinations into state *nL*

Radiative decays into *nL* from higher levels

Radiative decays from *nL* to lower levels

Emissivity from populations

- This is a set of *n* x *L* linear equations in *n* x *L* unknowns.
  - Choose N_e, T_e
  - Look up α_{nL} for this temperature
  - Solve for populations levels N_{nL} with linear algebra
- Once level populations are known, the emission produced by any transition is

$$j_{nn'} = \sum_{n'=1}^{n-1} \sum_{L'=L\pm1} N_{nL} A_{nL,n'L'}$$

- In practice, there are many routes into each level other than direct recombination
  - Hence level pops depend mainly on Einstein *A* values, only weakly on N_e, T_e.

So far we have neglected:

- Collisional excitation and deexcitation.
  - $kT << \Delta E$ for excitation n=1 to n=2 so that's OK
  - Critical density for collisional de-excitation is $N_{\mathrm{crit}} \sim 10^9$ cm^{-3}, so that's OK too
- Input to *nL* via absorption from lower levels
  - Hydrogen electrons decay to ground state (n=1) in 10^{-6} s, so that's more or less OK
  - However, absorptions from ground state can and do occur
  - In fact cross section for absorption in lower Lyman lines is greater than for photoionisation from n=1

Photoionization versus Lyman lines

- Cross sections for Lyman-line absorption:

Line	λ(Å)	A (sec^{-1})	a$_0$ (cm^2)	$\tau/\tau_{912Å}$
Ly α	**1215.67**	**6.26x10^8**	**5.90x10^{-14}**	**9366**
Ly β	**1025.72**	**1.67x10^8**	**9.46x10^{-15}**	**1501**
Ly γ	**972.54**	**6.82x10^7**	**3.29x10^{-15}**	**522**
Ly 10	**920.96**	**4.21x10^6**	**1.72x10^{-16}**	**27**
Ly 15	**915.82**	**1.24x10^6**	**5.00x10^{-17}**	**8**
Ly 20	**914.04**	**5.24x10^5**	**2.10x10^{-17}**	**3**

- At 912Å the cross section for photoionization from n=1 is 6.30x10^{-18} cm^2

Case A and Case B

- Traditionally nebulae are said to be either:
- Case A:
  - all Lyman line photons escape the nebula
  - No Lyman line absorptions occur
  - Nebula is optically thin, very faint
  - Earlier detailed balance relation is OK
- Case B:
  - All Lyman line photons re-absorbed by other atoms
  - Downward transitions to ground state aren't included in summations
  - In real life, almost everything is Case B or close to it:

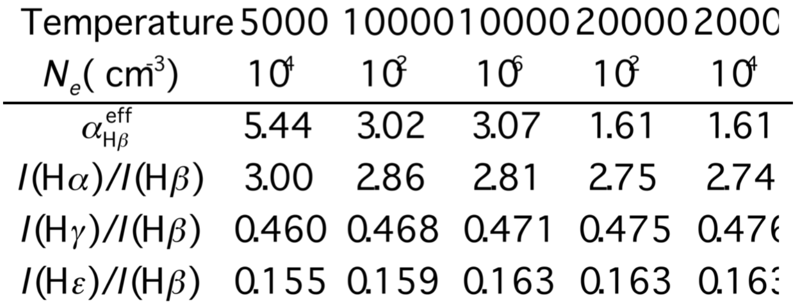
$$N_p N_e \alpha_{nL}(T) + \sum_{n'>n}^{\infty} \sum_{L'} N_{n'L'} A_{n'L',nL} = N_{nL} \sum_{n''=2}^{n-1} \sum_{L''} A_{nL,n''L''}$$

NB $\nearrow$

What’s good about Case B

- Decays to $n = 1$ don’t count, so every decay eventually goes to $n = 2$
- All transitions to $n = 2$ are in the optical
- So every ionization produces an optical Balmer-line photon
- *Number of optical Balmer-line photons = number of ionizing photons emitted by the central star*
- Level populations depend almost exclusively on the Einstein A coefficients, weakly on N_e, T_e.
- Line ratios (more or less) fixed by atomic physics
- *By measuring Hβ alone, we can derive the number of ionizing photons from the central star*

Line ratios for Balmer transitions



Temperature	5000	10000	10000	20000	2000
N_e (cm^{-3})	10^4	10^2	10^6	10^2	10^4
$\alpha^{\mathrm{eff}}_{\mathrm{H}\beta}$	5.44	3.02	3.07	1.61	1.61
$I(\mathrm{H}\alpha)/I(\mathrm{H}\beta)$	3.00	2.86	2.81	2.75	2.74
$I(\mathrm{H}\gamma)/I(\mathrm{H}\beta)$	0.460	0.468	0.471	0.475	0.47
$I(\mathrm{H}\varepsilon)/I(\mathrm{H}\beta)$	0.155	0.159	0.163	0.163	0.16

The fate of Lα photons

- In Case B, Balmer-series photons leave the nebula but Lα photons are trapped
- Eventually some event will destroy Lα photons
  - Random walk to edge of nebula and escape
  - Wavelength shift to optically-thin line wings and escape
  - Destruction by hitting a dust grain. A single grain can mop up many Lα photons. Grain heats up, re-emits in IR.
  - Resonance fluorescence: Lβ (or He Lα) photon excites a transition at the same wavelength in another species.
    - e.g., He II Lα 303.78Å can excite OIII 303.80Å

Lecture 18 revision quiz

- Use the information in the table on slide 5 to calculate the number density N_1 of hydrogen in the ground state, for which a Lα photon would have a mean free path of 1 pc.
- What fraction of all Balmer-line photons produced at typical nebular temperatures and densities emerge in the Hβ line?
- Why is it safe to ignore (a) collisional excitation and (b) collisional de-excitation of hydrogen at typical nebular densities and temperatures?