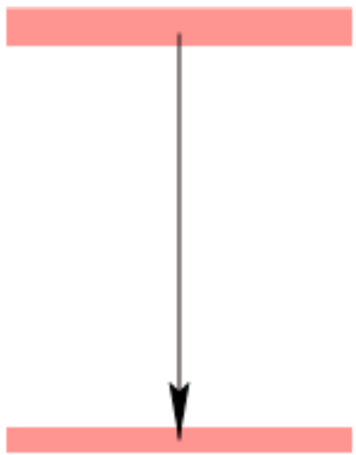
Widths of spectral lines

- **Real spectral lines are broadened because:**
  - Energy levels are not infinitely sharp.
  - Atoms are moving relative to observer.

- **3 mechanisms determine profile $\phi(\nu)$**
  - Quantum mechanical uncertainty in the energy E of levels with finite lifetimes. --> the natural width of a line (generally very small).
  - Collisional broadening. Collisions reduce the effective lifetime of a state, leading to broader lines.High pressure -> more collisions (eg stars).
  - Doppler or thermal broadening, due to the thermal (or large-scale turbulent) motion of individual atoms in the gas relative to the observer.



Natural width

- **Uncertainty principle. Energy level above ground state with energy *E* and lifetime *Δt*, has uncertainty in energy:**

$$\Delta E \Delta t \sim \hbar$$

ie short-lived states have large uncertainties in the energy.

- **A photon emitted in a transition from this level to the ground state will have a range of possible frequencies,**

$$\Delta \nu \sim \frac{\Delta E}{h} \sim \frac{1}{2\pi \Delta t}$$

Lorentzian profile

- **This effect is called *natural broadening*.**
- **If the spontaneous decay of an atomic state *n* (to all lower energy levels *n'*) proceeds at a rate,**

$$\gamma = \sum_{n'} A_{nn'}$$

it can be shown that the line profile for transitions to the ground state is of the form,

$$\phi(\nu) = \frac{\gamma / 4\pi^2}{(\nu - \nu_0)^2 + (\gamma / 4\pi)^2}$$

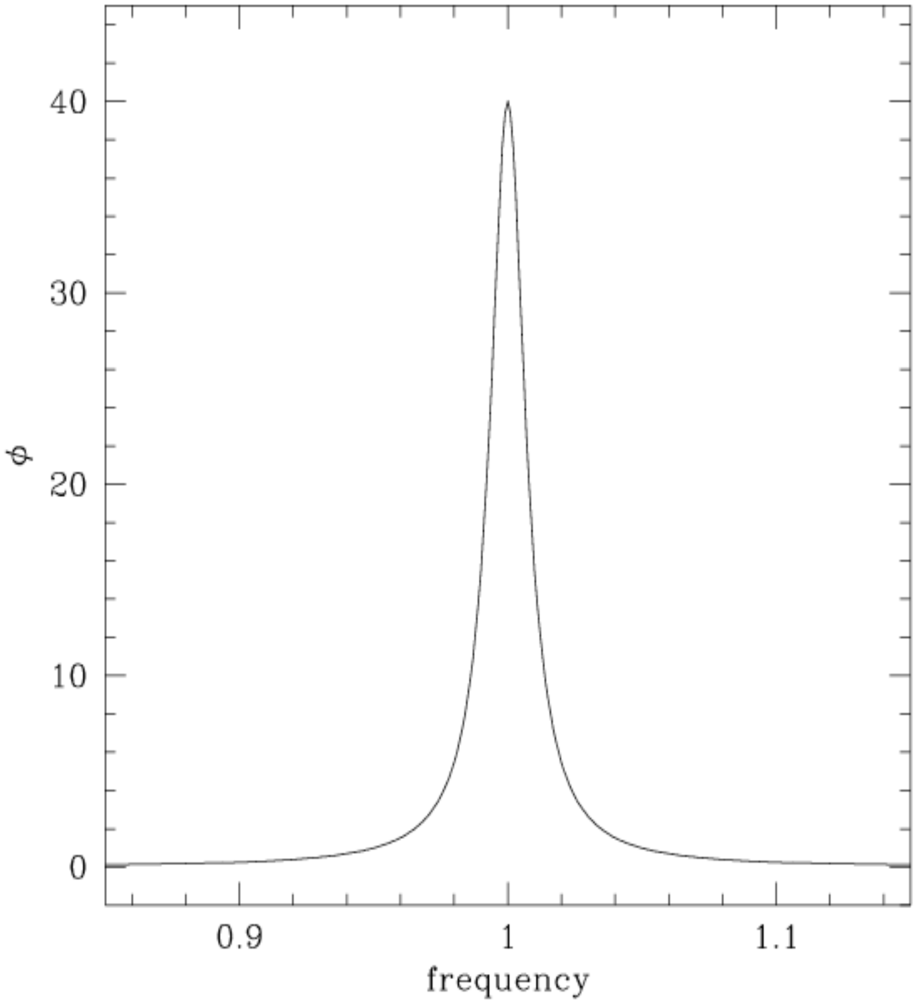
- **This is a *Lorentzian* or *natural* line profile.**
- **If both the upper and lower states are broadened, sum the γ values for the two states.**

Schematic Lorentz profile

- **NB:**
- **If radiation is present, add stimulated emission rates to spontaneous ones.**
- **Well away from the line centre,**

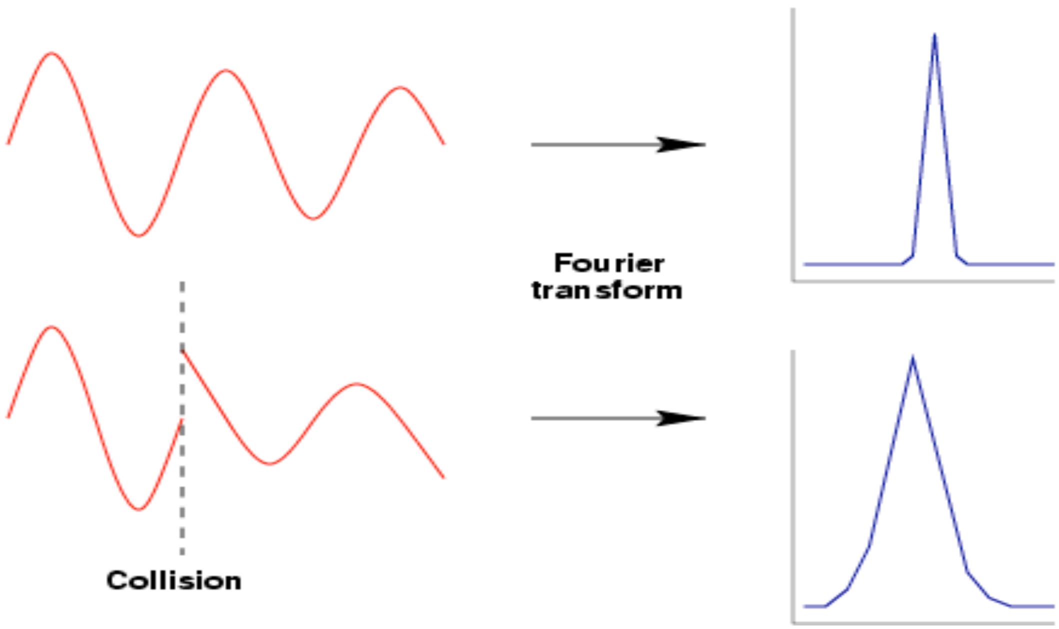
$$\phi(\nu) \propto \Delta\nu^{-2}$$

- **Decay is much slower than a gaussian.**
- **Natural linewidth isn't often directly observed, except in the line wings in low-pressure (nebular) environments. Other broadening mechanisms usually dominate.**



Collisional broadening

- **The natural linewidth arises because excited states have a finite lifetime.**
- **Collisions randomize the phase of the emitted radiation. If frequent enough they (effectively) shorten the lifetime further.**



Another Lorentz profile

- **If the frequency of collisions is ν_{col}, then the profile is**

$$\phi(\nu) = \frac{\Gamma / 4\pi^2}{(\nu - \nu_0)^2 + (\Gamma / 4\pi)^2}$$

where $\Gamma = \gamma + 2\nu_{col}$

- **Still a Lorentz profile. Collisions dominate in high density environments - hence get broader lines in dwarfs than giants of the same spectral type.**

Thermal broadening

- **Random motions of atoms in a gas depend on temperature.**
- ***Most probable speed u* is**

$$\frac{1}{2}mu^2 = kT$$

- **Maxwell's Law of velocity distribution gives number of atoms with a given speed or velocity. Important to distinguish between forms of this law for *speed* and for *one component of velocity*:**

$$dN(\upsilon_x) \propto \exp\left(-\frac{m\upsilon_x^2}{2kT}\right)d\upsilon_x$$

Speeds and velocities

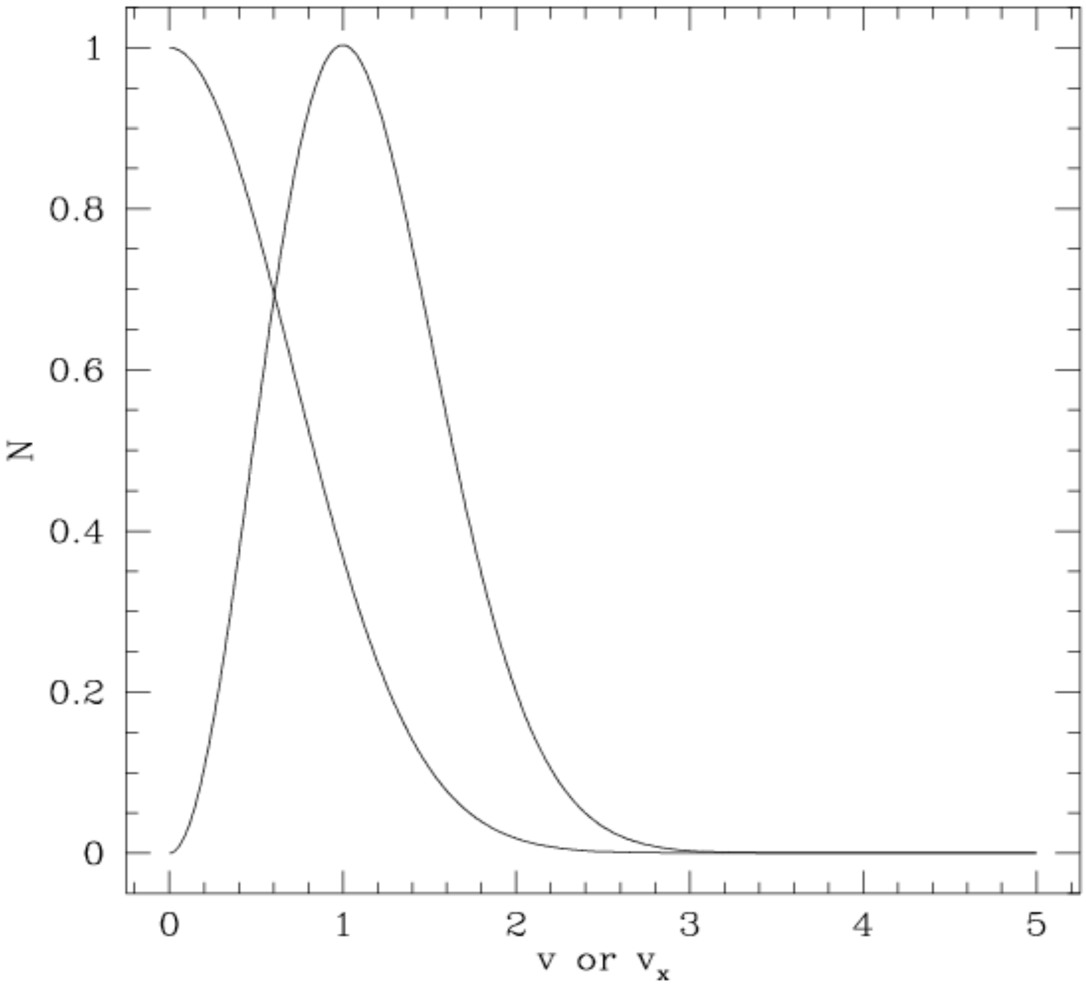
- **For any one component (say υ_x), the distribution of *velocities* is described by a Gaussian.**
- **Distribution law for *speeds* has an extra factor of υ^2,**

$$dN(\upsilon\ ) \propto \upsilon^2 \exp\left(-\frac{m\upsilon^2}{2kT}\right)d\upsilon$$

- **Prove this for yourself in tut sheet 3, Q3!**
- **The one-component version gives the thermal broadening, since we only care about the motions along the line of sight.**

Distributions of speed and velocity

- **Few particles have speeds much greater than the mean.**



Thermal Doppler effect

- **Doppler-shifted frequency ν for an atom moving at velocity υ_x along the line of sight differs from ν_0 in rest frame of atom:**
- $$\frac{\nu - \nu_0}{\nu_0} = \frac{\upsilon_x}{c}$$
- **Combine Doppler shift with the one-dimensional distribution of velocities to get profile function:**
- $$\phi(\nu) = \frac{1}{\Delta\nu_D \sqrt{\pi}} e^{-\left((\nu-\nu_0)^2/(\Delta\nu_D)^2\right)}$$
- **where Doppler width $\Delta\nu_D$ is defined as**

$$\Delta\nu_D = \frac{\nu_0}{c}\sqrt{\frac{2kT}{m}}$$

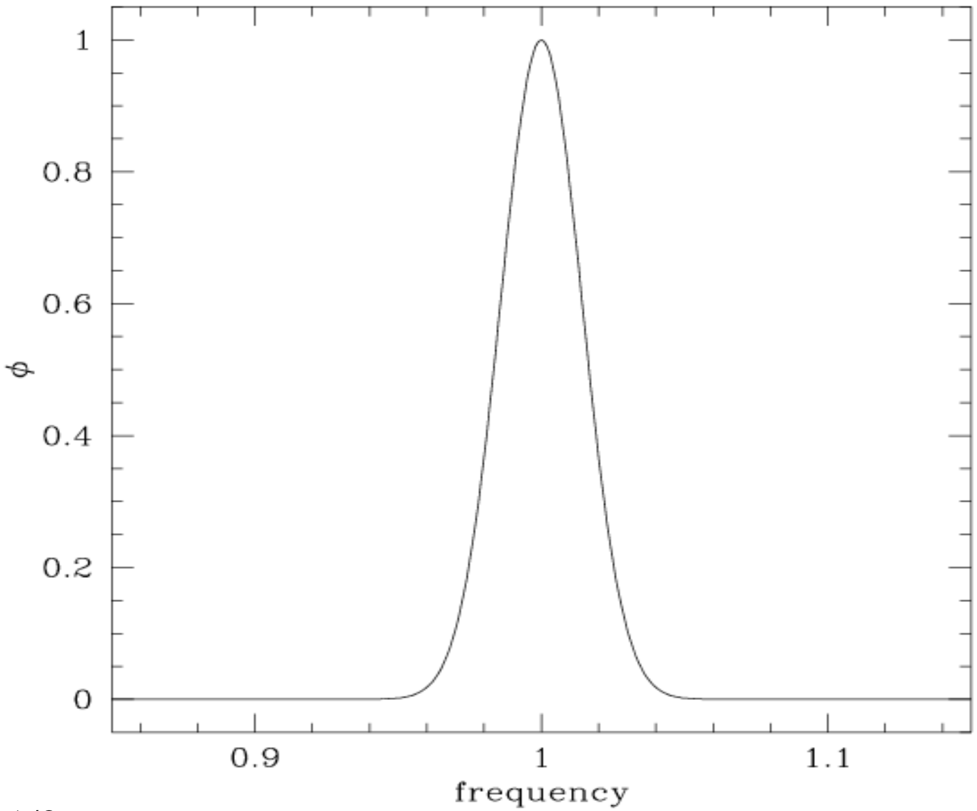
Including turbulent velocities

- **If turbulent motions in gas can be described by a similar velocity distribution, the effective Doppler width becomes**

$$\Delta\nu_D = \frac{\nu_0}{c}\left(\frac{2kT}{m} + \upsilon_{turb}^2\right)^{1/2}$$

- **where υ_{turb} is a root mean-square measure of the turbulent velocities. This situation occurs, e.g., in observations of star forming regions or in convective stellar photospheres.**

Thermal line profile



- **This is just a Gaussian, which falls off very rapidly away from the line centre.**
- **Numerically, for hydrogen the thermal broadening in velocity units is**

$$\frac{\Delta\nu_D c}{\nu_0} = \sqrt{\frac{2kT}{m}} \cong 13\left(\frac{T}{10^4 K}\right)^{1/2} \text{ km s}^{-1}$$

a rough number worth remembering.

Example

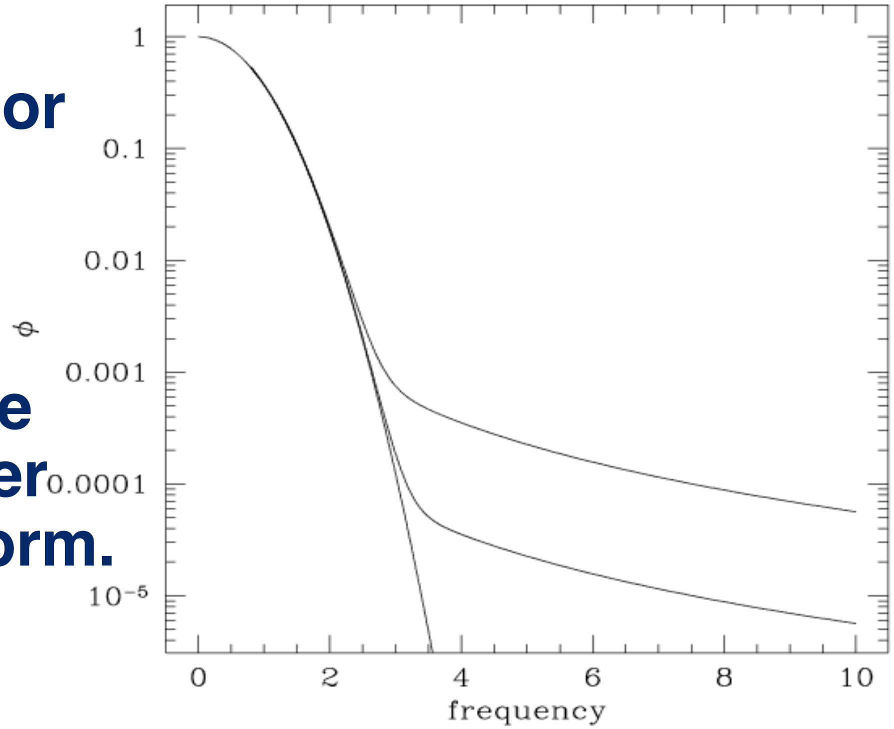
- **Show that the line width of hydrogen in the sun (T=6,000 K) is dominated by thermal velocities and that it is also wider than the line width of metals (use iron with atomic weight 56).**

$$\frac{\Delta\lambda}{\lambda} = \frac{\upsilon}{c} = \frac{1}{c}\sqrt{\frac{2kT}{m}} = \frac{1}{3\times 10^{8}}\sqrt{\frac{2\times 1.38\times 10^{-23}\times 6\times 10^{3}}{56\times 1.66\times 10^{-27}}} \approx \frac{1.33\times 10^{3}}{3\times 10^{8}} \approx 4\times 10^{-6}$$

- **Doppler width for λ=400 nm is then 0.0016 nm**
- **Larger than the damping width ~ 0.00011 nm for Balmer lines of hydrogen**
- **Hydrogen's thermal velocity is 10 km/s compared to 1.4 km/s for iron.**

Voigt profile

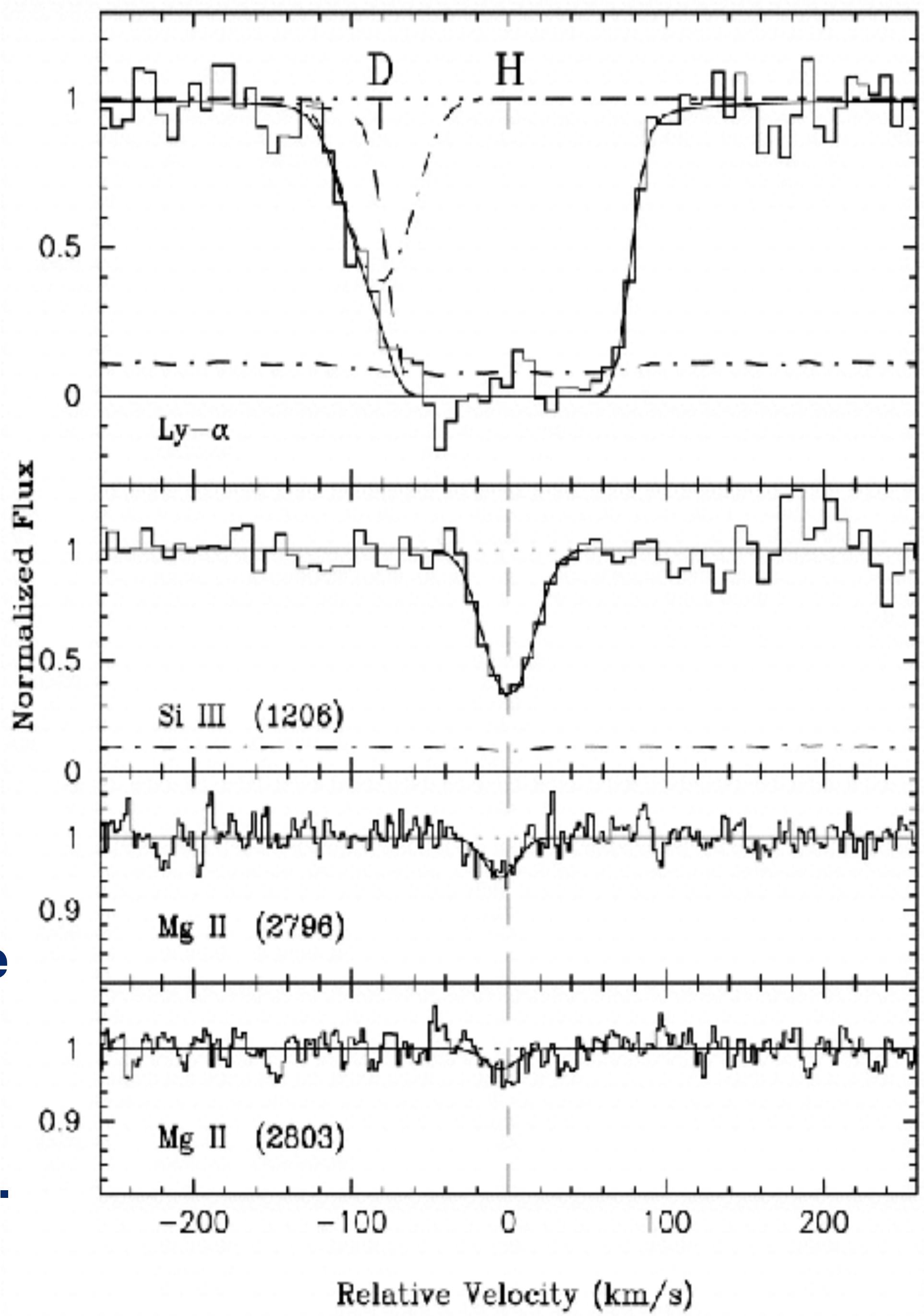
- **The combination of thermal broadening with the natural or collisionally broadened line profile is called the *Voigt profile*.**
- **This is the convolution of the Lorentz profile with a Doppler profile. No simple analytic form.**



Curves show the profile as the natural (or collisional) linewidth is increased. Lorentz profile falls off slower than Doppler profile, so core remains roughly Gaussian, while the wings look like a Lorentz profile.

Example of use of Voigt profile

- Voigt profiles are fit to observations of absorption towards quasars to measure temperatures and column densities of the gas along the line of sight.
- Deuterium absorbs at slightly different frequencies than hydrogen. Detecting a corresponding deuterium feature provides a limit on the primordial abundance of deuterium, since stellar processes always destroy it (figure from Tytler et al. 1999).



Lecture 8 revision quiz

- **Show that $\phi(\nu) \propto \Delta\nu^{-2}$ well away from the centre of a line with a Lorentzian broadening profile.**
- **Calculate the line-of-sight thermal velocity dispersion Δv_D of line photons emitted from a hydrogen cloud at a temperature of 10^4 K.**
- **Calculate the natural broadening linewidth of the Lyman α line, given that $A_{ul}=5\times10^8\ s^{-1}$. Convert to km/sec via the Doppler formula.**
- **Compute both profiles and plot them on the same velocity scale.**