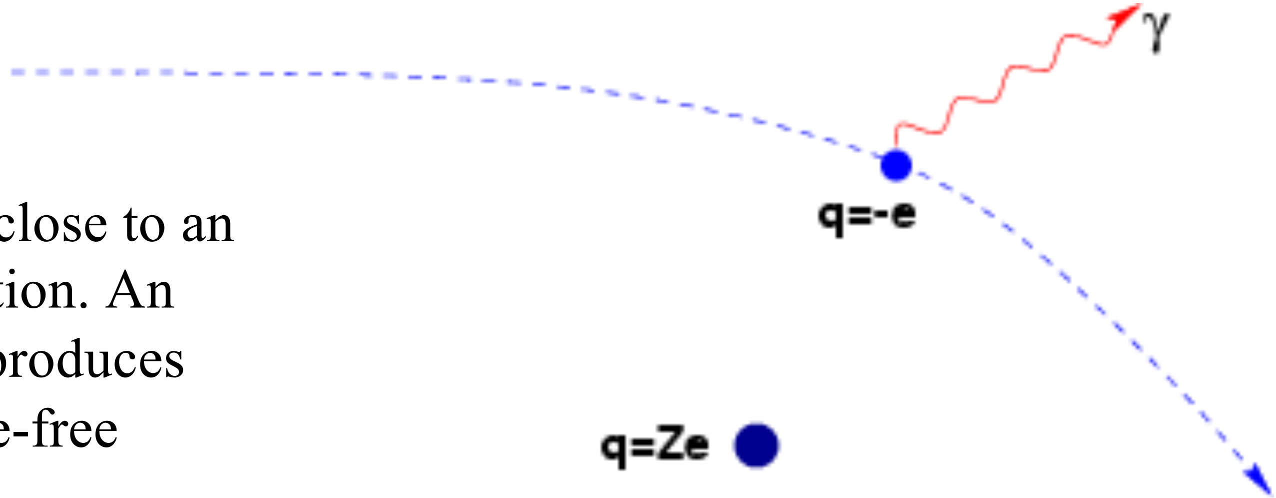
Free-free transitions and Scattering



- An electron passing close to an ion feels an acceleration. An accelerating charge produces radiation. This is free-free emission, also called *Bremsstrahlung*:
  - Important at high temperatures, where the plasma is highly ionized.
  - Depends upon temperature T, ion charge Z, and electron, ion densities n_e and n_i . The power per unit volume is proportional to,

$$\propto T^{1/2} n_e n_i Z^2$$

Free-Free absorption–I

- A free electron can also *gain* energy during a collision with an ion by absorbing a photon. This is *free-free absorption*.

- A free electron passing an ion can emit or absorb radiation while it is close enough. At temperature *T*, thermal velocity is

$$(1/2)m_e v^2 \approx kT$$

- and the time they are close enough will be proportional to

$$v^{-1} \propto T^{-1/2}$$

Free-Free absorption–II

- If the density is ρ, the number of systems able to participate is

$$\propto \rho T^{-1/2}$$

- A single system has an absorption coefficient proportional to $Z^2 \nu^{-3}$, where Z is the charge number of the ion. Then,

$$\kappa_\nu \propto Z^2 \rho T^{-1/2} \nu^{-3}$$

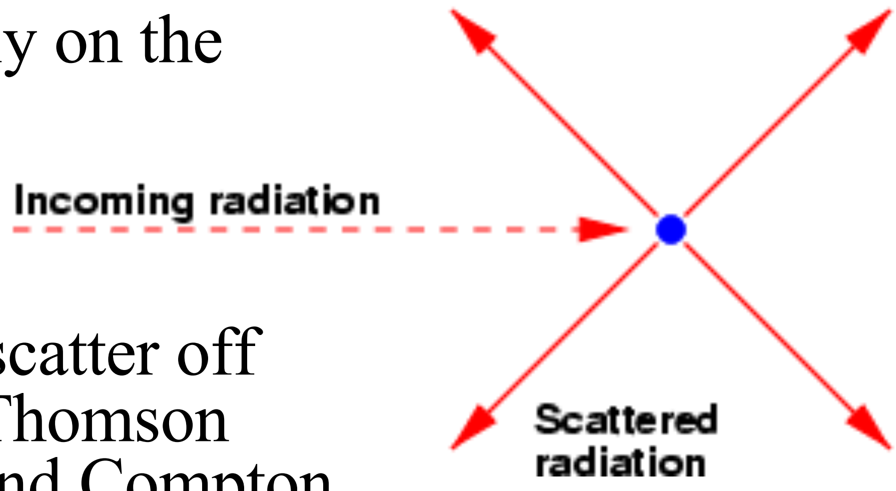
- Most absorption happens near the peak of $B_\nu(T)$, so since $\nu_{\max} \sim T$, an intensity-weighted integral over frequency gives *Kramers' Law*:

$$\kappa_{ff} \propto \rho T^{-7/2}$$

Scattering

- For thermal emission the source function $S_\nu = B_\nu(T)$ is:
  – Independent of the incident radiation field.
  – A function only of the local temperature.

- Another important emission process is *scattering*, which depends entirely on the incident radiation field.



- Example:
  – *Electron scattering*: photons scatter off free electrons. This is called Thomson scattering for low energies, and Compton scattering at high energies.

Electron scattering

- Free electrons scatter radiation with the same efficiency at all wavelengths. Absorption cross-section (m^2 per electron) is:

$$\sigma_{es} = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\varepsilon_0 m_e c^2}\right)^2$$

- which numerically is 6.7×10^{-29} m^2. The opacity is

$$\kappa_{es} = \frac{N}{\rho}\sigma_{es}$$

- which for pure hydrogen is,

$$\kappa_{es} = 0.04 m^2 kg^{-1}$$

- This process is called *Thomson scattering* or *electron scattering*. It becomes the most important opacity source at high temperatures.
- At high energies, when $h\nu \sim m_e c^2$, this description breaks down. Relativistic effects need to be taken into account -- this is the regime of *Compton scattering*.

Isotropic scattering

We consider the simplest case:

- Isotropic scattering: Scattered radiation is emitted uniformly across all angles $\rightarrow$ emission coefficient is independent of angle.
- Coherent scattering: Scattered radiation has the same frequency distribution as the incoming radiation (also called *elastic scattering*) *i.e.* no redistribution of energy across frequencies
- The astrophysically important case of electron scattering by non-relativistic electrons approximately meets these restrictions.

Transfer equation for pure scattering

- For coherent isotropic scattering, power j_ν emitted per unit volume per unit frequency range = power absorbed:

$$j_\nu = \sigma_\nu J_\nu$$

- Here, σ_ν is the absorption coefficient for the scattering process (the scattering coefficient), and J_ν is the mean intensity as before.
- Source function:

$$S_\nu \equiv \frac{j_\nu}{\sigma_\nu} = J_\nu \equiv \frac{1}{4\pi}\int I_\nu \, d\Omega$$

- For pure scattering, the transfer equation becomes,

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu = -\sigma_\nu \left(I_\nu - J_\nu \right)$$

- Notes:
  - J_ν is the integral of I_ν over all directions. The transfer equation for scattering no longer depends simply on local T.
  - scattering coefficient is usually called σ_ν. Don't confuse with the cross section per atom. σ_ν has the same units as the absorption coefficient α_ν, ie m^{-1} (area/volume).

Scattering and absorption

- Suppose we have a material in which there is *both* thermal emission with an absorption coefficient α_ν, and isotropic coherent scattering with scattering coefficient σ_ν. The transfer equation then has two terms,

$$\frac{dI_\nu}{ds} = -\alpha_\nu (I_\nu - B_\nu) - \sigma_\nu (I_\nu - J_\nu)$$

$$= -(\alpha_\nu + \sigma_\nu)(I_\nu - S_\nu)$$

- The source function is still the ratio of the emission to the absorption coefficients,

$$S_\nu = \frac{\alpha_\nu B_\nu + \sigma_\nu J_\nu}{\alpha_\nu + \sigma_\nu}$$

- The net absorption coefficient $(\alpha_\nu + \sigma_\nu)$ can be used to define the optical depth $d\tau_\nu = (\alpha_\nu + \sigma_\nu)\, ds$ exactly as before.

Lecture 10 revision quiz

- Evaluate the absorption cross-section (in m^2 per electron) for electron scattering.
- What type of source function (thermal or scattering) is most appropriate to compute the spectrum emanating from:
  - The Orion Nebula?
  - The solar white-light corona?
  - The solar photosphere?
  - A neon discharge tube?
  - A candle flame?
  - White clouds?
  - Blue sky?