Ionization fronts in HII regions

- Initial expansion of HII ionization front is supersonic, creating a shock front.
- Stationary frame: front advances into neutral material
- In frame where shock front is stationary, neutral gas flows into front at velocity $v_i$, with density $\rho_i$, and leaves as ionized gas with velocity $v_0$ and density $\rho_0$. 
Jump conditions

• To derive *jump conditions* across front, assume transition region is very narrow (a good approximation). Apply mass, momentum and energy conservation to get density jump.

• **Conservation of mass:**
  Mass flow (mass per area per time) into the front must equal the mass flow out:
  $$\rho_i v_i = \rho_o v_o$$

• **Conservation of Momentum:**
  Forces must balance on both sides of stationary front in reference frame of shock. Include momentum due to bulk flow, and pressure due to random motions:
  $$P_i + \rho_i v_i^2 = P_o + \rho_o v_o^2$$

  where $P_i$ and $P_o$ are the thermal pressures on the two sides.
Conservation of momentum

- Change of momentum equals impulse given by the net force in unit time
- Change of momentum per area per time equals force per area equals change in pressure
- Momentum flow in: \( \rho_i u_i u_i = \rho_i u_i^2 \)
- Momentum flow out, using conservation of mass:
  \[ \rho_i u_i u_o = \rho_o u_o^2 \]
- Equate change of momentum per area per time to change in pressure to get:
  \[ P_i + \rho_i u_i^2 = P_o + \rho_o u_o^2 \]
Conservation of Energy

- Normally, would also need to consider energy conservation. For an ionization front, can just assume that temperatures (hence sound speeds) in both neutral and ionized gas are fixed:

\[ P_i = \rho_i a_i^2 \]
\[ P_o = \rho_o a_o^2 \]

where \( a_i, a_o \) are the isothermal sound speeds, given by

\[ a_i = \sqrt{\frac{kT_i}{m_H}} \quad a_o = \sqrt{\frac{kT_o}{m_H}} \]
As HII region develops, velocity $\mathbf{U}_i$ of front depends on number of ionizing photons reaching it (i.e. on the optical depth to the front and number of recombinations inside HII region)

So solve for the density jump in terms of $\mathbf{U}_i$. Substituting for the pressures:

$$\rho_i (a_i^2 + \nu_i^2) = \rho_o (a_o^2 + \nu_o^2)$$

Using mass conservation we get: $\nu_o^2 = \left(\frac{\rho_i^2}{\rho_o^2}\right) \nu_i^2$

Substituting we get a quadratic equation for the density jump,

$$a_o^2 \left(\frac{\rho_o}{\rho_i}\right)^2 - (a_i^2 + \nu_i^2) \left(\frac{\rho_o}{\rho_i}\right) + \nu_i^2 = 0$$
This has solutions,

\[ a_o^2 \left( \frac{\rho_o}{\rho_i} \right)^2 - \left( a_i^2 + \nu_i^2 \right) \left( \frac{\rho_o}{\rho_i} \right) + \nu_i^2 = 0 \]

\[
\frac{\rho_0}{\rho_i} = \frac{\nu_i}{\nu_0} = \frac{1}{2a_o^2} \left\{ \left( a_i^2 + \nu_i^2 \right) \pm \sqrt{\left( a_i^2 + \nu_i^2 \right)^2 - 4a_o^2\nu_i^2} \right\}
\]

The temperature in the ionized gas is \( \sim 10^4 \text{K} \), whereas the temperature of the neutral gas is \( \sim 10^2 \text{K} \), so \( a_o^2 \sim 100 a_i^2 \).

Let’s explore the quantity in the square root; if this quantity is negative \( \rightarrow \) no physical solutions (no “single front” solution where ionisation, temperature, density change at same location).
The quantity in the square root is:

\[ f(\nu_i^2) = \left( \alpha_i^2 + \nu_i^2 \right)^2 - 4a_o^2 \nu_i^2 \]

Graphically, this looks like:

There are two critical velocities where this function passes through zero:

\[ \nu_R = a_o + \sqrt{a_o^2 - \alpha_i^2} \approx 2a_o \]
\[ \nu_D = a_o - \sqrt{a_o^2 - \alpha_i^2} \approx \frac{\alpha_i^2}{2a_o} \]

where we have used

\[ a_0^2 >> \alpha_i^2 \]

in the approximations.
• Two possible physical solutions:

\[ \nu_i \geq \nu_R \quad - \quad R \text{ (rarefied) - type ionization front} \]

\[ \nu_i \leq \nu_D \quad - \quad D \text{ (dense) - type front} \]

• Finally write the jump conditions as:

\[
\frac{\rho_o}{\rho_i} = \frac{\nu_i}{\nu_o} = \frac{1}{2a_o^2} \left\{ (\nu_R \nu_D + \nu_i^2) \pm \sqrt{(\nu_i^2 - \nu_R^2)(\nu_i^2 - \nu_D^2)} \right\}
\]

• If velocity is exactly \( \nu_R \) front is said to be \( R \)-critical; if velocity is exactly \( \nu_D \), \( D \)-critical.

• Otherwise there’s a choice of + or - sign.
  – Choice that gives smaller density contrast is “weak”, the larger “strong”.
Relation to the physical picture of the expansion of an HII region:

• If gas is rarefied, or ionizing flux is large, expect front to move rapidly.
• Expect an R-type ionization front during initial expansion of an HII region, when there are few recombinations in the interior and nearly all stellar photons reach the front.
• If gas is dense, or ionizing flux small, front moves more slowly. D-type fronts occur in late evolution of HII regions.
• In either case, the post-ionization gas may move either subsonically or supersonically with respect to the front.
Strong and weak R fronts

• Strong R-type front: velocity of ionized gas behind front is subsonic with respect to the front and the density ratio is large (does not exist in nature because disturbances in ionized gas continually catch up with the front and weaken it).

• So during initial growth of HII region a weak R-type front expands supersonically into the HI, leaving ionized gas only slightly compressed and moving out subsonically in a fixed reference frame.
Development of an HII region

(1) Early rapid expansion, weak R-type ionization front separates rarefied HI gas from rarefied HII gas.

(2) Expansion slows because of geometrical dilution and recombinations in interior. $v_i$ decreases until $v_i = v_R$, i.e. ionization front becomes R-critical, (velocity approaches sound speed and density contrast is $\approx 2$)

(3) For $v_D < v_i < v_R$ (see the “no physical solutions” region on previous diagram) there is no single front solution. Instead have a double front structure comprising an ionisation front with a shock front ahead of it. Shock wave breaks off from ionization front and moves into HI ahead of it. Ionization front slows further and becomes D-critical, because the shock compresses the HI gas to higher densities before the gas is ionized.

Detailed solutions show that the region between the shock and the ionization front remains fairly thin, (a small fraction of the radius of the HII region).
Observations

• Young HII regions are deeply embedded in gas and dust → need to go to the radio (free-free emission) or IR to observe them.
• The green colour in this false colour image denotes a compact HII region.
• Small HII regions (called compact or ultracompact HII regions), with sizes of 0.1 - 0.01 pc, or smaller, sometimes have roughly spherical shapes. However, there is a wide range of morphology, with some sources being cometary or irregular in appearance.

(Credit: http://astro.pas.rochester.edu/~jagoetz)
Possible explanations

- Density distribution around the young star is not spherically symmetric
  - HII region expands quickest towards low densities.
  - Can escape the cloud entirely → a `champagne' flow.
- Neutral gas is neither at rest nor uniform, but instead has a turbulent or chaotic structure on scales of a few parsecs
The Milky Way:
H II Regions & Diffuse Ionized Gas

WHAM: Wisconsin H α Mapper
SHASSA: Southern H α Sky Survey Atlas

WHAM PIs: Ron Reynolds, Matt Haffner
Turbulent ISM

- Supernovae driven turbulent density
- O stars near mid-plane
- Ionizing photons reach large heights due to 3D density structure
- But, may need B fields and pressure from cosmic rays to support gas at kpc heights above galactic midplane

Wood et al. (2010), Hill et al. (2012), Barnes et al. (2014), SILCC: Girichidis et al. (2017), Vandenbroucke et al. (2018), Kim et al. (2019)
Radiation hydrodynamics: Feedback from O Stars

Gravitational potential + photoionization feedback

Vandenbroucke & Wood (2019)
Lecture 16 revision quiz

• Assuming $P = \rho a^2$ on both sides of a shock, where $a$ is the isothermal sound speed, show that the density jump is given by

$$a_o^2 \left( \frac{\rho_o}{\rho_i} \right)^2 - \left( a_i^2 + v_i^2 \right) \left( \frac{\rho_o}{\rho_i} \right) + v_i^2 = 0$$

• Solve the quadratic and plot the quantity inside the square root sign as a function of inflow speed $v_i$ for the case where $a_o = 100a_i$.

• Show how the jump condition can be re-expressed as

$$\frac{\rho_o}{\rho_i} = \frac{v_i}{v_o} = \frac{1}{2a_o^2} \left\{ (v_R v_D + v_i^2) \pm \sqrt{(v_i^2 - v_R^2)(v_i^2 - v_D^2)} \right\}$$