Nebular emission spectra

- Four types of emission from a Stromgren sphere:
  - Recombination lines of H, He
  - Collisionally excited lines of metals. Optical: mostly forbidden lines of O,N,S,Ar,Ne. UV: C, Si, Mg.
  - Some continuum (f-f, f-b, 2-photon) emission. f-f and f-b significant only at radio wavelengths.
  - Other emission lines: resonance-fluorescence, di-electronic recombination, other exotica.
Hydrogen recombination spectrum

- **Einstein A-coeffs typically** \( A \sim 10^6 \text{ s}^{-1} \).
- **Collision rates typically** \( q \sim 10^{-4} \text{ cm}^3 \text{ s}^{-1} \).
- **Hence collisions for H become important only when**
  \[
  N_e \geq \frac{A}{q} \sim 10^{10} \text{ cm}^{-3}.
  \]
- **So collisions are unimportant at nebular densities.**
- **Detailed balance for H gives level pops** \( N_{nL} \):
  \[
  N_p N_e \alpha_{nL} (T) + \sum_{n' > n} \sum_{L'} N_{n'L'} A_{n'L',nL} = N_{nL} \sum_{n'' = 1}^{n-1} \sum_{L''} A_{nL,n''L''}
  \]

- **Recombinations into state nL**
- **Radiative decays into nL from higher levels**
- **Radiative decays from nL to lower levels**
Emissivity from populations

- This is a set of $n \times L$ linear equations in $n \times L$ unknowns.
  - Choose $N_e, T_e$
  - Look up $\alpha_{nL}$ for this temperature
  - Solve for populations levels $N_{nL}$ with linear algebra
- Once level populations are known, the emission produced by any transition is
  \[ j_{nn'} = \sum_{n'=1}^{n-1} \sum_{L'=L\pm 1} N_{nL} A_{nL,n'L'} \]
- In practice, there are many routes into each level other than direct recombination
  - Hence level pops depend mainly on Einstein $A$ values, only weakly on $N_e, T_e$. 
So far we have neglected:

- **Collisional excitation and deexcitation.**
  - $kT \ll \Delta E$ for excitation $n=1$ to $n=2$ so that’s OK.
  - Critical density for collisional de-excitation is $N_{\text{crit}} \sim 10^9 \text{ cm}^{-3}$, so that’s OK too.

- **Input to $nL$ via absorption from lower levels.**
  - Hydrogen electrons decay to ground state ($n=1$) in $10^{-6}$ s, so that’s more or less OK.
  - However, absorptions from ground state can and do occur.
  - In fact cross section for absorption in lower Lyman lines is greater than for photoionisation from $n=1$. 
Photoionization versus Lyman lines

- Cross sections for Lyman-line absorption:

<table>
<thead>
<tr>
<th>Line</th>
<th>$\lambda$(Å)</th>
<th>$A$ (sec$^{-1}$)</th>
<th>$a_0$ (cm$^2$)</th>
<th>$\tau/\tau_{912Å}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ly $\alpha$</td>
<td>1215.67</td>
<td>6.26x10$^8$</td>
<td>5.90x10$^{-14}$</td>
<td>9366</td>
</tr>
<tr>
<td>Ly $\beta$</td>
<td>1025.72</td>
<td>1.67x10$^8$</td>
<td>9.46x10$^{-15}$</td>
<td>1501</td>
</tr>
<tr>
<td>Ly $\gamma$</td>
<td>972.54</td>
<td>6.82x10$^7$</td>
<td>3.29x10$^{-15}$</td>
<td>522</td>
</tr>
<tr>
<td>Ly 10</td>
<td>920.96</td>
<td>4.21x10$^6$</td>
<td>1.72x10$^{-16}$</td>
<td>27</td>
</tr>
<tr>
<td>Ly 15</td>
<td>915.82</td>
<td>1.24x10$^6$</td>
<td>5.00x10$^{-17}$</td>
<td>8</td>
</tr>
<tr>
<td>Ly 20</td>
<td>914.04</td>
<td>5.24x10$^5$</td>
<td>2.10x10$^{-17}$</td>
<td>3</td>
</tr>
</tbody>
</table>

- Cross section from photoionization from $n=1$ is 6.30x10$^{-18}$ cm$^2$ at 912Å.
Case A and Case B

• Traditionally nebulae are said to be either:
  • Case A:
    – all Lyman line photons escape the nebula.
    – No Lyman line absorptions occur.
    – Nebula is optically thin, very faint
    – Earlier detailed balance relation is OK.
  • Case B:
    – All Lyman line photons re-absorbed by other atoms.
    – Downward transitions to ground state aren’t included in summations.
    – In real life, almost everything is Case B or close to it:

\[
N_p N_e \alpha_{nL}(T) + \sum_{n' > n} \sum_{L'} N_{n'L'} A_{n'L',nL} = N_{nL} \sum_{n'' = 2}^{n-1} \sum_{L''} A_{nL,n''L''}
\]

NB
What’s good about Case B

- Decays to $n = 1$ don’t count, so every decay eventually goes to $n = 2$.
- All transitions to $n = 2$ are in the optical.
- So every ionization produces an optical Balmer-line photon.
- *Number of optical Balmer-line photons = number of ionizing photons emitted by the central star.*
- Level populations depend almost exclusively on the $A$ coeffs, weakly on $N_e$, $T_e$.
- Line ratios (more or less) fixed by atomic physics
- *By measuring H$\beta$ alone, we can derive the number of ionizing photons from central star.*
### Line ratios for Balmer transitions

<table>
<thead>
<tr>
<th>Temperature</th>
<th>$N_e$ (cm$^{-3}$)</th>
<th>$5000$</th>
<th>$10000$</th>
<th>$10000$</th>
<th>$20000$</th>
<th>$20000$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$10^4$</td>
<td>$10^2$</td>
<td>$10^6$</td>
<td>$10^2$</td>
<td>$10^4$</td>
<td></td>
</tr>
<tr>
<td>$\alpha_{H\beta}^{\text{eff}}$</td>
<td>$5.44$</td>
<td>$3.02$</td>
<td>$3.07$</td>
<td>$1.61$</td>
<td>$1.61$</td>
<td></td>
</tr>
<tr>
<td>$I(H\alpha)/I(H\beta)$</td>
<td>$3.00$</td>
<td>$2.86$</td>
<td>$2.81$</td>
<td>$2.75$</td>
<td>$2.74$</td>
<td></td>
</tr>
<tr>
<td>$I(H\gamma)/I(H\beta)$</td>
<td>$0.460$</td>
<td>$0.468$</td>
<td>$0.471$</td>
<td>$0.475$</td>
<td>$0.476$</td>
<td></td>
</tr>
<tr>
<td>$I(H\epsilon)/I(H\beta)$</td>
<td>$0.155$</td>
<td>$0.159$</td>
<td>$0.163$</td>
<td>$0.163$</td>
<td>$0.163$</td>
<td></td>
</tr>
</tbody>
</table>
The fate of Lα photons

- In Case B, Balmer-series photons leave the nebula but Lα photons are trapped.
- Eventually some event will destroy Lα photons
  - Random walk to edge of nebula and escape
  - Wavelength shift to optically-thin line wings and escape
  - Destruction by hitting a dust grain. A single grain can mop up many Lα photons. Grain heats up, re-emits in IR.
  - Resonance fluorescence: Lβ (or He Lα) photon excites a transition at the same wavelength in another species.
    » E.g. He II Lα 303.78Å versus OIII 303.80Å
Lecture 19 revision quiz

• Use the information in the table on slide 5 to calculate the number density $N_1$ of hydrogen in the ground state, for which a $L\alpha$ photon would have a mean free path of 1 pc.

• What fraction of all Balmer-line photons produced at typical nebular temperatures and densities emerge in the $H\beta$ line?

• Why is it safe to ignore (a) collisional excitation and (b) collisional de-excitation of hydrogen at typical nebular densities and temperatures?