JH ASTRONOMY AND ASTROPHYSICS : AS 3015

Nebulae: Tutorial Questions 2

1. Given that the spectral energy density of radiation in a blackbody cavity is $u_\nu = 4\pi B_\nu / c$, show that the number density n_ν of photons of frequency ν can be written as
$$n_\nu = \frac{8\pi\nu^2}{c^3}\frac{1}{e^{h\nu/kT}-1}.$$

   Derive asymptotic expressions for n_ν in the limits $h\nu << kT$ and $h\nu >> kT$.

   Show that n_ν peaks at a frequency $\nu_{\max} = 1.593\ kT/h$, and evaluate this frequency for the microwave background, given that the blackbody temperature of the background radiation is 2.7 K. In what part of the electromagnetic spectrum does this frequency lie?

   Show that the total number density of photons in a blackbody cavity can be expressed in the form $n = bT^3$, and verify that $b = 2.03\text{x}10^7\ \text{m}^{-3}\ \text{K}^{-3}$.

   Use this result to calculate the number density of microwave background photons in the present-day Universe.

   The following integral may prove useful:
$$\int_0^\infty \frac{x^2 dx}{e^x - 1} \approx 2.40411$$

2. Consider a nebula with constant density ρ, mass absorption coefficient κ_ν and source function S_ν throughout its volume. A beam of light of specific intensity $I_\nu(0)$ enters the nebula at $s=0$ and exits at $s=x$.

   For the special case of a dark cloud with $S_\nu=0$ at visible wavelengths, show that
$$I_\nu(x) = I_\nu(0)e^{-\kappa_\nu \rho x}.$$

   For the more general case where S_ν is nonzero but constant throughout the cloud, integrate the equation of radiative transfer to show that the intensity of the emergent beam is
$$I_\nu(x) = I_\nu(0)e^{-\kappa_\nu \rho x} + S_\nu(1 - e^{-\kappa_\nu \rho x}).$$

   By replacing the specific intensity with the brightness temperature T_b and the source function of the nebula with the cloud temperature T_c, show that the optical depth τ_L in a radio line can be expressed as
$$\tau_L = -\ln\left[1 - (\Delta T_{\text{off}} - \Delta T_{\text{on}})/T_{\text{b}}\right]$$

where ΔT_{on} is the difference between the brightness temperature in the line and the adjacent continuum, when the cloud is observed against against a constant background source of temperature T_b, while ΔT_{off} is the same quantity when the cloud is observed away from the background source. Specify clearly any assumptions made.

When the background source is observed through the cloud, the brightness temperature in the centre of the line is found to be 3698 K, while that in the adjacent continuum is 12000 K. Away from the direction of the background source the brightness temperature at the centre of the line is 84 K while that in the adjacent continuum is now essentially 0 K. Sketch the observed spectrum in the region of the line in each case. Calculate the temperature of the cloud and its optical thickness in the line centre.